Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

Algorithms

FOURTH EDITION

ROBERT SEDGEWICK | KEVIN WAYNE

https://algs4.cs.princeton.edu

1.3 Stacks and Queues

- *stacks*
- *resizing arrays*
- *queues*
- *generics*
- *iterators* ← see precept
- *applications*

Last updated on 2/12/19 7:33 AM

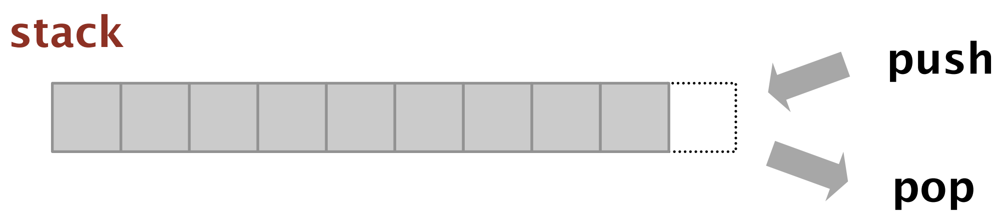
Stacks and queues: fundamental data types

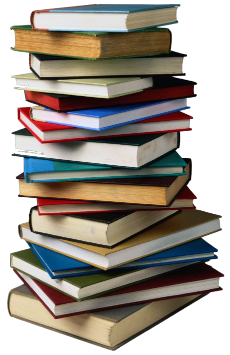
Both are collections of objects.

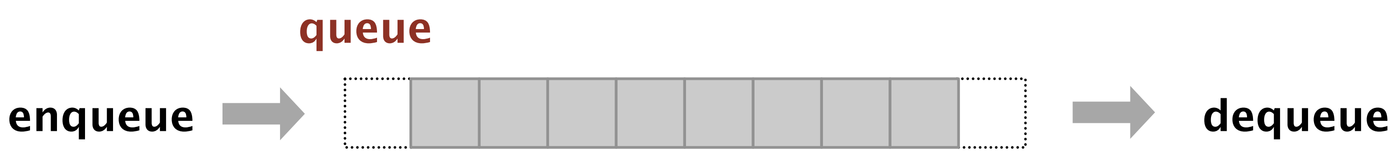
Both support add, remove, iterate, test if empty.

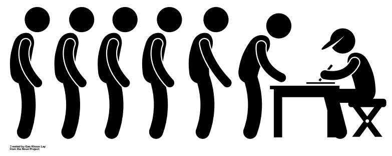
Intent is clear when we add.

Difference between stack and queue: which item to remove.







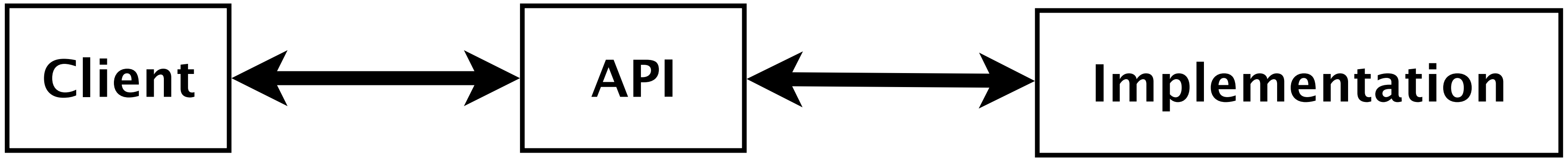


Stack. Remove the item most recently added. ← LIFO = "last in first out"

Queue. Remove the item least recently added. ← FIFO = "first in first out"

Client, implementation, API

Separate client and implementation via API.



API: operations that characterize the behavior of a data type.

Client: program that uses the API operations.

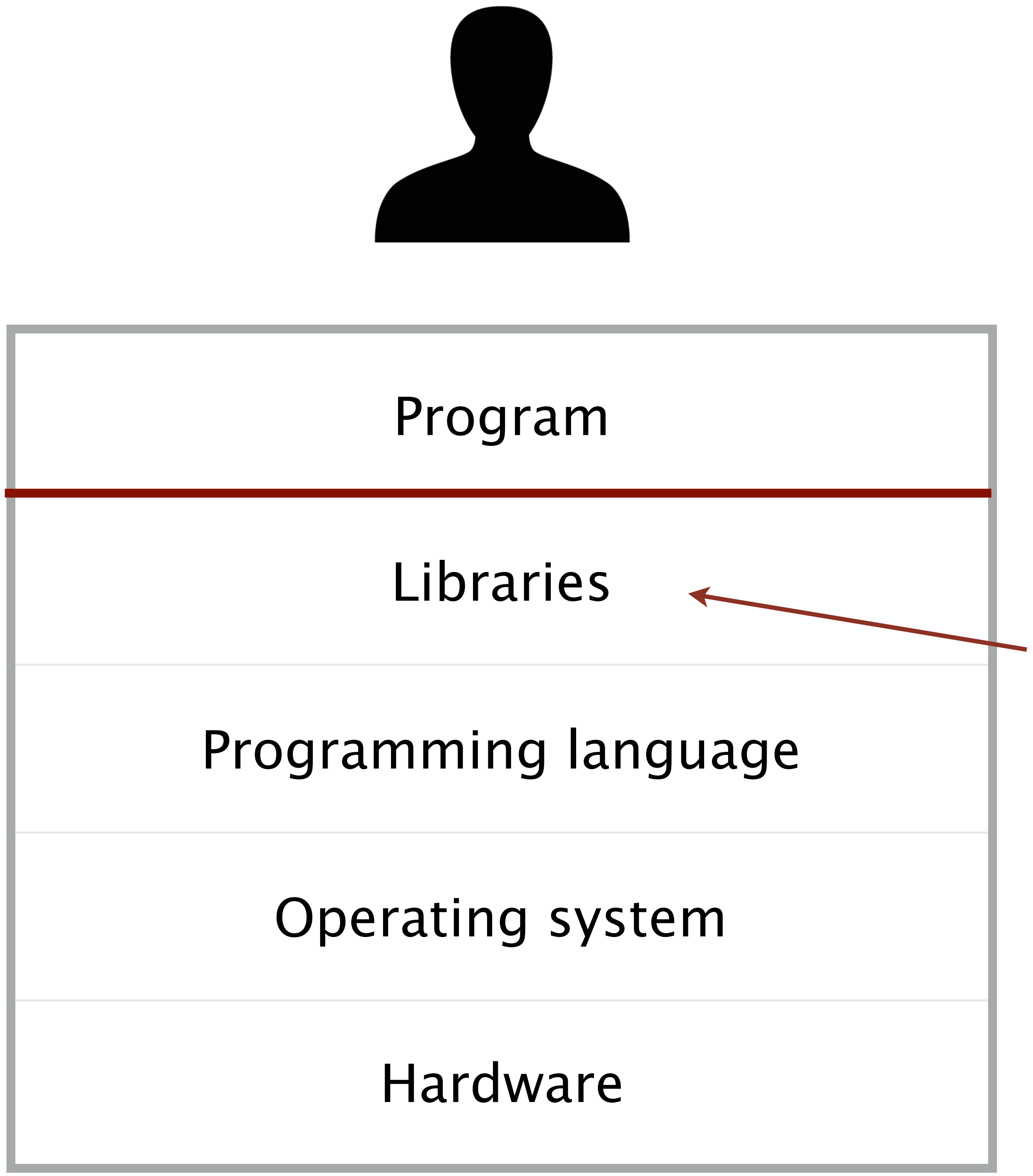
Implementation: code that implements the API operations.

Benefits.

- Design: create modular, reusable libraries.
- Performance: substitute faster implementations.

Ex. Stack, queue, bag, priority queue, symbol table, union-find, ….

Layers in a computer system



Program

Libraries

Programming language

Operating system

Hardware

API

Java libraries include stacks and queues but in this course we'll prefer our own implementations

Algorithms

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1.3 Stacks and Queues

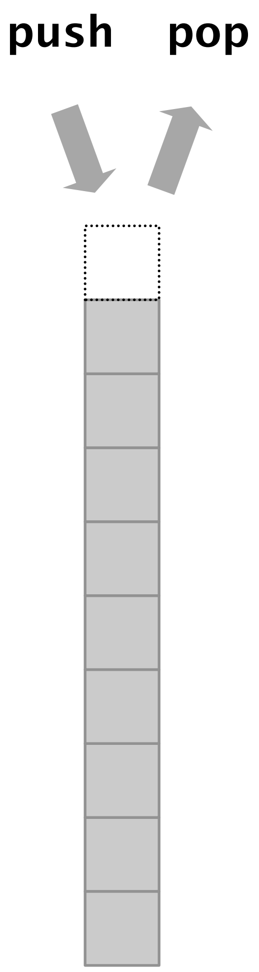
- *stacks*
- *resizing arrays*
- *queues*
- *generics*
- *iterators*
- *applications*

Stack API

Warmup API. Stack of strings data type.

public class StackOfStrings

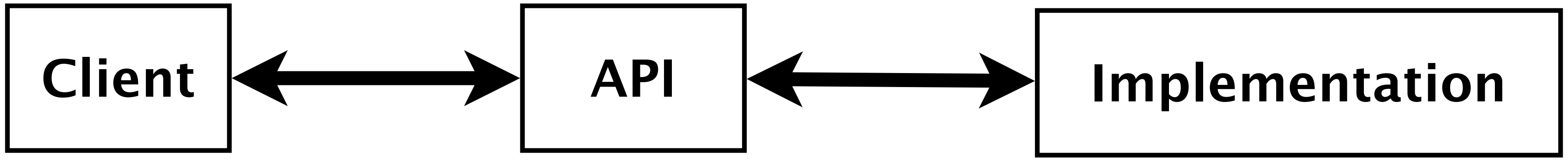
	StackOfStrings()	*create an empty stack*
void	push(String item)	*add a new string to stack*
String	pop()	*remove and return the string most recently added*
boolean	isEmpty()	*is the stack empty?*
int	size()	*number of strings on the stack*



Performance requirements. All operations must take constant time.

Either data type can be implemented using either data structure

	Linked list	Array
Stack	✓	✓
Queue	✓	✓



Client

API

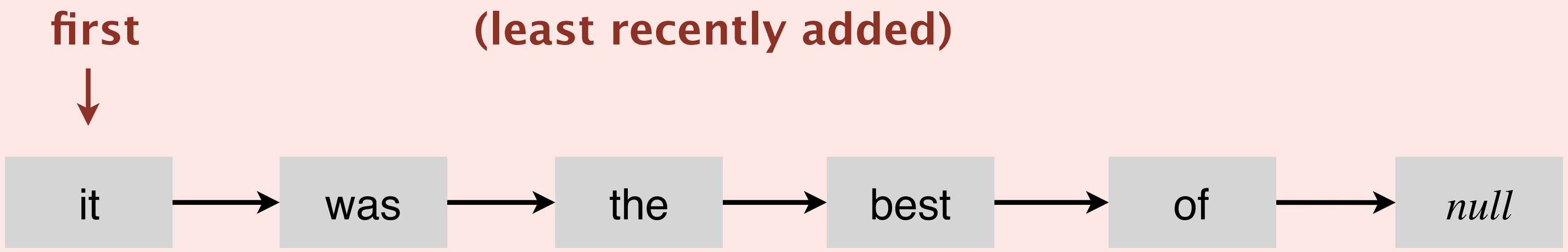
Implementation

Stacks and queues: quiz 1

How to implement a stack with a singly linked list?

Recall: we only keep track of the head of the list.

A.

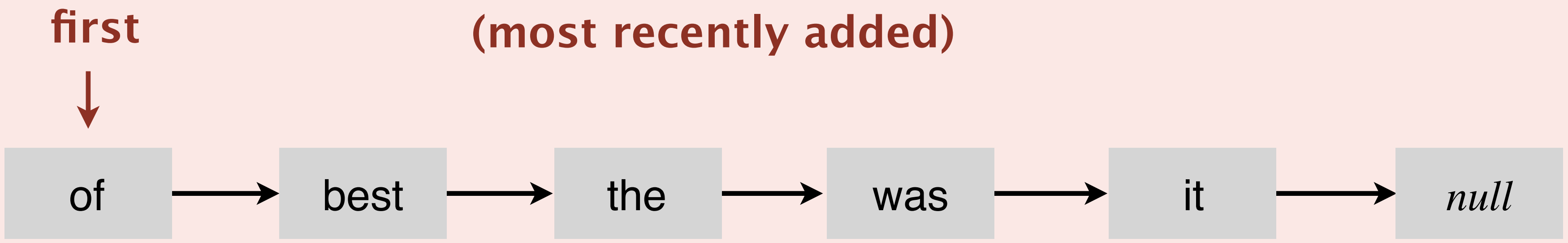


first

(least recently added)

it → was → the → best → of → *null*

B.



first

(most recently added)

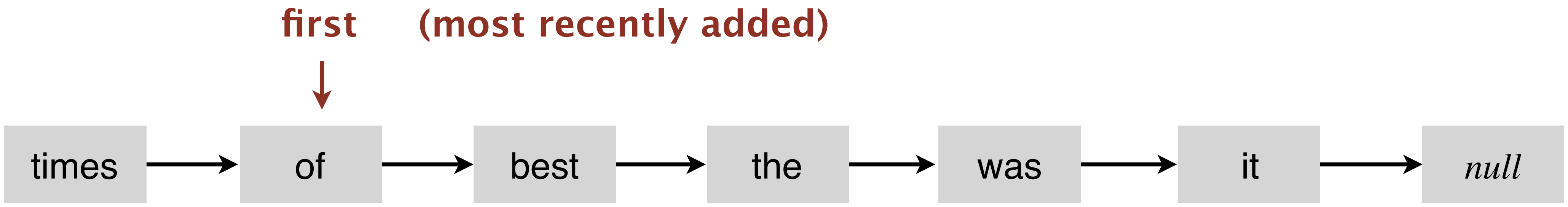
of → best → the → was → it → *null*

C. *Both A and B.*

D. *Neither A nor B.*

Stack: linked-list implementation

- Maintain pointer `first` to first node in a singly linked list.
- Push new item before `first`.
- Pop item from `first`.



Stack: linked-list implementation

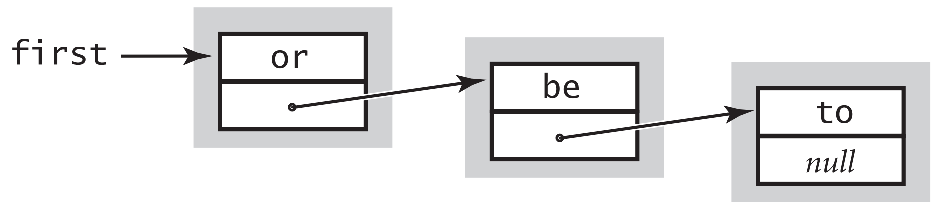
```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      private String item;
      private Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

private inner class
(access modifiers for instance
variables of such a class don’t matter)

Stack pop: linked-list implementation

inner class

```
private class Node
{
   String item;
   Node next;
}
```

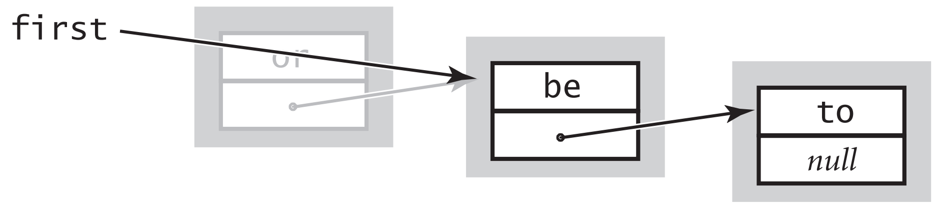


save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```



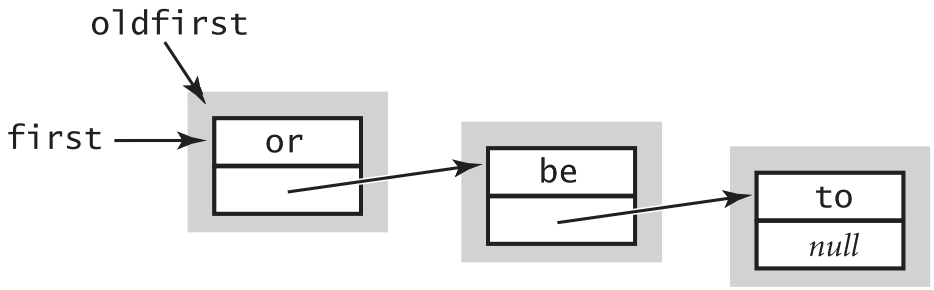
return saved item

```
return item;
```

Stack push: linked-list implementation

save a link to the list

```
Node oldfirst = first;
```

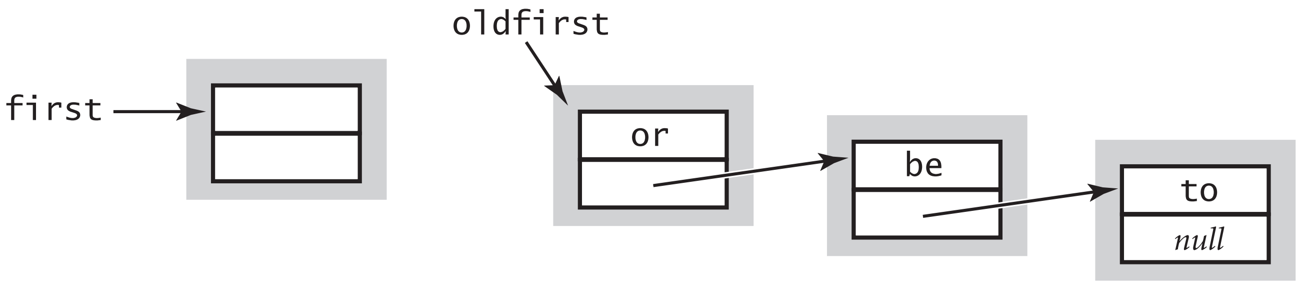


inner class

```
private class Node
{
   String item;
   Node next;
}
```

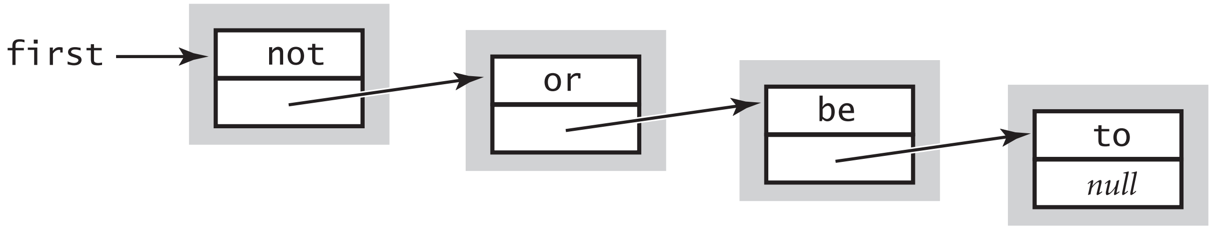
create a new node for the beginning

```
first = new Node();
```



set the instance variables in the new node

```
first.item =  item ;
first.next = oldfirst;
```



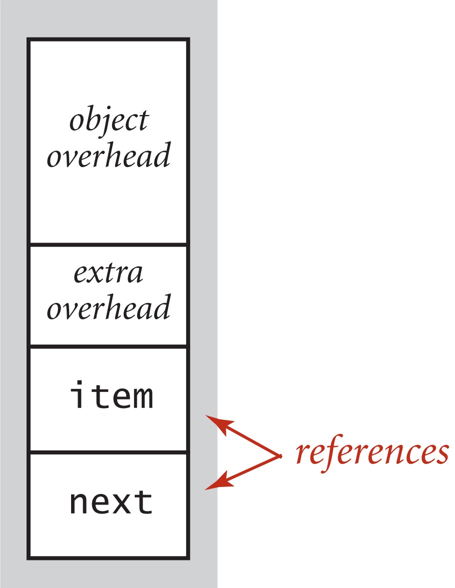
Stack: linked-list implementation performance

Proposition. Every operation takes constant time in the worst case.

Proposition. A stack with n items uses $\sim 40\, n$ bytes.

inner class

```
private class Node
{
   String item;
   Node next;
}
```



16 bytes (object overhead)

8 bytes (inner class extra overhead)

8 bytes (reference to String)

8 bytes (reference to Node)

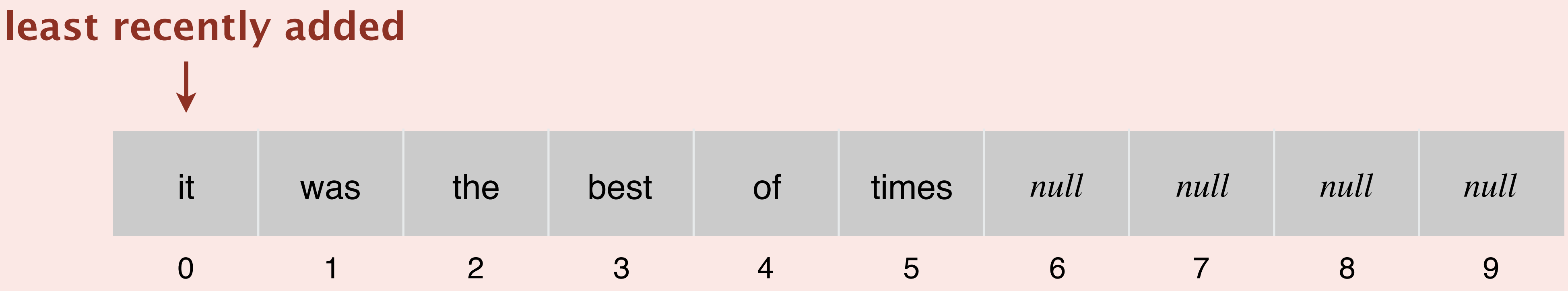
40 bytes per stack Node

Remark. This counts the memory for the stack
(but not the memory for the strings themselves, which the client owns).

Stacks and queues: quiz 2

How to implement a fixed-capacity stack with an array?

A.

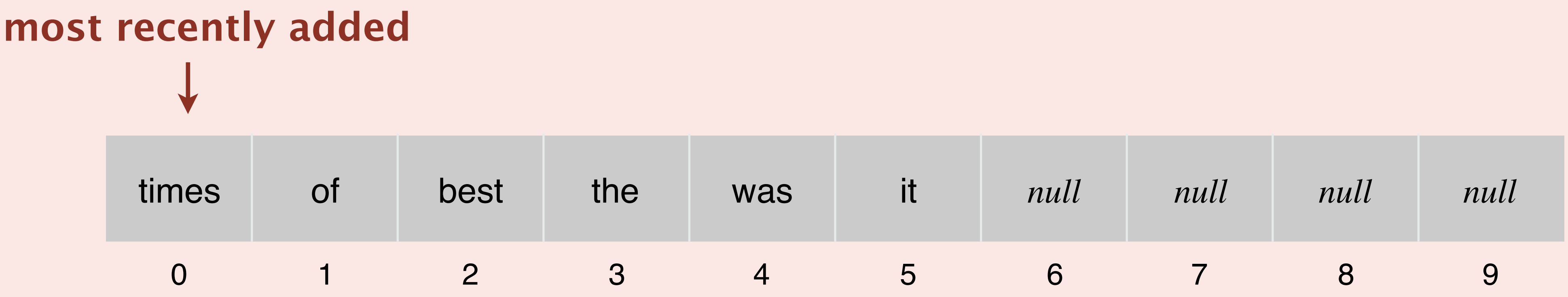


least recently added

↓

it	was	the	best	of	times	*null*	*null*	*null*	*null*
0	1	2	3	4	5	6	7	8	9

B.



most recently added

↓

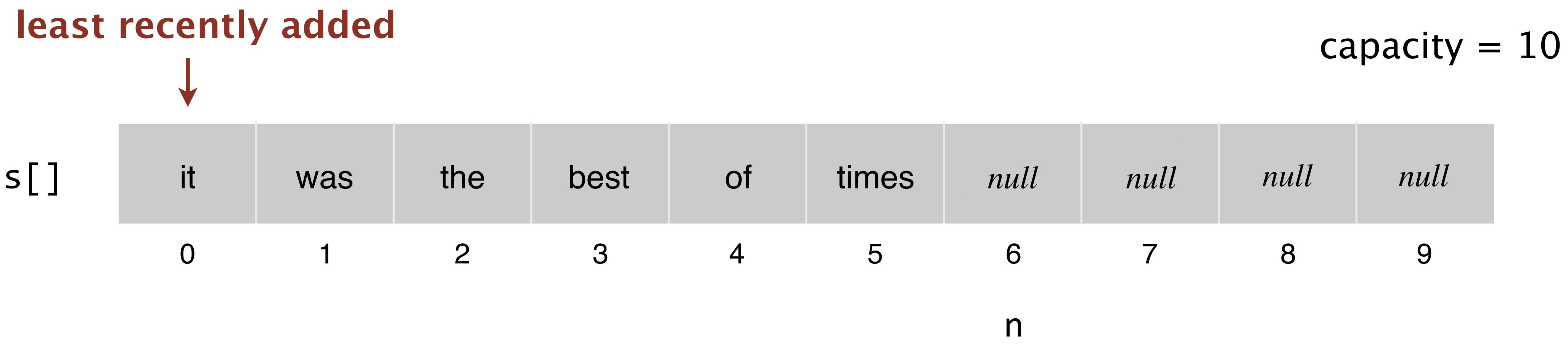
times	of	best	the	was	it	*null*	*null*	*null*	*null*
0	1	2	3	4	5	6	7	8	9

C. *Both A and B.*

D. *Neither A nor B.*

Fixed-capacity stack: array implementation

- Use array `s[]` to store `n` items on stack.
- `push()`: add new item at `s[n]`.
- `pop()`: remove item from `s[n-1]`.



least recently added

capacity = 10

s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9
							n			

Defect. Stack overflows when `n` exceeds `capacity`. [stay tuned]

Fixed-capacity stack: array implementation

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int n = 0;

   public FixedCapacityStackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(String item)
   {
      s[n] = item;
      n++;
   }

   public String pop()
   {
      n--;
      return s[n];
   }
}
```

a cheat
(stay tuned)

Stack considerations

Overflow and underflow.

- Underflow: throw exception if `pop()` from an empty stack.
- Overflow: use "resizing array" for array implementation. [stay tuned]

Null items. We allow `null` items to be added.

Duplicate items. We allow an item to be added more than once.

Loitering. Holding a reference to an object when it is no longer needed.

```
public String pop()
{
   n--;
   return s[n];
}
```

loitering

```
public String pop()
{
   n--;
   String item = s[n];
   s[n] = null;
   return item;
}
```

no loitering

Common source of bugs

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1.3 Stacks and Queues

- stacks
- *resizing arrays*
- queues
- generics
- iterators
- applications

Stack: resizing-array implementation

Problem. Requiring client to provide capacity does not implement API!

Q. How to grow and shrink array?

First try.

- `push()`: increase size of array `s[]` by 1.
- `pop()`: decrease size of array `s[]` by 1.

Too expensive.

- Need to copy all items to a new array, for each operation.
- Array accesses to add first n items = $\underset{\uparrow}{n} + (\underset{\uparrow}{2 + 4} + \ldots + 2(n-1)) \sim \overset{\downarrow}{n^2}$.

infeasible for large n

1 array access per push

$2(k-1)$ array accesses to expand to size k (ignoring cost to create new array)

Challenge. Ensure that array resizing happens infrequently.

Stack: resizing-array implementation

Q. How to grow array?

A. If array is full, create a new array of twice the size, and copy items.

“repeated doubling”

```
public ResizingArrayStackOfStrings()
{  s = new String[1]; }

public void push(String item)
{
   if (n == s.length) resize(2 * s.length);
   n++;
   s[n] = item;
}

private void resize(int capacity)
{
   String[] copy = new String[capacity];
   for (int i = 0; i < n; i++)
      copy[i] = s[i];
   s = copy;
}
```

Array accesses to add first $n = 2^i$ items. $n + (2 + 4 + 8 + \ldots + n) \sim 3\,n$.

feasible for large n

1 array access per push

k array accesses to double to size k (ignoring cost to create new array)

Stack: resizing-array implementation

Q. How to shrink array?

First try.

- `push()`: double size of array `s[]` when array is full.
- `pop()`: halve size of array `s[]` when array is one-half full.

Too expensive in worst case.

- Consider push-pop-push-pop-… sequence when array is full.
- Each operation takes time proportional to n.

full	to	be	or	not				
push("to")	to	be	or	not	to	*null*	*null*	*null*
pop()	to	be	or	not				
push("be")	to	be	or	not	be	*null*	*null*	*null*

Stack: resizing-array implementation

Q. How to shrink array?

Efficient solution.

- `push()`: double size of array `s[]` when array is full.
- `pop()`: halve size of array `s[]` when array is one-quarter full.

```
public String pop()
{
   n--;
   String item = s[n];
   s[n] = null;
   if (n > 0 && n == s.length/4) resize(s.length/2);
   return item;
}
```

Invariant. Array is between 25% and 100% full.

Stack resizing-array implementation: performance

Amortized analysis. Starting from an empty data structure, average running time per operation over a worst-case sequence of operations.

Proposition. Starting from an empty stack, any sequence of m push and pop operations takes time proportional to m.

	typical	worst	amortized
construct	1	1	1
push	1	n	1
pop	1	n	1
size	1	1	1

doubling and halving operations

order of growth of running time for resizing array stack with n items

Stack resizing-array implementation: memory usage

Proposition. A `ResizingArrayStackOfStrings` uses between $\sim 8n$ and $\sim 32n$ bytes of memory for a stack with n items.

- $\sim 8n$ when full.
- $\sim 32n$ when one-quarter full.

```
public class ResizingArrayStackOfStrings
{
    private String[] s;
    private int n = 0;

    ⋮
}
```

8 bytes × array length

Remark. This counts the memory for the stack
(but not the memory for the strings themselves, which the client owns).

Stack implementations: resizing array vs. linked list

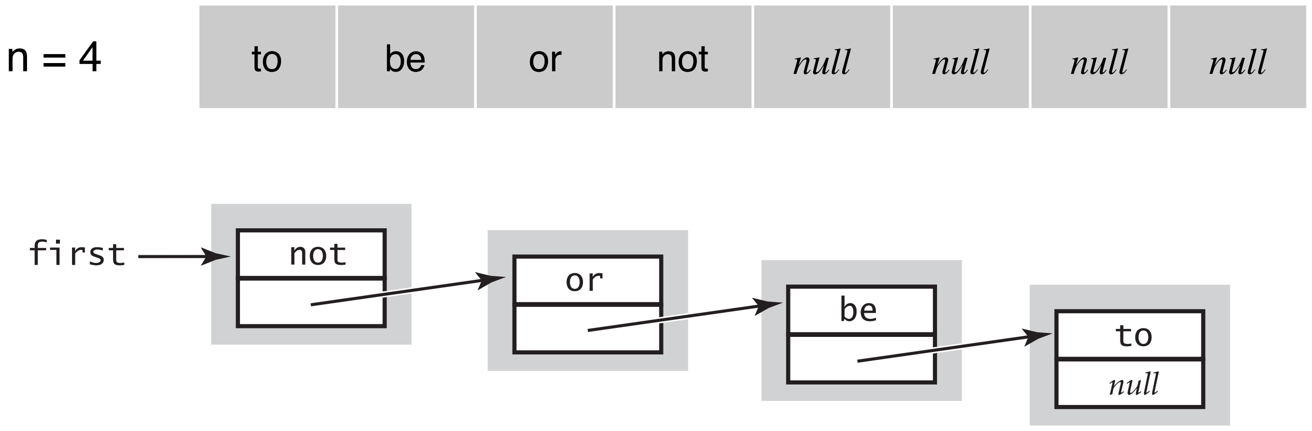
Tradeoffs. Can implement a stack with either resizing array or linked list; client can use interchangeably. Which one is better?

Linked-list implementation.

- Every operation takes constant time in the worst case.
- Uses extra time and space to deal with the links.

Resizing-array implementation.

- Every operation takes constant amortized time.
- Less wasted space.



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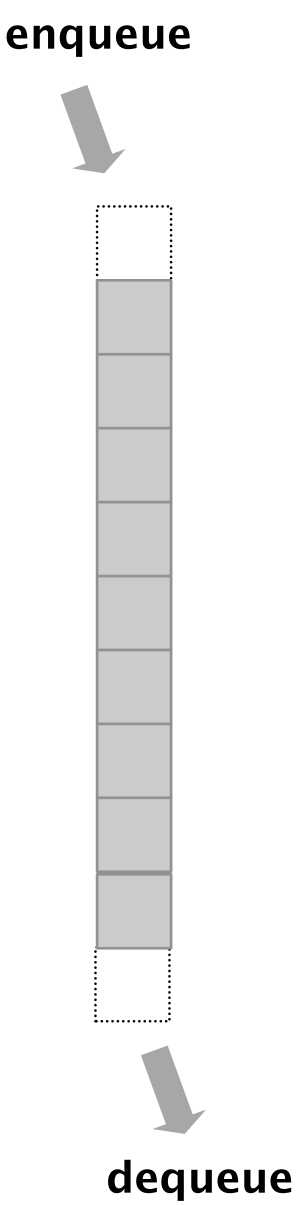
1.3 Stacks and Queues

- *stacks*
- *resizing arrays*
- *queues*
- *generics*
- *iterators*
- *applications*

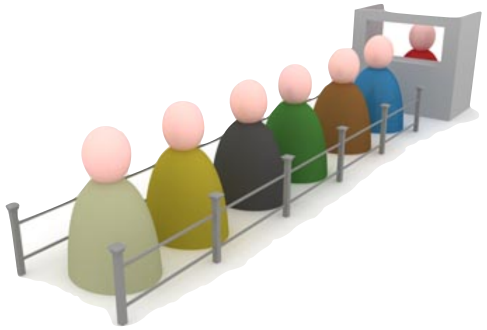
Queue API

Warmup API. Queue of strings data type.

public class QueueOfStrings		
	QueueOfStrings()	*create an empty queue*
void	enqueue(String item)	*add a new string to queue*
String	dequeue()	*remove and return the string least recently added*
boolean	isEmpty()	*is the queue empty?*
int	size()	*number of strings on the queue*



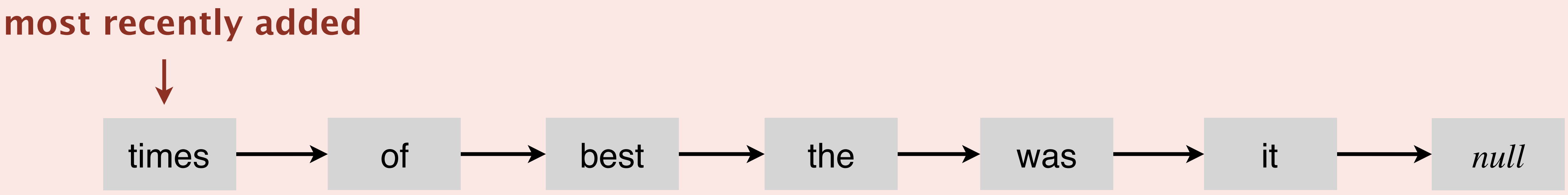
Performance requirements. All operations take constant time.



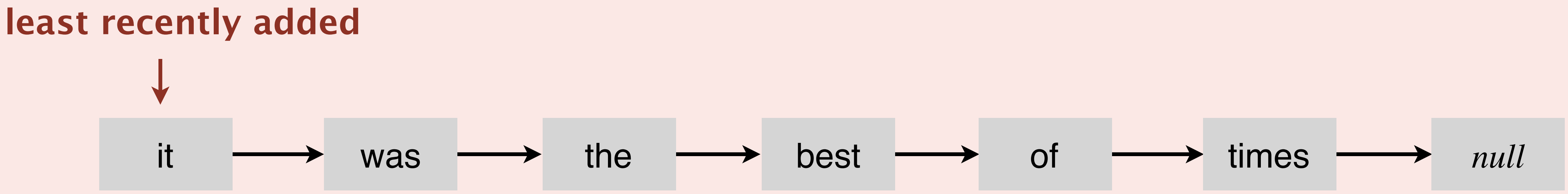
Stacks and queues: quiz 3

How to implement a queue with a singly linked list?

A.



B.

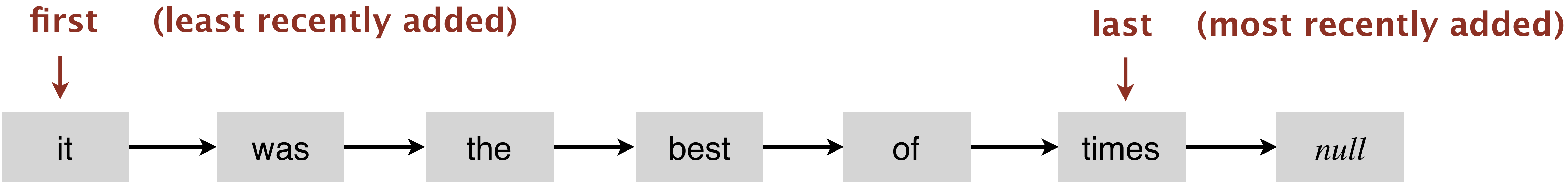


C. *Both A and B.*

D. *Neither A nor B.*

Queue: linked-list implementation

- Maintain one pointer `first` to first node in a singly linked list.
- Maintain another pointer `last` to last node.
- Dequeue from `first`.
- Enqueue after `last`.



Queue dequeue: linked-list implementation

inner class

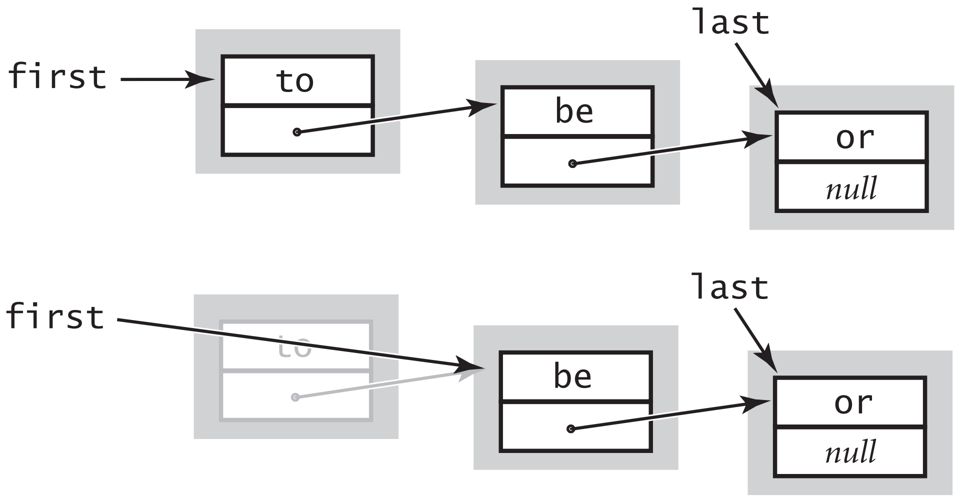
```
private class Node
{
   String item;
   Node next;
}
```

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```



return saved item

```
return item;
```

Remark. Identical code to linked-list stack `pop()`.

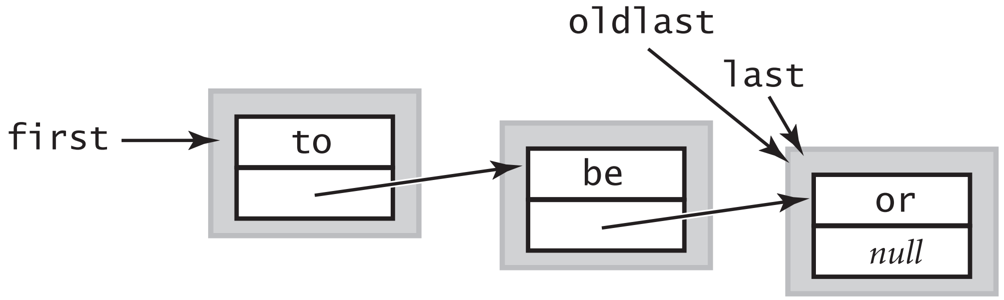
Queue enqueue: linked-list implementation

inner class

```
private class Node
{
   String item;
   Node next;
}
```

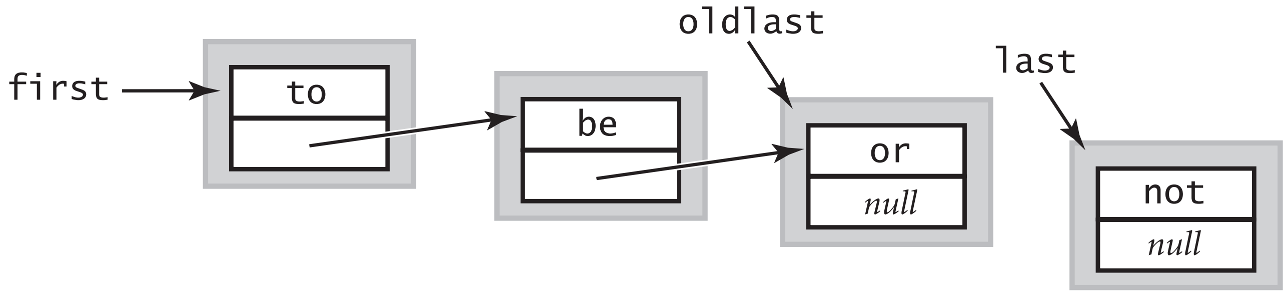
save a link to the last node

```
Node oldlast = last;
```



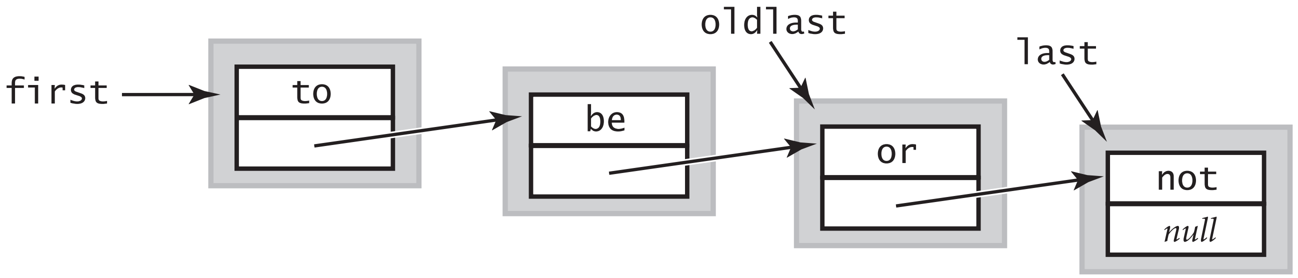
create a new node for the end

```
last = new Node();
last.item = "not";
```



link the new node to the end of the list

```
oldlast.next = last;
```



Queue: linked-list implementation

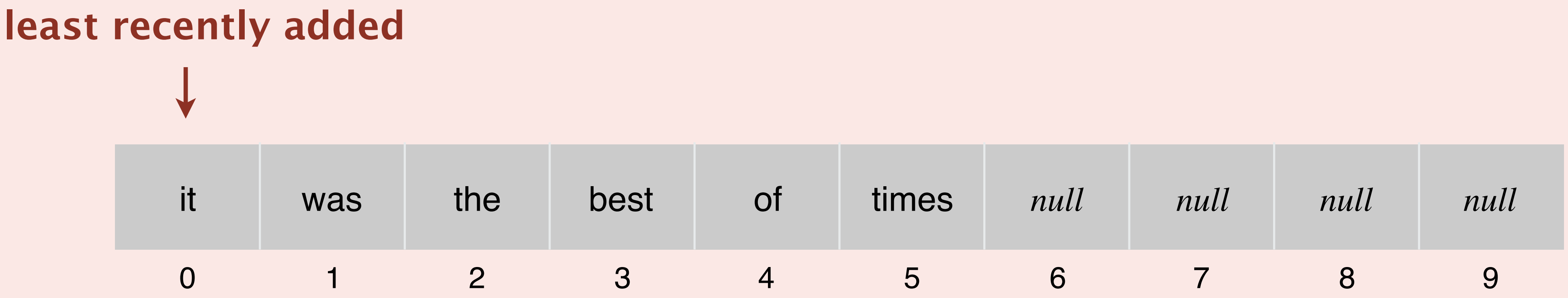
```
public class LinkedQueueOfStrings
{
   private Node first, last;

   private class Node
   {  /* same as in LinkedStackOfStrings */  }

   public boolean isEmpty()
   {  return first == null;  }

   public void enqueue(String item)
   {
      Node oldlast = last;
      last = new Node();
      last.item = item;
      last.next = null;
      if (isEmpty()) first = last;
      else           oldlast.next = last;
   }

   public String dequeue()
   {
      String item = first.item;
      first       = first.next;
      if (isEmpty()) last = null;
      return item;
   }
}
```

special cases for empty queue

Stacks and queues: quiz 4

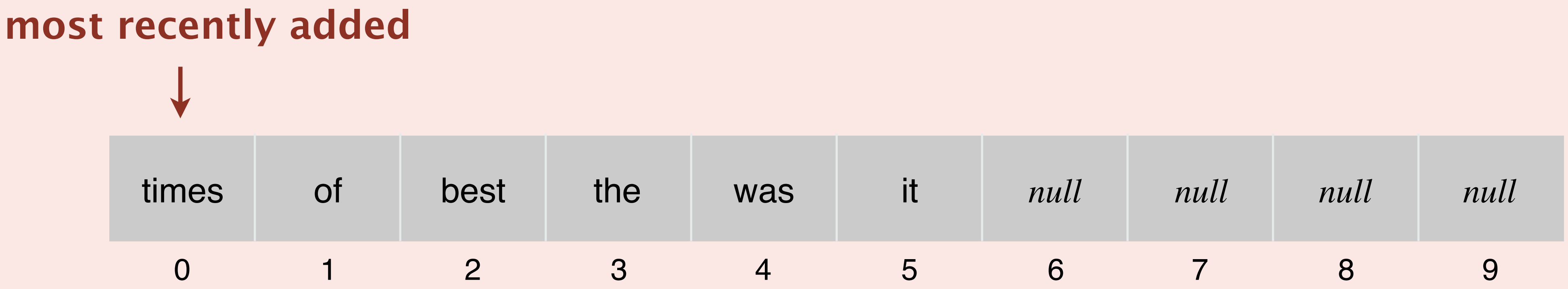
How to implement a fixed-capacity queue with an array?

A.



it	was	the	best	of	times	*null*	*null*	*null*	*null*
0	1	2	3	4	5	6	7	8	9

B.



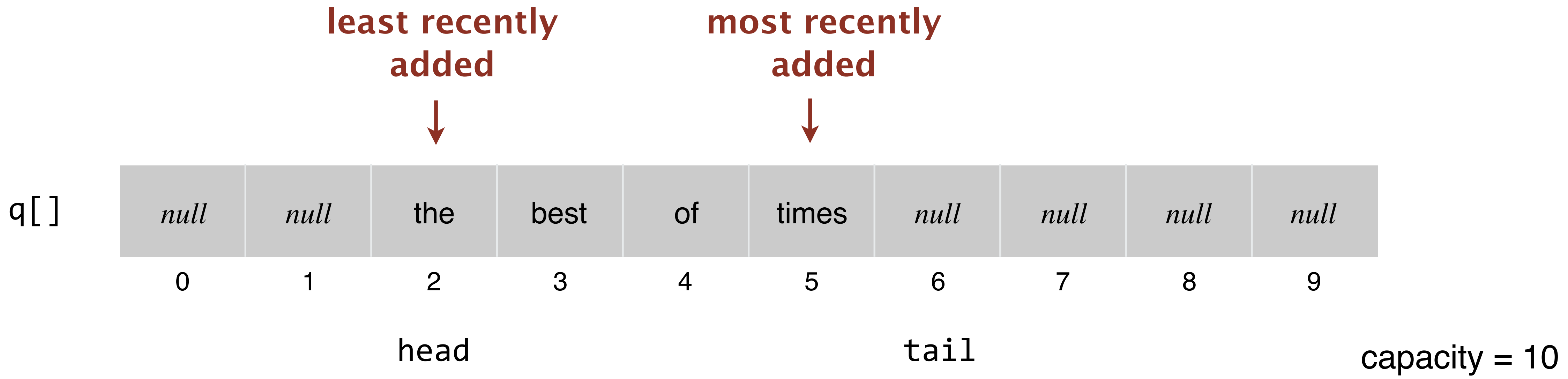
times	of	best	the	was	it	*null*	*null*	*null*	*null*
0	1	2	3	4	5	6	7	8	9

C. *Both A and B.*

D. *Neither A nor B.*

Queue: resizing-array implementation

- Use array `q[]` to store items in queue.
- `enqueue()`: add new item at `q[tail]`.
- `dequeue()`: remove item from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.



Q. How to resize?

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1.3 STACKS AND QUEUES

- stacks
- resizing arrays
- queues
- generics
- iterators
- applications

Parameterized stack

We implemented: `StackOfStrings`.

We also want: `StackOfURLs`, `StackOfInts`, `StackOfApples`, `StackOfVans`, ….

Solution in Java: generics.

type parameter
(use syntax both to specify type and to call constructor)

```
Stack<Apple> stack = new Stack<Apple>();
Apple apple = new Apple();
stack.push(apple);
Van van = new Van();
stack.push(van);
...
```

compile-time error

Generic stack: linked-list implementation

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

stack of strings (linked list)

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      Item item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

generic type name

generic stack (linked list)

Generic stack: array implementation

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int n = 0;

   public ..StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(String item)
   {  s[n++] = item;  }

   public String pop()
   {  return s[--n];  }
}
```

stack of strings (fixed-length array)

```
public class FixedCapacityStack<Item>
{
   private Item[] s;
   private int n = 0;

   public FixedCapacityStack(int capacity)
   {  s = new Item[capacity];  }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(Item item)
   {  s[n++] = item;  }

   public Item pop()
   {  return s[--n];  }
}
```

generic stack (fixed-length array) ?

@#$*! generic array creation not allowed in Java

Generic stack: array implementation

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int n = 0;

   public ..StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(String item)
   {  s[n++] = item;  }

   public String pop()
   {  return s[--n];  }
}
```

stack of strings (fixed-length array)

```
public class FixedCapacityStack<Item>
{
   private Item[] s;
   private int n = 0;

   public FixedCapacityStack(int capacity)
   {  s = (Item[]) new Object[capacity]; }

   public boolean isEmpty()
   {  return n == 0;  }

   public void push(Item item)
   {  s[n++] = item;  }

   public Item pop()
   {  return s[--n];  }
}
```

generic stack (fixed-length array)

the ugly cast

Stacks and queues: quiz 5

Which of the following is the correct way to declare and initialize an empty stack of integers?

A. `Stack stack = new Stack<int>();`

B. `Stack<int> stack = new Stack();`

C. `Stack<int> stack = new Stack<int>();`

D. *None of the above.*

Generic data types: autoboxing and unboxing

Q. What to do about primitive types?

Wrapper type.

- Each primitive type has a wrapper object type.
- Ex: `Integer` is wrapper type for `int`.

Autoboxing. Automatic cast from primitive type to wrapper type.

Unboxing. Automatic cast from wrapper type to primitive type.

```
Stack<Integer> stack = new Stack<Integer>();
stack.push(17);          // stack.push(Integer.valueOf(17));
int a = stack.pop();     // int a = stack.pop().intValue();
```

Bottom line. Client code can use generic stack for any type of data.

Algorithms

Robert Sedgewick | Kevin Wayne

https://algs4.cs.princeton.edu

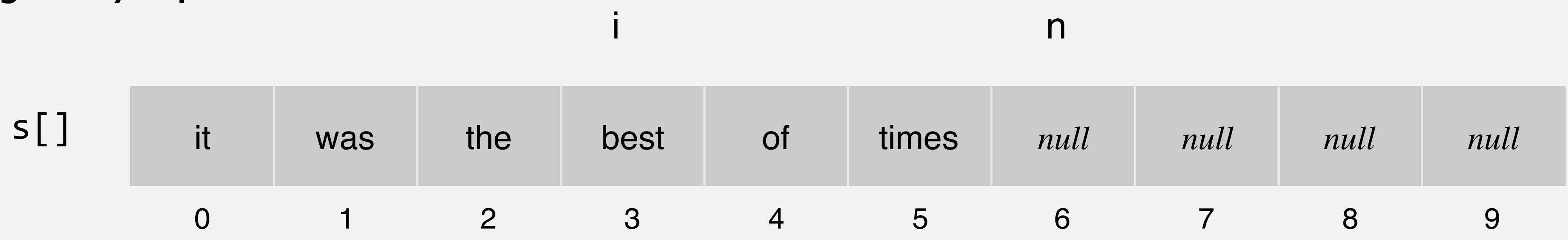
1.3 Stacks and Queues

- *stacks*
- *resizing arrays*
- *queues*
- *generics*
- *iterators* ← **see precept**
- *applications*

Iteration

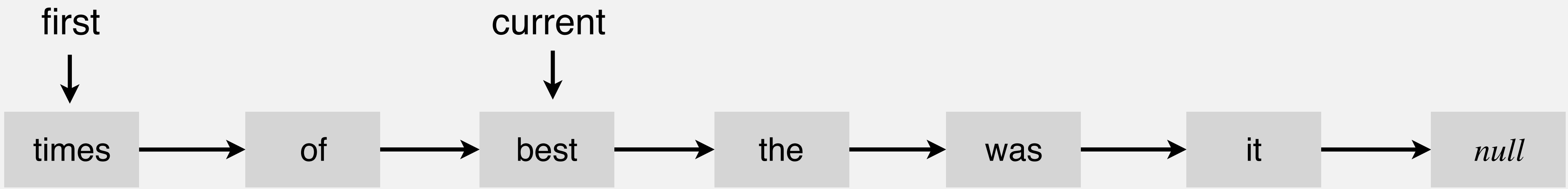
Design challenge. Support iteration over stack items by client, without revealing the internal representation of the stack.

resizing-array representation



it	was	the	best	of	times	*null*	*null*	*null*	*null*
0	1	2	3	4	5	6	7	8	9

linked-list representation



Java solution. Use a foreach loop.

Foreach loop

Java provides elegant syntax for iteration over collections.

“foreach” loop (shorthand)

```
Stack<String> stack;
...

for (String s : stack)
    ...
```

equivalent code (longhand)

```
Stack<String> stack;
...

Iterator<String> i = stack.iterator();
while (i.hasNext())
{
   String s = i.next();
   ...
}
```

To make user-defined collection support foreach loop:

- Data type must have a method named iterator().
- The iterator() method returns an object that has two core method.
  - the hasNext() methods returns false when there are no more items
  - the next() method returns the next item in the collection

Iterators

To support foreach loops, Java provides two interfaces.

- Iterator interface: next() and hasNext() methods.
- Iterable interface: iterator() method that returns an Iterator.
- Both should be used with generics.

java.util.Iterator interface

```
public interface Iterator<Item>
{
   boolean hasNext();
   Item next();
   void remove();    ←   optional; use
                         at your own risk
}
```

java.lang.Iterable interface

```
public interface Iterable<Item>
{
   Iterator<Item> iterator();
}
```

Type safety.

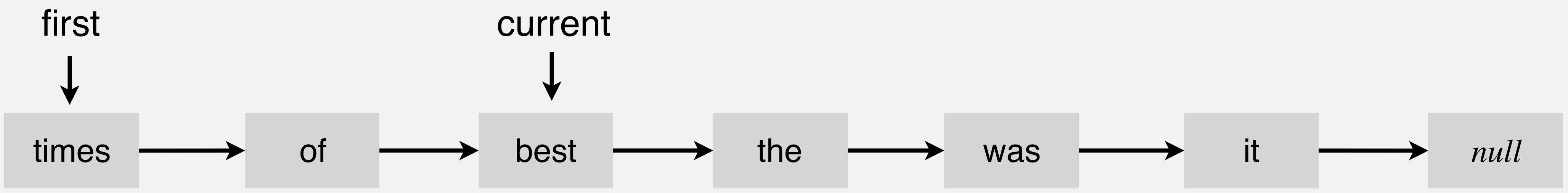
- Implementation must use these interfaces to support foreach loop.
- Client program won't compile unless implementation do.

Stack iterator: linked-list implementation

```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    ...

   public Iterator<Item> iterator() { return new ListIterator(); }

   private class ListIterator implements Iterator<Item>
   {
      private Node current = first;

      public boolean hasNext() {  return current != null;  }
      public void remove()     {  /* not supported */      }
      public Item next()
      {
         Item item = current.item;
         current   = current.next;
         return item;
      }
   }
}
```

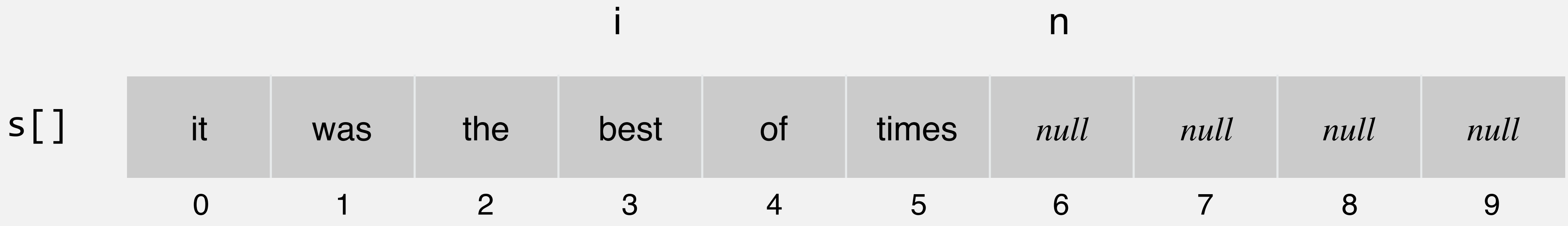
throw UnsupportedOperationException

throw NoSuchElementException
if no more items in iteration



Stack iterator: array implementation

```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    ...

   public Iterator<Item> iterator()
   { return new ReverseArrayIterator(); }

   private class ReverseArrayIterator implements Iterator<Item>
   {
      private int i = n;

      public boolean hasNext() {  return i > 0;         }
      public void remove()     {  /* not supported */   }
      public Item next()       {  return s[--i];        }
   }
}
```



				i			n			
s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

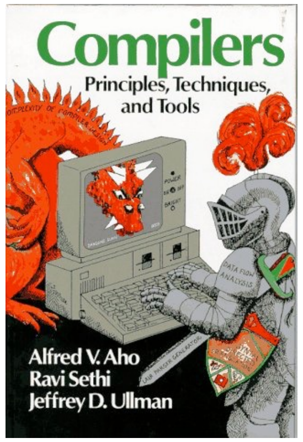
https://algs4.cs.princeton.edu

1.3 Stacks and Queues

- *stacks*
- *resizing arrays*
- *queues*
- *generics*
- *iterators*
- *applications*

Stack applications

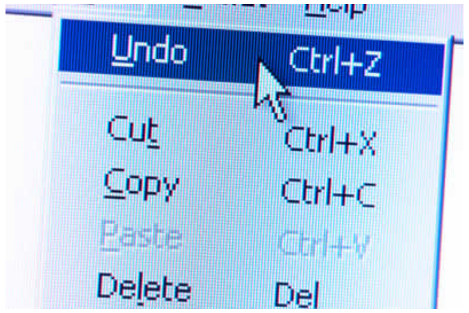
- Java virtual machine.
- Parsing in a compiler.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.
- ...



Compilers

Principles, Techniques, and Tools

Alfred V. Aho
Ravi Sethi
Jeffrey D. Ullman



Adobe® PostScript®

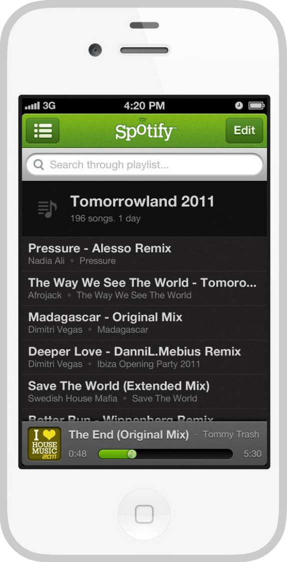
Queue applications

Familiar applications.

- Spotify playlist.
- Data buffers (iPod, TiVo, sound card, streaming video, …).
- Asynchronous data transfer (file IO, pipes, sockets, …).
- Dispensing requests on a shared resource (printer, processor, …).

Simulations of the real world.

- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.



Tomorrowland 2011
196 songs, 1 day

Pressure - Alesso Remix
Nadia Ali • Pressure

The Way We See The World - Tomoro...
Afrojack • The Way We See The World

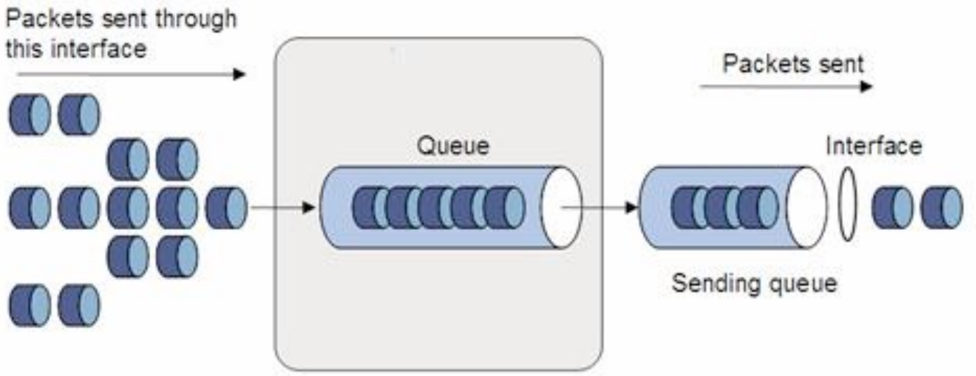
Madagascar - Original Mix
Dimitri Vegas • Madagascar

Deeper Love - DanniL.Mebius Remix
Dimitri Vegas • Ibiza Opening Party 2011

Save The World (Extended Mix)
Swedish House Mafia • Save The World

Better Run - Wippenberg Remix

The End (Original Mix) - Tommy Trash
0:48 5:30



Java collections library

List interface. `java.util.List` is API for a sequence of items.

public interface List<Item> extends Iterable<Item>

	List()	*create an empty list*
boolean	isEmpty()	*is the list empty?*
int	size()	*number of items*
void	add(Item item)	*add item to the end*
Iterator<Item>	iterator()	*iterator over all items in the list*
Item	get(int index)	*return item at given index*
Item	remove(int index)	*return and delete item at given index*
boolean	contains(Item item)	*does the list contain the given item?*
	⋮	

Implementations. `java.util.ArrayList` uses a resizing array; `java.util.LinkedList` uses a doubly linked list.

Caveat: not all operations are efficient!

Java collections library

`java.util.Stack`.

- Supports `push()`, `pop()`, and iteration.
- Inherits from `java.util.Vector`, which implements `java.util.List` interface.

Java 1.3 bug report (June 27, 2001)

> The iterator method on java.util.Stack iterates through a Stack from the bottom up. One
> would think that it should iterate as if
> it were popping off the top of the Stack.

status (closed, will not fix)

> It was an incorrect design decision to have Stack extend Vector ("is-a" rather than "has-a").
> We sympathize with the submitter
> but cannot fix this because of compatibility.

Java collections library

`java.util.Stack`.

- Supports `push()`, `pop()`, and iteration.
- Inherits from `java.util.Vector`, which implements `java.util.List` interface.

`java.util.Queue`. An interface, not an implementation of a queue.

Best practices. Use our Stack and Queue for stacks and queues; use `java.util.ArrayList` or `java.util.LinkedList` when appropriate.

Unchecked cast

```
% javac -Xlint:unchecked FixedCapacityStack.java
FixedCapacityStack.java:26: warning: [unchecked] unchecked cast
        s = (Item[]) new Object[capacity];
                     ^
  required: Item[]
  found:    Object[]
  where Item is a type-variable:
    Item extends Object declared in class FixedCapacityStack
1 warning
```

Q. Why does Java require a cast (or reflection)?

Short answer. Backward compatibility.

Long answer. Need to learn about type erasure and covariant arrays.