https://algs4.cs.princeton.edu

2.1 ELEMENTARY SORTS

- *rules of the game*
- *selection sort*
- *insertion sort*
- *shuffling*
- *comparators*

https://algs4.cs.princeton.edu

2.1 Elementary Sorts

- *rules of the game*
- *selection sort*
- *insertion sort*
- *shuffling*
- *comparators*

Sorting problem

Ex. Student records in a university.

	Chen	3	A	(991) 878-4944	308 Blair
	Rohde	2	A	(232) 343-5555	343 Forbes
	Gazsi	4	B	(800) 867-5309	101 Brown
item →	Furia	1	A	(766) 093-9873	101 Brown
	Kanaga	3	B	(898) 122-9643	22 Brown
	Andrews	3	A	(664) 480-0023	097 Little
key →	Battle	4	C	(874) 088-1212	121 Whitman

Sort. Rearrange array of n items in ascending order by key.

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

Total order

Sorting is well defined if and only if there is a total order.

A total order is a binary relation $\leq$ that satisfies:

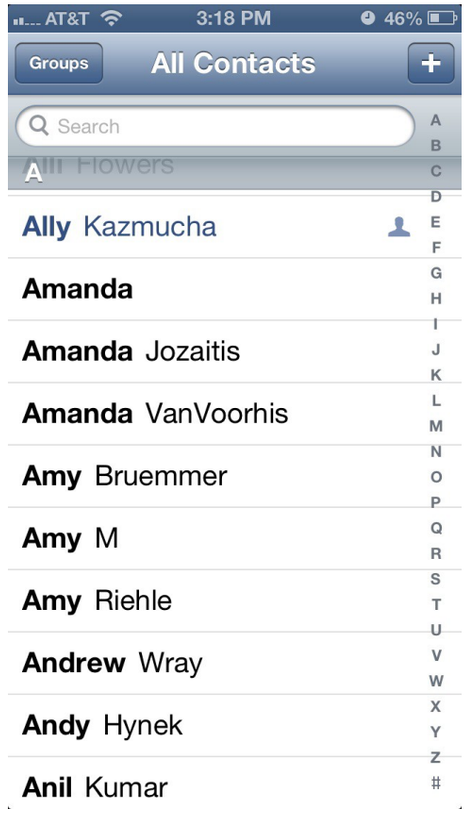
- Totality: either $v \leq w$ or $w \leq v$ or both.
- Transitivity: if both $v \leq w$ and $w \leq x$, then $v \leq x$.
- Antisymmetry: if both $v \leq w$ and $w \leq v$, then $v = w$.

Examples.

Video name	Views*
"Despacito"[6]	2,993,700,000
"See You Again"[11]	2,894,000,000
"Gangnam Style"[17]	803,700,000
"Baby"[41]	245,400,000
"Bad Romance"[146]	178,400,000
"Charlie Bit My Finger"[136]	128,900,000
"Evolution of Dance"[131]	118,900,000

numerical order (descending)

chronological order



lexicographic order

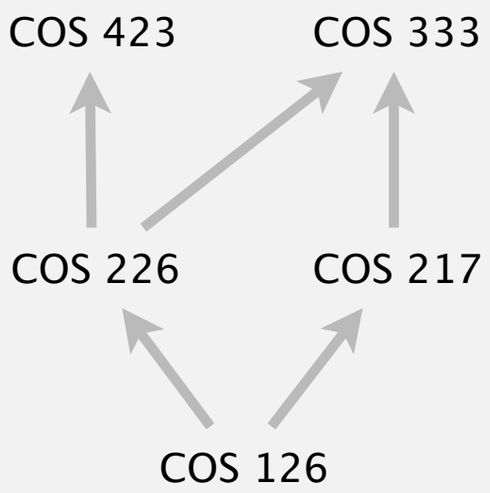
Non-examples of total order

Sorting is well defined if and only if there is a total order.

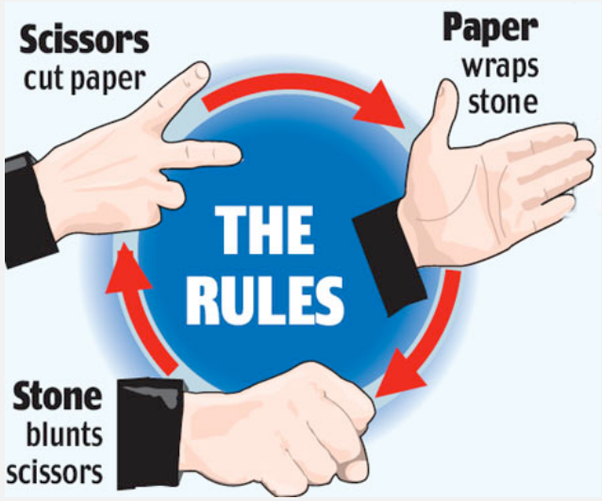
A total order is a binary relation $\leq$ that satisfies:

- Totality: either $v \leq w$ or $w \leq v$ or both.
- Transitivity: if both $v \leq w$ and $w \leq x$, then $v \leq x$.
- Antisymmetry: if both $v \leq w$ and $w \leq v$, then $v = w$.

Hogwarts houses
(no ≤; different sense of the word sort)



course prerequisites
(violates totality)



Ro-sham-bo order
(violates transitivity)

predator-prey
(violates antisymmetry)

How can a single algorithm/implementation sort any type of data?

Please sort these Japanese words for me

But I don't speak Japanese and I don't know how words are ordered

No problem. Every time you need to compare two words, give me a call back.

Answer: callbacks

Callbacks

Goal. Sort any type of data (for which sorting is well defined).

Q. How can a `sort()` function compare data of type `String`, `Double`, and `java.io.File` without hardwiring type-specific information.

Sort the files in a directory by filename

Callback = reference to executable code.

- Client passes array of objects to `sort()` function.
- The `sort()` method calls object's `compareTo()` function as needed.

Implementing callbacks.

- Java: interfaces.
- C: function pointers.
- C++: class-type functors.
- C#: delegates.
- Python, Perl, ML, Javascript: first-class functions.

Callbacks: Java interfaces

Interface. A type that defines a set of methods that a class can provide.

```
public interface Comparable<Item>
{
   public int compareTo(Item that);
}
```

contract: one method with this signature (and prescribed behavior)

Class that implements interface. Must implement all interface methods.

```
public class String implements Comparable<String>
{
   ...
   public int compareTo(String that)
   {
      ...
   }
```

class promises to honor the contract

class abides by the contract

Impact.

- You can invoke the `compareTo()` method on any `String` object
- Enables callbacks.

Callbacks: roadmap

client (StringSorter.java)

```
public class StringSorter
{
   public static void main(String[] args)
   {
      String[] a = StdIn.readAllStrings();
      Insertion.sort(a);
      for (int i = 0; i < a.length; i++)
         StdOut.println(a[i]);
   }
}
```

java.lang.Comparable interface

```
public interface Comparable<Item>
{
   public int compareTo(Item that);
}
```

sort implementation (Insertion.java)

```
public static void sort(Comparable[] a)
{
   int n = a.length;
   for (int i = 0; i < n; i++)
      for (int j = i; j > 0; j--)
         if (a[j].compareTo(a[j-1]) < 0)
               exch(a, j, j-1);
         else break;
}
```

callback

data type implementation (String.java)

```
public class String
implements Comparable<String>
{
   ...
   public int compareTo(String that)
   {
      ...
   }
}
```

key point: no dependence on type of data to be sorted

Elementary sorts: quiz 1

Suppose that the Java architects leave out `implements Comparable<String>` **in the class declaration for** `String`**. What would be the effect?**

A. `String.java` won't compile.

B. `StringSorter.java` won't compile.

C. `Insertion.java` won't compile.

D. `Insertion.java` will throw an exception.

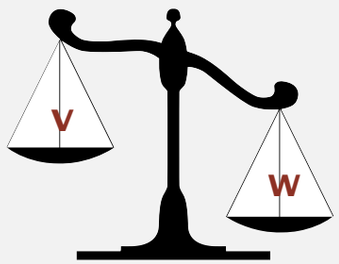
java.lang.Comparable API

Implement `compareTo()` so that `v.compareTo(w)`

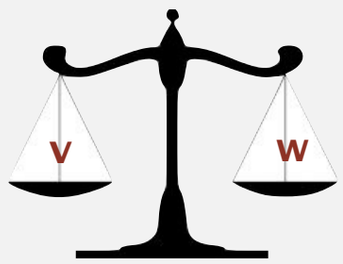
- Returns a
  - negative integer if v is less than w
  - positive integer if v is greater than w
  - zero if v is equal to w

  } `v.compareTo(w) <= 0` means v is less than or equal to w

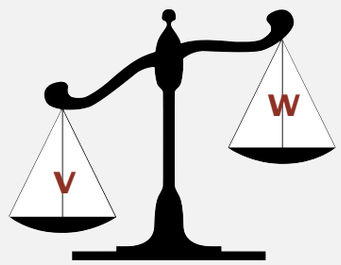
- Defines a total order.
- Throws an exception if incompatible types (or either is `null`).



less than
(return negative integer)



equal to
(return 0)



greater than
(return positive integer)

Built-in comparable types. `Integer`, `Double`, `String`, `Date`, `File`, ...

User-defined comparable types. Implement the `Comparable` interface.

Implementing the Comparable interface

Date data type. Simplified version of `java.util.Date`.

```
public class Date implements Comparable<Date>
{
   private final int month, day, year;

   public Date(int m, int d, int y)
   {
      month = m;
      day   = d;
      year  = y;
   }

   public int compareTo(Date that)
   {
      if (this.year  < that.year ) return -1;
      if (this.year  > that.year ) return +1;
      if (this.month < that.month) return -1;
      if (this.month > that.month) return +1;
      if (this.day   < that.day  ) return -1;
      if (this.day   > that.day  ) return +1;
      return 0;
   }
}
```

can compare Date objects only to other Date objects

https://algs4.cs.princeton.edu/12oop/Date.java.html

https://algs4.cs.princeton.edu

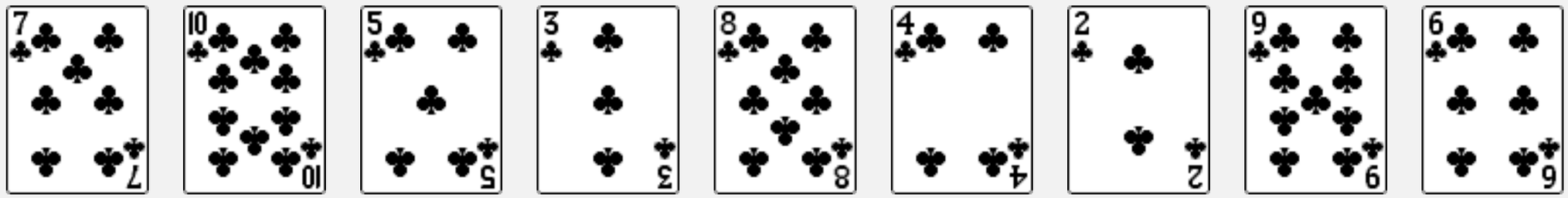
2.1 Elementary Sorts

- *rules of the game*
- *selection sort*
- *insertion sort*
- *shuffling*
- *comparators*

Selection sort

- In iteration `i`, find index `min` of smallest remaining entry.
- Swap `a[i]` and `a[min]`.

initial array



Example: in iteration 0, swap 2♣ and 7♣.

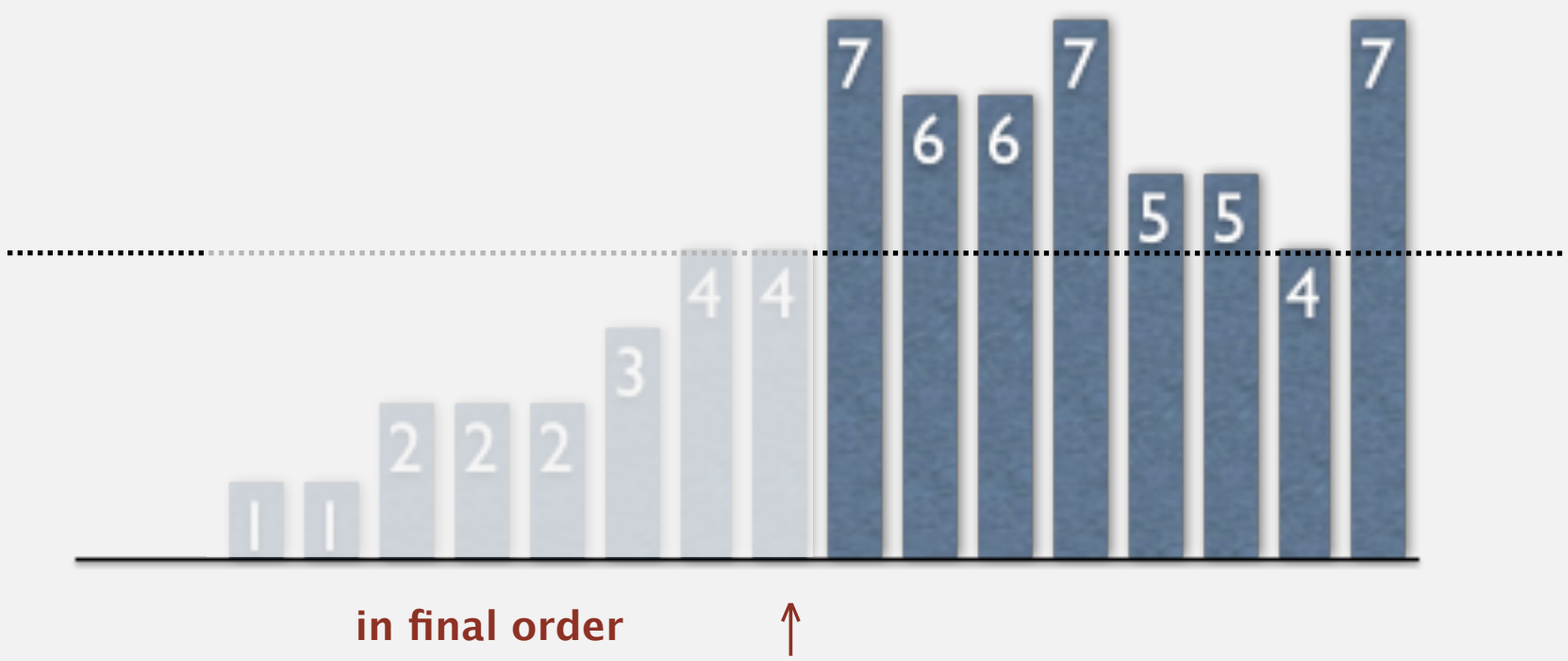
Exercise: what observations can you make about the left half of the array after half the iterations have completed? What about the right half?

Selection sort

Algorithm. ↑ scans from left to right.

Invariants.

- Entries the left of ↑ (including ↑) fixed and in ascending order.
- No entry to right of ↑ is smaller than any entry to the left of ↑.



Two useful sorting abstractions

Helper functions. Refer to data only through compares and exchanges.

Less. Is item `v` less than `w` ?

```
private static boolean less(Comparable v, Comparable w)
{  return v.compareTo(w) < 0;  }
```

Exchange. Swap item in array `a[]` at index `i` with the one at index `j`.

```
private static void exch(Object[] a, int i, int j)
{
   Object swap = a[i];
   a[i] = a[j];
   a[j] = swap;
}
```

Selection sort: Java implementation

```
public class Selection
{
   public static void sort(Comparable[] a)
   {
      int n = a.length;
      for (int i = 0; i < n; i++)
      {
         int min = i;
         for (int j = i+1; j < n; j++)
            if (less(a[j], a[min]))
               min = j;
         exch(a, i, min);
      }
   }

   private static boolean less(Comparable v, Comparable w)
   {  /* see previous slide */  }

   private static void exch(Object[] a, int i, int j)
   {  /* see previous slide */  }
}
```

In iteration *i* ...

Find the index `min` of the smallest remaining entry

Swap `a[i]` and `a[min]`

https://algs4.cs.princeton.edu/21elementary/Selection.java.html

Generic methods

Oops. The compiler complains.

```
% javac-algs4 Selection.java
Selection.java:83: warning: [unchecked] unchecked call to
compareTo(T) as a member of the raw type java.lang.Comparable
       return (v.compareTo(w) < 0);
                           ^
1 warning
```

Q. How to silence the compiler?

Generic methods

Pedantic (type-safe) version. Compiles without any warnings.

generic type variable (static method)
(type inferred from argument; must be Comparable)

```
public class SelectionPedantic
{
   public  static <Key extends Comparable<Key>> void sort(Key[] a)
   {  /* as before */  }

   private static <Key extends Comparable<Key>> boolean less(Key v, Key w)
   {  /* as before */  }

   private static Object void exch(Object[] a, int i, int j)
   {  /* as before */  }
}
```

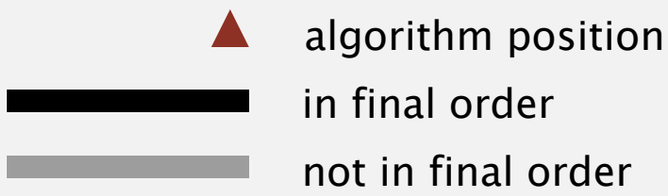
https://algs4.cs.princeton.edu/21elementary/SelectionPedantic.java.html

and Assignment 3

Remark. Use type-safe version in system code (but not in lecture).

Selection sort: animations

20 random items



http://www.sorting-algorithms.com/selection-sort

Elementary sorts: quiz 2

How many compares does selection sort make to sort an array of n distinct items?

A. $\sim n$

B. $\sim 1/4\ n^2$

C. $\sim 1/2\ n^2$

D. $\sim n^2$

Selection sort: mathematical analysis

Proposition. Selection sort makes $(n-1) + (n-2) + \ ... + 1 + 0 \ \sim \ n^2/2$ compares and n exchanges to sort any array of n items.

i	min	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
0	6	S	O	R	T	E	X	A	M	P	L	E
1	4	A	O	R	T	E	X	S	M	P	L	E
2	10	A	E	R	T	O	X	S	M	P	L	E
3	9	A	E	E	T	O	X	S	M	P	L	R
4	7	A	E	E	L	O	X	S	M	P	T	R
5	7	A	E	E	L	M	X	S	O	P	T	R
6	8	A	E	E	L	M	O	S	X	P	T	R
7	10	A	E	E	L	M	O	P	X	S	T	R
8	8	A	E	E	L	M	O	P	R	S	T	X
9	9	A	E	E	L	M	O	P	R	S	T	X
10	10	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

Running time insensitive to input. Quadratic time, even if input is sorted.

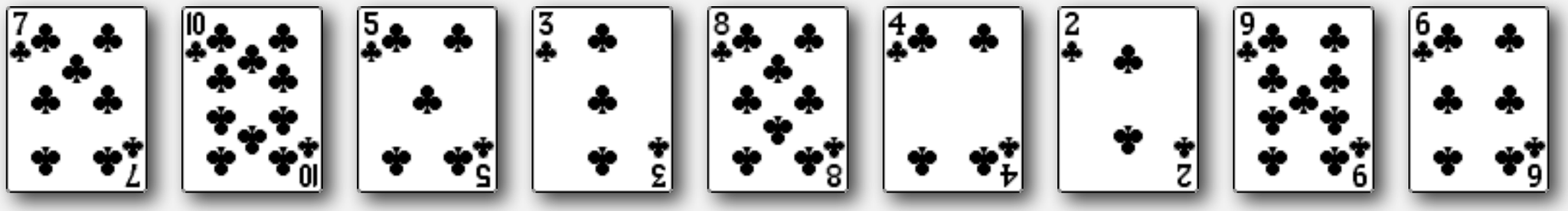
https://algs4.cs.princeton.edu

2.1 Elementary Sorts

- *rules of the game*
- *selection sort*
- *insertion sort*
- *shuffling*
- *comparators*

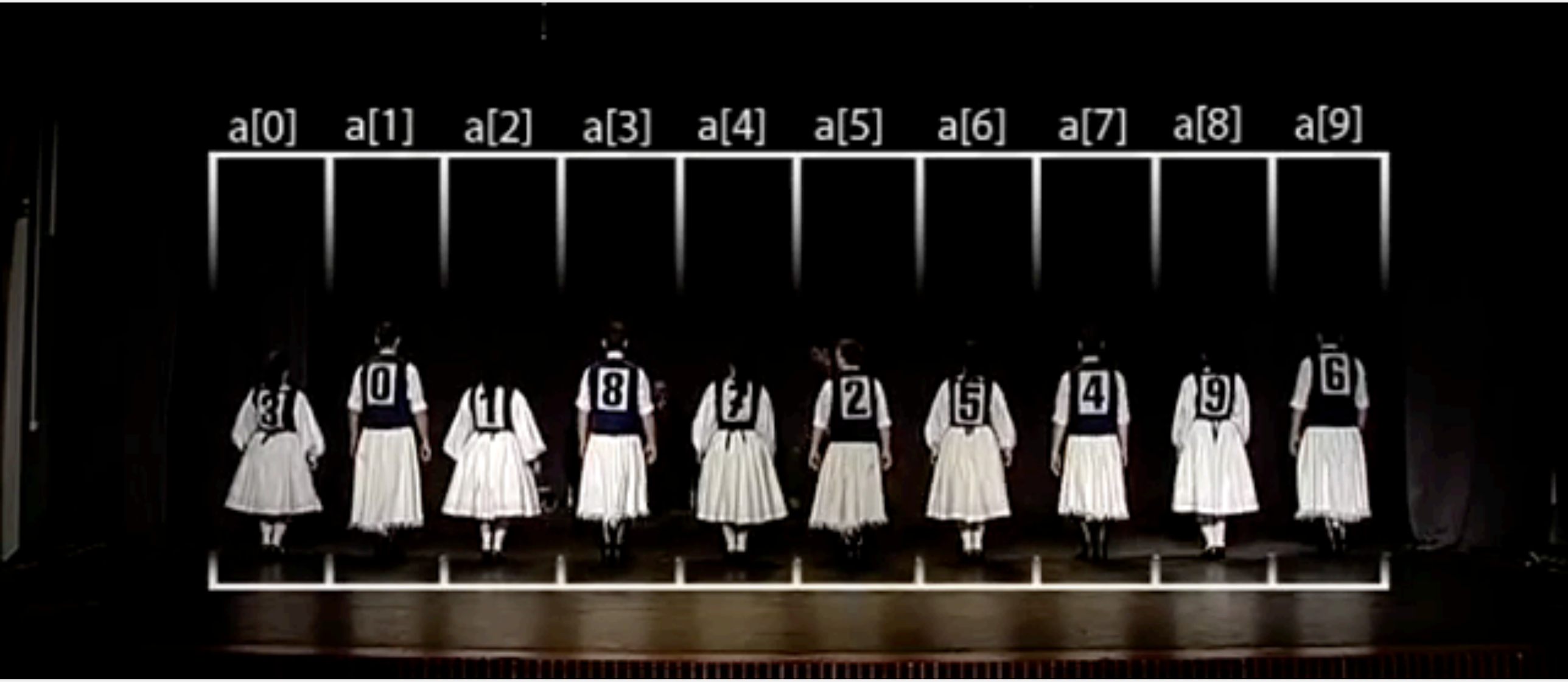
Insertion sort demo

- In iteration `i`, swap `a[i]` with each larger entry to its left.



Insertion sort demo

- In iteration `i`, swap `a[i]` with each larger entry to its left.



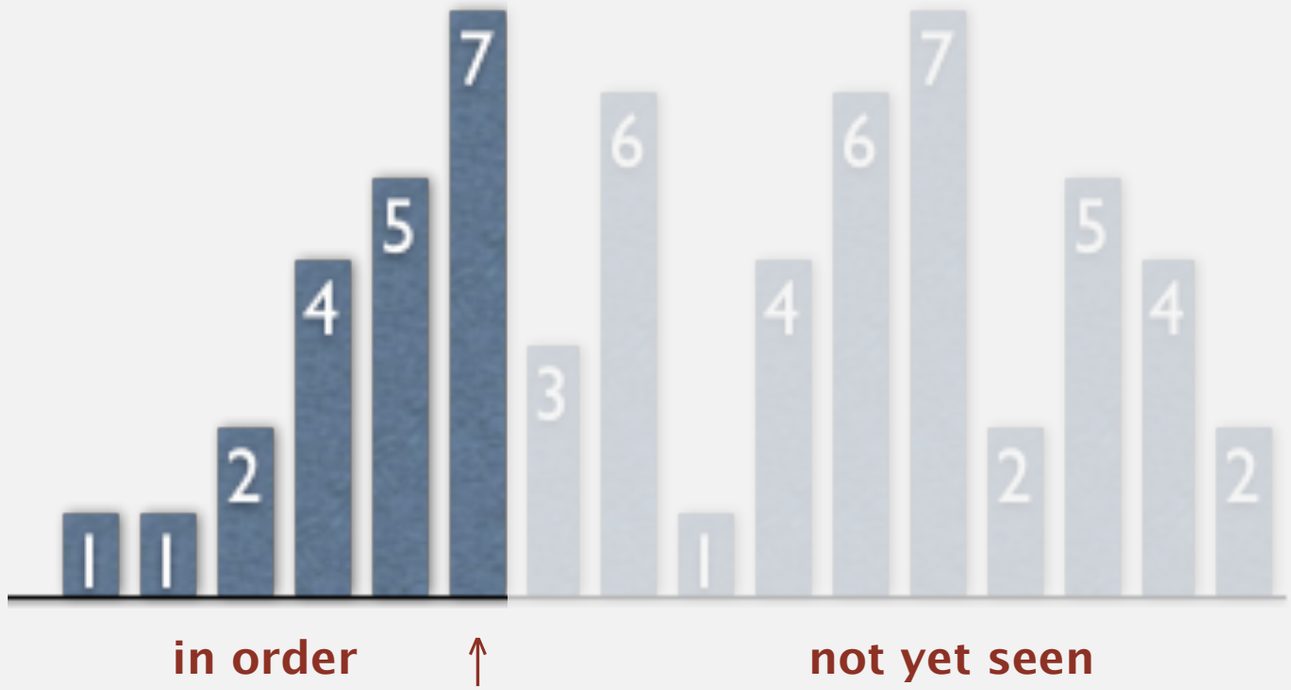
https://www.youtube.com/watch?v=ROalU379l3U

Insertion sort

Algorithm. ↑ scans from left to right.

Invariants.

- Entries to the left of ↑ (including ↑) are in ascending order.
- Entries to the right of ↑ have not yet been seen.



Insertion sort: inner loop

To maintain algorithm invariants:

- Move the pointer to the right.

```
i++;
```

- Moving from right to left, exchange `a[i]` with each larger entry to its left.

```
for (int j = i; j > 0; j--)
   if (less(a[j], a[j-1]))
         exch(a, j, j-1);
   else break;
```

Insertion sort: Java implementation

```
public class Insertion
{
   public static void sort(Comparable[] a)
   {
      int n = a.length;
      for (int i = 0; i < n; i++)
         for (int j = i; j > 0; j--)
            if (less(a[j], a[j-1]))
               exch(a, j, j-1);
            else break;
   }

   private static boolean less(Comparable v, Comparable w)
   {  /* as before */  }

   private static void exch(Object[] a, int i, int j)
   {  /* as before */  }
}
```

https://algs4.cs.princeton.edu/21elementary/Insertion.java.html

Elementary sorts: quiz 3

How many compares does insertion sort make to sort an array of n distinct keys in the worst case?

Hint: what is the worst case input?

A. $\sim n$

B. $\sim 1/4 \ n^2$

C. $\sim 1/2 \ n^2$

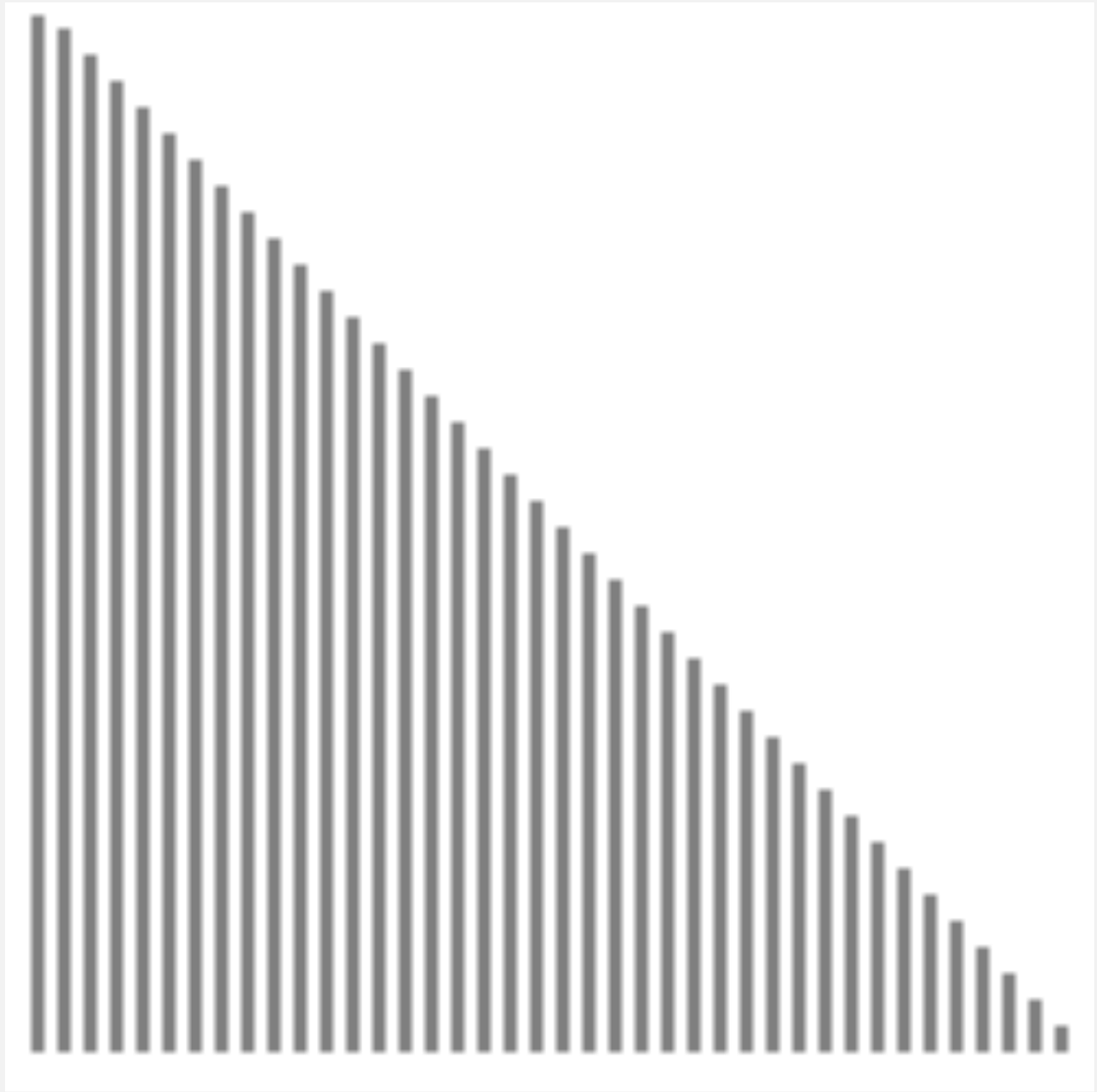
D. $\sim n^2$

Insertion sort: analysis

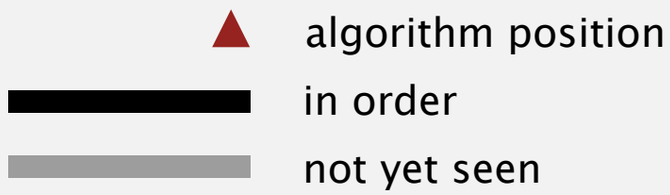
Worst case. Insertion sort makes $\sim \frac{1}{2} n^2$ compares and $\sim \frac{1}{2} n^2$ exchanges to sort an array of n distinct keys in reverse order.

Pf. Exactly i compares and exchanges in iteration i.

$0 + 1 + 2 + \ldots + (n - 1)$

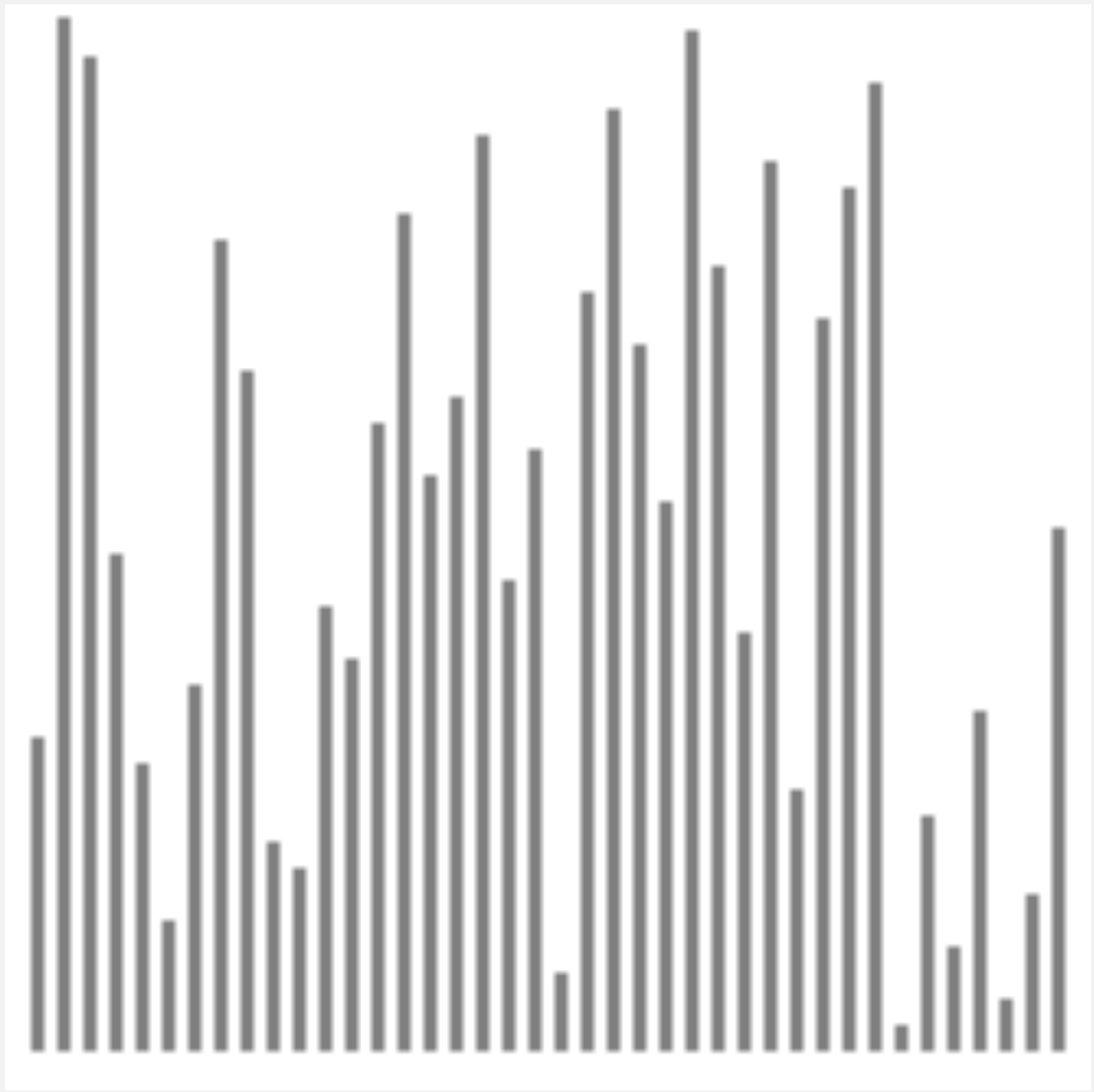


http://www.sorting-algorithms.com/insertion-sort



Insertion sort: mathematical analysis

Average case. To sort a randomly ordered array with n distinct keys, insertion sort makes $\sim \frac{1}{4} n^2$ compares and $\sim \frac{1}{4} n^2$ exchanges on average.



http://www.sorting-algorithms.com/insertion-sort

Insertion sort: mathematical analysis

Average case. To sort a randomly ordered array with n distinct keys, insertion sort makes $\sim \frac{1}{4} n^2$ compares and $\sim \frac{1}{4} n^2$ exchanges on average.

Pf. Expect $\sim \frac{1}{2} i$ compares and $\sim \frac{1}{2} i$ exchanges in iteration i.

$$\frac{0 + 1 + 2 + \ldots + (n-1)}{2}$$

		a[]										
i	j	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
1	0	O	S	R	T	E	X	A	M	P	L	E
2	1	O	R	S	T	E	X	A	M	P	L	E
3	3	O	R	S	T	E	X	A	M	P	L	E
4	0	E	O	R	S	T	X	A	M	P	L	E
5	5	E	O	R	S	T	X	A	M	P	L	E
6	0	A	E	O	R	S	T	X	M	P	L	E
7	2	A	E	M	O	R	S	T	X	P	L	E
8	4	A	E	M	O	P	R	S	T	X	L	E
9	2	A	E	L	M	O	P	R	S	T	X	E
10	2	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

entries in gray do not move

entry in red is `a[j]`

entries in black moved one position right for insertion

Trace of insertion sort (array contents just after each insertion)

Elementary sorts: quiz 4

Which is faster in practice to sort an array of n random items, selection sort or insertion sort?

A. Selection sort.

B. Insertion sort.

C. No significant difference.

D. It depends.

Recall:

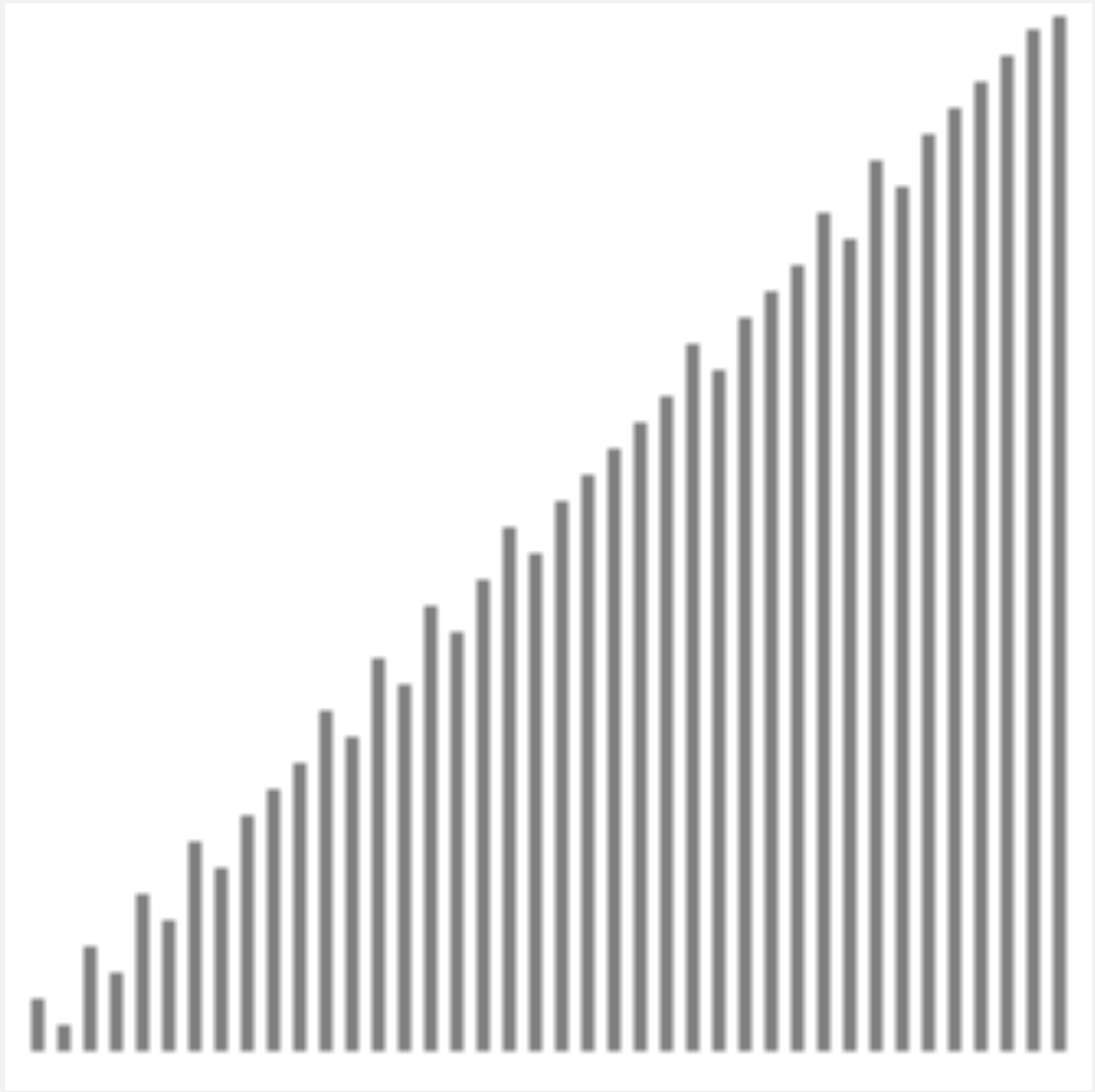
Selection sort [any input]: $\sim \frac{1}{2}\, n^2$ compares and $\sim n$ exchanges

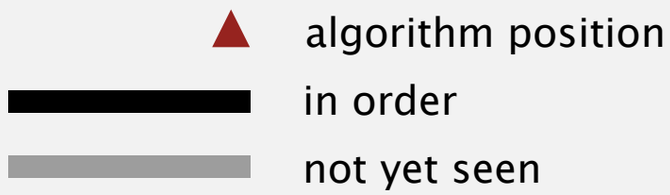
Insertion sort [worst case]: $\sim \frac{1}{2}\, n^2$ compares and $\sim \frac{1}{2}\, n^2$ exchanges

Insertion sort [average case]: $\sim \frac{1}{4}\, n^2$ compares and $\sim \frac{1}{4}\, n^2$ exchanges

Insertion sort: analysis

Best case. Insertion sort makes $n-1$ compares and 0 exchanges to sort an array of n distinct keys in ascending order.





http://www.sorting-algorithms.com/insertion-sort

Insertion sort: partially sorted arrays

Def. An inversion is a pair of keys that are out of order.

A E E L M O T R X P S

1	2	3	4	5	6
T–R	T–P	T–S	R–P	X–P	X–S

(6 inversions)

Def. A family of arrays is partially sorted if the number of inversions is $\leq c\,n$.

- Ex 1. A sorted array.
- Ex 2. A subarray of length 10 appended to a sorted subarray of length n.

At most $10n + 45$ inversions

Proposition. Insertion sort runs in linear time on partially sorted arrays.

Pf.

exchange decreases number of inversions by 1

- Number of exchanges in insertion sort = number of inversions.
- Number of compares $\leq$ number of exchanges + $(n - 1)$.

each compare in iteration i triggers an exchange (except possibly last one)

Improvement: binary insertion sort

Use binary search to find insertion point.

- Number of compares $\sim n \log_2 n$.
- But still a quadratic number of array accesses.

https://algs4.cs.princeton.edu

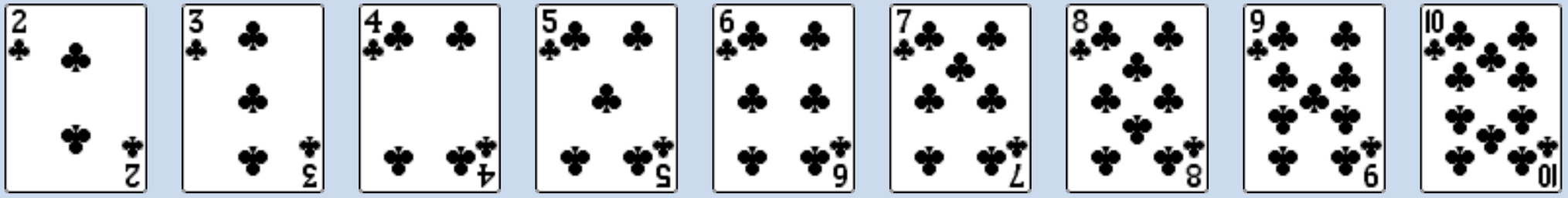
2.1 Elementary Sorts

- *rules of the game*
- *selection sort*
- *insertion sort*
- *shuffling*
- *comparators*

INTERVIEW QUESTION: SHUFFLE AN ARRAY

Goal. Rearrange array so that result is a uniformly random permutation.

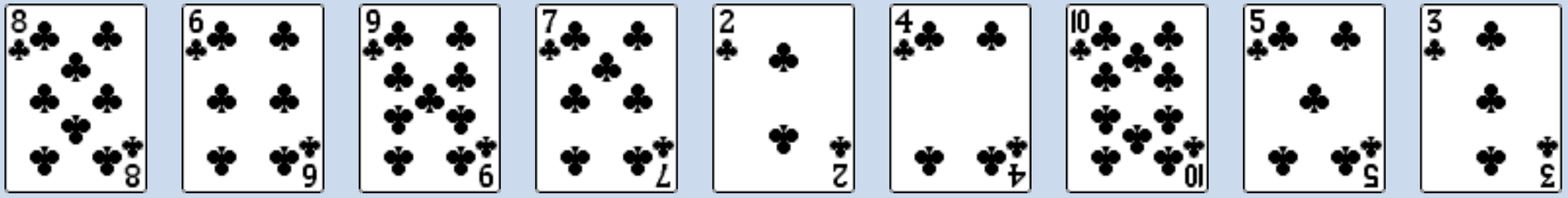
all $n!$ permutations
equally likely



INTERVIEW QUESTION: SHUFFLE AN ARRAY

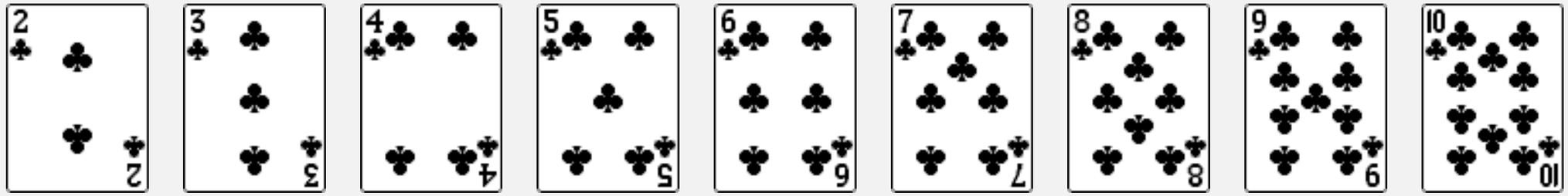
Goal. Rearrange array so that result is a uniformly random permutation.

all $n!$ permutations
equally likely



Shuffling by sorting

- Generate a random real number for each array entry.
- Sort the array using the real numbers as keys.

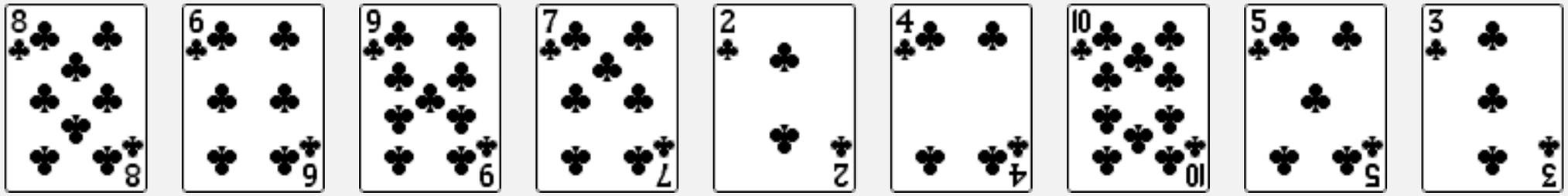


0.8003 **0.9706** **0.9157** **0.9649** **0.1576** **0.4854** **0.1419** **0.4218** **0.9572**

Shuffling by sorting

- Generate a random real number for each array entry.
- Sort the array using the real numbers as keys.

assume no ties

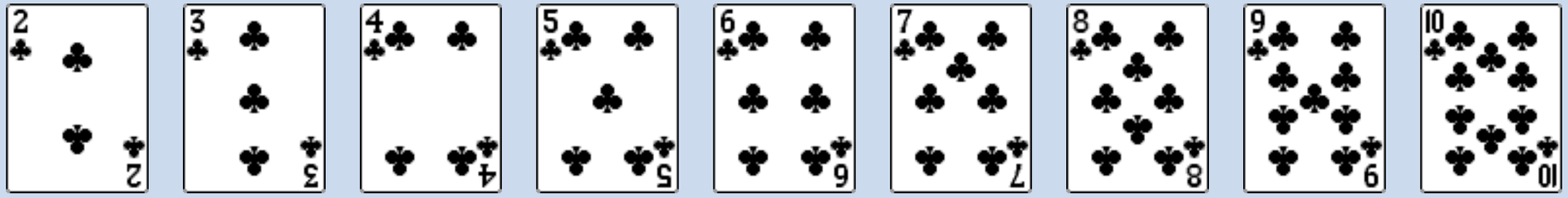


0.1419 0.1576 0.4218 0.4854 0.8003 0.9157 0.9572 0.9649 0.9706

INTERVIEW QUESTION: SHUFFLE AN ARRAY

Goal. Rearrange array so that result is a uniformly random permutation.

all $n!$ permutations
equally likely



Shuffling by sorting is quadratic (using insertion or selection sort)
or linearithmic (using mergesort or quicksort).

Exercise. Devise with a linear-time algorithm for shuffling.

War story (Microsoft)

Microsoft antitrust probe by EU. Microsoft agreed to provide a randomized ballot screen for users to select browser.

appeared last 50% of the time

War story (Microsoft)

Microsoft antitrust probe by EU. Microsoft agreed to provide a randomized ballot screen for users to select browser.

Solution? Implement shuffle sort by making comparator always return a random answer.

```
public int compareTo(Browser that)
{
   double r = Math.random();
   if (r < 0.5) return -1;
   if (r > 0.5) return +1;
   return 0;
}
```

browser comparator

(fails to implement a total order)

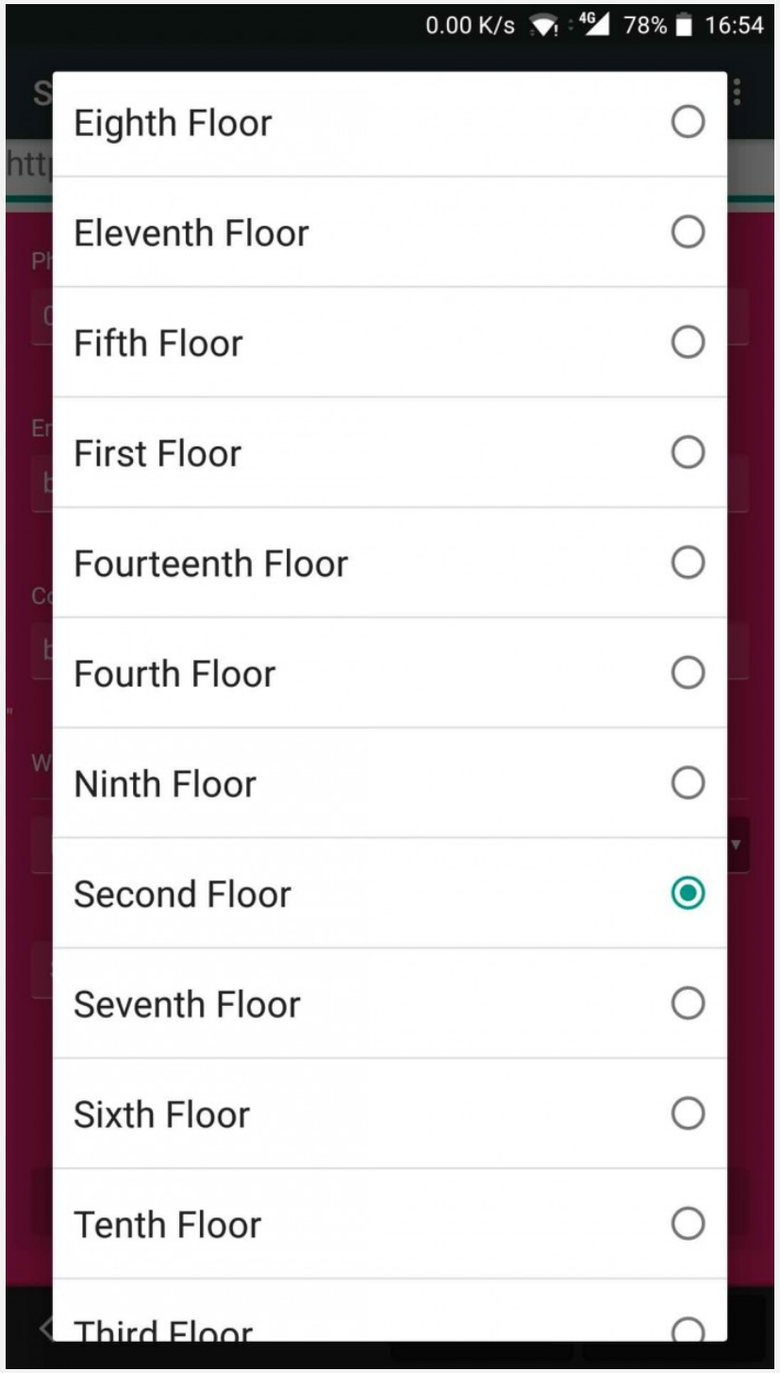
https://algs4.cs.princeton.edu

2.1 Elementary Sorts

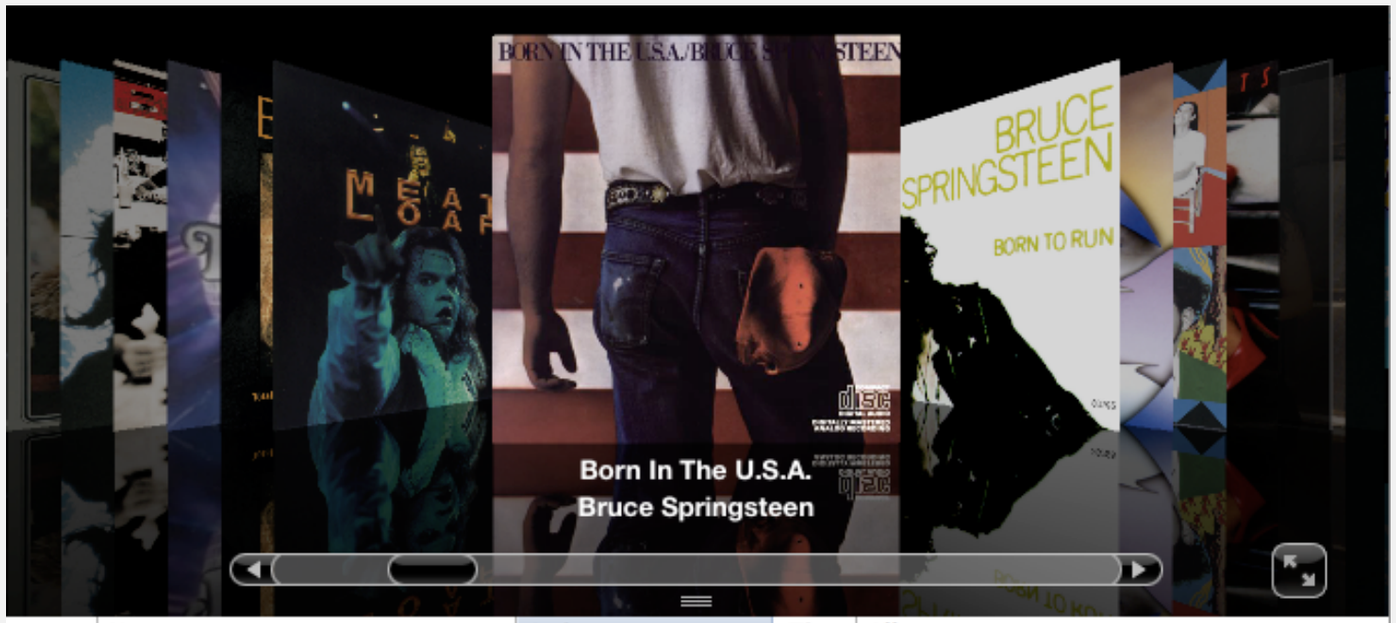
- *rules of the game*
- *selection sort*
- *insertion sort*
- *shuffling*
- *comparators*

Different orderings

Q. When might we need to define different sort orderings?

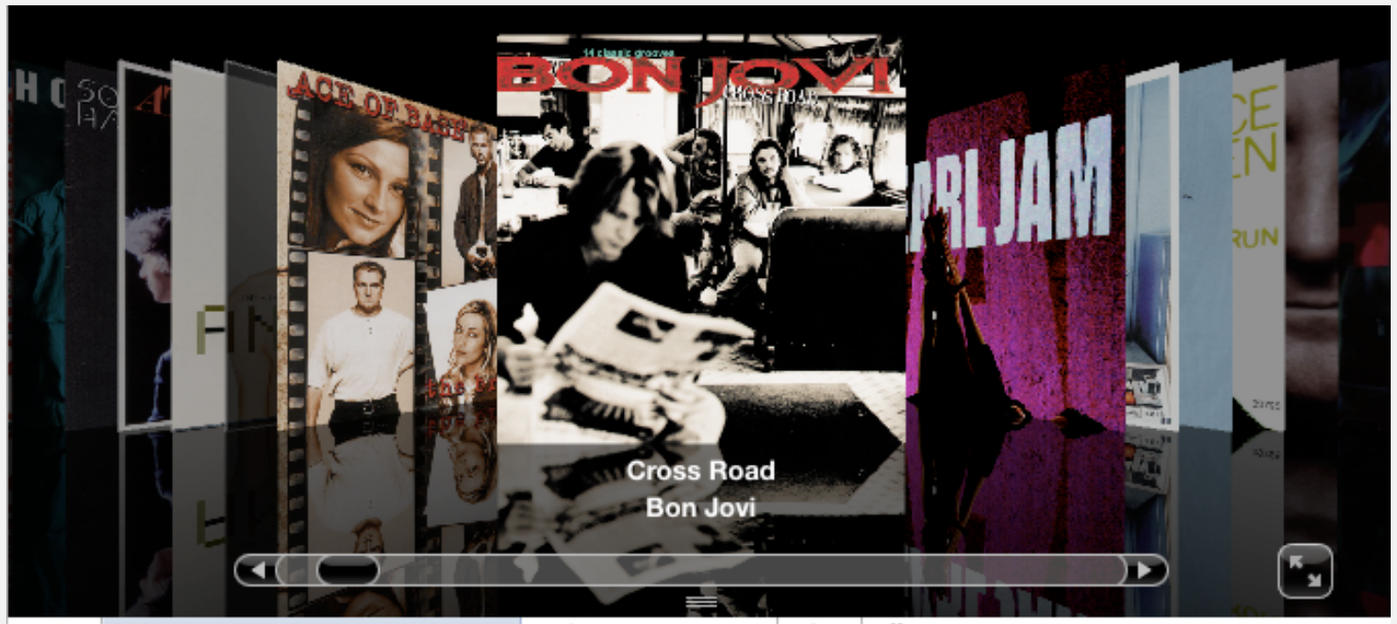


Sort music library by artist



	Name	Artist	Time	Album
12	Let It Be	The Beatles	4:03	Let It Be
13	Take My Breath Away	BERLIN	4:13	Top Gun – Soundtrack
14	Circle Of Friends	Better Than Ezra	3:27	Empire Records
15	Dancing With Myself	Billy Idol	4:43	Don't Stop
16	Rebel Yell	Billy Idol	4:49	Rebel Yell
17	Piano Man	Billy Joel	5:36	Greatest Hits Vol. 1
18	Pressure	Billy Joel	3:16	Greatest Hits, Vol. II (1978 – 1985) (Disc 2)
19	The Longest Time	Billy Joel	3:36	Greatest Hits, Vol. II (1978 – 1985) (Disc 2)
20	Atomic	Blondie	3:50	Atomic: The Very Best Of Blondie
21	Sunday Girl	Blondie	3:15	Atomic: The Very Best Of Blondie
22	Call Me	Blondie	3:33	Atomic: The Very Best Of Blondie
23	Dreaming	Blondie	3:06	Atomic: The Very Best Of Blondie
24	Hurricane	Bob Dylan	8:32	Desire
25	The Times They Are A-Changin'	Bob Dylan	3:17	Greatest Hits
26	Livin' On A Prayer	Bon Jovi	4:11	Cross Road
27	Beds Of Roses	Bon Jovi	6:35	Cross Road
28	Runaway	Bon Jovi	3:53	Cross Road
29	Rasputin (Extended Mix)	Boney M	5:50	Greatest Hits
30	Have You Ever Seen The Rain	Bonnie Tyler	4:10	Faster Than The Speed Of Night
31	Total Eclipse Of The Heart	Bonnie Tyler	7:02	Faster Than The Speed Of Night
32	Straight From The Heart	Bonnie Tyler	3:41	Faster Than The Speed Of Night
33	Holding Out For A Hero	Bonny Tyler	5:49	Meat Loaf And Friends
34	Dancing In The Dark	Bruce Springsteen	4:05	Born In The U.S.A.
35	Thunder Road	Bruce Springsteen	4:51	Born To Run
36	Born To Run	Bruce Springsteen	4:30	Born To Run
37	Jungleland	Bruce Springsteen	9:34	Born To Run

Sort music library by song name



	Name	Artist	Time	Album
1	Alive	Pearl Jam	5:41	Ten
2	All Over The World	Pixies	5:27	Bossanova
3	All Through The Night	Cyndi Lauper	4:30	She's So Unusual
4	Allison Road	Gin Blossoms	3:19	New Miserable Experience
5	Ama, Ama, Ama Y Ensancha El ...	Extremoduro	2:34	Deltoya (1992)
6	And We Danced	Hooters	3:50	Nervous Night
7	As I Lay Me Down	Sophie B. Hawkins	4:09	Whaler
8	Atomic	Blondie	3:50	Atomic: The Very Best Of Blondie
9	Automatic Lover	Jay-Jay Johanson	4:19	Antenna
10	Baba O'Riley	The Who	5:01	Who's Better, Who's Best
11	Beautiful Life	Ace Of Base	3:40	The Bridge
12	Beds Of Roses	Bon Jovi	6:35	Cross Road
13	Black	Pearl Jam	5:44	Ten
14	Bleed American	Jimmy Eat World	3:04	Bleed American
15	Borderline	Madonna	4:00	The Immaculate Collection
16	Born To Run	Bruce Springsteen	4:30	Born To Run
17	Both Sides Of The Story	Phil Collins	6:43	Both Sides
18	Bouncing Around The Room	Phish	4:09	A Live One (Disc 1)
19	Boys Don't Cry	The Cure	2:35	Staring At The Sea: The Singles 1979-1985
20	Brat	Green Day	1:43	Insomniac
21	Breakdown	Deerheart	3:40	Deerheart
22	Bring Me To Life (Kevin Roen Mix)	Evanescence Vs. Pa...	9:48	
23	Californication	Red Hot Chili Pepp...	1:40	
24	Call Me	Blondie	3:33	Atomic: The Very Best Of Blondie
25	Can't Get You Out Of My Head	Kylie Minogue	3:50	Fever
26	Celebration	Kool & The Gang	3:45	Time Life Music Sounds Of The Seventies - C

Comparable interface: review

Comparable interface: sort using a type's natural order.

```
public class Date implements Comparable<Date>
{
   private final int month, day, year;

   public Date(int m, int d, int y)
   {
      month = m;
      day   = d;
      year  = y;
   }
   ...

   public int compareTo(Date that)
   {
      if (this.year  < that.year ) return -1;
      if (this.year  > that.year ) return +1;
      if (this.month < that.month) return -1;
      if (this.month > that.month) return +1;
      if (this.day   < that.day  ) return -1;
      if (this.day   > that.day  ) return +1;
      return 0;
   }
}
```

natural order

https://algs4.cs.princeton.edu/12oop/Date.java.html

Comparator interface

Comparator interface: sort using an alternate order.

```
public interface Comparator<Item>
{
   public int compare(Item v, Item w);
}
```

Required property. Must be a total order.

string order	example
natural order	Now is the time
case insensitive	is Now the time
Spanish language	café cafetero cuarto churro nube ñoño
British phone book	McKinley Mackintosh

Comparator interface: system sort

To use with Java system sort:

- Create `Comparator` object.
- Pass as second argument to `Arrays.sort()`.

```
String[] a;
...
Arrays.sort(a);
...
Arrays.sort(a, String.CASE_INSENSITIVE_ORDER);
...
Arrays.sort(a, Collator.getInstance(new Locale("es")));
...
Arrays.sort(a, new BritishPhoneBookOrder());
...
```

uses natural order

uses alternate order defined by Comparator<String> object

Bottom line. Decouples the definition of the data type from the definition of what it means to compare two objects of that type.

Comparator interface: using with our sorting libraries

To support comparators in our sort implementations:

- Pass `Comparator` to both `sort()` and `less()`, and use it in `less()`.
- Use `Object` instead of `Comparable`.


```
import java.util.Comparator;

public class Insertion
{
   ...

   public static void sort(Object[] a, Comparator comparator)
   {
      int n = a.length;
      for (int i = 0; i < n; i++)
         for (int j = i; j > 0 && less(comparator, a[j], a[j-1]); j--)
            exch(a, j, j-1);
   }

   private static boolean less(Comparator comparator, Object v, Object w)
   {  return comparator.compare(v, w) < 0;  }
}
```

https://algs4.cs.princeton.edu/21elementary/Insertion.java.html
https://algs4.cs.princeton.edu/21elementary/InsertionPedantic.java.html

Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the `Comparator` interface.
- Define it as `static` because there is one comparator for the whole class.
- Implement the `compare()` method.
- Provide client access to `Comparator`.

```
import java.util.Comparator;

public class Student
{
   private final String name;
   private final int section;
   ...

   private static class NameOrder implements Comparator<Student>
   {
      public int compare(Student v, Student w)
      {  return v.name.compareTo(w.name);  }
   }
   public static Comparator<Student> byNameOrder()
   {  return new NameOrder();  }
}
```

https://algs4.cs.princeton.edu/12oop/Student.java.html

Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the `Comparator` interface.
- Implement the `compare()` method.
- Provide client access to `Comparator`.

```
Insertion.sort(a, Student.byNameOrder());
```

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

```
Insertion.sort(a, Student.bySectionOrder());
```

Furia	1	A	(766) 093-9873	101 Brown
Rohde	2	A	(232) 343-5555	343 Forbes
Andrews	3	A	(664) 480-0023	097 Little
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Kanaga	3	B	(898) 122-9643	22 Brown
Battle	4	C	(874) 088-1212	121 Whitman
Gazsi	4	B	(800) 867-5309	101 Brown

Stability

A typical application. First, sort by name; then sort by section.

```
Selection.sort(a, Student.byNameOrder());
```

Andrews	3	A	(664) 480-0023	097 Little
Battle	4	C	(874) 088-1212	121 Whitman
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Furia	1	A	(766) 093-9873	101 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Kanaga	3	B	(898) 122-9643	22 Brown
Rohde	2	A	(232) 343-5555	343 Forbes

```
Selection.sort(a, Student.bySectionOrder());
```

Furia	1	A	(766) 093-9873	101 Brown
Rohde	2	A	(232) 343-5555	343 Forbes
Chen	3	A	(991) 878-4944	308 Blair
Fox	3	A	(884) 232-5341	11 Dickinson
Andrews	3	A	(664) 480-0023	097 Little
Kanaga	3	B	(898) 122-9643	22 Brown
Gazsi	4	B	(800) 867-5309	101 Brown
Battle	4	C	(874) 088-1212	121 Whitman

@#%&@! Students in section 3 no longer sorted by name.

A stable sort preserves the relative order of items with equal keys.

Elementary sorts: quiz 5

Which sorting algorithm(s) are stable?

A. Selection sort.

B. Insertion sort.

C. Both A and B.

D. Neither A nor B.

Stability: insertion sort

Proposition. Insertion sort is stable.

```
public class Insertion
{
   public static void sort(Comparable[] a)
   {
      int n = a.length;
      for (int i = 0; i < n; i++)
         for (int j = i; j > 0 && less(a[j], a[j-1]); j--)
            exch(a, j, j-1);
   }
}
```

i	j	0	1	2	3	4
0	0	B_1	A_1	A_2	A_3	B_2
1	0	A_1	B_1	A_2	A_3	B_2
2	1	A_1	A_2	B_1	A_3	B_2
3	2	A_1	A_2	A_3	B_1	B_2
4	4	A_1	A_2	A_3	B_1	B_2
		A_1	A_2	A_3	B_1	B_2

Pf. Equal items never move past each other.

Stability: selection sort

Proposition. Selection sort is not stable.

```
public class Selection
{
   public static void sort(Comparable[] a)
   {
      int n = a.length;
      for (int i = 0; i < n; i++)
      {
         int min = i;
         for (int j = i+1; j < n; j++)
            if (less(a[j], a[min]))
               min = j;
         exch(a, i, min);
      }
   }
}
```

i	min	0	1	2
0	2	B_1	B_2	A
1	1	A	B_2	B_1
2	2	A	B_2	B_1
		A	B_2	B_1

Pf by counterexample. Long-distance exchange can move an equal item past another one.