MIMO II: Physical Channel Modeling, Spatial Multiplexing

COS 463: Wireless Networks

Lecture 17

Kyle Jamieson

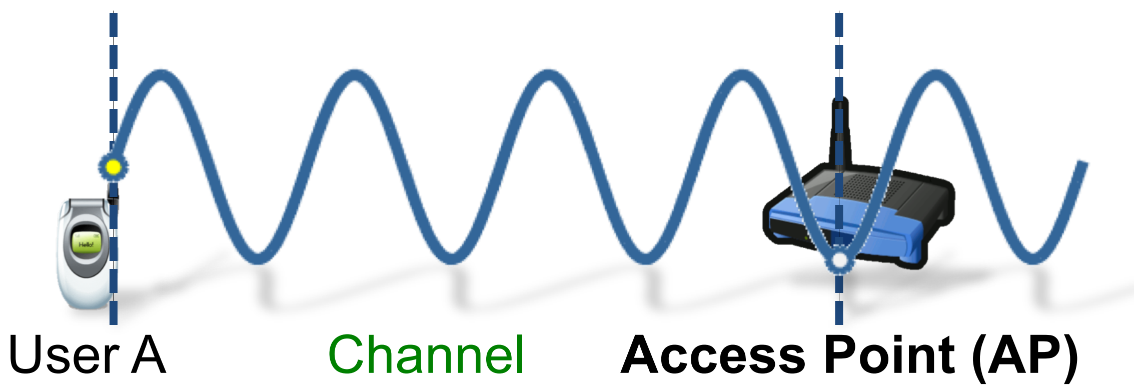
Today

1. **Graphical intuition in the I-Q plane**

2. Physical modeling of the SIMO channel

3. Physical modeling of the MIMO channel

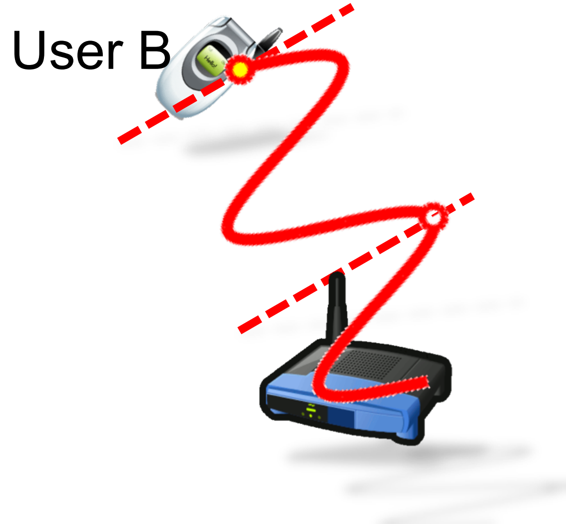
The problem of wireless interference



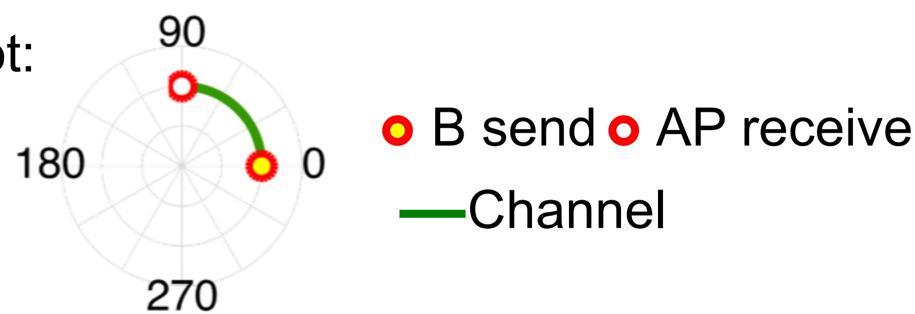
I-Q plot:

AP can estimate the channel, so can decode User A's signal ( )

The problem of wireless interference

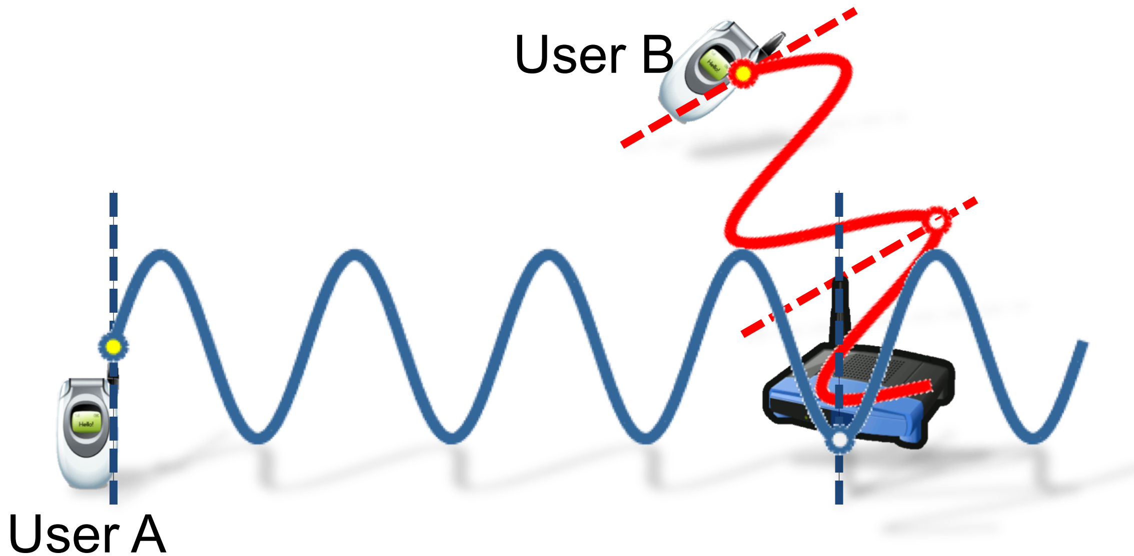


I-Q plot:

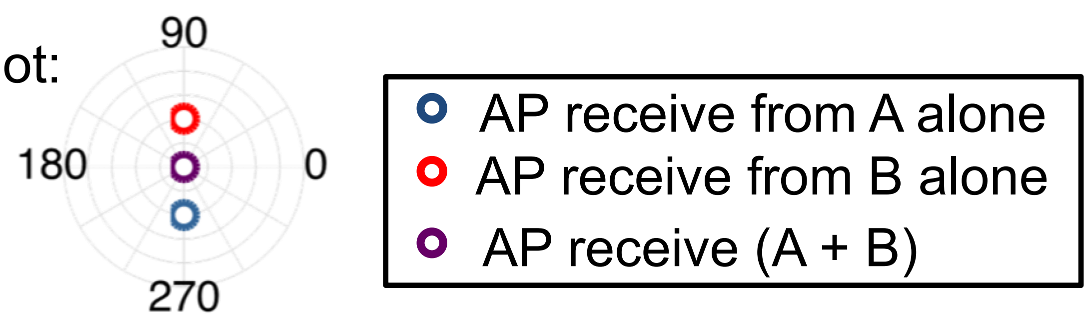


AP can estimate the channel, so can decode User B's signal ( )

The problem of wireless interference



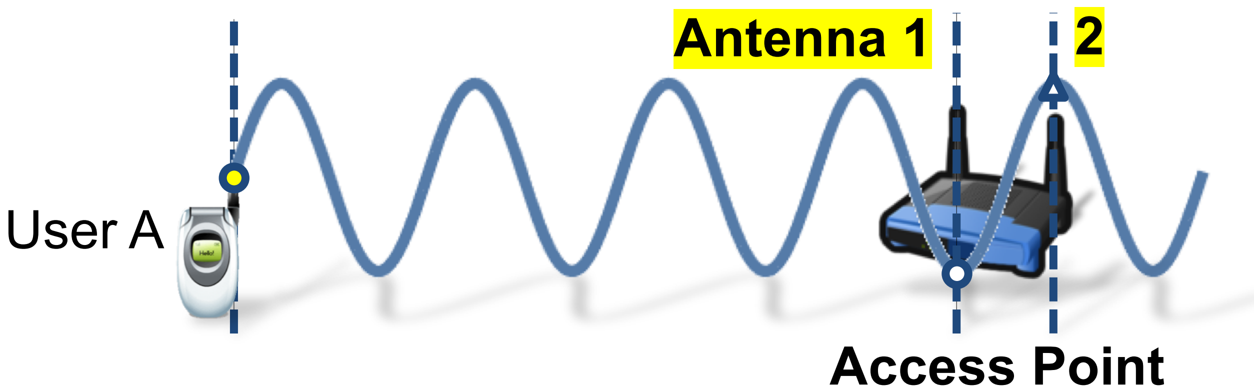
I-Q plot:

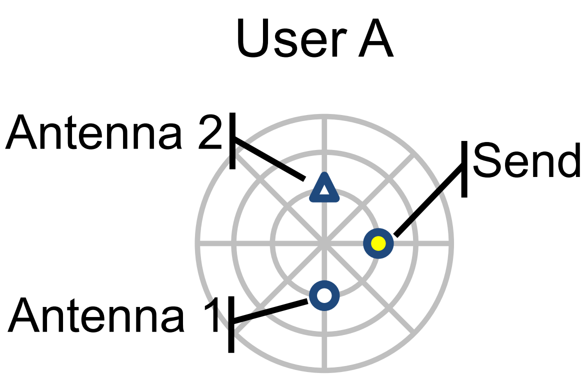


One received signal ( ), two sent (,), so **AP can't decode**

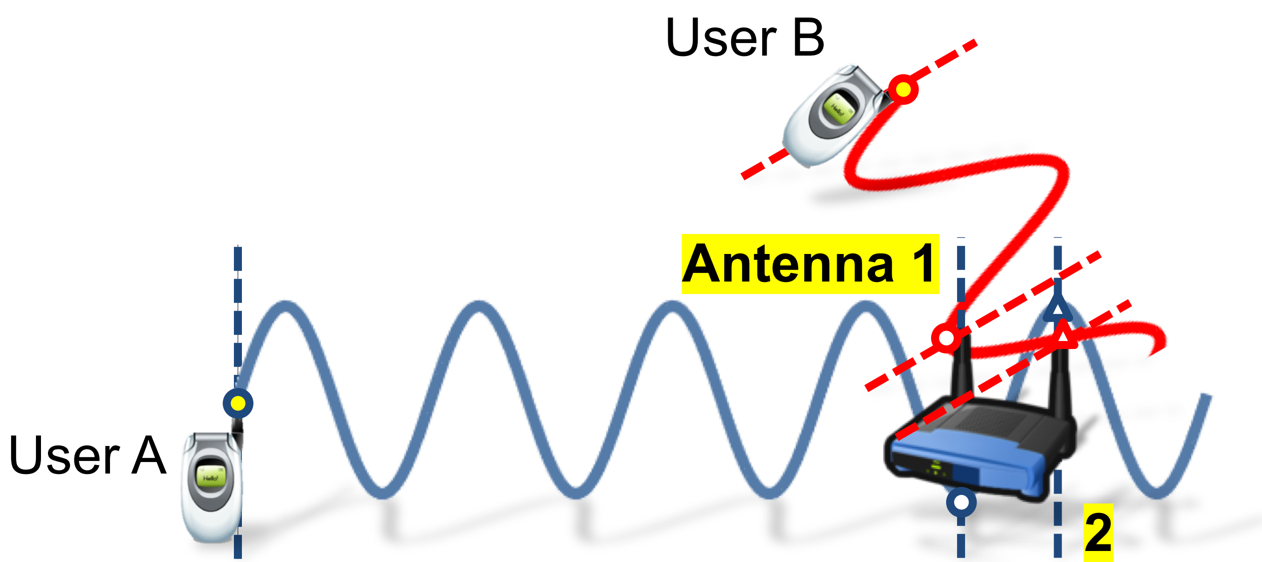
Leveraging Multiple Antennas

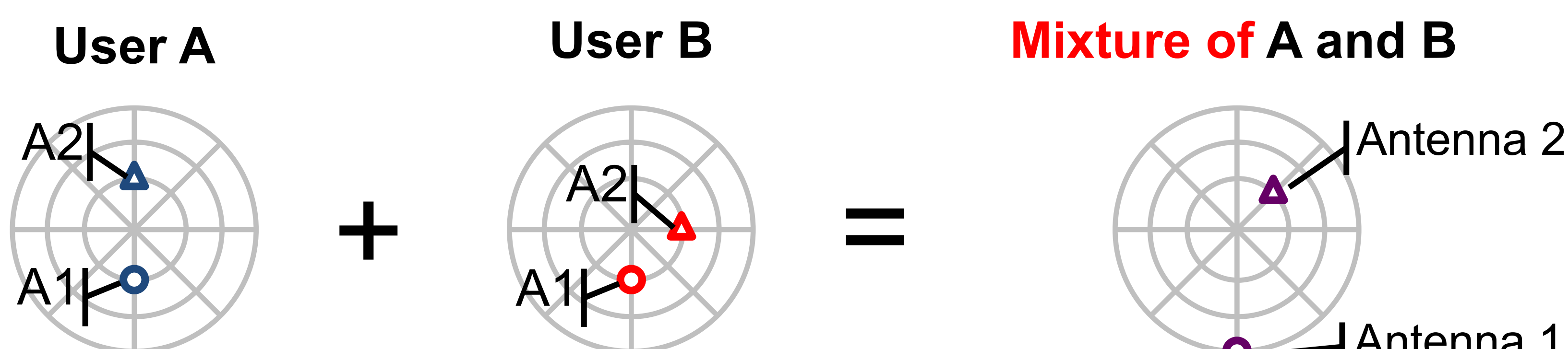
- Now, the AP hears **two** received signals, one on each antenna:





Leveraging Multiple Antennas

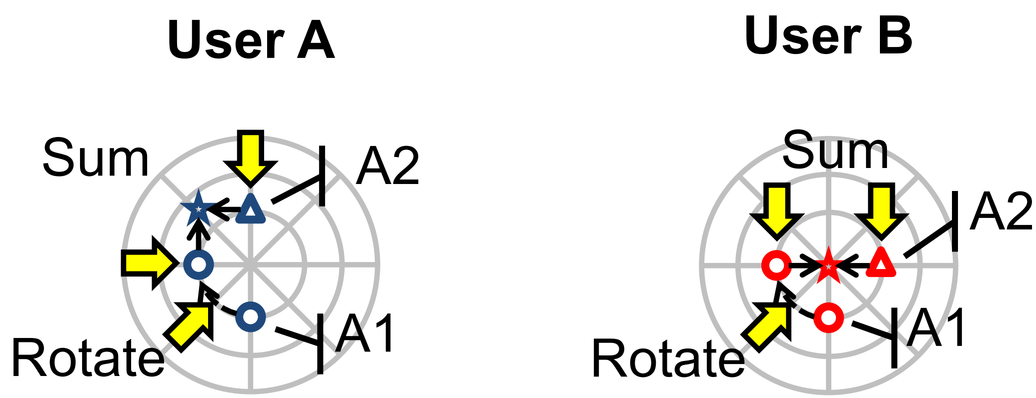




Intuition: Zero-Forcing Receiver

- MIMO ***zero-forcing receiver***

1. **Rotate** one antenna's signal (○)
2. **Sum** the two antennas' signals together (○ + △)



Zero-forcing **cancels** B, **revealing** A

Can re-run to cancel A, revealing B

Spatial Multiplexing: More "Streams"

- Send **multiple streams of information** **over each of the spatial paths** between sender and receiver
  - This is called ***spatial multiplexing***

- **Potential for** **increased capacity by a factor of *N*** (minimum number of send or receive antennas):

$$C = BN \log\left(1 + \mathrm{SNR}\right) \mathrm{bits/s/Hz}$$

- **Potential for** a **multiplicative rate speed-up**

Today

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2. **Physical modeling of the SIMO channel**

3. Physical modeling of the MIMO channel

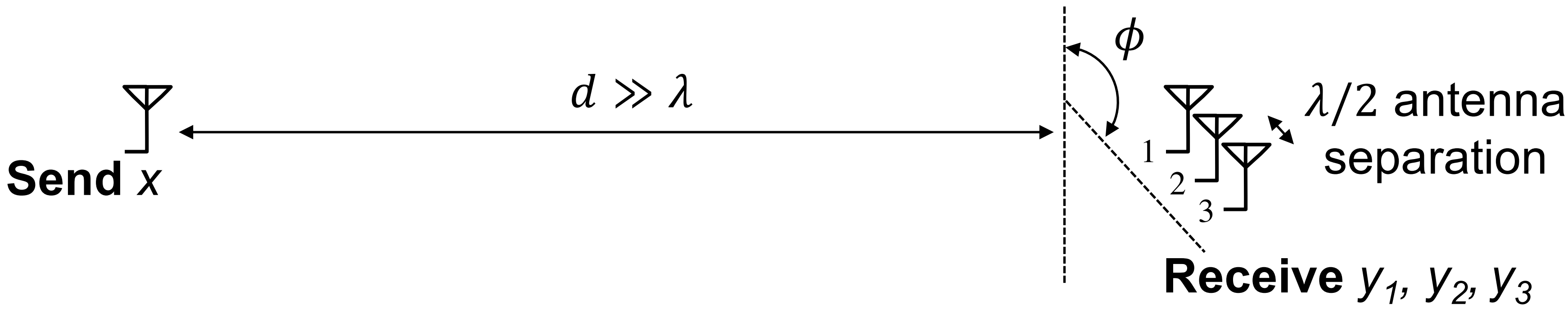
Physical Modeling of Multi-Antenna Channels

- Gain intuition as to how the RF channel (ambient environment) impacts capacity

- Many **physical antenna arrangement geometries** possible

- Limit discussion today to ***linear*** **antenna arrays, half-wavelength antenna spacing**
  - Details vary with more sophisticated antenna arrangements, but concepts do not

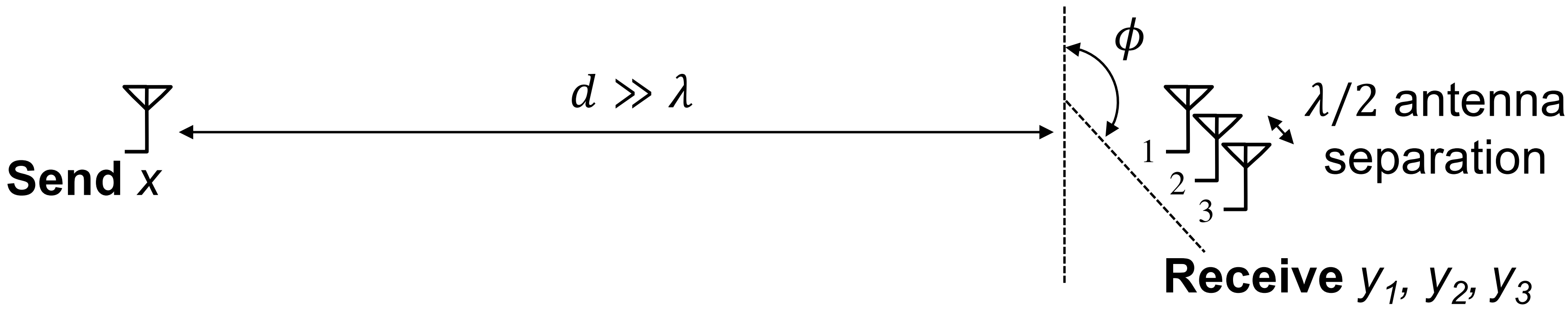
Line-of-Sight SIMO Channel: A Second Look



- **Vector notation** for the system: $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \vec{y} = \vec{h}x + \vec{w}$

- **Wireless channel** is now a **three-tuple vector:** $\vec{h} = \begin{bmatrix} ae^{j2\pi d_1} \\ ae^{j2\pi d_2} \\ ae^{j2\pi d_3} \end{bmatrix}$

Line-of-Sight SIMO Channel: A Second Look



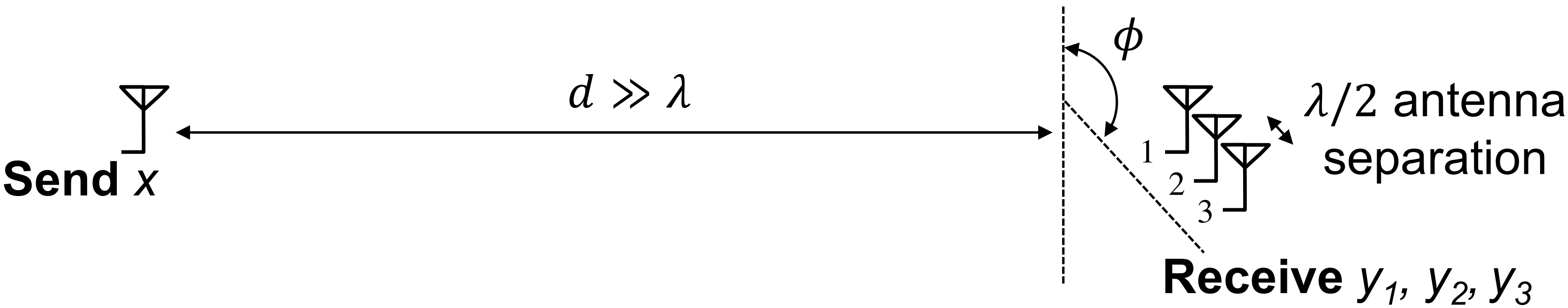
- **Wireless channel** is now a **three-tuple vector:** $\vec{h} = \begin{bmatrix} ae^{j2\pi d_1/\lambda} \\ ae^{j2\pi d_2/\lambda} \\ ae^{j2\pi d_3/\lambda} \end{bmatrix}$

- **Antenna separations:**
  - Assume $d_1 = d$
  - $d_2 \approx d + \frac{1}{2}\lambda \cos\phi$
  - $d_3 \approx d + \lambda \cos\phi$

- **Wireless channel:**

$$\vec{h} = ae^{j2\pi d/\lambda} \begin{bmatrix} 1 \\ e^{j\pi \cos\phi} \\ e^{j2\pi \cos\phi} \end{bmatrix}$$

Line-of-Sight SIMO Channel: The Spatial Signature

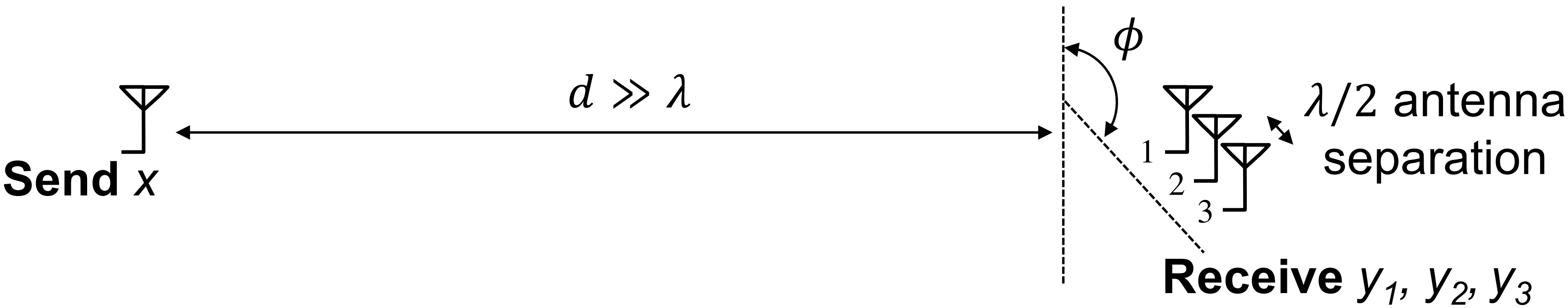


- The wireless channel **decomposes into two components:**

$$\vec{h} = \underbrace{a e^{j2\pi d/\lambda}}_{\textbf{Path component}} \underbrace{\begin{bmatrix} 1 \\ e^{j\pi \cos\phi} \\ e^{j2\pi \cos\phi} \end{bmatrix}}_{\textbf{\textit{Spatial Signature}}}$$

- The **angle of arrival** of the sender's signal **at the receive array** **determines the spatial signature**

Line-of-Sight SIMO Channel: Maximal Ratio Combining (Review)



- Maximal ratio combining **"projects" the received signals $\vec{y}$ onto the receive spatial signature:**
  - $\tilde{y} = \vec{h}^* \vec{y}$
- **Reverses the phases** in the spatial signature to **align** each antenna's component of the above sum
  - **SNR improvement** but **no multiplexing**

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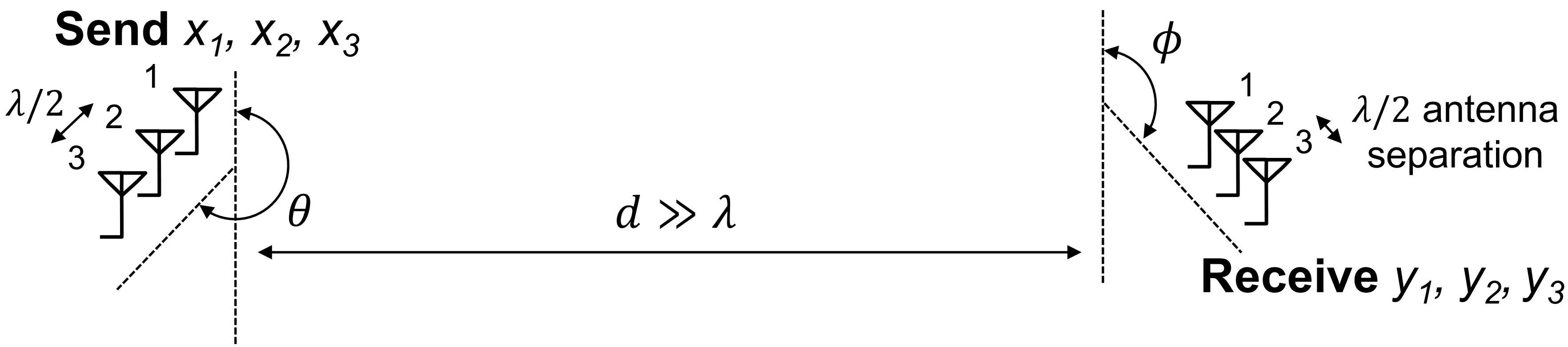
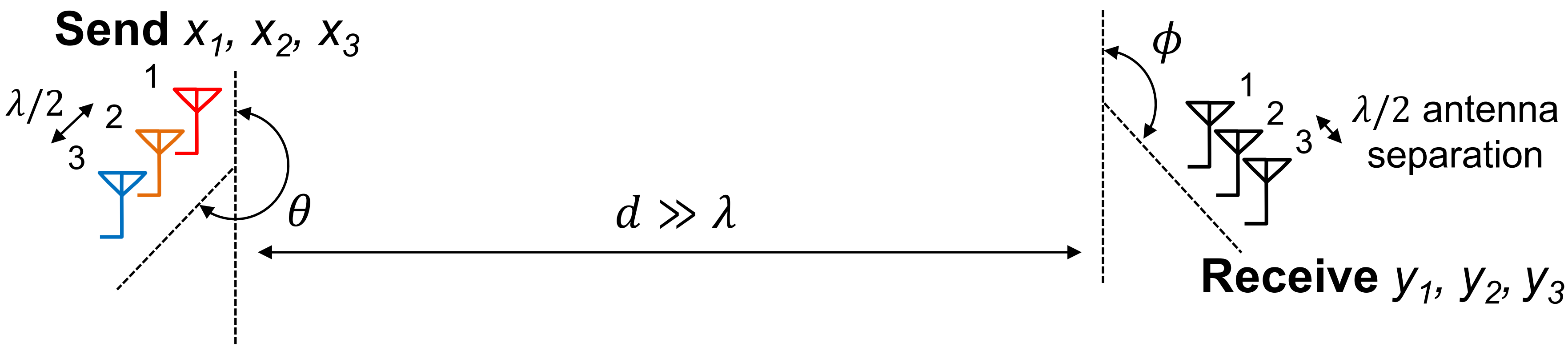
3. **Physical modeling of the MIMO channel**
   - **Line-of-Sight MIMO Channel**
   - Geographically-Separated Transmit Antennas
   - Geographically-Separated Receive Antennas
   - MIMO Link in Multipath

The Line-of-Sight MIMO Channel



- Want to transmit **three symbols** per symbol time: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
- $\boldsymbol{h_{kl}}$: channel between *k* th receive and *l* th transmit antenna
- $\vec{y} = \mathrm{H}\vec{x}$, where $\mathrm{H} = \begin{bmatrix} h_{11} & h_{12} & h_{11} \\ h_{21} & h_{22} & h_{11} \\ h_{31} & h_{32} & h_{11} \end{bmatrix}$ is the MIMO ***channel matrix***

The Line-of-Sight MIMO Channel: Channel Matrix

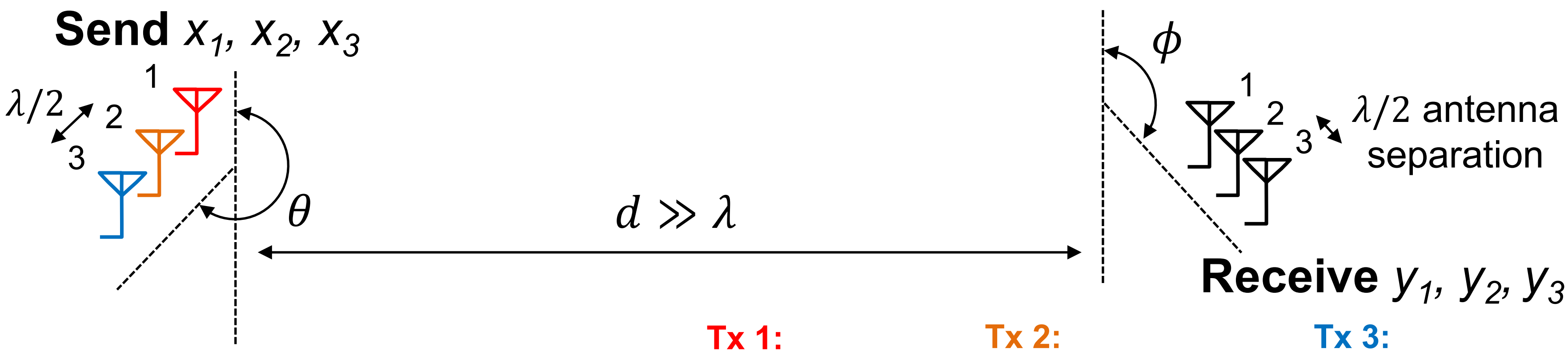


- $\boldsymbol{h_{kl}}$: channel between k th receive and l th transmit antenna
- Suppose as before, $d_{11} = d$
  - Then $d_{kl} = d + \frac{1}{2}(k-1)\cos\phi + \frac{1}{2}(l-1)\cos\theta$
- **Channel matrix** $\mathbf{H} = ae^{j2\pi d/\lambda}$

$$\begin{array}{ccc} \textbf{Tx 1:} & \textbf{Tx 2:} & \textbf{Tx 3:} \end{array}$$

$$\mathbf{H} = ae^{j2\pi d/\lambda} \begin{bmatrix} 1 & e^{j\pi \cos\theta} & e^{j\pi\, 2\cos\theta} \\ e^{j\pi \cos\phi} & e^{j\pi\, (\cos\phi + \cos\theta)} & e^{j\pi\, (\cos\phi + 2\cos\theta)} \\ e^{j2\pi \cos\phi} & e^{j\pi\, (2\cos\phi + \cos\theta)} & e^{j\pi\, (2\cos\phi + 2\cos\theta)} \end{bmatrix}$$

The Line-of-Sight MIMO Channel: Identical Spatial Signatures



- **Channel matrix** $\mathbf{H} = a e^{j2\pi d/\lambda} \begin{array}{ccc} \textbf{Tx 1:} & \textbf{Tx 2:} & \textbf{Tx 3:} \\ \end{array}$
$$\mathbf{H} = a e^{j2\pi d/\lambda} \begin{bmatrix} 1 & e^{j\pi \cos\theta} & e^{j\pi\, 2\cos\theta} \\ e^{j\pi\cos\phi} & e^{j\pi\,(\cos\phi + \cos\theta)} & e^{j\pi\,(\cos\phi + 2\cos\theta)} \\ e^{j2\pi\cos\phi} & e^{j\pi\,(2\cos\phi + \cos\theta)} & e^{j\pi\,(2\cos\phi + 2\cos\theta)} \end{bmatrix}$$

- **Transmit antenna 2's** channel and **spatial signature:**

$$\begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = a e^{j2\pi\left(\frac{d}{\lambda} + \cos\theta\right)} \begin{bmatrix} 1 \\ e^{j\pi\cos\phi} \\ e^{j2\pi\cos\phi} \end{bmatrix}$$

The Line-of-Sight MIMO Channel: Takeaways

- Spatial signature tells us how to phase-shift the received signals in order to align them

- **Spatial signature** of **Transmit antenna 1**
  - Equals spatial signature of **Transmit antenna 2**
  - Equals spatial signature of **Transmit antenna 3**

- So any **receiver attempt** to align signal from **Transmit antenna 1**
  - **Also aligns transmit antennas 2 and 3**

- Result is **interference between x_1, x_2, x_3**
  - **Can send same single symbol x** on all transmit antennas
    - Results in **same power gain as MRC**

Spatial mux fail

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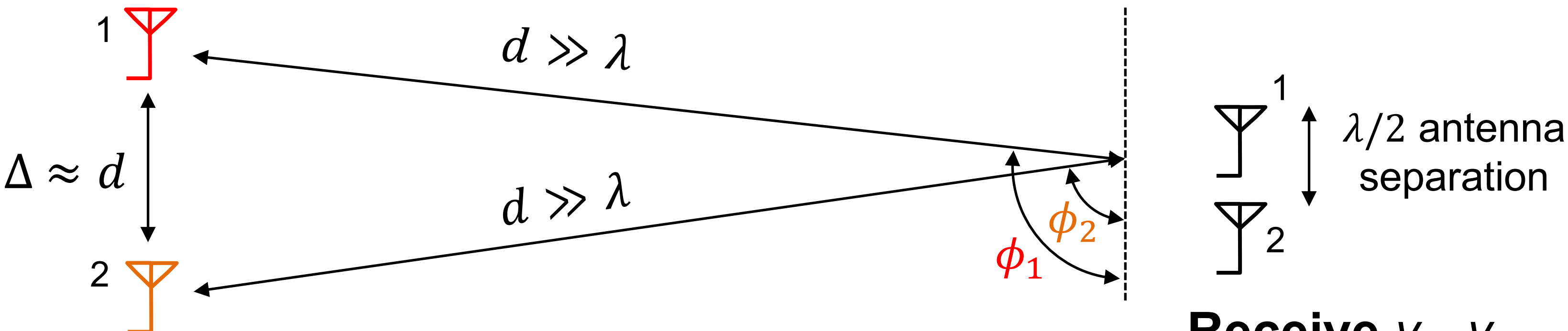
Geographically-Separated Transmit Antennas

Send x_1, x_2

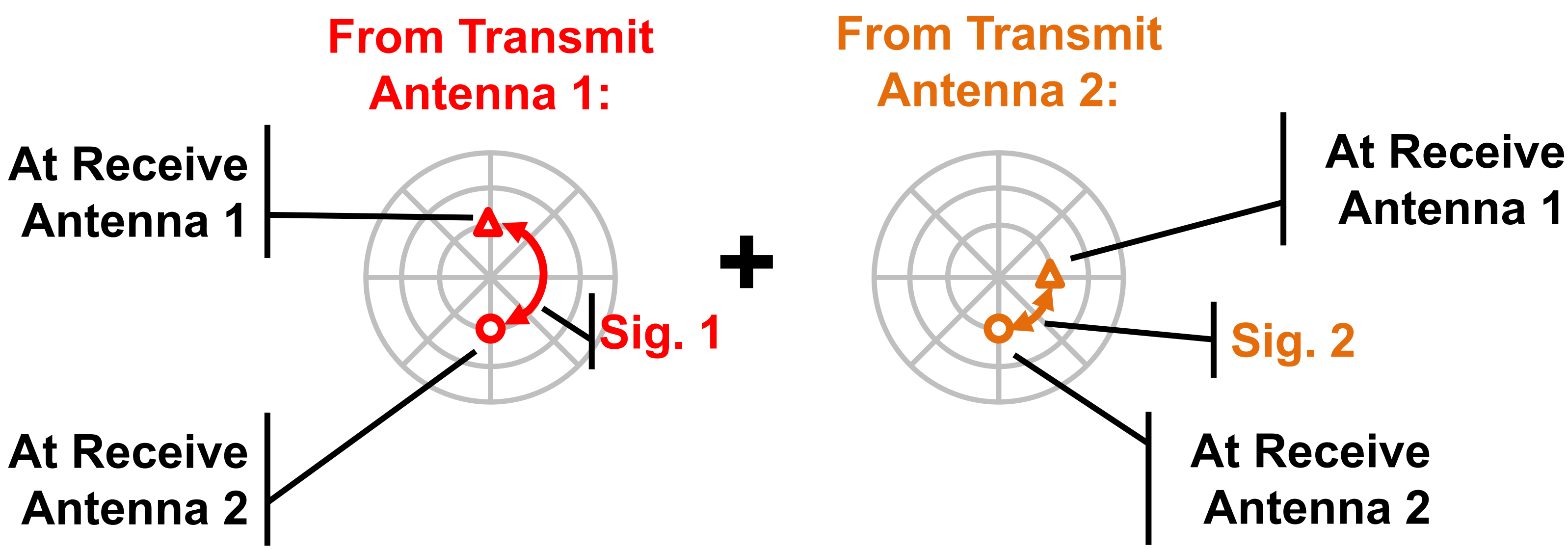


Receive y_1, y_2

- **Channel matrix** $\mathbf{H} = ae^{j2\pi d/\lambda} \begin{array}{cc} \textbf{Tx 1:} & \textbf{Tx 2:} \\ \left[\begin{array}{cc} 1 & 1 \\ e^{j\pi\cos\phi_1} & e^{j\pi\cos\phi_2} \end{array}\right] & \\ \underbrace{\hphantom{e^{j\pi\cos\phi_1}}}_{\textbf{Sig. 1}} & \underbrace{\hphantom{e^{j\pi\cos\phi_2}}}_{\textbf{Sig. 2}} \end{array}$
- **Different spatial signatures** for Transmit Antenna **1, 2**

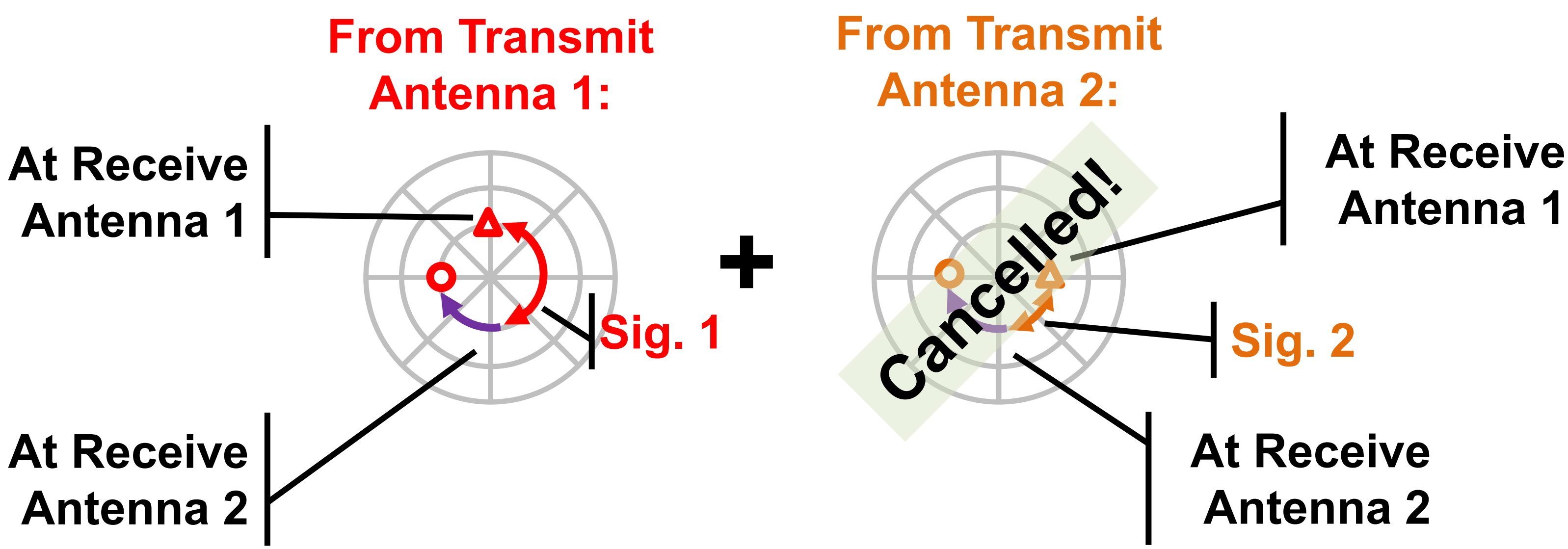
Spatial Signature = Series of Phase Differences

- **Channel matrix** $\mathrm{H} = ae^{j2\pi d/\lambda} \begin{array}{cc} \text{Tx 1:} & \text{Tx 2:} \\ \left[\begin{array}{c} 1 \\ e^{j\pi\cos\phi_1} \end{array}\right. & \left.\begin{array}{c} 1 \\ e^{j\pi\cos\phi_2} \end{array}\right] \\ \underbrace{\hphantom{e^{j\pi\cos\phi_1}}}_{\text{Sig. 1, } h_{\phi_1}} & \underbrace{\hphantom{e^{j\pi\cos\phi_2}}}_{\text{Sig. 2, } h_{\phi_2}} \end{array}$



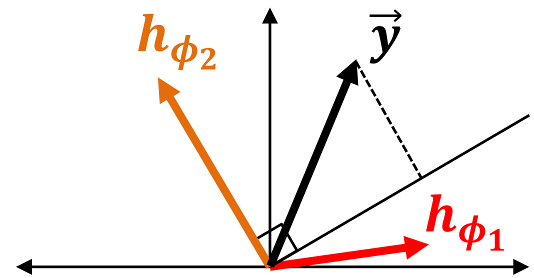
The Zero-Forcing Receiver
(via Spatial Signatures)

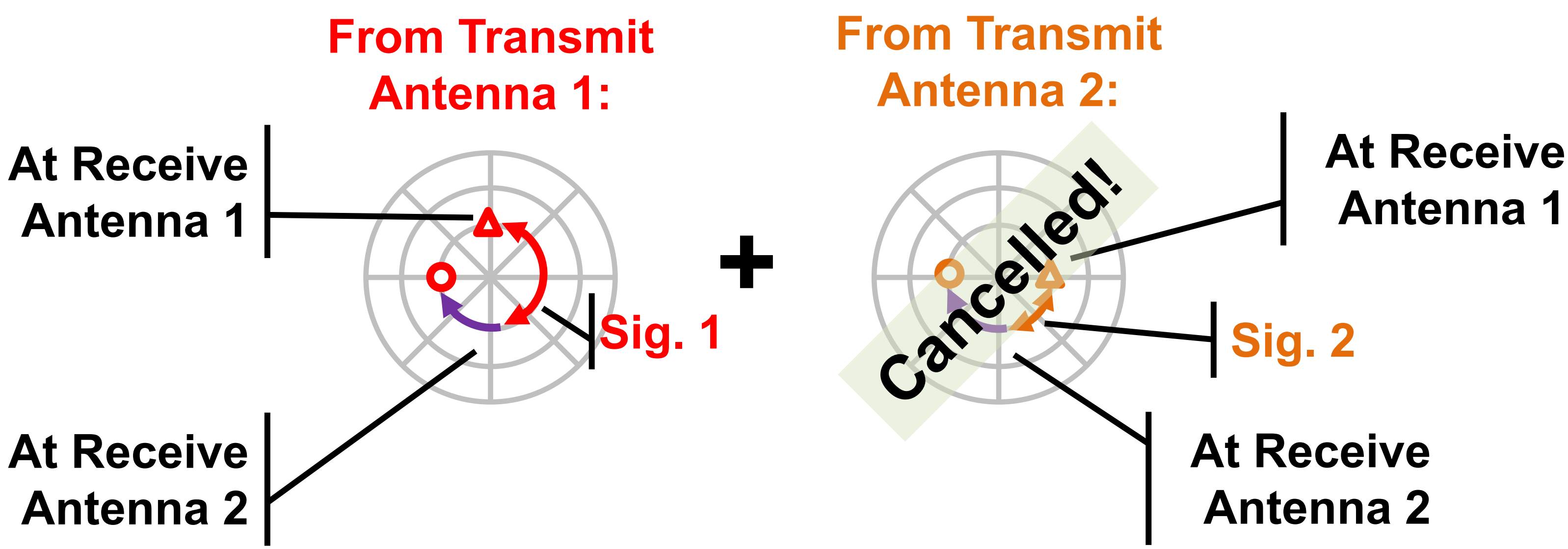
- Suppose want to receive from **Transmit Antenna 1**
  - (Recall:) **Rotate Receive Antenna 2's** signal so that **Signature 2** **cancels itself**



The Zero-Forcing Receiver
(via Spatial Signatures)

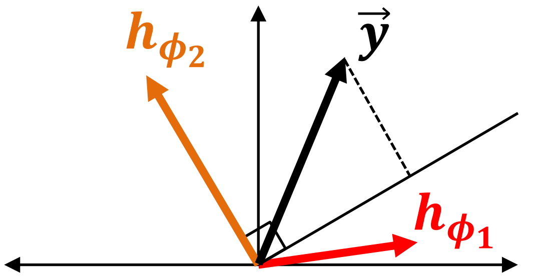
- One **spatial signature** = One **direction**
- **Zero forcing Antenna 2** is **projection**
  - Onto **subspace ⊥ to** $\boldsymbol{h_{\phi_2}}$



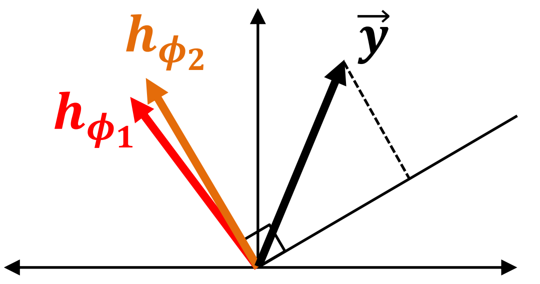


MIMO Separability: Discussion

- **Transmit antenna** separation →
  - **Spatial signature** separation →
    - **Better projection,**
    - **Better performance**



- **MIMO antenna array without multipath**
  - No transmit antenna separation
  - No spatial signature separation
    - Cancel **Tx Ant 2:** cancels **Tx Ant 1**
  - **No spatial multiplexing**



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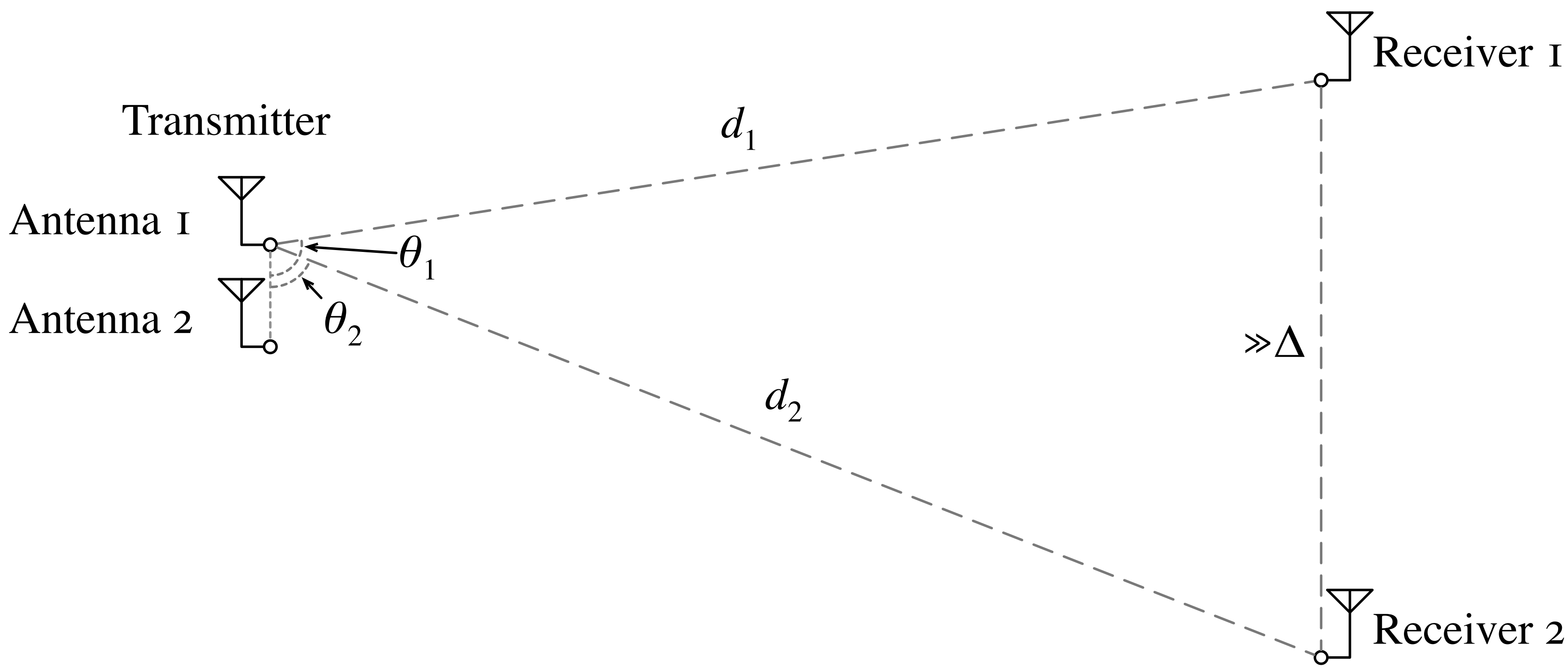
Geographically-Separated Receive Antennas



- **Different spatial signatures** for Receive Antennas **1, 2**
  - **Rows, instead of columns** in the MIMO matrix

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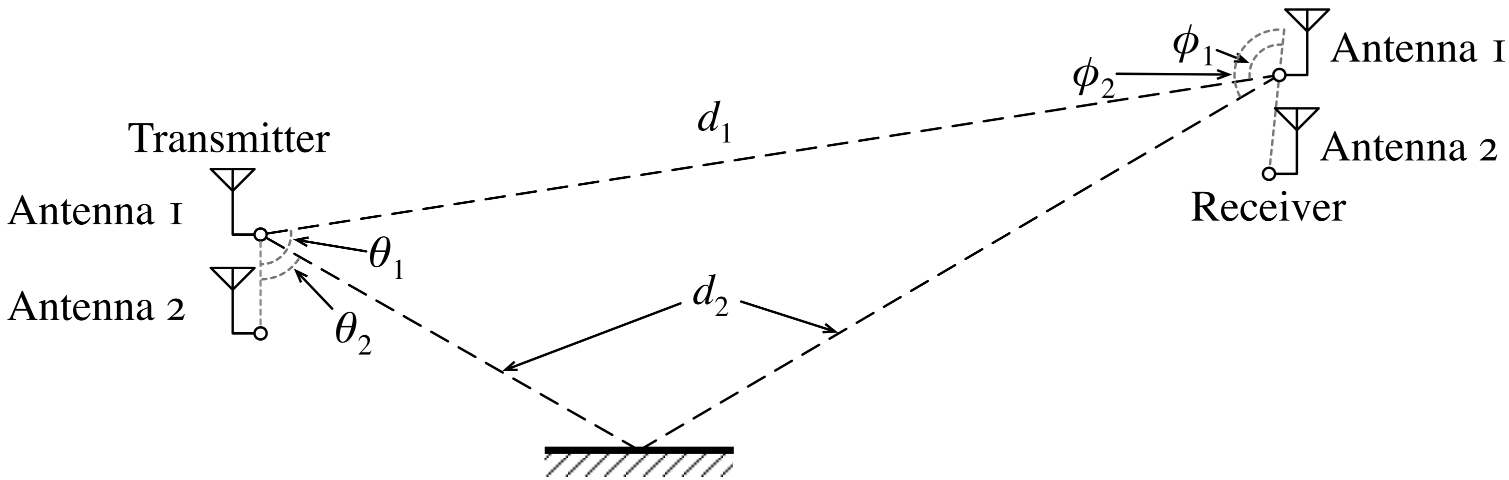
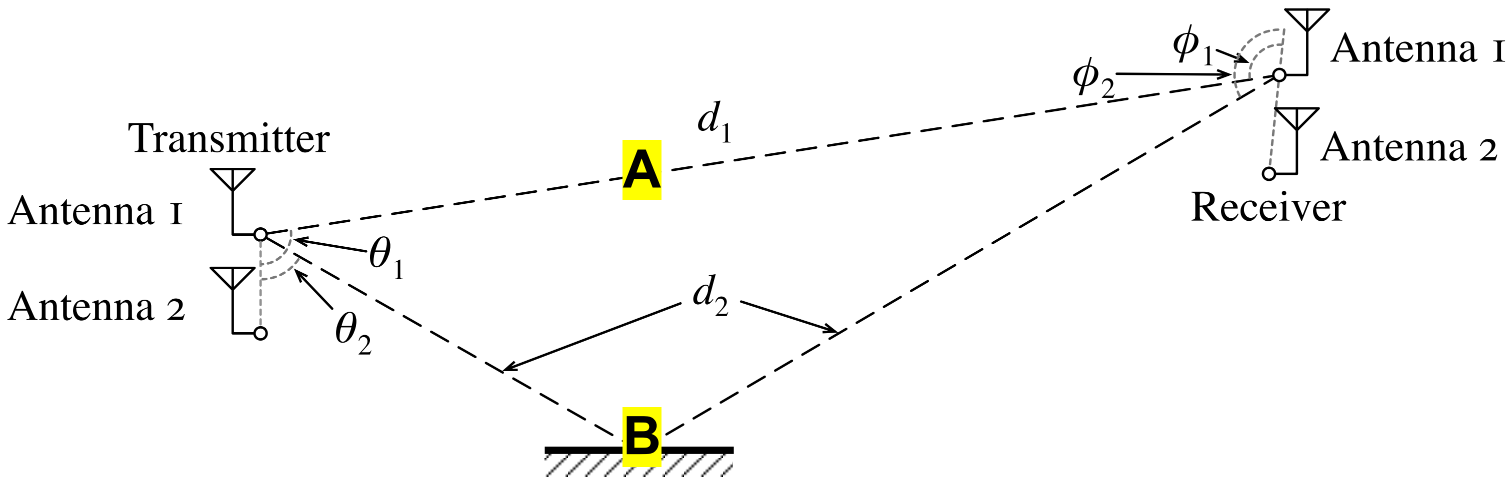
MIMO in Multipath



$$\mathbf{H} = \begin{bmatrix} a_1 e^{j2\pi d_1/\lambda} + a_2 e^{j2\pi d_2/\lambda} & a_1 e^{j2\pi\left(\frac{d_1}{\lambda}+\cos\theta_1\right)} + a_2 e^{j2\pi\left(\frac{d_2}{\lambda}+\cos\theta_2\right)} \\ a_1 e^{j2\pi\left(\frac{d_1}{\lambda}+\cos\phi_1\right)} + a_2 e^{j2\pi\left(\frac{d_2}{\lambda}+\cos\phi_2\right)} & a_1 e^{j2\pi\left(\frac{d_1}{\lambda}+\cos\theta_1+\cos\phi_1\right)} + a_2 e^{j2\pi\left(\frac{d_2}{\lambda}+\cos\theta_2+\cos\phi_2\right)} \end{bmatrix}$$

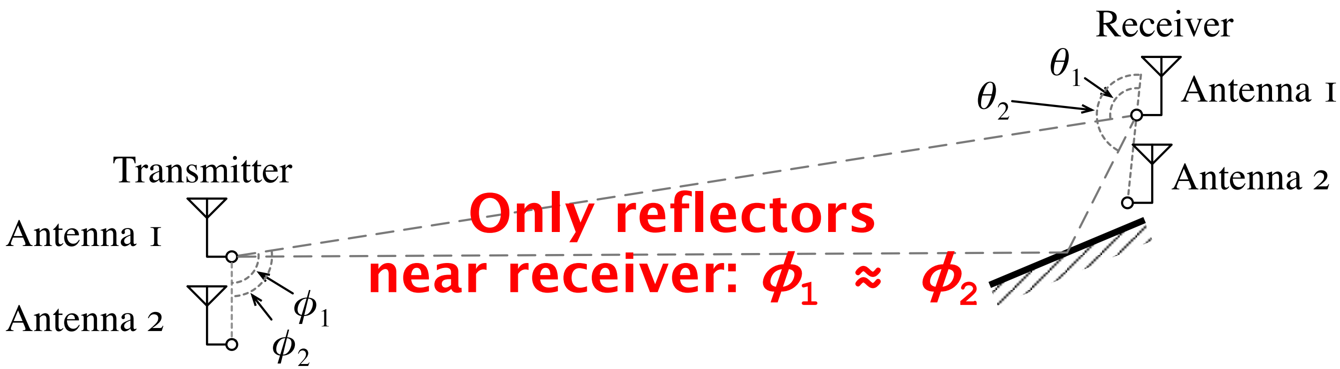
- **Channel matrix H** has **two different transmitter spatial signatures**

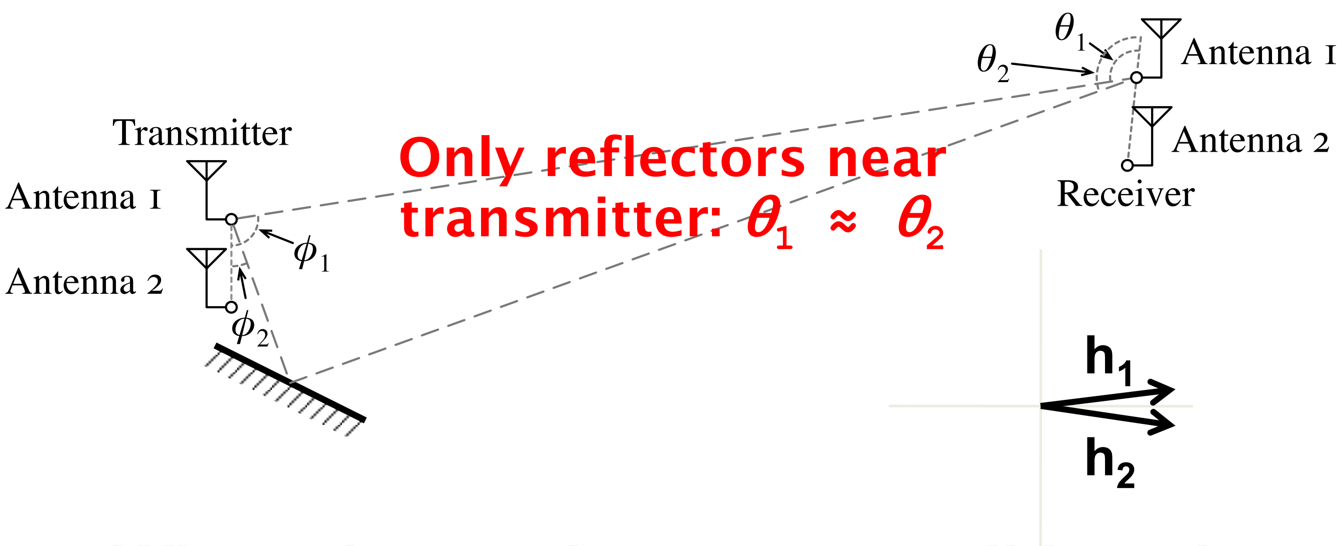
Different Spatial Signatures: Intuition



- **Channel matrix H** has **two different spatial signatures**
- Imagine perfect **signal "relays" A, B**
  - **This H is the product of:**
    - Geographically-separated **receive** antenna channel
    - Geographically-separated **transmit** antenna channel

“Poorly-Conditioned” MIMO channels





When channel is poorly conditioned, spatial signatures are closer aligned

Friday Precept:
Exploiting Doppler

Tuesday Topic:
MIMO III: MIMO Channel Capacity, Interference Alignment