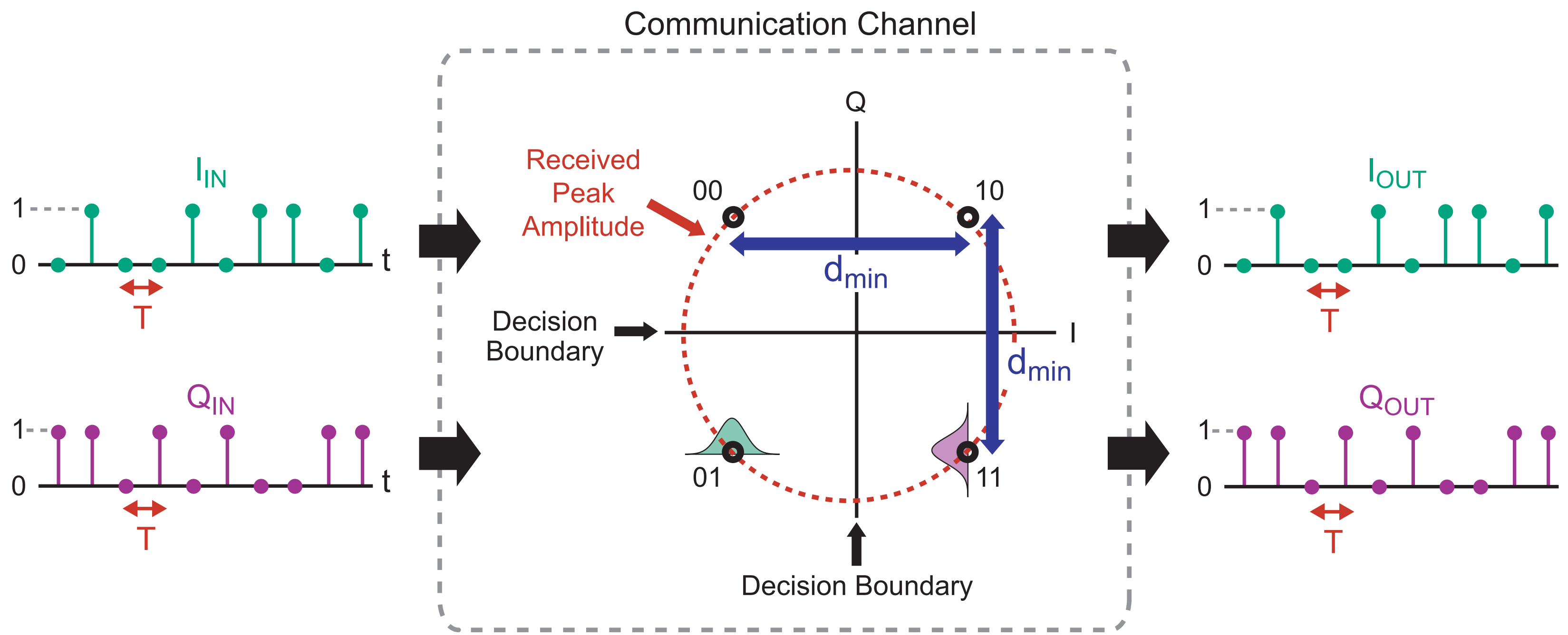
Multiuser Communication and Capacity

COS 463 *Wireless Networks*
Lecture 19
Kyle Jamieson

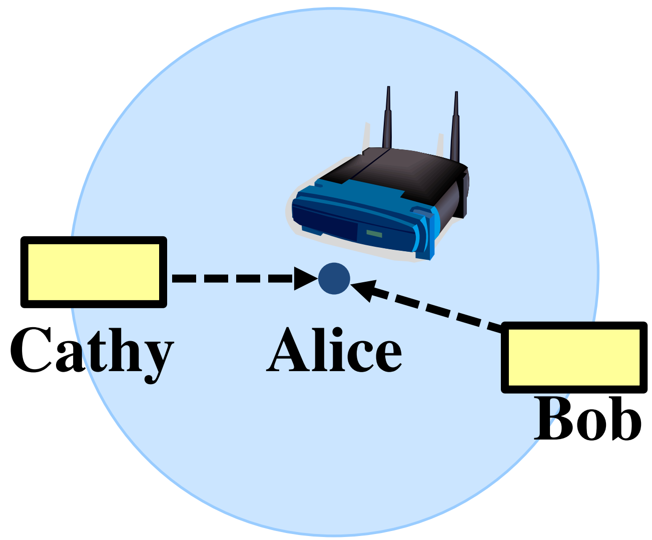
Review: Shannon Capacity



- ***Capacity,*** or **maximum rate** that a Gaussian channel **supports arbitrarily-low bit error rates** is $C = BW\log_2(1 + SNR)$ bits/s
- Reliable communication **possible** at **rates *R* < *C***, **not possible** when ***R* > *C***

Review: Sharing the Wireless Medium

- **Uplink to Alice,** Cathy and Bob both want to deliver information



- **Contention-based:** Unstructured, taking turns in time
- **TDMA:** Scheduled turns in time
- **FDMA:** Sharing via Different Frequency Bands
- **CDMA:** Sharing via Different Codes

What's the best we can do?

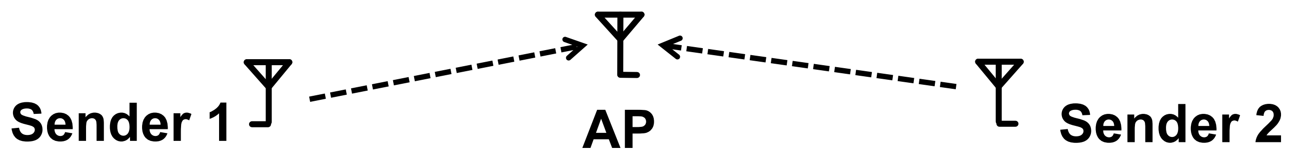
Today: Multiuser Capacity

- **Building atop Shannon capacity** of a **single link:**

1. What's the best we can do in the multiuser uplink channel?

2. What's the best we can do in the multiuser downlink channel?

Two-User Interference Channel



- On the **same frequency channel** at the **same time:**
  - **Sender 1** sends signal x_1 with power P_1
  - **Sender 2** sends signal x_2 with power P_2

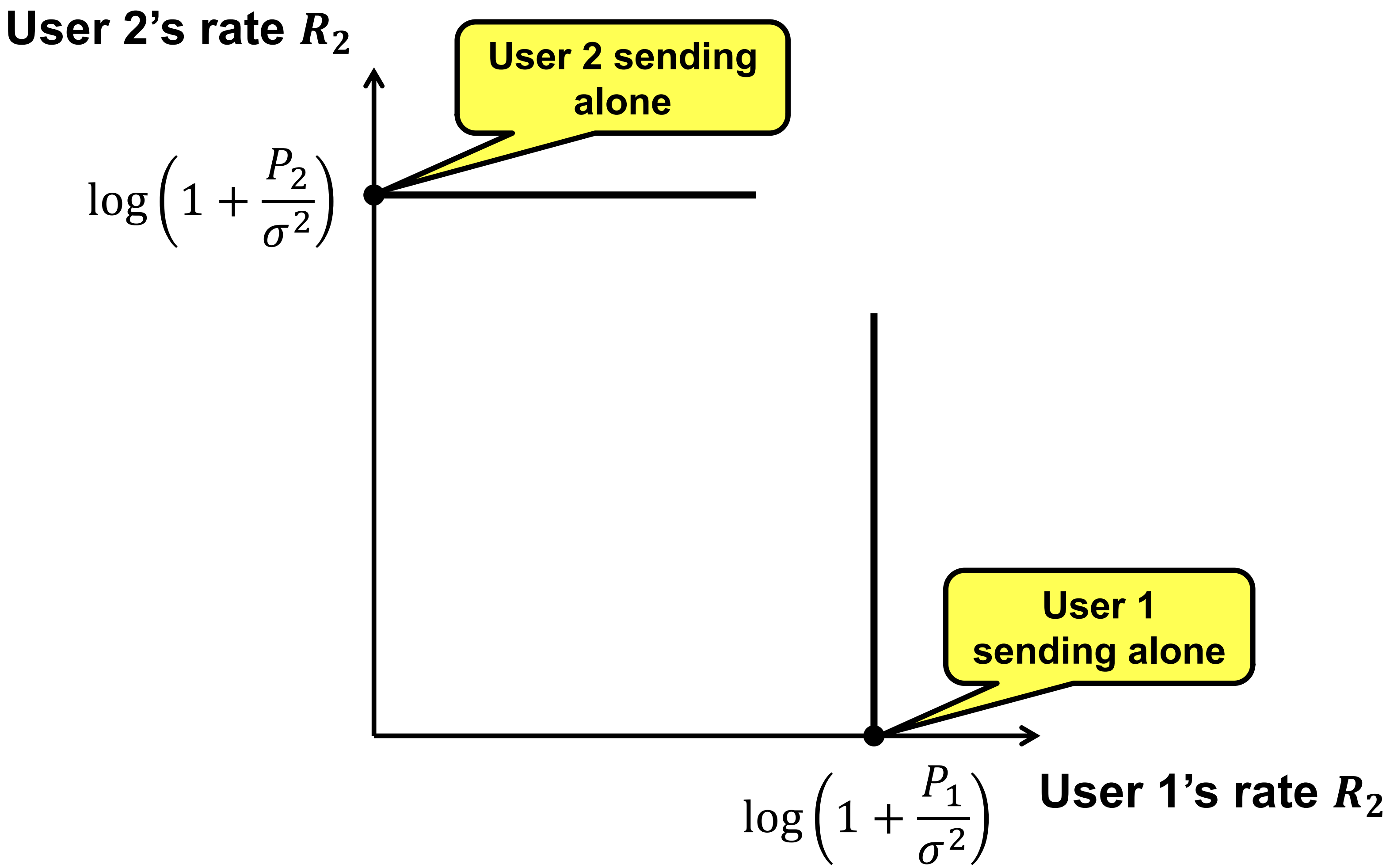
- **AP receives:** $y[m] = x_1[m] + x_2[m] + w[m]$
  - $w[m]$ is background Gaussian Noise with variance σ^2

Extension of Capacity to Multiple Users

- **Single-channel** Shannon capacity is a **single rate** (bits/s/Hz)

- Generalizing for two users **capacity** becomes a ***region***:
  - Set of all pairs (R_1, R_2) such that simultaneously,
    - User 1 can communicate at rate R_1 and
    - User 2 can communicate at rate R_2
  - **Tradeoff between reliable communication rates:**
    - If **User 1** wants to **increase** its rate, **User 2** may need to **decrease** its rate

Two-User Interference Channel: Single-User Bounds



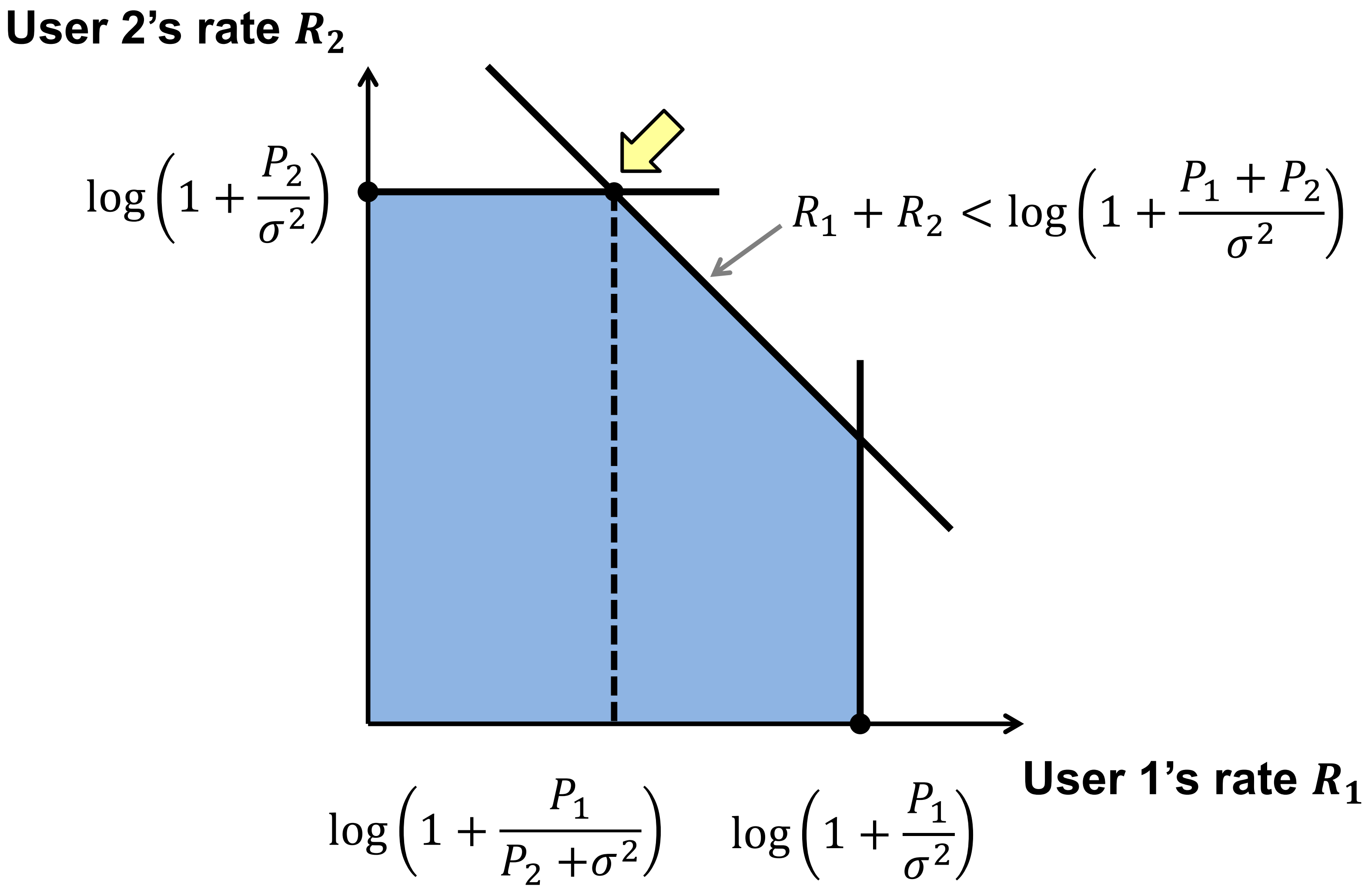
Interference Doesn't Help

- **Assumption:** User 1's data are completely independent from User 2's data, and vice-versa

- **Thought exercise:** Point-to-point link sending with power $P_1 + P_2$
  - Must outperform interfering link (otherwise interference helps)

- So therefore, $R_1 + R_2 < \log\left(1 + \frac{P_1+P_2}{\sigma^2}\right)$

Two-User Interference Channel: Capacity Region



Successive Interference Cancellation (SIC)

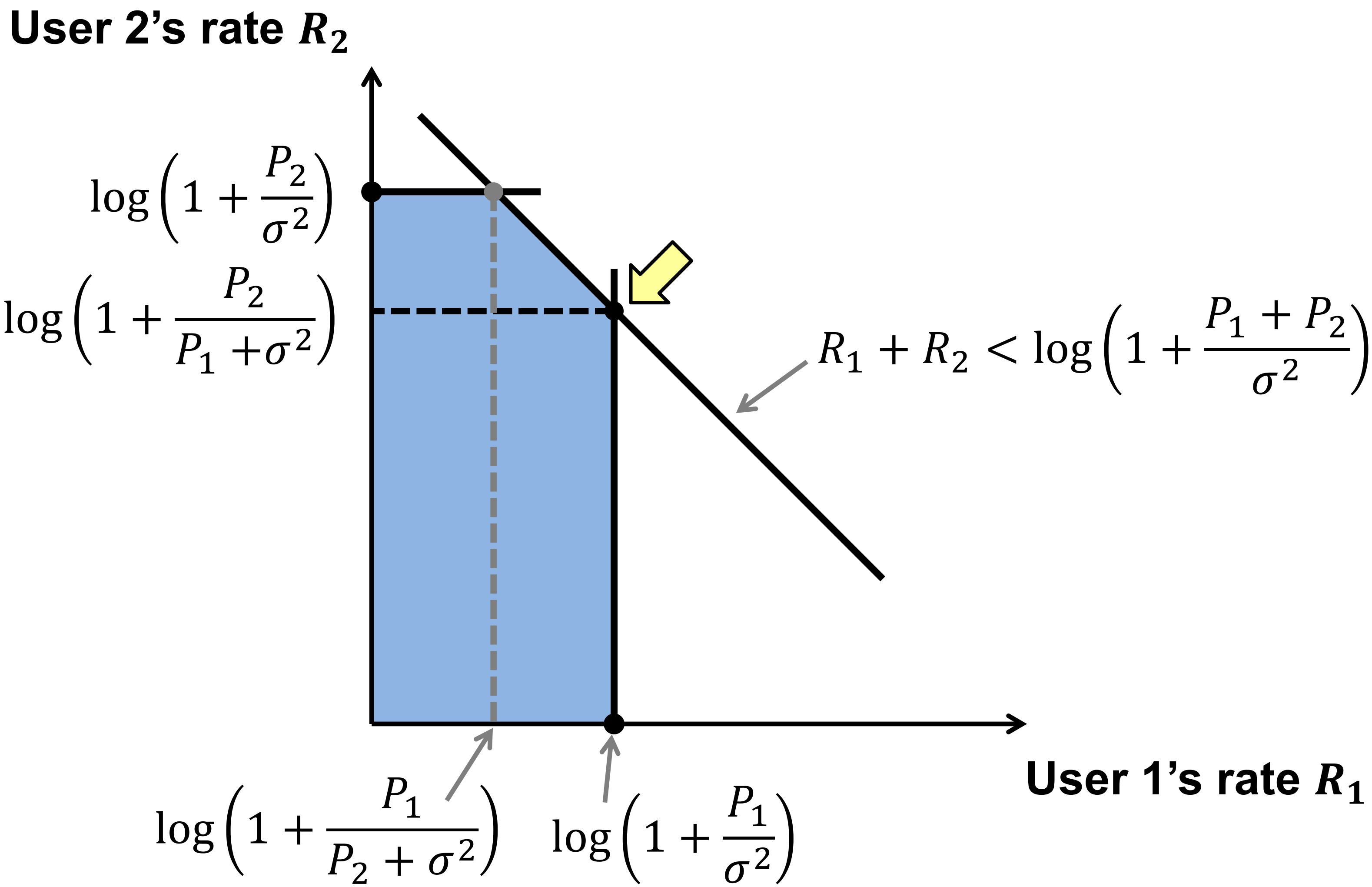
- Receiver decodes information from both senders in three stages:

1. Decode data of user 2, **treating signal from user 1 as noise**
2. Reconstruct user 2's signal ($x'_2[m]$) from decoded data and subtract from aggregate received signal $y[m]$, **cancelling it:**

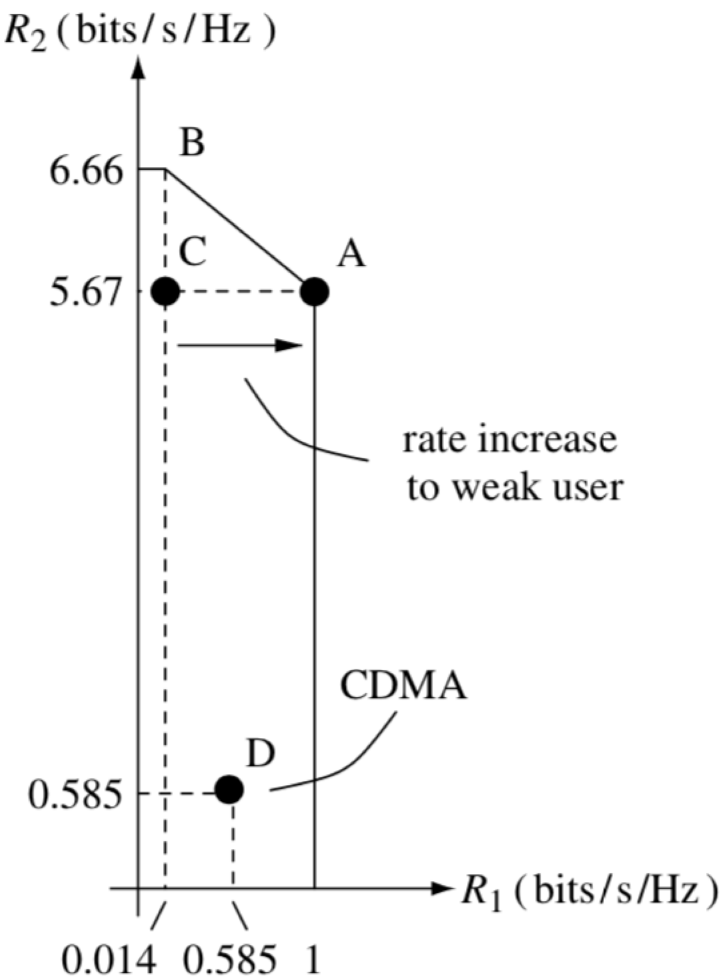
$$y'[m] = y[m] - x'_2[m]$$
$$= x_1[m] + (x_2[m] - x'_2[m]) + w[m]$$

3. Decode user 1's signal from y'[m]

SIC: Choice of User Order

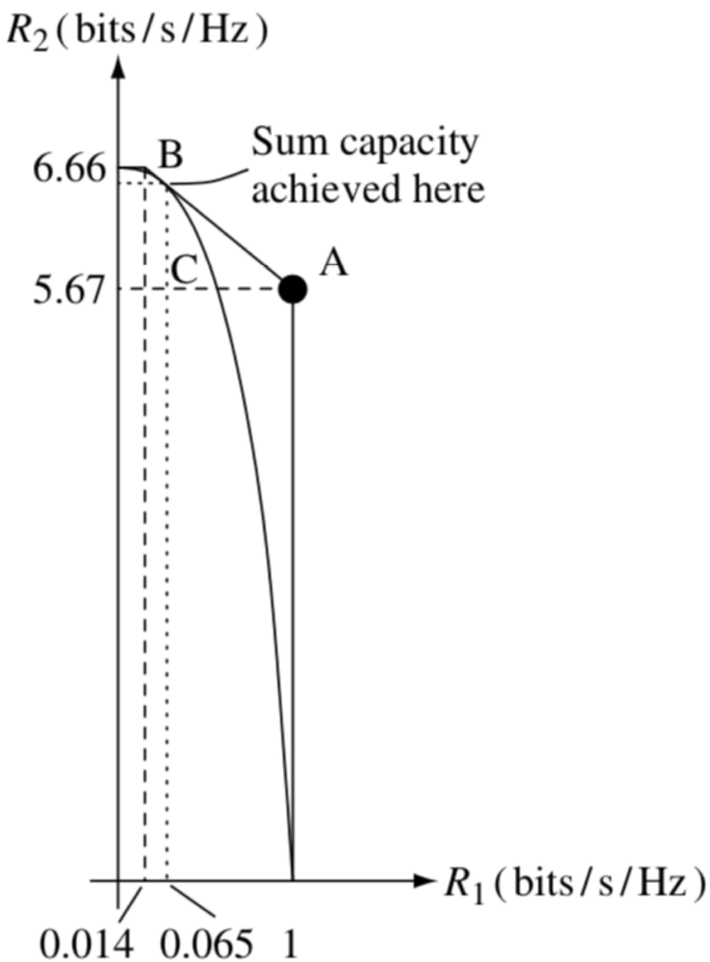


Comparison with CDMA



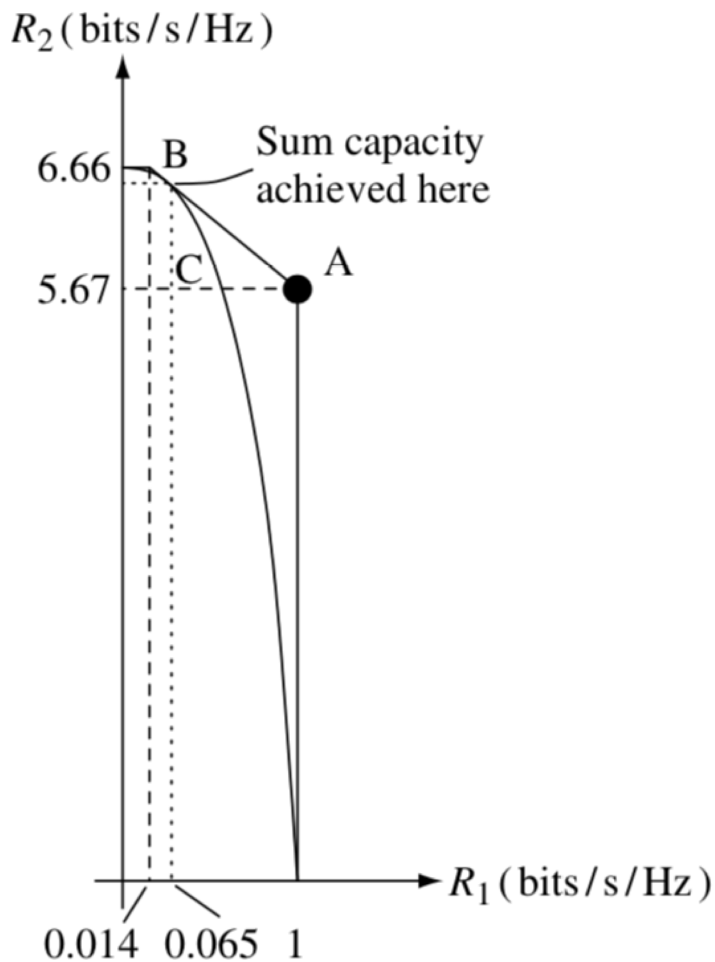
- **CDMA:** Every user decoded treating the other users as noise
  - Achieves **Point C**
    - **But, User 1 starves**
- **CDMA power control:** Reduce power of the strong user
  - **Achieves Point D**

Comparison with TDMA/FDMA



- Allocate **α** time- or frequency-fraction to **User 1; 1 − α** to **User 2**
- Scale each user's power according to allocated proportion
- User 1 maximum rate: $\alpha \log\left(1 + \frac{P_1}{\alpha\sigma^2}\right)$
- User 2 maximum rate: $(1-\alpha)\log\left(1 + \frac{P_2}{(1-\alpha)\sigma^2}\right)$

Comparison with TDMA/FDMA



- Allocate as follows:
  - **α** time- or frequency-fraction to **User 1;**
  - **1 − α** to **User 2**

- Tuning α (time/frequency and power) to P_1, P_2 can achieve a point on the A-B curve (**optimal**)

Today: Multiuser Capacity

- **Building atop Shannon capacity** of a **single link:**

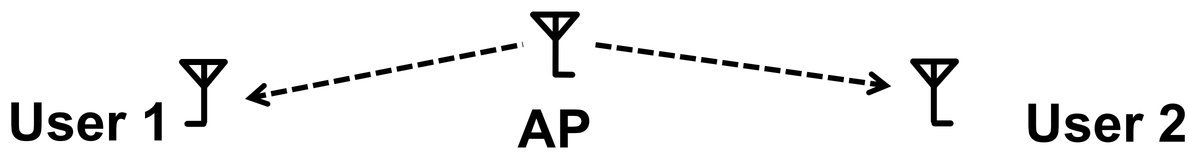
1. What's the best we can do in the multiuser uplink channel?

2. What's the best we can do in the multiuser downlink channel?

   – **Unknown information streams**

   – Taking content of the data into account

Two-User Downlink Channel



- On the **same frequency channel** at the **same time:**
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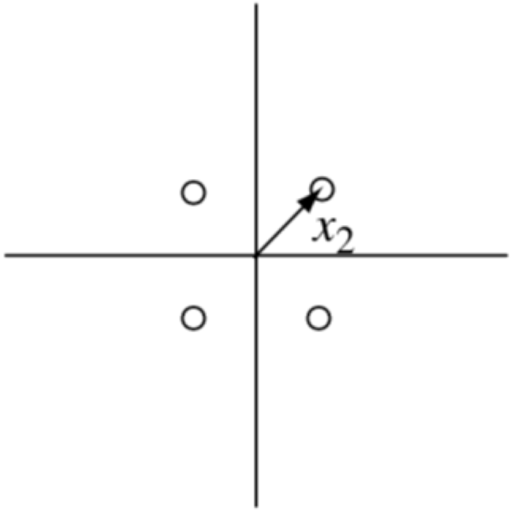
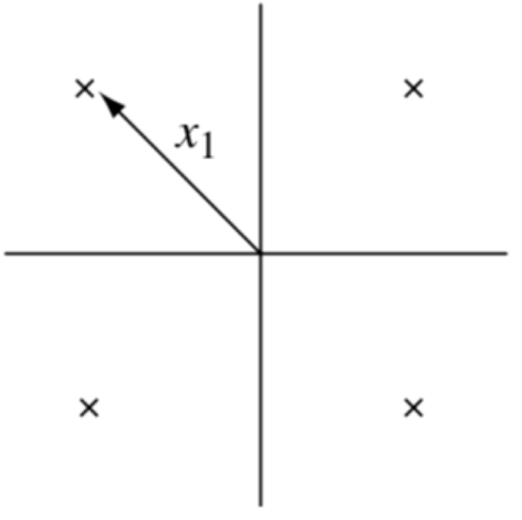
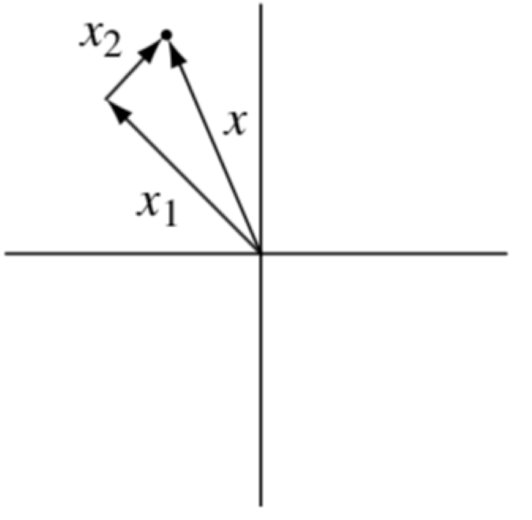
Superposition Coding



User 2 alone
QPSK signal

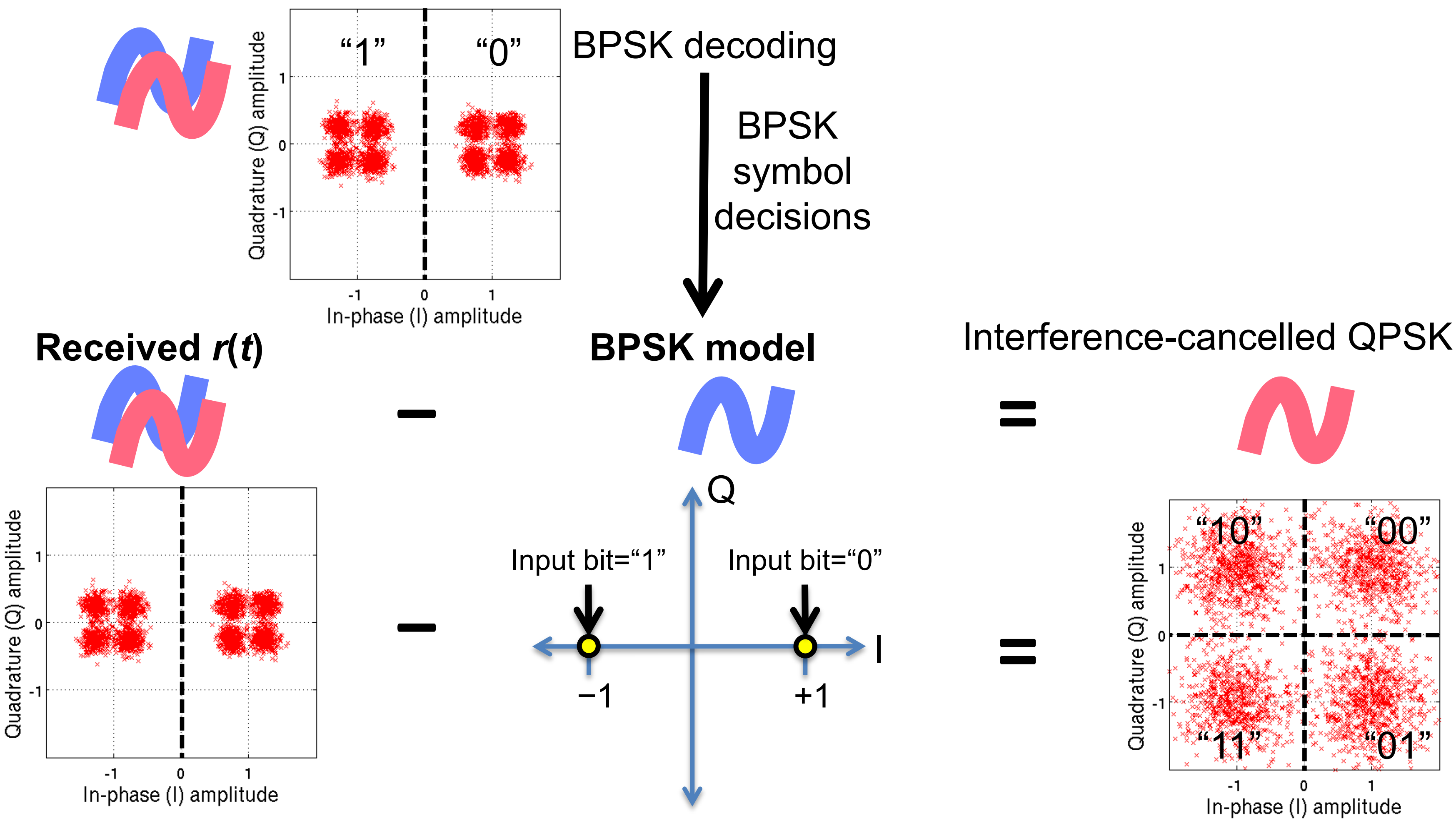


User 1 alone
QPSK signal

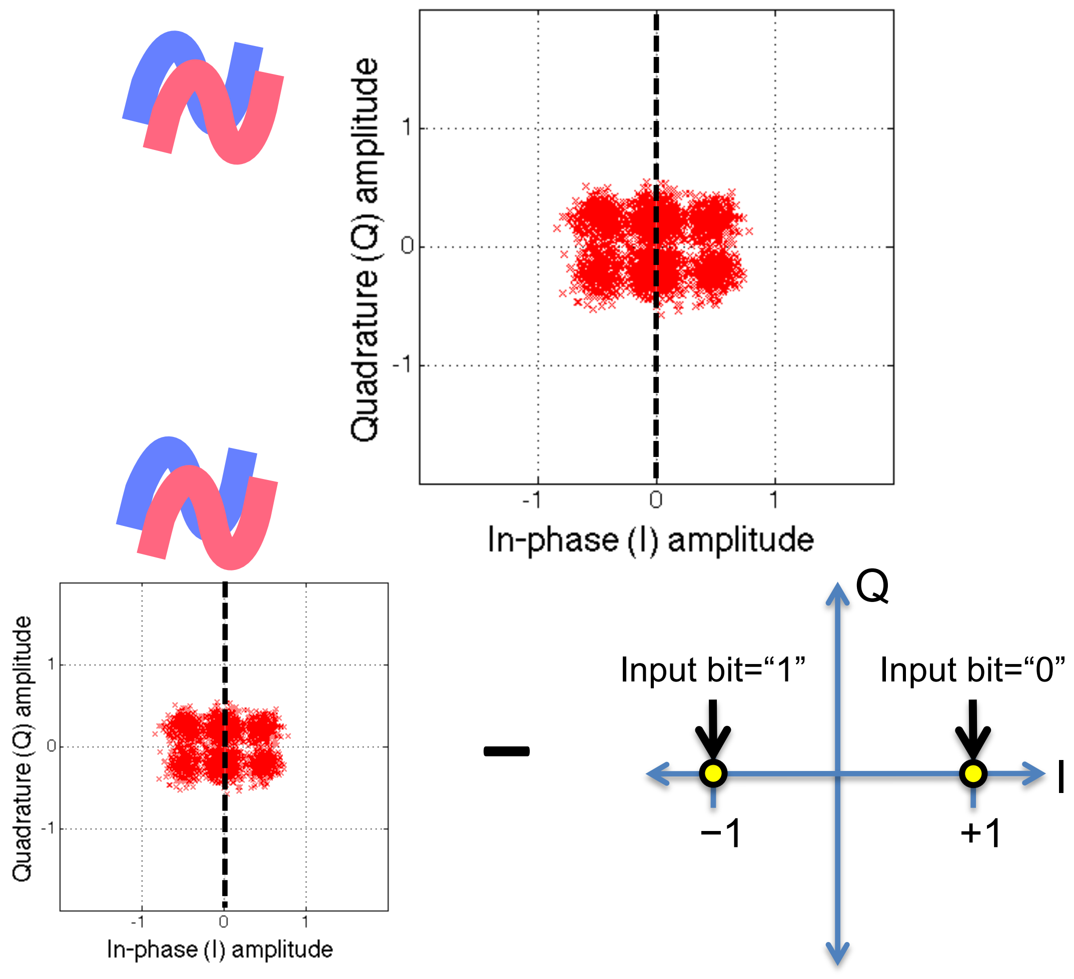


User 1 and 2
Superposition

Decoding Strong BPSK with weak QPSK



Power Difference Helps Superposition Coding



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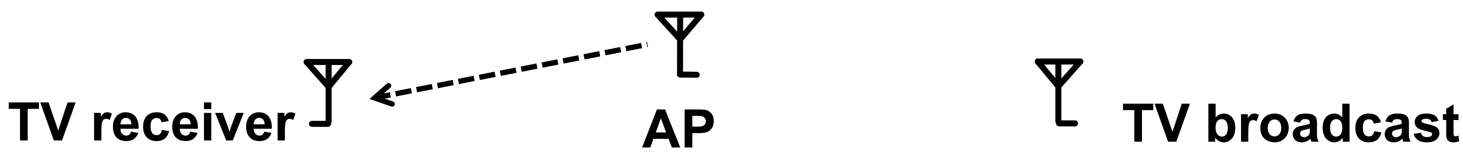
Precoding for Known Interference

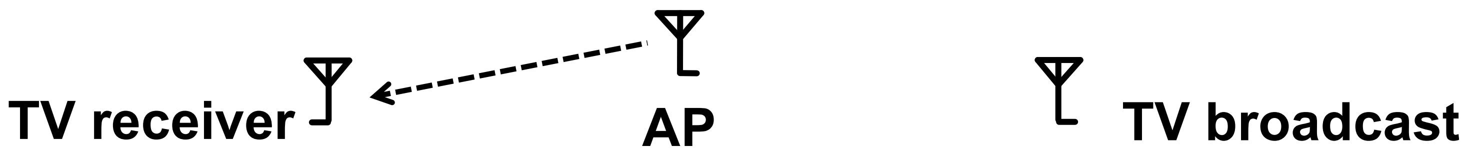


Receives:

$$y = x + s + w$$

- **Scenario: Hide digital data** in an **analog TV broadcast**
- AP transmitting on downlink to user **overhears broadcast**
  - Sends digital data ***x*** intended for User 1
  - ***s*** is the stronger analog TV broadcast signal
- Background noise ***w***

Precoding for Known Interference

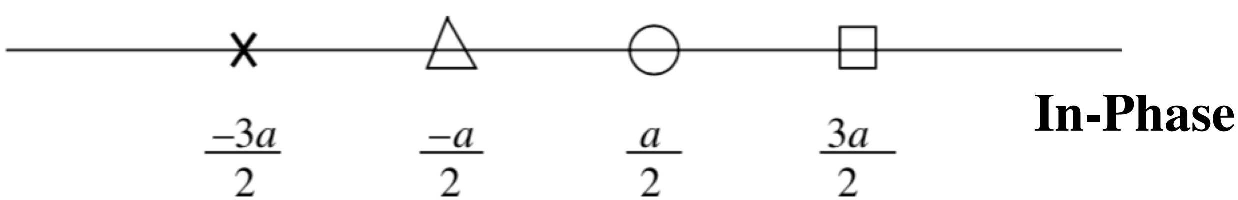


Receives:

$y = x + s + w$

- Want to distort TV broadcast signal *s* minimally

- Suppose we want to send one of four in-phase symbols (no Q):



Precoding for Known Interference

- Want to distort TV broadcast ***s*** minimally

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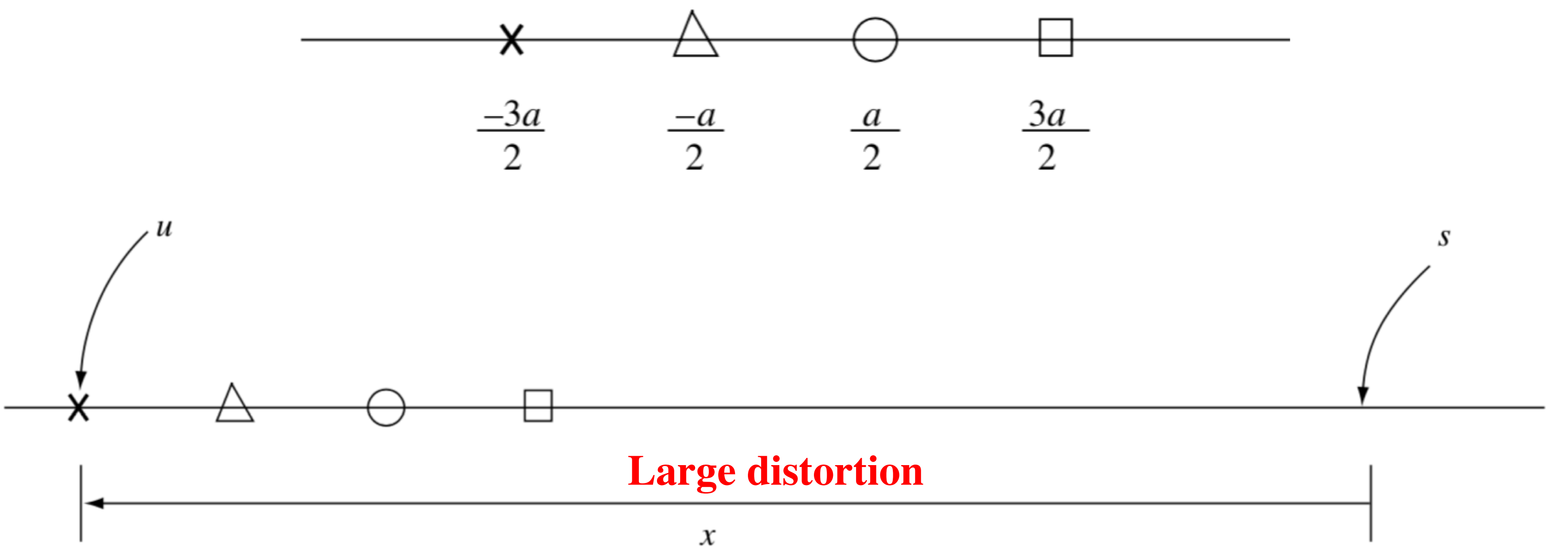
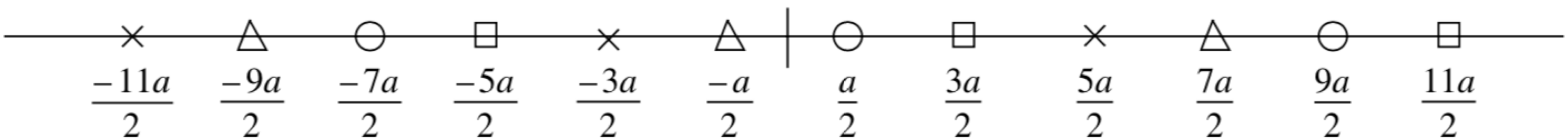


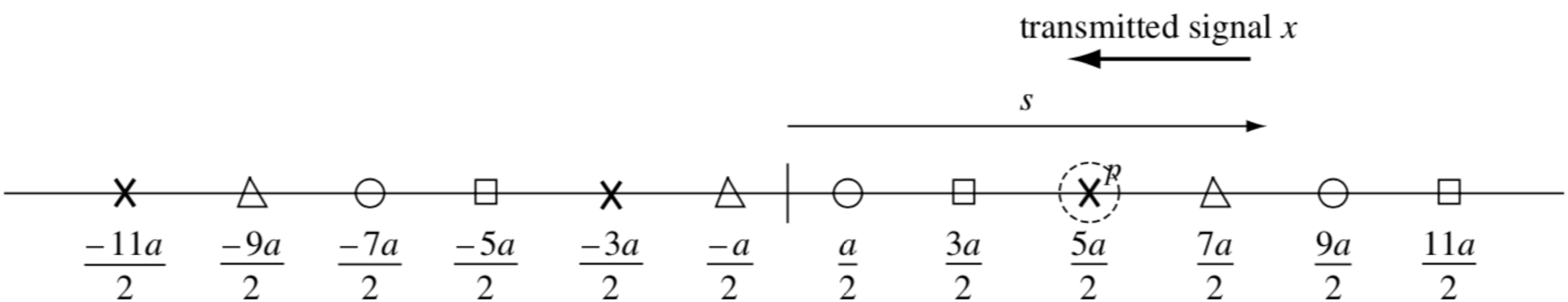
Tomlinson-Harashima Precoding

- Suppose we want to send one of four in-phase symbols (no Q):

- **Idea:** Replicate constellation across amplitudes:



- Transmit **closest replicated constellation point:**



Tomlinson-Harashima: Summary

- Effect is like superposition coding
- Choosing closest replicated constellation group minimizes power
  - Transmitted signal depends on the interfering signal
- Result: Hide digital information in the analog transmission

Friday Precept:
Lab 4 Hackathon

Tuesday Topic:
Low-Power Wireless Communication