

# *COS320: Compiling Techniques*

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# *Compiling with Types*

- Intrinsic view: a term that cannot be typed is not a term at all
  - Compiler cannot translate terms that cannot be typed
- If *target* language is typed, this imposes an additional burden:
  - Well-typed programs in the source language translate to well-typed programs in the target language

- Intrinsic view: a term that cannot be typed is not a term at all
  - Compiler cannot translate terms that cannot be typed
- If *target* language is typed, this imposes an additional burden:
  - Well-typed programs in the source language translate to well-typed programs in the target language
- Can think of compilation as translation of (derivations of) judgements from a source language to a target language
  - Each kind of judgement has a different translation category. E.g.,
    - Well-formed types in source become well-formed types in target
    - Expressions in source become (operand, instruction list) pairs in target
    - ...
  - Each inference rule corresponds to a case within that category

## Oat v1 (HW4) – well-formed types

Judgements take the form:

- $\vdash t$ : “ $t$  is a well-formed type”
- $\vdash_r ref$ : “ $ref$  is a well-formed reference type”
- $\vdash_{rt} rt$ : “ $rt$  is a well-formed return type”

TINT

$$\frac{}{\vdash \text{int}}$$

TBOOL

$$\frac{}{\vdash \text{bool}}$$

TREF

$$\frac{\vdash_r ref}{\vdash ref}$$

RSTRING

$$\frac{}{\vdash_r \text{string}}$$

RARRAY

$$\frac{\vdash t}{\vdash_r t[]}$$

RFUN

$$\frac{\vdash t_1 \quad \dots \quad \vdash t_n \quad \vdash_{rt} rt}{\vdash_r (t_1, \dots, t_n) \rightarrow rt}$$

RTVOID

$$\frac{}{\vdash_{rt} \text{void}}$$

RTTYP

$$\frac{\vdash t}{\vdash_{rt} t}$$

## LLVMlite well-formed types

Judgements take the form:

- $T \vdash t$ : With named types  $T$ ,  $t$  is a well-formed type
- $T \vdash_s t$ : With named types  $T$ ,  $t$  is a well-formed simple type
- $T \vdash_r t$ : With named types  $T$ ,  $t$  is a well-formed reference type
- $T \vdash_{rt} t$ : With named types  $T$ ,  $t$  is a well-formed return type

$\frac{}{T \vdash_s \text{i1}}$	$\frac{}{T \vdash_s \text{i64}}$	$\frac{T \vdash_r \text{ref}}{T \vdash_s \text{ref*}}$	$\frac{\text{LLTUPLE} \quad T \vdash t_1 \quad \dots \quad T \vdash t_n}{T \vdash \{t_1, \dots, t_n\}}$	$\frac{\text{LLARRAY} \quad T \vdash t}{T \vdash [n \times t]} \quad n \in \mathbb{N}$	$\frac{\text{LLSIMPLE} \quad \vdash_s t}{\vdash t}$
$\frac{\text{LLRTVOID}}{T \vdash_{rt} \text{void}}$	$\frac{\text{LLRTSIMPLE} \quad T \vdash_s t}{T \vdash_{rt} t}$	$\frac{\text{LLRCHAR}}{T \vdash_r \text{i8}}$	$\frac{\text{LLRTYPE} \quad T \vdash t}{T \vdash_r t}$	$\frac{\text{LLRFUN} \quad T \vdash_{rt} \text{rt} \quad T \vdash_s t_1 \quad \dots \quad T \vdash_s t_n}{T \vdash_r \text{rt}(t_1, \dots, t_n)}$	
$\frac{\text{LLNAMED}}{T \vdash \%uid} \quad \%uid \in T$					

## Translating well-formed types

- Each well-formed Oat type is translated to a well-formed LLVM type
  - types  $\rightarrow$  simple types
  - reference types  $\rightarrow$  reference types
  - return types  $\rightarrow$  return types
- Use  $\llbracket \cdot \rrbracket$  to denote translation
  - I.e.,  $\llbracket \vdash \text{int} \rrbracket = \vdash_s \text{i64}$  denotes that the Oat type `int` is translated to the (simple) LLVMlite type `i64`

## Translating well-formed types

Suppose we have a well-formed type Oat type,  $\vdash t$ . There are three inference rules:

TINT

$$\frac{}{\vdash \text{int}}$$

TBOOL

$$\frac{}{\vdash \text{bool}}$$

TREF

$$\frac{\vdash_r \text{ref}}{\vdash \text{ref}}$$

Each has a corresponding case:

- Case TINT:  $\llbracket \vdash \text{int} \rrbracket = \vdash_s \text{i64}$  (well-formed by LLInt)
- Case TBOOL:  $\llbracket \vdash \text{bool} \rrbracket = \vdash_s \text{i1}$  (well-formed by LLBool)



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- Case TBOOL:  $\llbracket \vdash \text{bool} \rrbracket = \vdash_s \text{i1}$  (well-formed by LLBool)
- Case TREF: By induction on the derivation,  $\llbracket \vdash_r \text{ref} \rrbracket$  is a valid judgement of an LLVM reference type, say  $\vdash_r t$

$$\frac{\text{TREF} \quad \vdash_r \text{ref}}{\vdash \text{ref}} \rightsquigarrow \frac{\text{LLPTR} \quad \vdash_r t (= \llbracket \vdash_r \text{ref} \rrbracket)}{\vdash t^*}$$

- I.e.,  $\llbracket \vdash \text{ref} \rrbracket = \vdash_s t^*$ , where  $\vdash_r t = \llbracket \vdash_r \text{ref} \rrbracket$

## Translating well-formed array types

- In Oat v2, arrays accesses are checked at runtime
- **Recall:** Can implement run-time array access checking by allocating additional memory at the beginning of the array to store its size
- In Oat v1, arrays accesses are unchecked, but for forwards-compatibility we represent arrays in the same way.

## Translating well-formed array types

- In Oat v2, arrays accesses are checked at runtime
- Recall:** Can implement run-time array access checking by allocating additional memory at the beginning of the array to store its size
- In Oat v1, arrays accesses are unchecked, but for forwards-compatibility we represent arrays in the same way.

$$\begin{array}{c}
 \text{RARRAY} \\
 \frac{\vdash t}{\vdash_r t[]} \\
 \hline
 \end{array}
 \rightsquigarrow
 \begin{array}{c}
 \text{LLRTYPE} \\
 \frac{\text{LLTUPLE} \frac{\text{LLSIMPLE} \frac{\text{LLSIMPLE} \frac{\vdash_s i64}{\vdash i64} \quad \text{LLARRAY} \frac{\text{LLSIMPLE} \frac{\vdash_s t' (= \llbracket \vdash t \rrbracket)}{\vdash t'}}{\vdash [0xt']}}{\vdash \{i64, [0xt']\}}}{\vdash_r \{i64, [0xt']\}}
 \end{array}$$

## Summary of type translation

- $\llbracket \vdash \text{int} \rrbracket = \vdash_s \text{i64}$
- $\llbracket \vdash \text{bool} \rrbracket = \vdash_s \text{i1}$
- $\llbracket \vdash \text{ref} \rrbracket = \vdash_s t^*$ , where  $\vdash_r t = \llbracket \vdash_r \text{ref} \rrbracket$
- $\llbracket \vdash_r \text{string} \rrbracket = \vdash_r \text{i8}$
- $\llbracket \vdash_r t[\ ] \rrbracket = \vdash_r \{ \text{i64}, [\text{0x}t'] \}$ , where  $\vdash_s t' = \llbracket \vdash t \rrbracket$
- $\llbracket \vdash_r (t_1, \dots, t_n) \rightarrow \text{rt} \rrbracket = \vdash_{rt} \mathbf{rt}'(t'_1, \dots, t'_n)$ , where
  - $\vdash_{rt} \mathbf{rt} = \llbracket \vdash_{rt} \text{rt} \rrbracket$ ,
  - $\vdash_s t'_1 = \llbracket \vdash t_1 \rrbracket, \dots, \vdash_s t'_n = \llbracket \vdash t_n \rrbracket$
- $\llbracket \vdash_{rt} \text{void} \rrbracket = \vdash_{rt} \text{void}$
- $\llbracket \vdash_{rt} t \rrbracket = \vdash_{rt} t$ , where  $\vdash_s t = \llbracket \vdash t \rrbracket$

(see: `cmp_ty`, `cmp_rty`, `cmp_ret_ty` in HW4)

## Well-formed codestreams

Judgements take the form

- $\Gamma \vdash s \Rightarrow \Gamma'$ : “under type environment  $\Gamma$ , code stream  $s$  is well-formed and results in type environment  $\Gamma'$ ”
- $\Gamma \vdash \text{opn} : t$ : “under type environment  $\Gamma$ , operand  $\text{opn}$  has type  $t$ ”

**ID**

$$\frac{}{\Gamma \vdash \text{id} : \Gamma(\text{id})}$$

**NUM**

$$\frac{}{\Gamma \vdash n : \text{i64}} \quad n \in \mathbb{Z}$$

**ADD**

$$\frac{\Gamma \vdash \text{opn}_1 : \text{i64} \quad \Gamma \vdash \text{opn}_2 : \text{i64}}{T, \Gamma \vdash \%uid = \text{add i64 } \text{opn}_1, \text{opn}_2 \Rightarrow \Gamma\{\%uid \mapsto \text{i64}\}} \quad \%uid \notin \text{dom}(\Gamma)$$

**SEQ**

$$\frac{T, \Gamma \vdash s_1 \Rightarrow \Gamma' \quad T, \Gamma' \vdash s_2 \Rightarrow \Gamma''}{T, \Gamma \vdash s_1, s_2 \Rightarrow \Gamma''}$$

**BASE**

$$\frac{}{T, \Gamma \vdash \epsilon \Rightarrow \Gamma}$$

...lots more

## Well-typed expressions

$$\text{VAR} \quad \frac{}{\Gamma \vdash x : \Gamma(x)}$$

$$\text{ADD} \quad \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}$$

...

Expression compilation (`cmp_exp`) translates a type judgement  $\Gamma \vdash e : t$  to

- A **codestream judgement**  $\Gamma_u \vdash s \Rightarrow \Gamma'_u$ , and
- An **operand judgement**  $\Gamma'_u \vdash \text{opn} : t_u$

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- Need a symbol table  $ctxt$ , which maps Oat identifiers to LLVMlite operand judgements
  - The operand associated with a variable  $x$  is a *pointer* to the memory location associated with  $x$
- To compute  $\llbracket \Gamma \vdash x : t \rrbracket(ctxt)$ , first let  $(id, t^*) = ctxt(x)$ , then:
  - Define  $\llbracket ctxt \rrbracket$  to be the type environment associated with  $ctxt$
  - Codestream:  $\llbracket ctxt \rrbracket \vdash \%uid = \text{load } t^* \text{ opn} \Rightarrow \llbracket ctxt \rrbracket \{ \%uid \mapsto t \}$
  - Operand:  $\llbracket ctxt \rrbracket \{ \%uid \mapsto t \} \vdash \%uid : t$



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How can translate  $\Gamma \vdash e_1 + e_2 : \text{int}$  (i.e., **ADD**)?

- Let  $(\llbracket ctxt \rrbracket \vdash s_1 \Rightarrow \Gamma_1, \Gamma_1 \vdash \text{opn}_1 : \text{i64}) = \llbracket e_1 \rrbracket(ctxt)$
- Let  $(\llbracket ctxt \rrbracket \vdash s_2 \Rightarrow \Gamma_2, \Gamma_2 \vdash \text{opn}_2 : \text{i64}) = \llbracket e_2 \rrbracket(ctxt)$
- Codestream:  $\Gamma_1 + \Gamma_2 \vdash s_1, s_2, \%uid = \text{add i64 } \text{opn}_1, \text{opn}_2 \Rightarrow (\Gamma_1 + \Gamma_2) \{ \%uid \mapsto \text{i64} \}$
- Operand:  $(\Gamma_1 + \Gamma_2) \{ \%uid \mapsto \text{i64} \} \vdash \%uid : \text{i64}$

## Global initializers

One would expect the following coherence property:

*If  $\Gamma \vdash e : t$  translates to the codestream judgement  $\Gamma_u \vdash s \Rightarrow \Gamma'_u$  and the operand judgement  $\Gamma'_u \vdash opn : t_u$ , then  $\llbracket t \rrbracket = t_u$*

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- There is some subtlety in making this work!
  - Global declaration of string / array constants **must** compile to types with known length
    - E.g., `var x = int[] {0,1}` translates to `@x = global { 2, {0, 1} }`
    - Oat: x has type `int[]`
    - LLVM: `@x` has type `{ i64, [2xi64] }` ( $\neq \llbracket \text{int}[] \rrbracket = \{ i64, [0xi64] \}$ )
  - `cmp_exp_as` is a variant of `cmp_exp` that ensures type preservation via bitcast.

## Oat v2 (HW5)

- Specified by a (fairly large) type system
  - Invest some time in making sure you understand how to read the judgements and inference rules
- Adds several features to the Oat language:
  - Memory safety
    - *nullable* and *non-null* references. Type system enforces no null pointer dereferences.
    - Run-time array bounds checking (like Java, OCaml)
  - Mutable record types
  - Subtyping
    - *ref <: ref?*: non-null references are a subtype of nullable references
    - Record subtyping: width but not depth (*why?*)

## Subtyping and type inference

$$\text{SUBSUMPTION} \\ \frac{\Gamma \vdash e : s \quad \vdash s <: t}{\Gamma \vdash e : t}$$

- Challenge:
  - In the presence of the subsumption rule, a term may have more than one type (how can we infer types for a declaration like `var x = exp?`)
  - Subsumption destroys **syntax-directed** quality of the type system
- Solution:
  - Do not use subsumption! Integrate subtyping into other inference rules. E.g.,

$$\text{TYP\_CARR} \\ \frac{H \vdash t \quad H; G; L \vdash e_1 : t \quad \dots \quad H; G; L \vdash e_n : t}{H; G; L \vdash \text{new } t[]\{e_1, \dots, e_n\}}$$

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