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5.3 Substring Search

- *introduction*
- *brute force*
- *Knuth–Morris–Pratt*
- *Boyer–Moore*

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Substring search

Goal. Find pattern of length m in a text of length n.

typically $n \gg m$

pattern → N E E D L E

text → I N A H A Y S T A C K N E E D L E I N A

match

Substring search applications

Goal. Find pattern of length m in a text of length n.

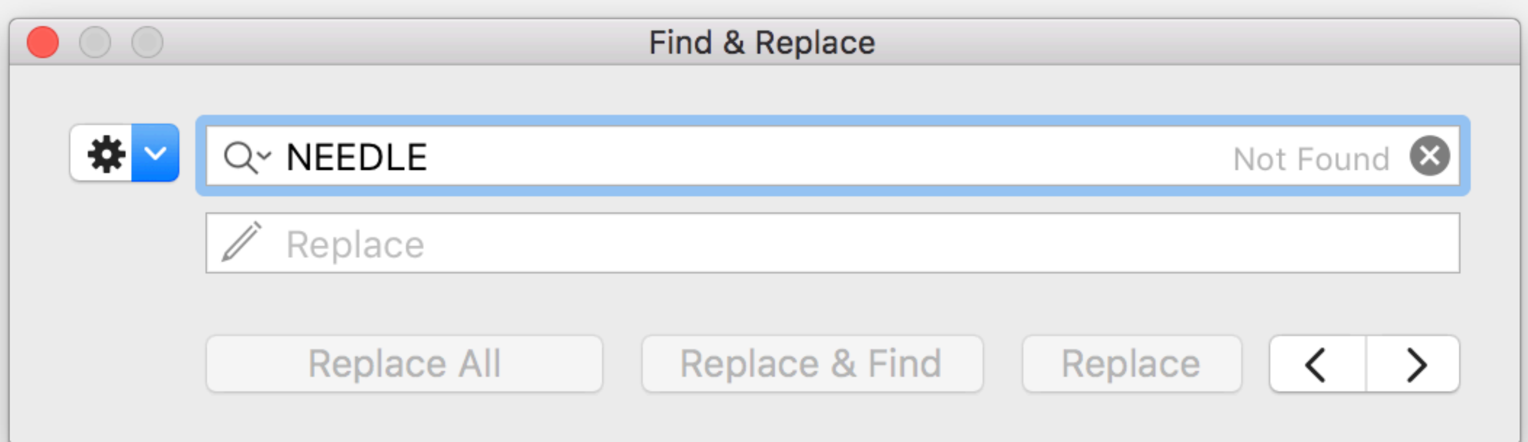
typically $n \gg m$

pattern → N E E D L E

text → I N A H A Y S T A C K <u>N E E D L E</u> I N A

match

Search in a word processor or IDE.



Substring search applications

Goal. Find pattern of length m in a text of length n.

typically $n \gg m$

pattern → N E E D L E

text → I N A H A Y S T A C K N E E D L E I N A

match

Computer forensics.
Search memory or disk for signatures,
e.g., all URLs or RSA keys that the user has entered.

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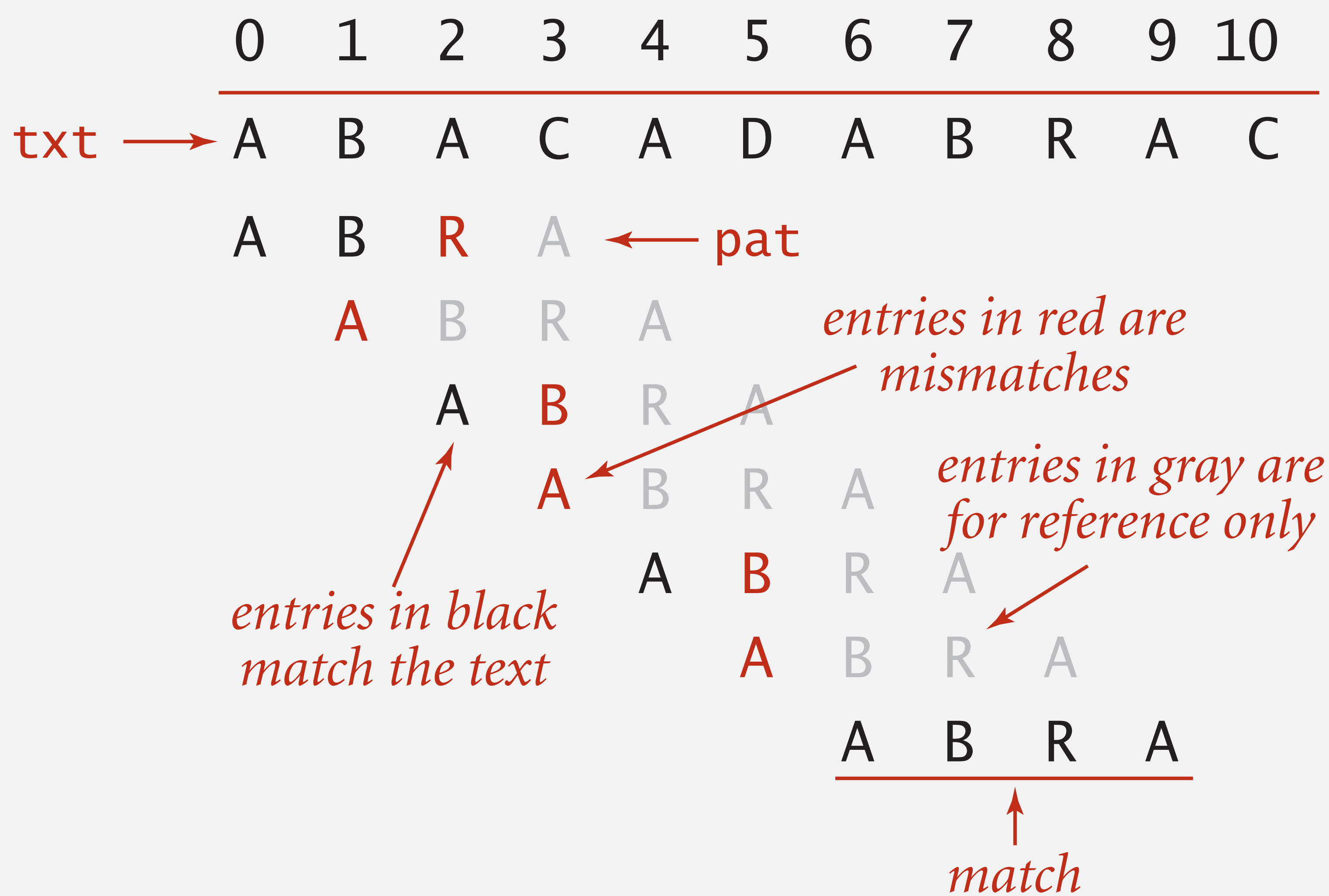
Brute-force substring search

Check for pattern starting at each text position.



Brute-force substring search: Java implementation

Check for pattern starting at each text position.

i	j	i+j	0	1	2	3	4	5	6	7	8	9	10
			A	B	A	C	A	D	A	B	R	A	C
⋮													
4	3	7					A	D	A	C	R		
5	0	5						A	D	A	C	R	

```
public static int search(String pat, String txt)
{
   int m = pat.length();
   int n = txt.length();
   for (int i = 0; i <= n - m; i++)        for each possible offset
   {
      int j;                                number of characters that match
      for (j = 0; j < m; j++)
         if (txt.charAt(i+j) != pat.charAt(j))
            break;
      if (j == m) return i;                 index in text where pattern starts
   }
   return n;                                not found
}
```

Substring search: quiz 1

What is the worst-case running time of brute-force substring search as a function of the pattern length m and text length n ?

A. $m + n$

B. m^2

C. $m\,n$

D. n^2

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5.3 Substring Search

- *introduction*
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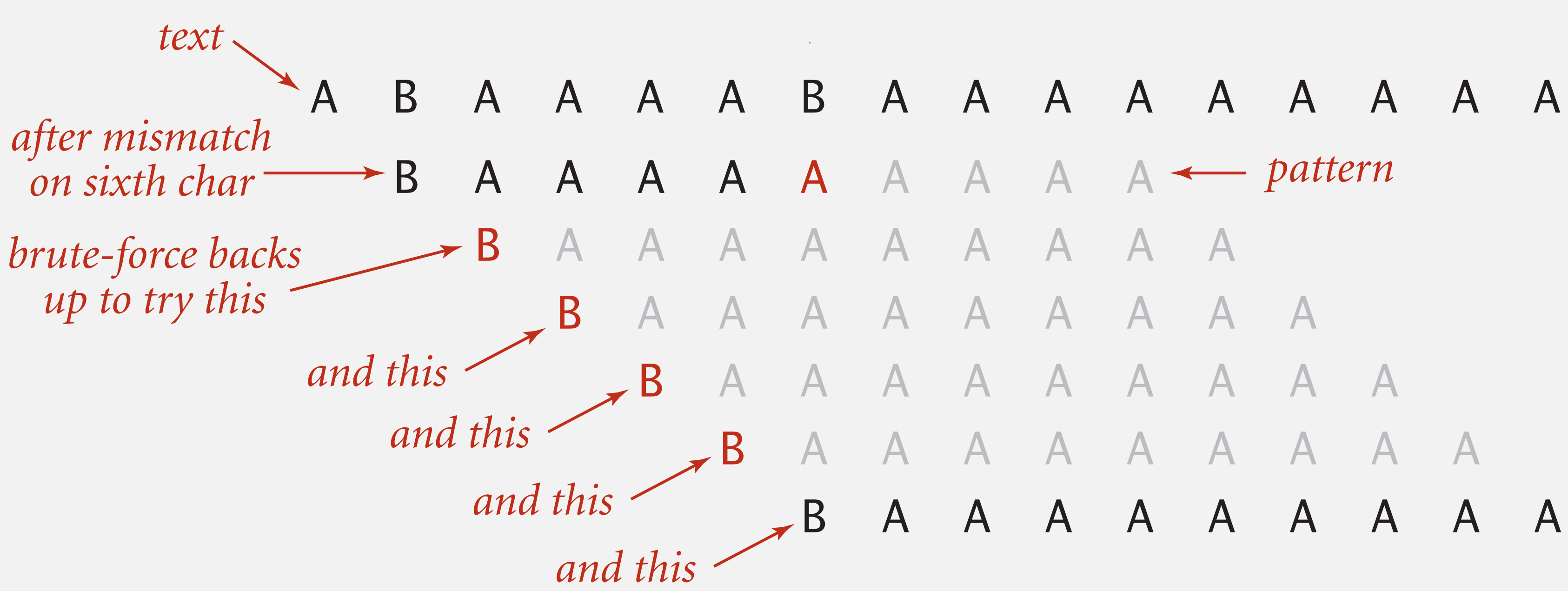
Knuth–Morris–Pratt substring search

Intuition. Suppose we are searching in text for pattern `BAAAAAAAAA`.

- Suppose we match 5 chars in pattern, with mismatch on 6th char.



Knuth–Morris–Pratt substring search

Intuition. Suppose we are searching in text for pattern B A A A A A A A A A.

- Suppose we match 5 chars in pattern, with mismatch on 6th char.
- We know previous 6 chars in text must be B A A A A B.
- Don't need to compare any text character twice.

assuming { A, B } alphabet

text

A B A A A A B A A A A A A A A A

after mismatch on sixth char

B A A A A A A A A A

pattern

but no backup is needed

B A A A A A A A A A

Knuth–Morris–Pratt algorithm. Clever method to always avoid comparing a text character more than once!

Deterministic finite state automaton (DFA)

A DFA is an abstract string-searching machine.

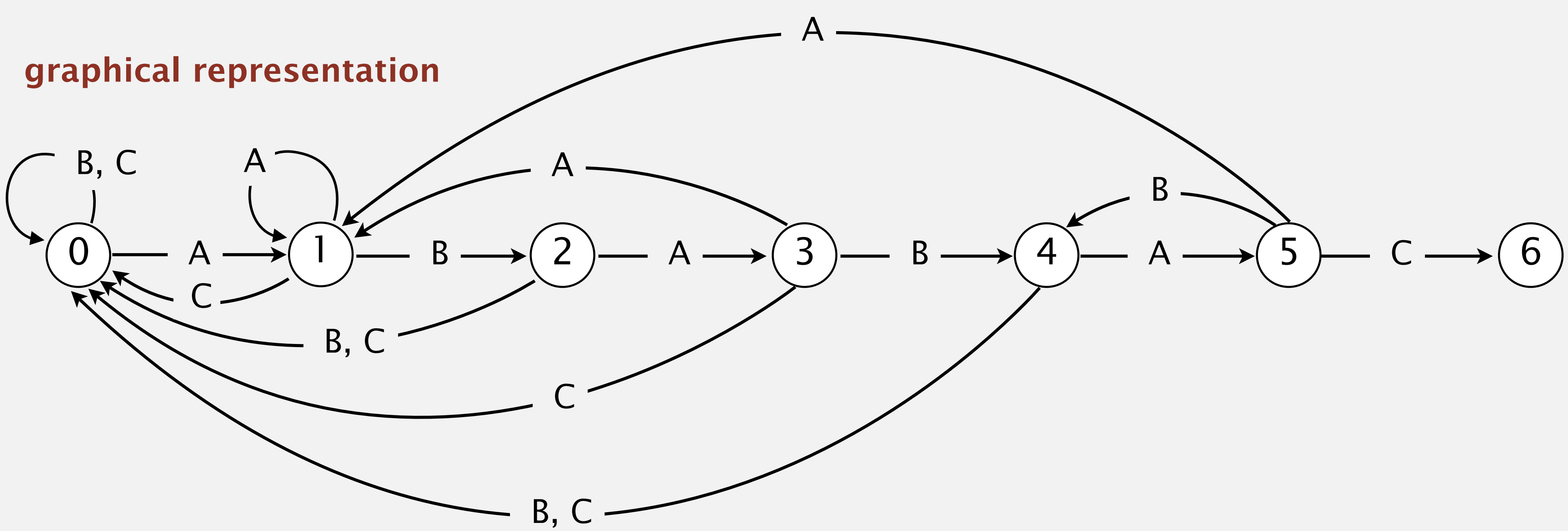
- Finite number of states (including start and halt).
- Exactly one state transition for each char in alphabet.
- Accept if sequence of state transitions leads to halt state.

internal representation

		0	1	2	3	4	5
`pat.charAt(j)`		A	B	A	B	A	C
`dfa[][j]`	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

If in state `j` reading char `c`:
- if `j` is 6 halt and accept
- else move to state `dfa[c][j]`

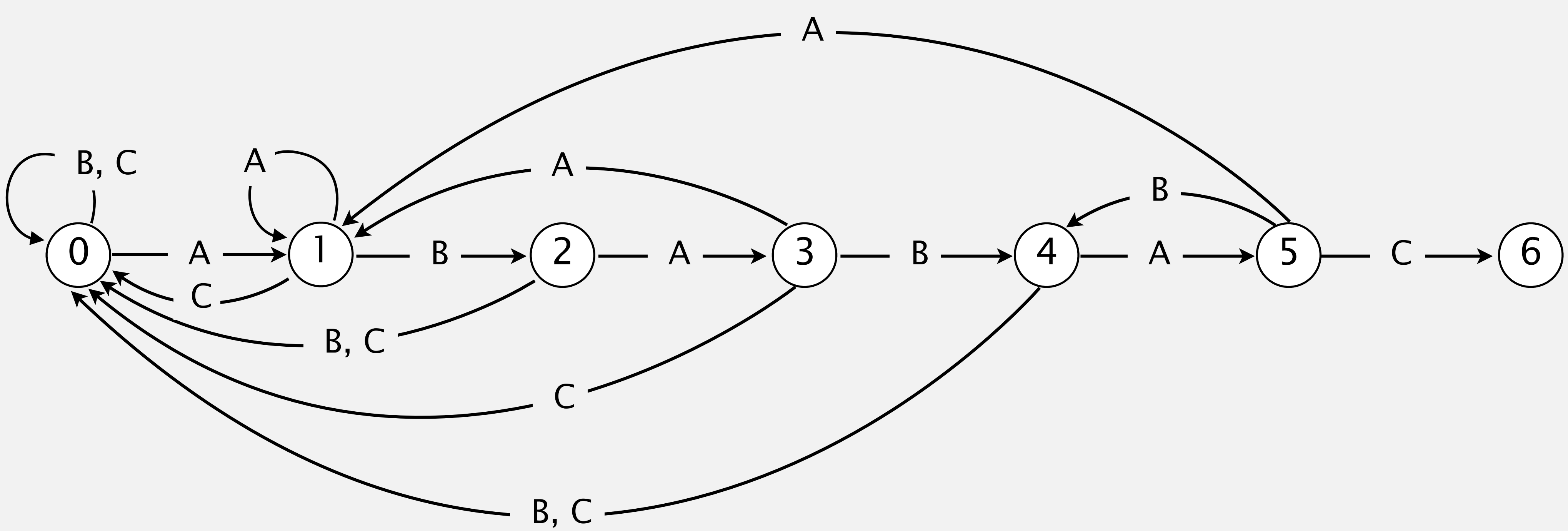
graphical representation



Knuth–Morris–Pratt demo: DFA simulation

A A B A C A A B A B A C A A

		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6



Interpretation of Knuth-Morris-Pratt DFA

Q. What is interpretation of DFA state after reading in `txt[i]`?

A. State = number of characters in pattern that have been matched.

length of longest prefix of `pat[]` that is a suffix of `txt[0..i]`

Ex. DFA is in state 3 after reading in `txt[0..6]`.

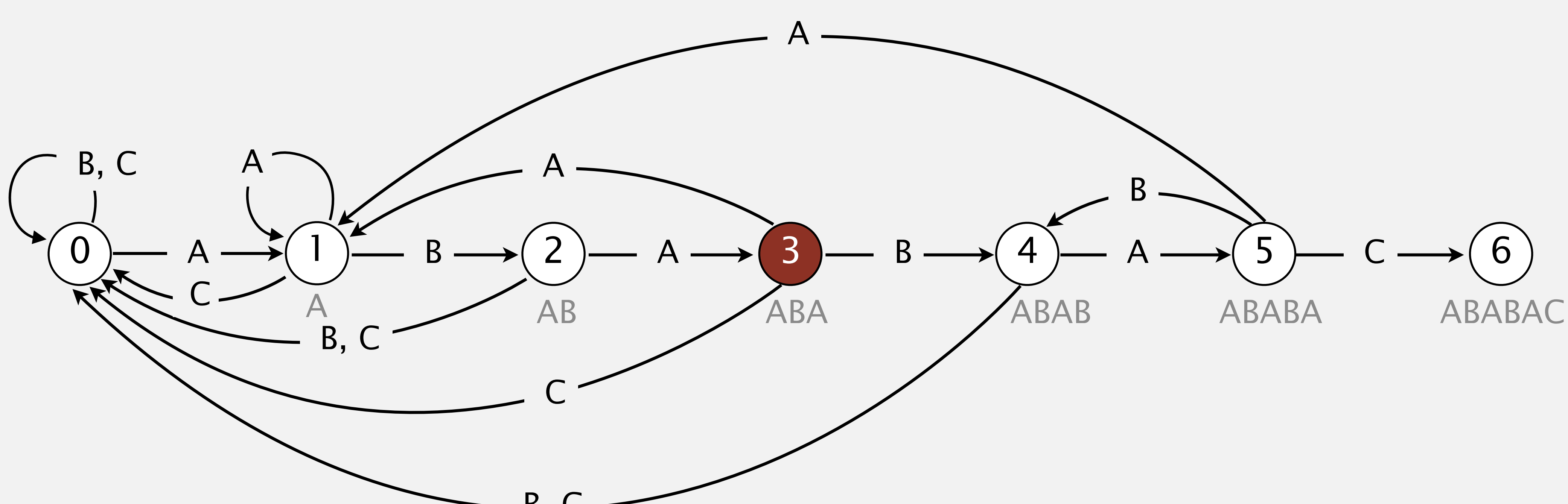
i

	0	1	2	3	4	5	6	7	8
txt →	B	C	B	A	A	B	A	C	A

suffix of `txt[0..6]`

	0	1	2	3	4	5
pat →	A	B	A	B	A	C

prefix of `pat[]`



Substring search: quiz 2

Which state is the DFA in after processing the following input?

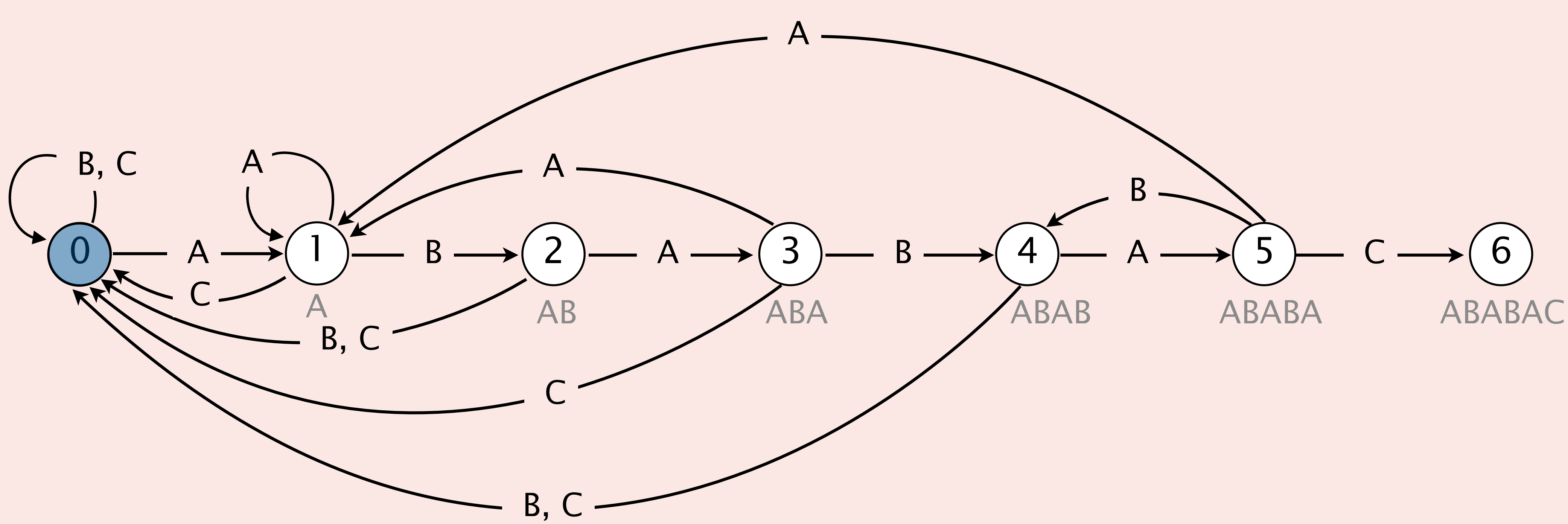
B A A B A B A B

A. 0

B. 1

C. 3

D. 4



Substring search: quiz 3

Which state is the DFA in after processing the following input?

A B A A B B A B A B B A B A A B A A B A A A B A B A B A A B A A B A A B A B A B

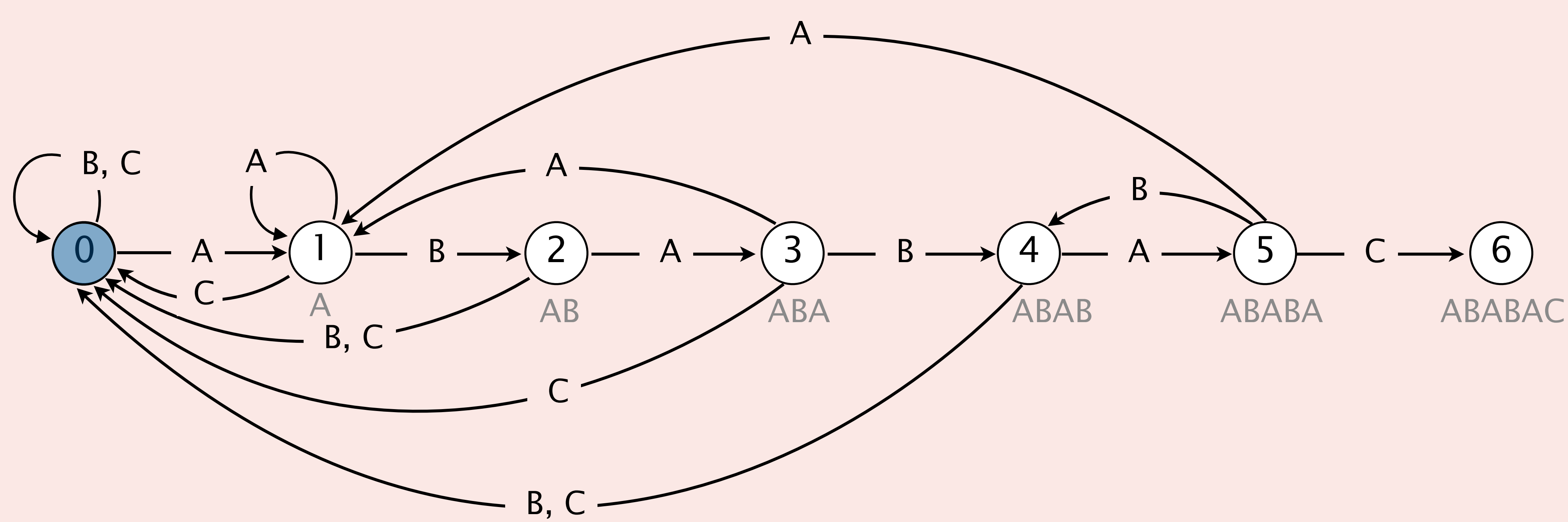
A. 0

B. 1

C. 3

D. 4

E. 5



Knuth–Morris–Pratt substring search: Java implementation

Key differences from brute-force implementation.

- Need to precompute `dfa[][]` from pattern.
- Each text character compared (at most) once.

stop on first match

```
public int search(String txt)
{
   int i, j, n = txt.length();
   for (i = 0, j = 0; i < n && j < m; i++)
      j = dfa[txt.charAt(i)][j];
   if (j == m) return i - m;
   else        return n;
}
```

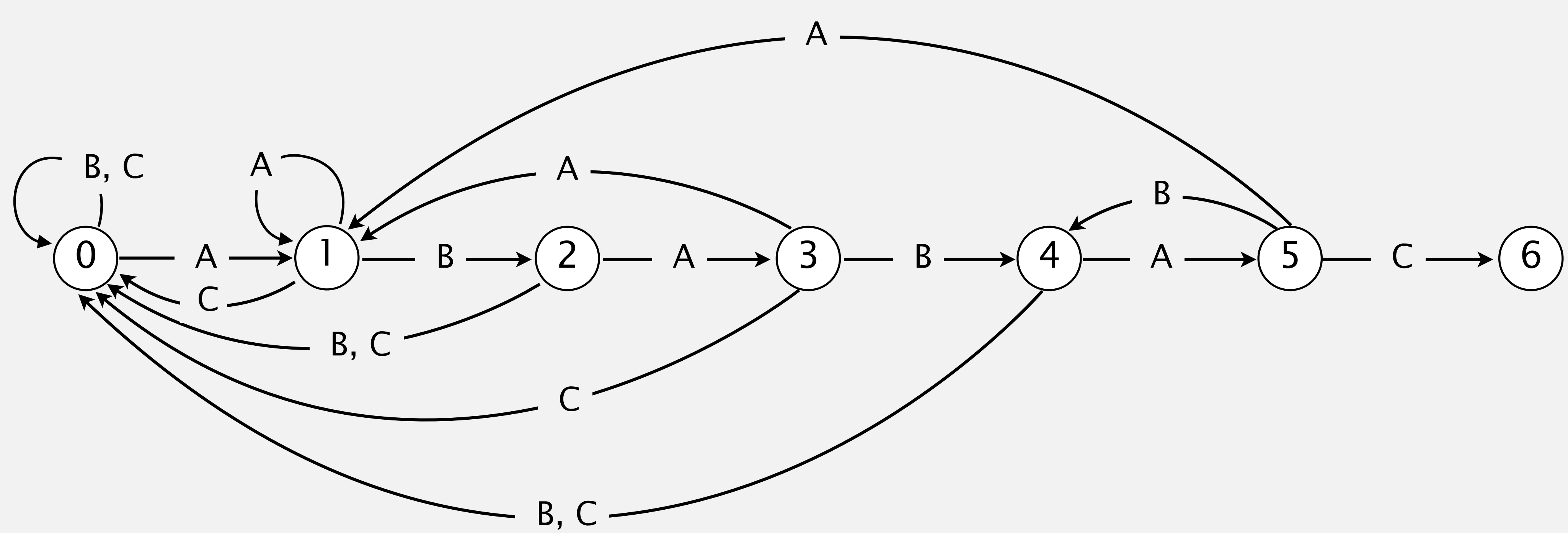
Running time.

- Simulate DFA on text: at most n character accesses.
- Build DFA: how to do efficiently? [tricky algorithm ahead]

Knuth–Morris–Pratt demo: DFA construction

		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
	A	1	1	3	1	5	1
dfa[][j]	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

Constructing the DFA for KMP substring search for A B A B A C



How to build DFA from pattern?

Include one state for each character in pattern (plus accept state).

		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A						
	B						
	C						

How to build DFA from pattern?

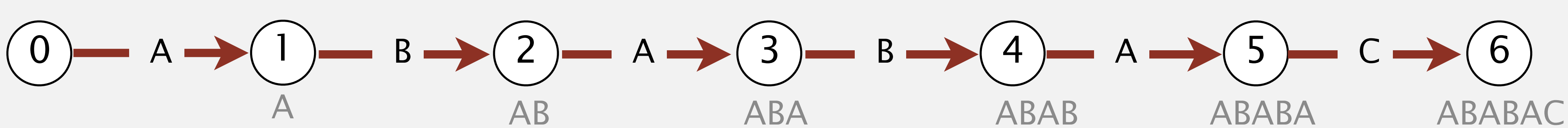
Match transition. If in state `j` and next char `c == pat.charAt(j)`, go to `j+1`.

first j characters of pattern have already been matched

next char matches

now first j +1 characters of pattern have been matched

		0	1	2	3	4	5
`pat.charAt(j)`		A	B	A	B	A	C
`dfa[][j]`	A	1		3		5	
	B		2		4		
	C						6



How to build DFA from pattern?

Mismatch transition. If in state `j` and next char `c != pat.charAt(j)`, then the last `j-1` characters of input are `pat[1..j-1]`, followed by `c`.

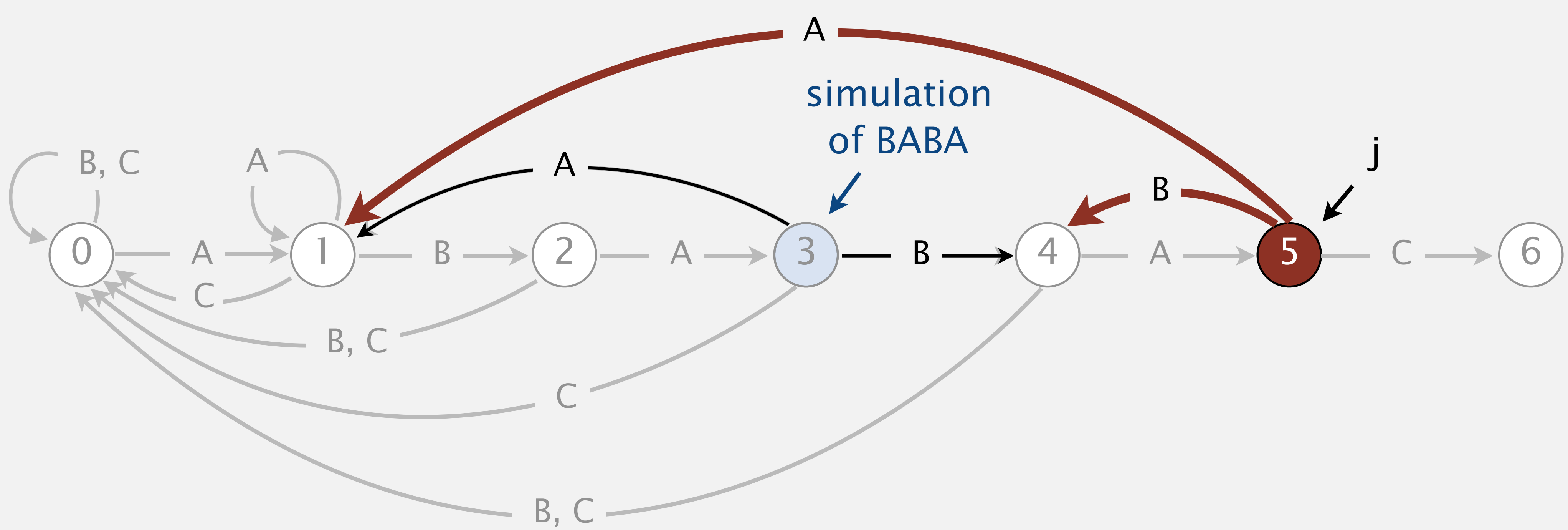
To compute `dfa[c][j]`: Simulate `pat[1..j-1]` on DFA and take transition `c`.

still under construction (!)

Ex. `dfa['A'][5] = 1`   `dfa['B'][5] = 4`

simulate BABAA   simulate BABAB

j	0	1	2	3	4	5
pat.charAt(j)	A	B	A	B	A	C



How to build DFA from pattern?

Mismatch transition. If in state `j` and next char `c != pat.charAt(j)`, then the last `j-1` characters of input are `pat[1..j-1]`, followed by `c`.

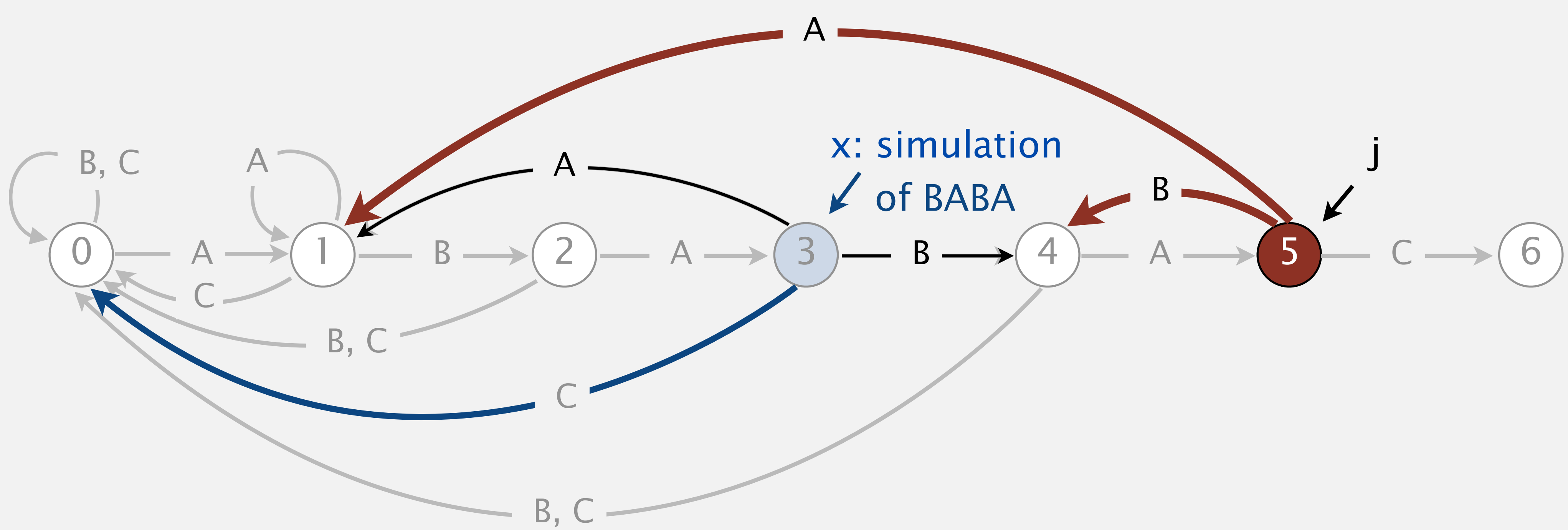
state x

To compute `dfa[c][j]`: Simulate `pat[1..j-1]` on DFA and take transition `c`.

Ex. `dfa['A'][5] = 1` `dfa['B'][5] = 4` `x' = 0`

`dfa['A'][5] = 1`	`dfa['B'][5] = 4`	`x' = 0`
from state x, take transition 'A' = dfa['A'][x]	from state x, take transition 'B' = dfa['B'][x]	from state x, take transition 'C' = dfa['C'][x]

0	1	2	3	4	5
A	B	A	B	A	C

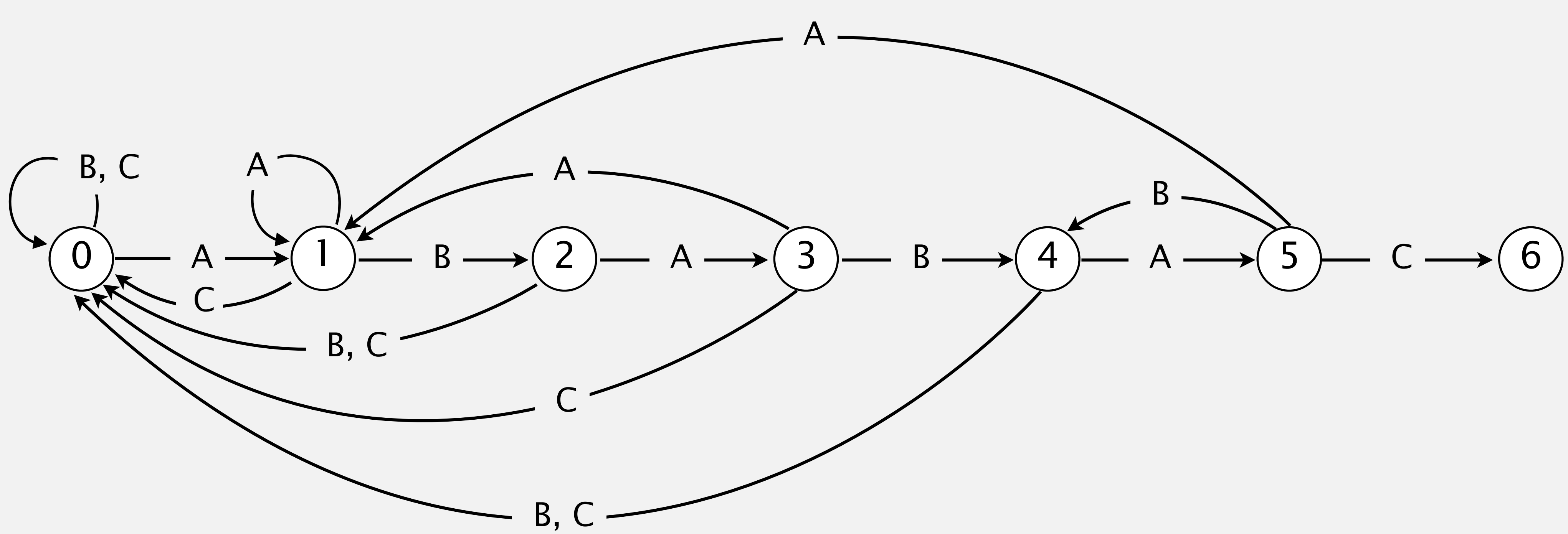


Knuth–Morris–Pratt demo: DFA construction in linear time

Linear in the size of the DFA, which is Rm.

		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

Constructing the DFA for KMP substring search for A B A B A C



Constructing the DFA for KMP substring search: Java implementation

For each state `j`:

- Copy `dfa[][x]` to `dfa[][j]` for mismatch case.
- Set `dfa[pat.charAt(j)][j]` to `j+1` for match case.
- Update `x`.

```
public KMP(String pat)
{
   this.pat = pat;
   m = pat.length();
   dfa = new int[R][m];
   dfa[pat.charAt(0)][0] = 1;
   for (int x = 0, j = 1; j < m; j++)
   {
      for (int c = 0; c < R; c++)
         dfa[c][j] = dfa[c][x];        ← copy mismatch cases
      dfa[pat.charAt(j)][j] = j+1;     ← set match case
      x = dfa[pat.charAt(j)][x];       ← update restart state
   }
}
```

Running time. m character accesses (but space/time proportional to $R\,m$).

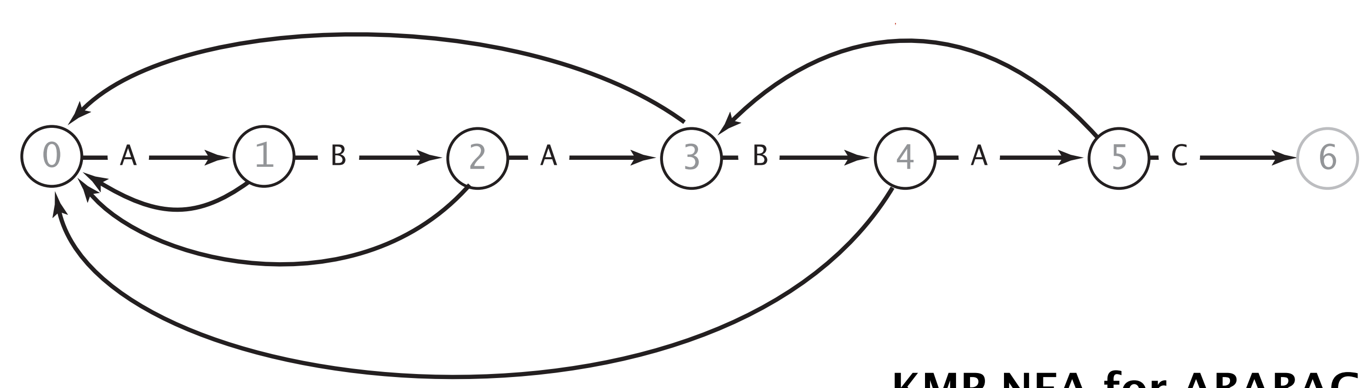
KMP substring search analysis

Proposition. KMP substring search accesses no more than $m + n$ chars to search for a pattern of length m in a text of length n.

Pf. Each pattern character accessed once when constructing the DFA; each text character accessed (at most) once when simulating the DFA.

Proposition. KMP constructs `dfa[][]` in time and space proportional to $R\,m$.

Larger alphabets. Improved version of KMP constructs `nfa[]` in time and space proportional to m.



KMP NFA for ABABAC

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5.3 Substring Search

Boyer-Moore: mismatched character heuristic

Intuition.

- Scan characters in pattern from right to left.
- Can skip as many as m text chars when finding one not in the pattern.

i	j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	text →	F	I	N	D	I	N	A	H	A	Y	S	T	A	C	K	N	E	E	D	L	E	I	N	A
0	5	N	E	E	D	L	E ← *pattern*																		
5	5						N	E	E	D	L	E													
11	4												N	E	E	D	L	E							
15	0																N	E	E	D	L	E			

return i = 15

no S in pattern

align N in text with N in pattern

align N in text with N in pattern

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 1. Mismatch character not in pattern.

```
                   i
before             ↓
txt    . . . . . . T L E . . . . . .
pat          N E E D L E

                           i
after                      ↓
txt    . . . . . . T L E . . . . . .
pat                  N E E D L E
```

mismatch character T not in pattern: increment i one character beyond T

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2a. Mismatch character in pattern.

```
                    i
before              ↓
txt   .  .  .  .  .  .  N  L  E  .  .  .  .  .  .
pat                  N  E  E  D  L  E

                             i
after                        ↓
txt   .  .  .  .  .  .  N  L  E  .  .  .  .  .  .
pat                     N  E  E  D  L  E
```

mismatch character N in pattern: align text N with rightmost (why?) pattern N

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2b. Mismatch character in pattern (but heuristic no help).

before

```
        i
        ↓
txt  . . . . . . E L E . . . . . .
pat        N E E D L E
```

aligned with rightmost E?

```
      i
      ↓
txt  . . . . . . E L E . . . . . .
pat    N E E D L E
```

mismatch character E in pattern: align text E with rightmost pattern E ?

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2b. Mismatch character in pattern (but heuristic no help).

before

				i													
txt	.	.	.	.	.	.	E	L	E	.	.	.	.	.	.		
pat				N	E	E	D	L	E								

after

					i												
txt	.	.	.	.	.	.	E	L	E	.	.	.	.	.	.		
pat					N	E	E	D	L	E							

mismatch character E in pattern: increment i by 1

Substring search: quiz 5

Which text character is compared with the E next in Boyer–Moore?

A. R (index 5)

B. O (index 6)

C. O (index 12)

D. O (index 13)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
text →	B	O	O	Y	E	R	O	B	E	R	T	M	O	O	R	E	J	S
pattern →	M	O	O	R	E													
					M	O	O	R	E									

Substring search: quiz 6

Which text character is compared with the E next in Boyer–Moore?

A. O

B. R

C. E

D. J

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
text →	B	O	O	Y	E	R	O	B	E	R	T	M	O	O	R	E	J	S
pattern →	M	O	O	R	E													
					M	O	O	R	E									
									M	O	O	R	E					

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

A. Precompute index of rightmost occurrence of character c in pattern.
(−1 if character not in pattern)

```
right = new int[R];
for (int c = 0; c < R; c++)
   right[c] = -1;
for (int j = 0; j < m; j++)
   right[pat.charAt(j)] = j;
```

		N	E	E	D	L	E	
c		0	1	2	3	4	5	right[c]
A	-1	-1	-1	-1	-1	-1	-1	-1
B	-1	-1	-1	-1	-1	-1	-1	-1
C	-1	-1	-1	-1	-1	-1	-1	-1
D	-1	-1	-1	-1	3	3	3	3
E	-1	-1	1	2	2	2	5	5
...								-1
L	-1	-1	-1	-1	-1	4	4	4
M	-1	-1	-1	-1	-1	-1	-1	-1
N	-1	0	0	0	0	0	0	0
...								-1

Boyer-Moore skip table computation

Boyer-Moore: Java implementation

```
public int search(String txt)
{
   int n = txt.length();
   int m = pat.length();
   int skip;
   for (int i = 0; i <= n-m; i += skip)
   {
      skip = 0;
      for (int j = m-1; j >= 0; j--)
      {
         if (pat.charAt(j) != txt.charAt(i+j))
         {
            skip = Math.max(1, j - right[txt.charAt(i+j)]);
            break;
         }
      }
      if (skip == 0) return i;
   }
   return n;
}
```

compute skip value

in case other term is zero or negative

match

Boyer-Moore: analysis

Property. Substring search with the Boyer–Moore mismatched character heuristic takes about $\sim n / m$ character compares to search for a pattern of length m in a text of length n.

sublinear!

Worst-case. Can be as bad as $\sim m\, n$.

i	skip	0	1	2	3	4	5	6	7	8	9	
	txt →	B	B	B	B	B	B	B	B	B	B	
0	0	A	B	B	B	B						← pat
1	1		A	B	B	B	B					
2	1			A	B	B	B	B				
3	1				A	B	B	B	B			
4	1					A	B	B	B	B		
5	1						A	B	B	B	B	

Boyer–Moore variant. Can improve worst case to $\sim 3\, n$ character compares by adding a KMP-like rule to guard against repetitive patterns.

Substring search: quiz 7

Which substring search algorithm does Java's indexOf() method use?

A. Brute-force search

B. Knuth–Morris–Pratt

C. Boyer–Moore

D. None of the above

indexOf

```
public int indexOf(String str)
```

Returns the index within this string of the first occurrence of the specified substring.

The returned index is the smallest value k for which:

```
this.startsWith(str, k)
```

If no such value of k exists, then -1 is returned.

Parameters:

str - the substring to search for.

Returns:

the index of the first occurrence of the specified substring, or -1 if there is no such occurrence.

Substring search cost summary

Cost of searching for an m-character pattern in an n-character text.

algorithm	version	operation count		extra space
		guarantee	typical	
brute force	—	MN	$1.1\,N$	1
Knuth-Morris-Pratt	*full DFA (Algorithm 5.6)*	$2N$	$1.1\,N$	MR
	mismatch transitions only	$3N$	$1.1\,N$	M
Boyer-Moore	*full algorithm*	$3N$	N/M	R
	mismatched char heuristic only (Algorithm 5.7)	MN	N/M	R

Out of scope