Receiver Designs for the Radio Channel

COS 463: Wireless Networks

Lecture 15

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[Parts adapted from C. Sodini, W. Ozan, J. Tan]

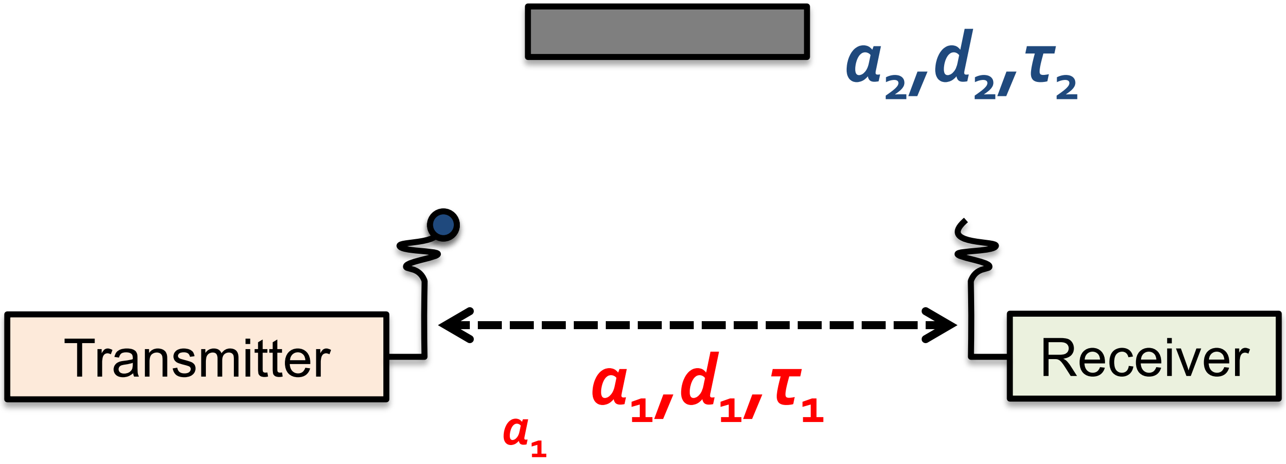
Today

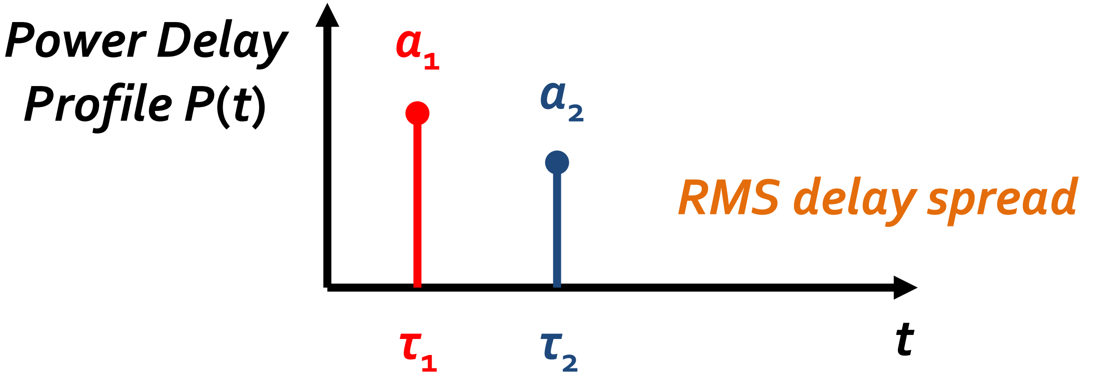
1. **Delay Spread and Frequency-Selective Fading**

2. Time-Domain Equalization

3. Orthogonal Frequency Division Multiplexing

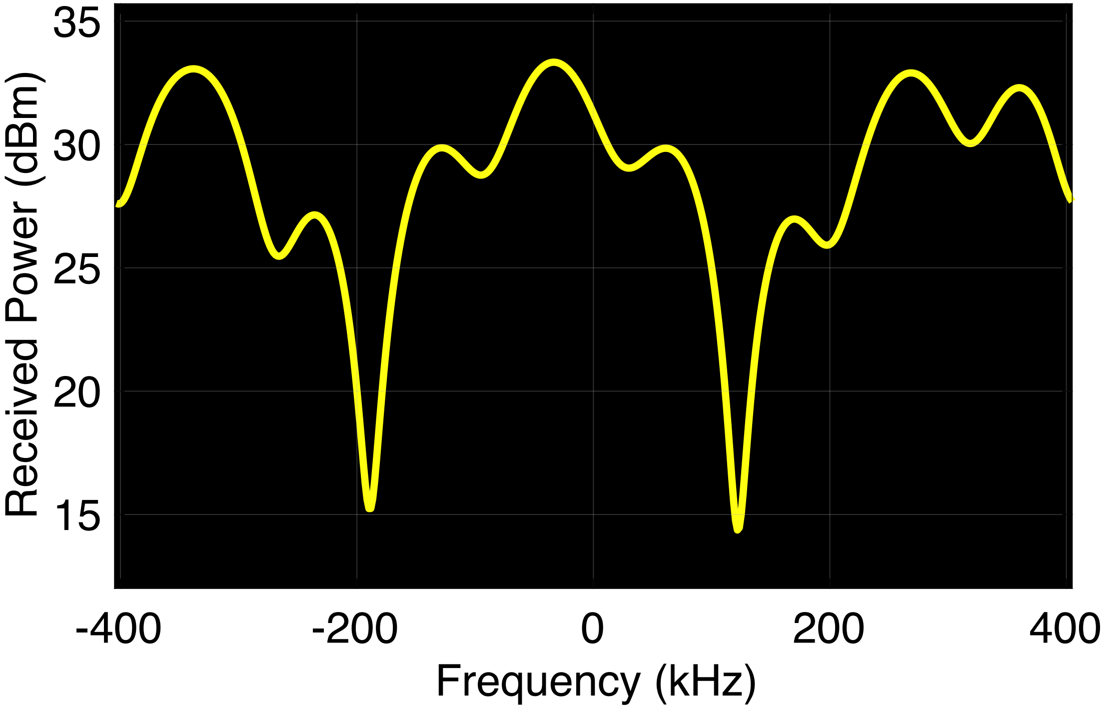
Last Time: Power Delay Profile, Delay Spread, Excess Delay



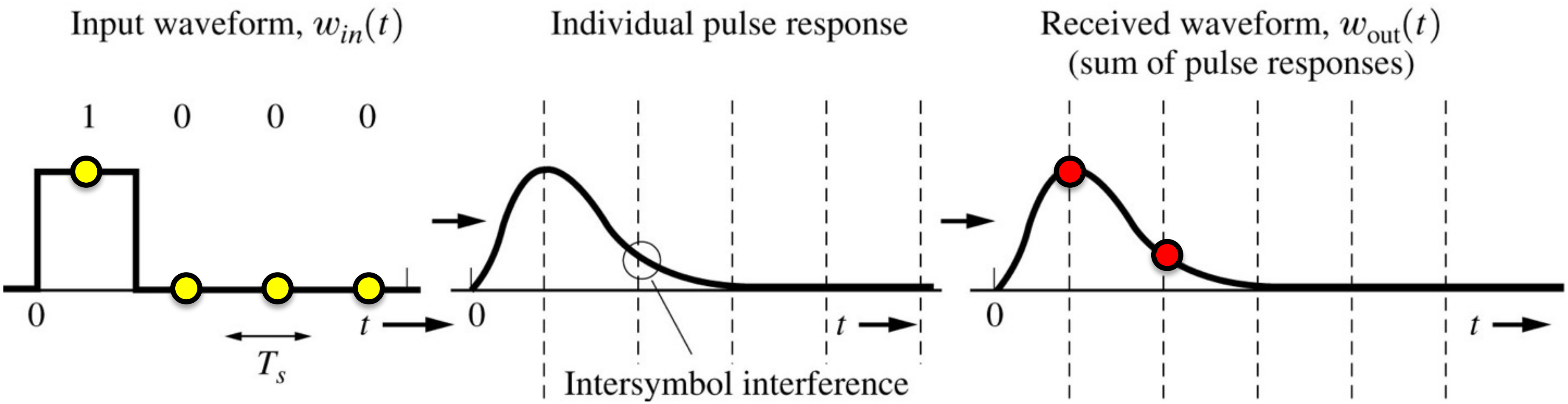


Last Time: Multipath causes Frequency *Selectivity*

- Interference between reflected and line-of-sight radio waves results in **frequency dependent fading**

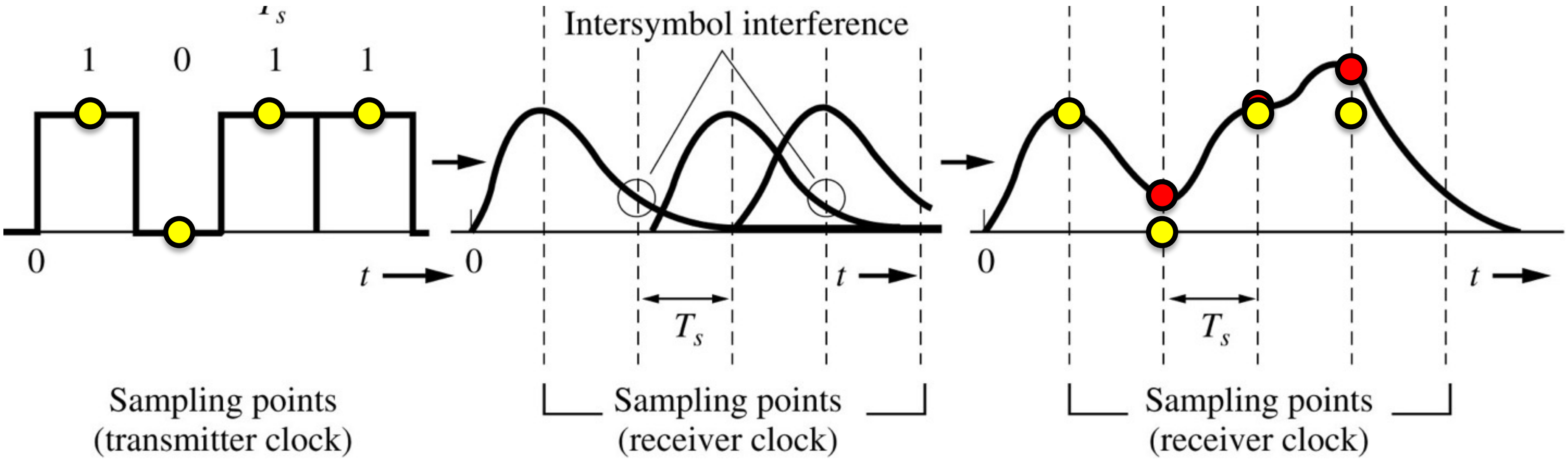


Problem: Inter-symbol interference (ISI)



- **Transmitted signal** 🟡
- **Received signal with ISI** 🔴

Problem: Inter-symbol interference (ISI)



- Transmitted signal 🟡
- Received signal with ISI 🔴

- ISI at one symbol **depends on** the value of **other** symbols

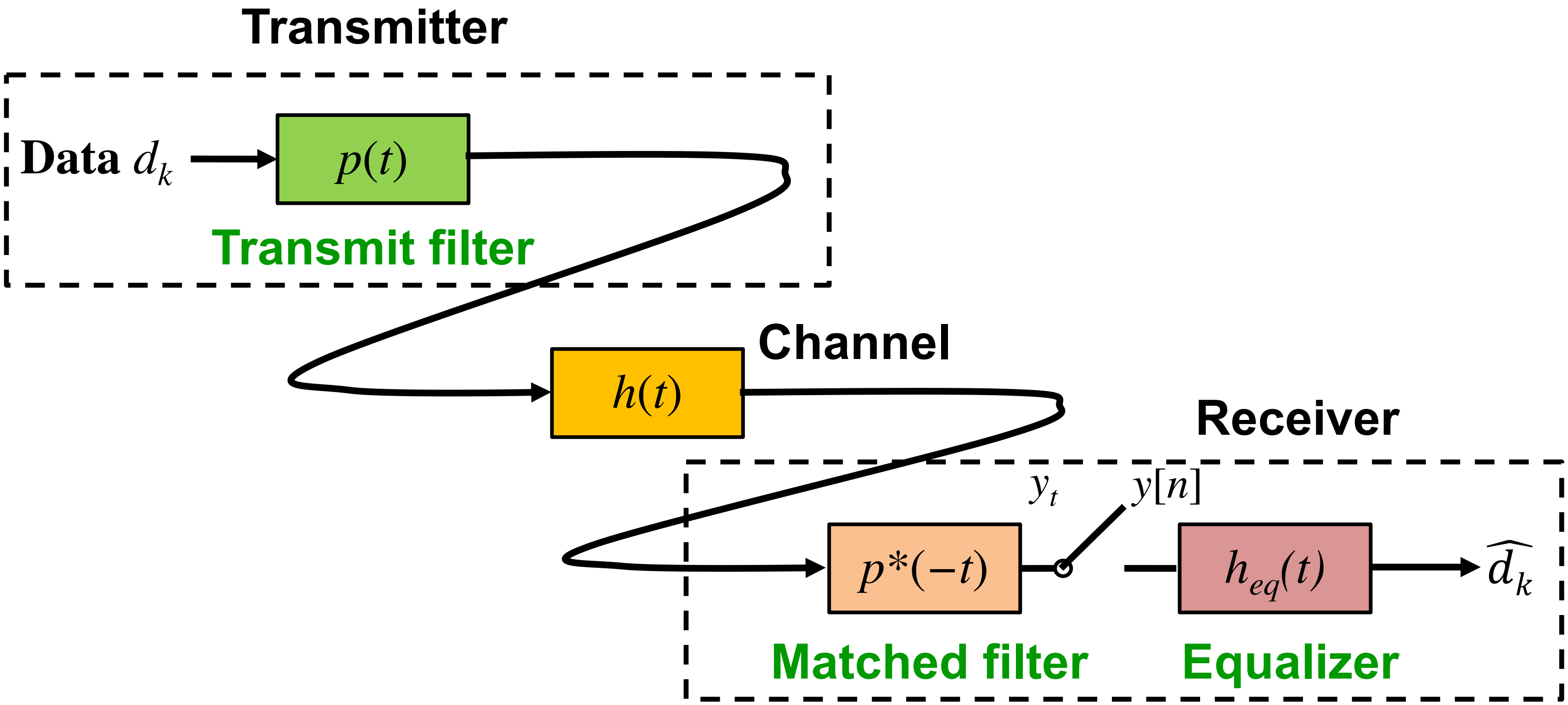
Today

1. Delay Spread and Frequency-Selective Fading

2. **Time-Domain Equalization**

3. Orthogonal Frequency Division Multiplexing

Wideband System Design



Transmitter

Data d_k

$p(t)$

Transmit filter

$h(t)$

Channel

Receiver

y_t

$y[n]$

$p^*(-t)$

$h_{eq}(t)$

$\widehat{d}_k$

Matched filter

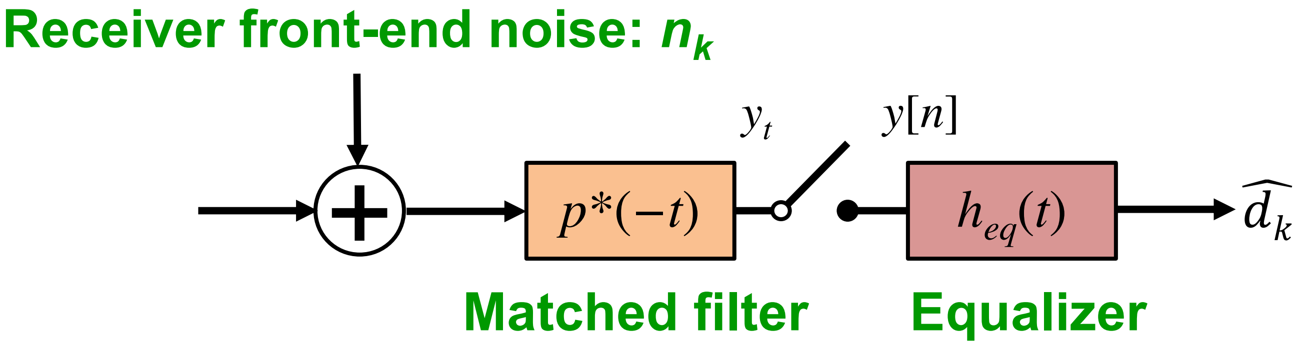
Equalizer

$$\boldsymbol{f(t) = (p * h)(t) * p^*(-t)}$$

- ***Composite channel*** ***f*** (made up of pulse shape, radio channel, and matched filter)

Zero-Forcing Equalizer

Receiver:

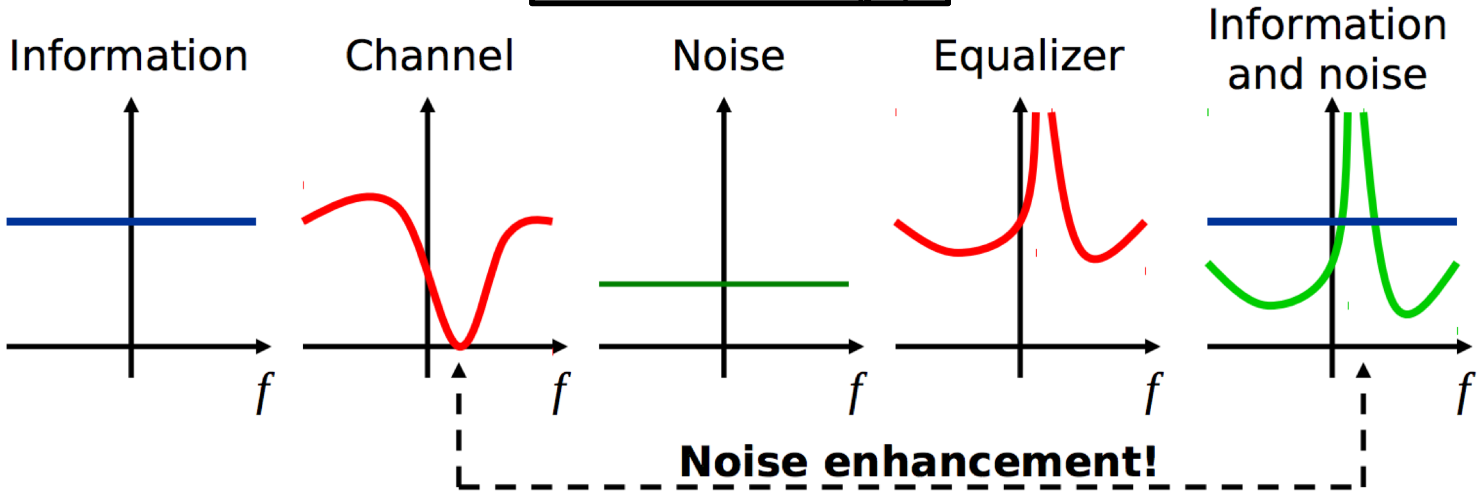


Receiver front-end noise: n_k

Matched filter

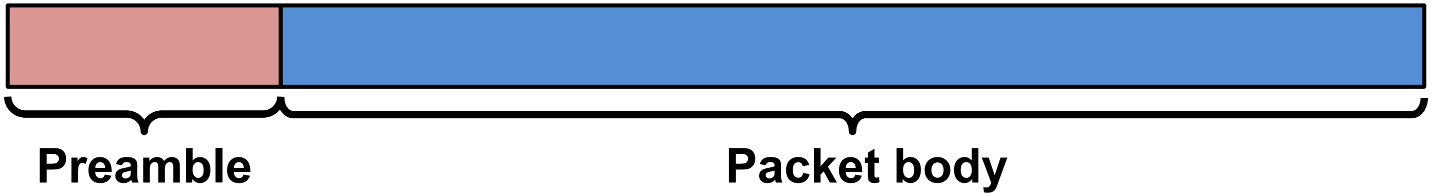
Equalizer

$$\boxed{H_{eq}(f) = \frac{1}{F(f)}}$$



Noise enhancement!

Physical Layer Preamble



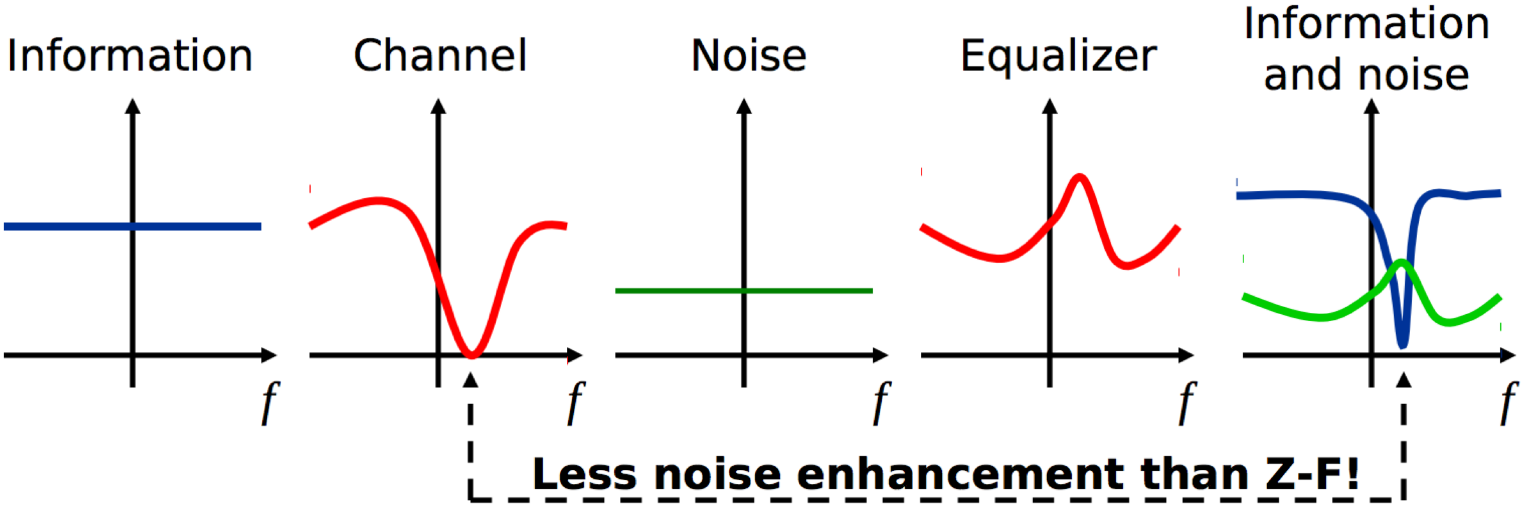
- Sequence of symbols known to both transmitter & receiver

Minimal Mean-Squared Error Equalizer

- **Goal**: ***h_{eq}*** that minimizes mean-squared error (MSE) between **received** and **transmitted symbols from preamble**

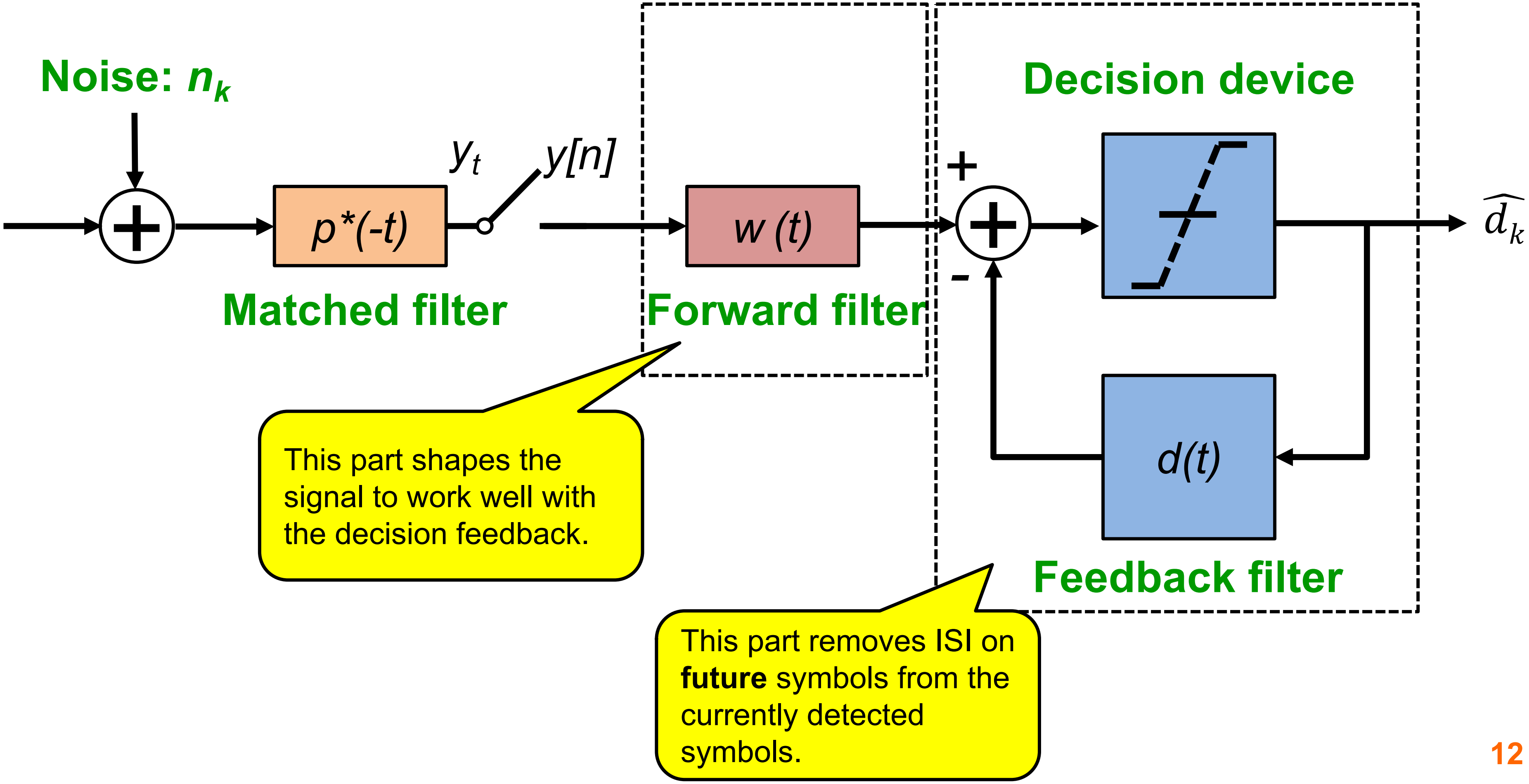
$$MSE = \sum_{k=0}^{K} |d_k - \widehat{d_k}|^2$$

- Assumes packet has a **preamble**, channel **stays same** over a packet time



Decision-feedback Equalizer

- **Idea:** Subtract the **interference** caused by already detected data (symbols)

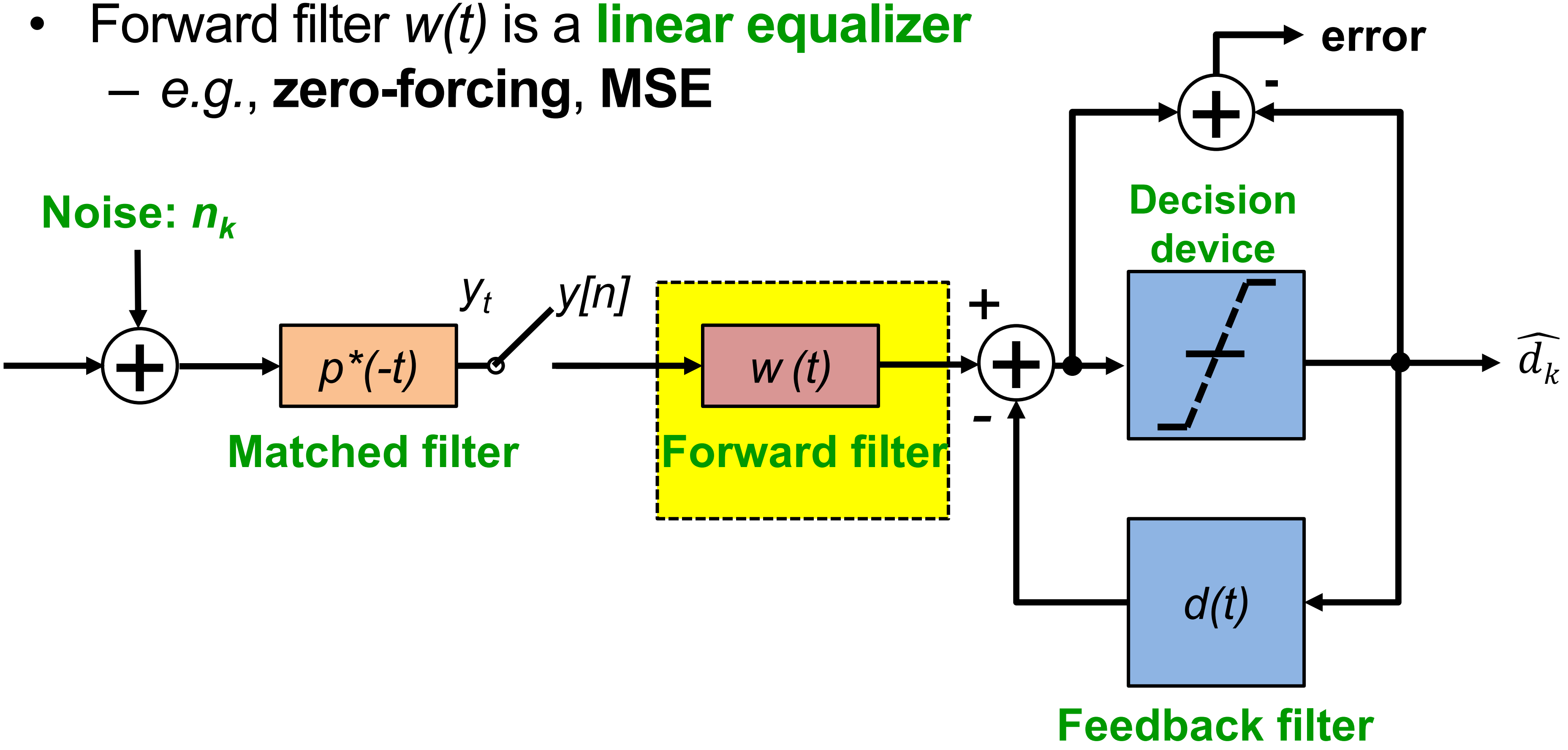


This part shapes the signal to work well with the decision feedback.

This part removes ISI on **future** symbols from the currently detected symbols.

Decision-feedback Equalizer

- Forward filter *w(t)* is a **linear equalizer**
  - *e.g.*, **zero-forcing**, **MSE**

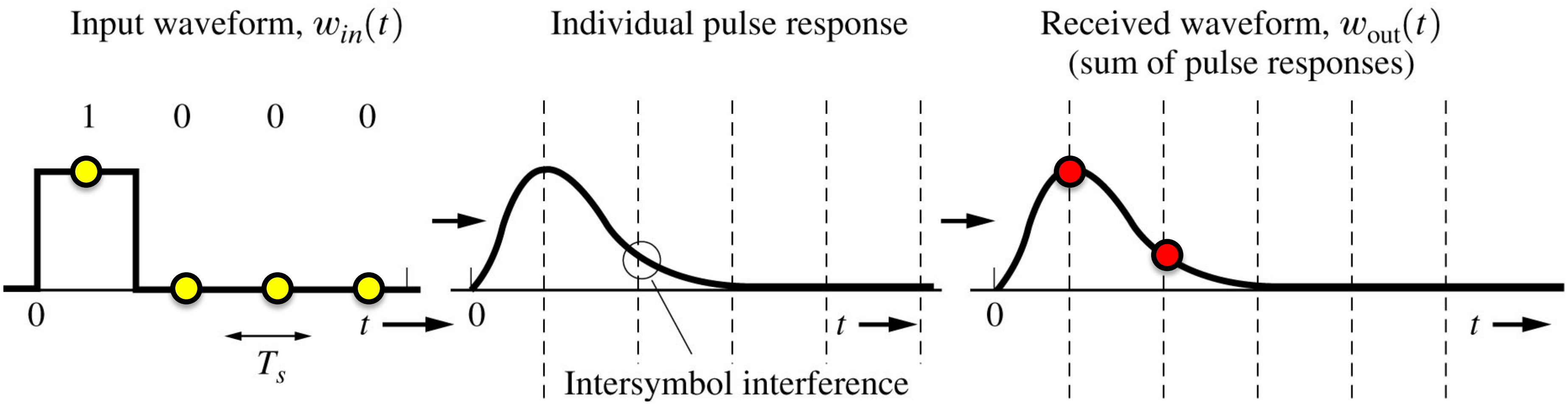


- The DFE has access to the symbol decisions, computes **error signal** to **update** feedback filter (**complex!**)

Today

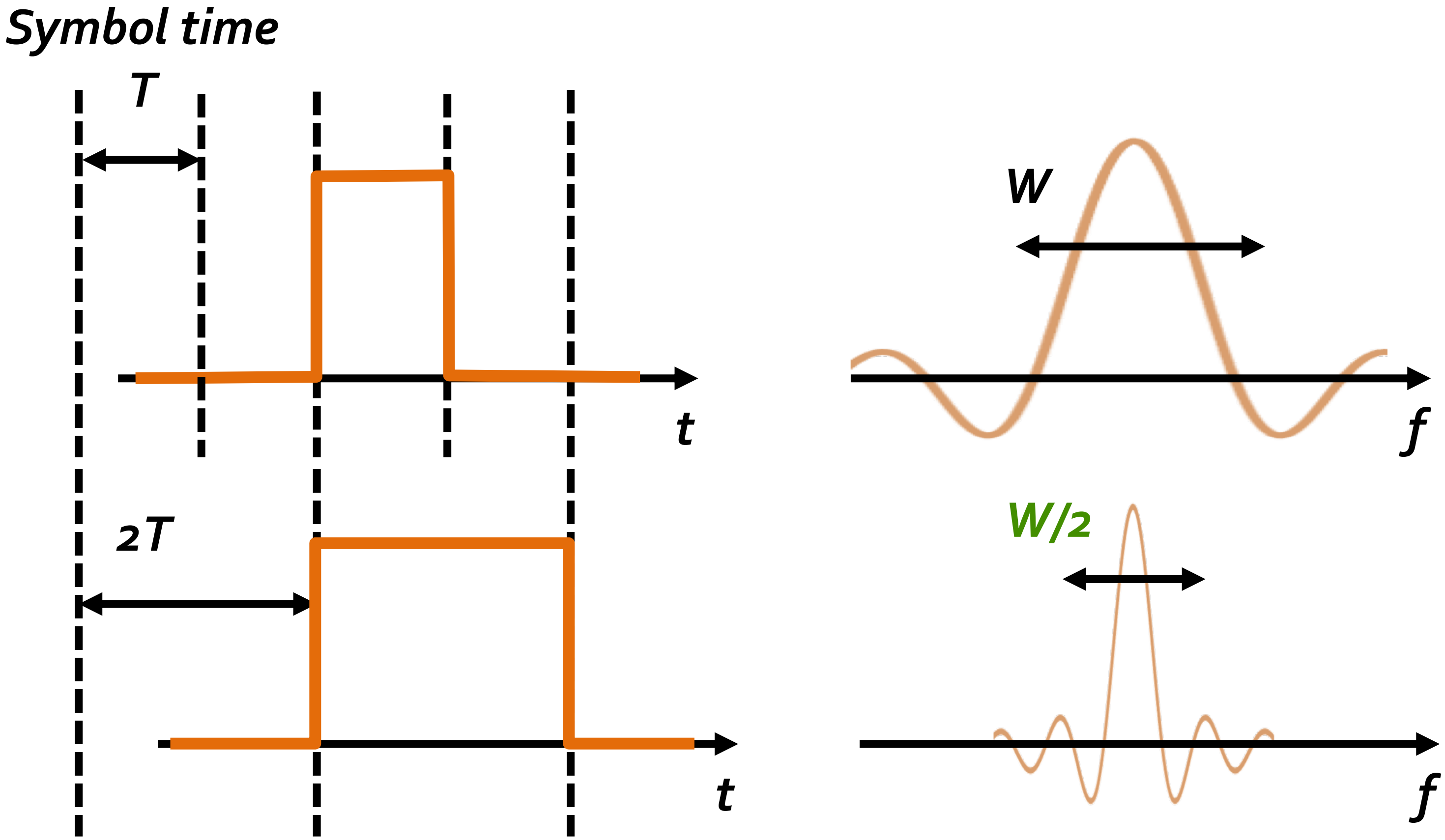
1. The Culprits: Delay Spread and Frequency-Selective Fading
2. Time-Domain Equalization
3. **Orthogonal Frequency Division Multiplexing**

Problem: Inter-symbol interference (ISI)



- **Transmitted signal** 🟡
- **Received signal with ISI** 🔴

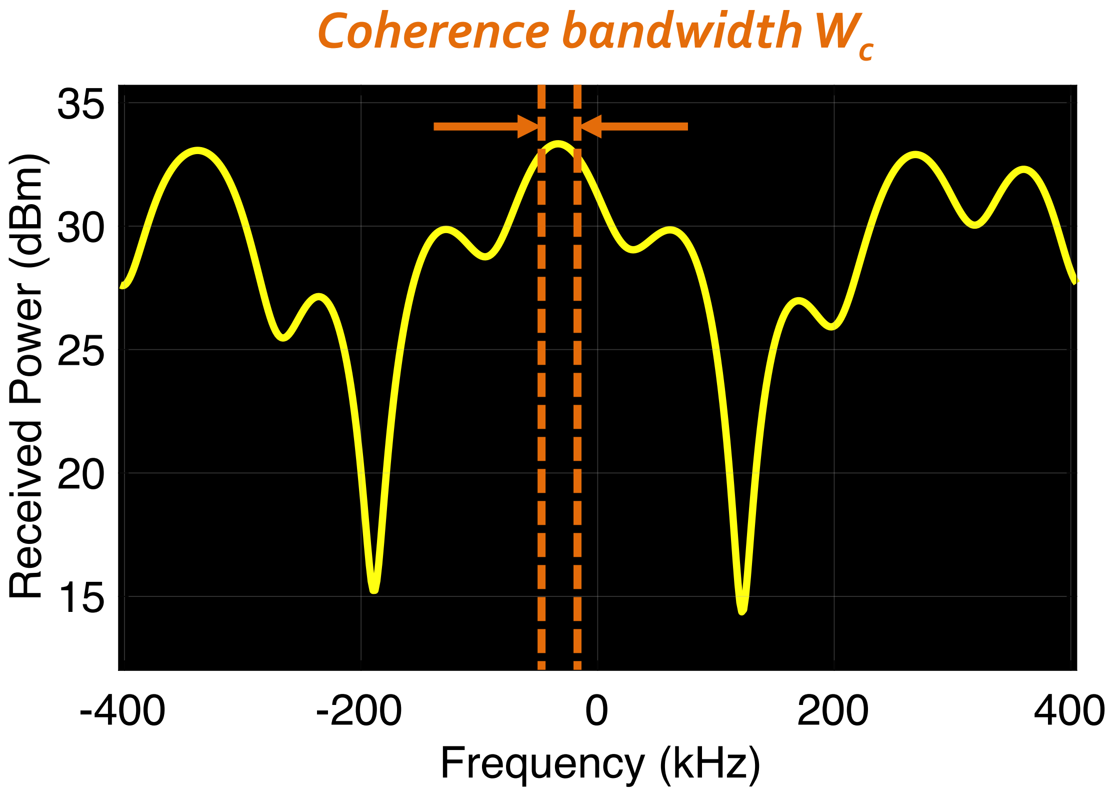
Symbol time determines frequency bandwidth



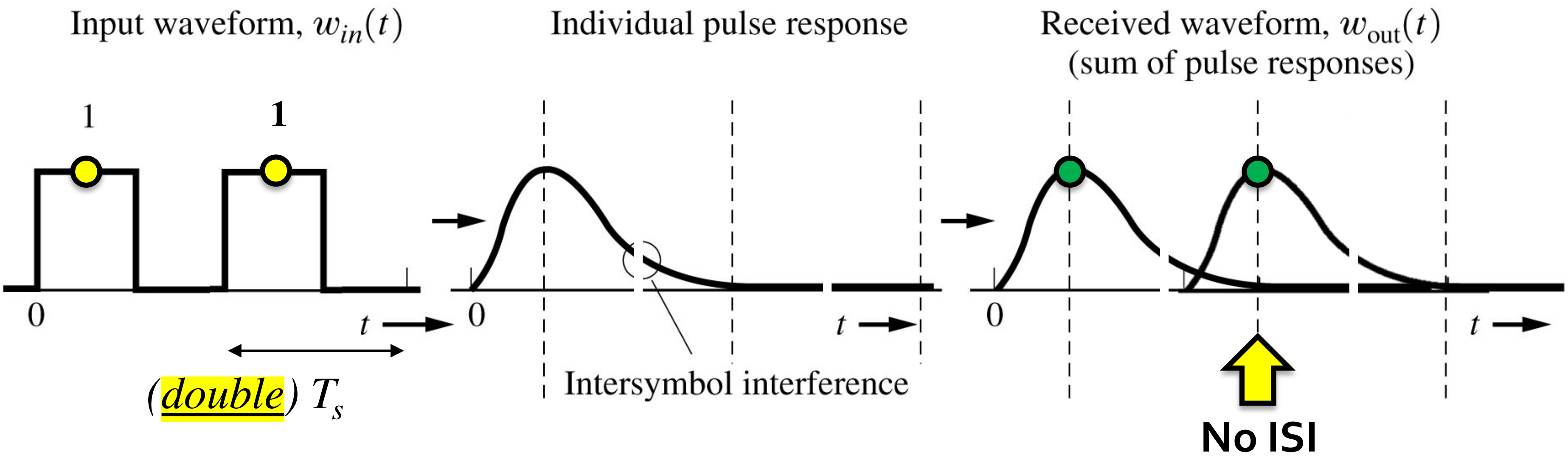
Slowing down by a factor of two **halves the frequency bandwidth** of the sender's signal

A narrowband signal “fits into” the coherence bandwidth

- Over what frequency range is the channel approximately the same? This is the ***coherence bandwidth*** $W_c \approx \frac{1}{2T_d}$

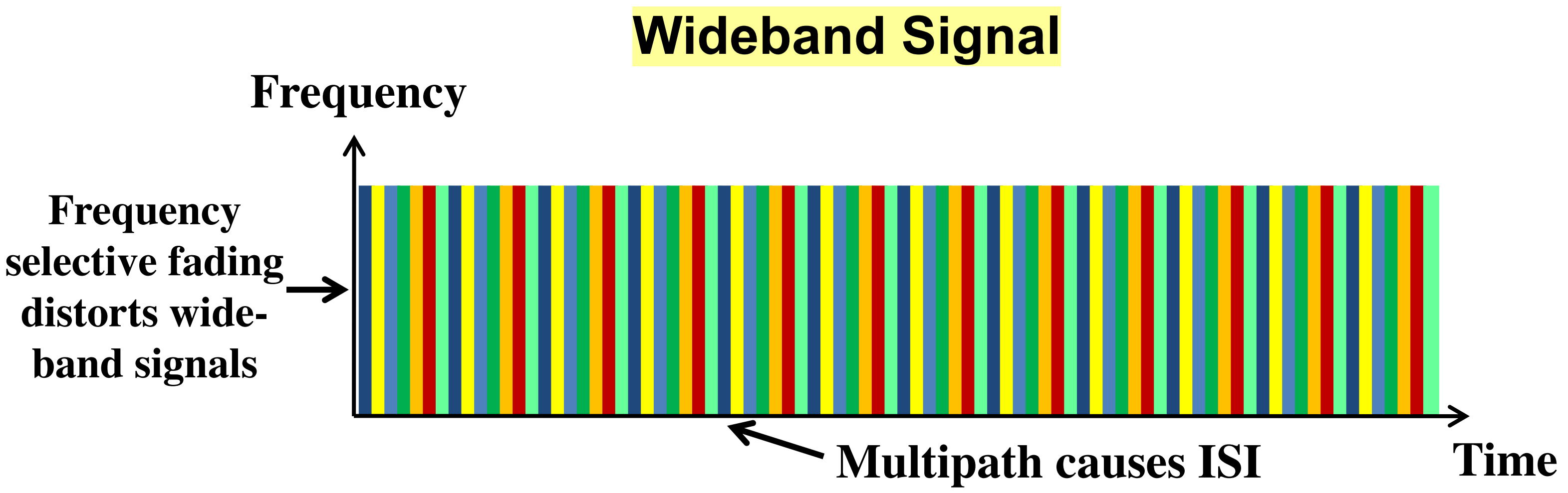


Simple Solution: Slow down



- **Transmitted signal** 🟡
- **Received signal** 🟢

Wideband versus OFDM



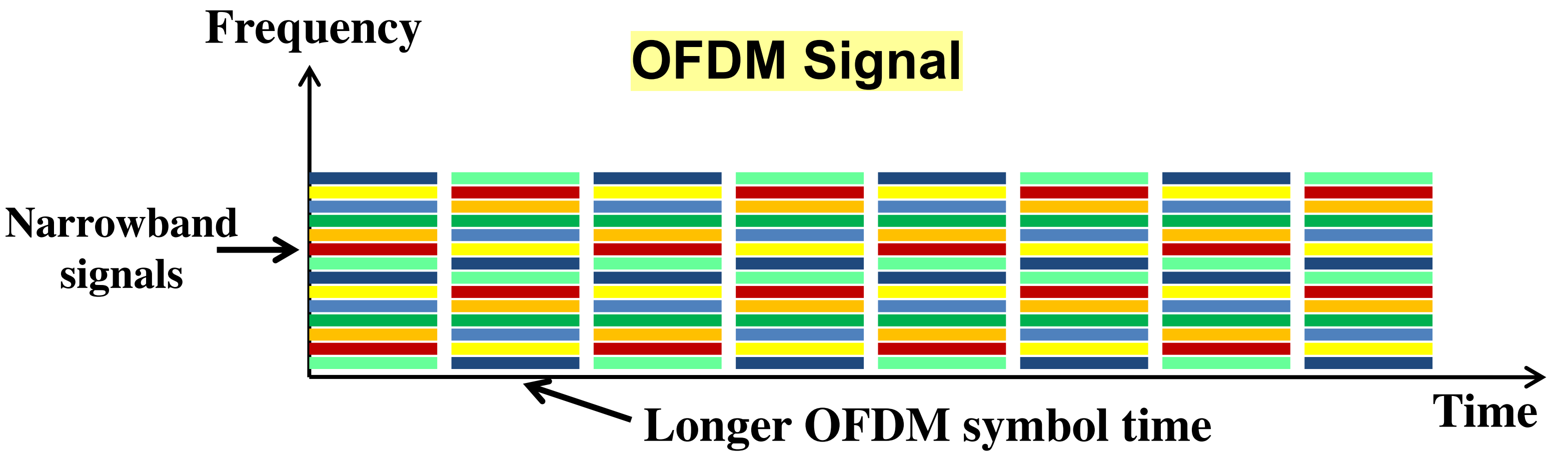
Wideband Signal

Frequency

Frequency selective fading distorts wide-band signals

Multipath causes ISI

Time



Frequency

OFDM Signal

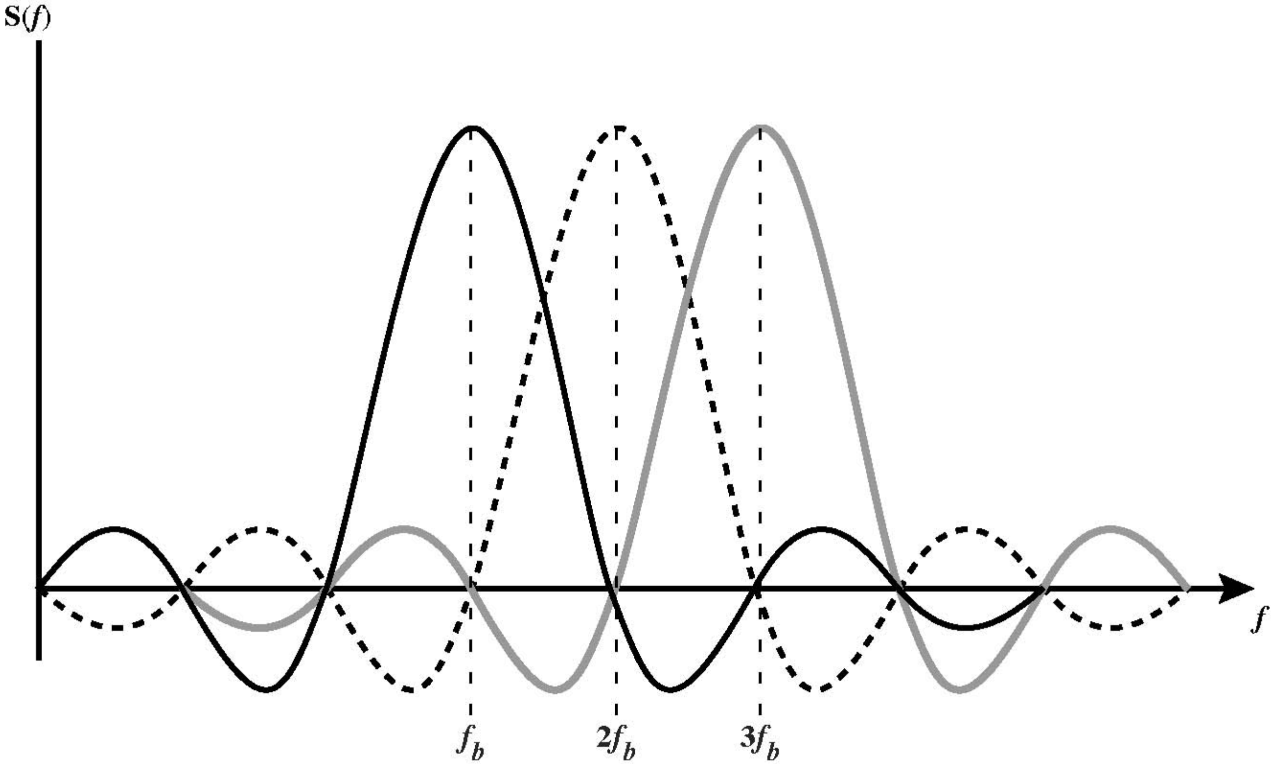
Narrowband signals

Longer OFDM symbol time

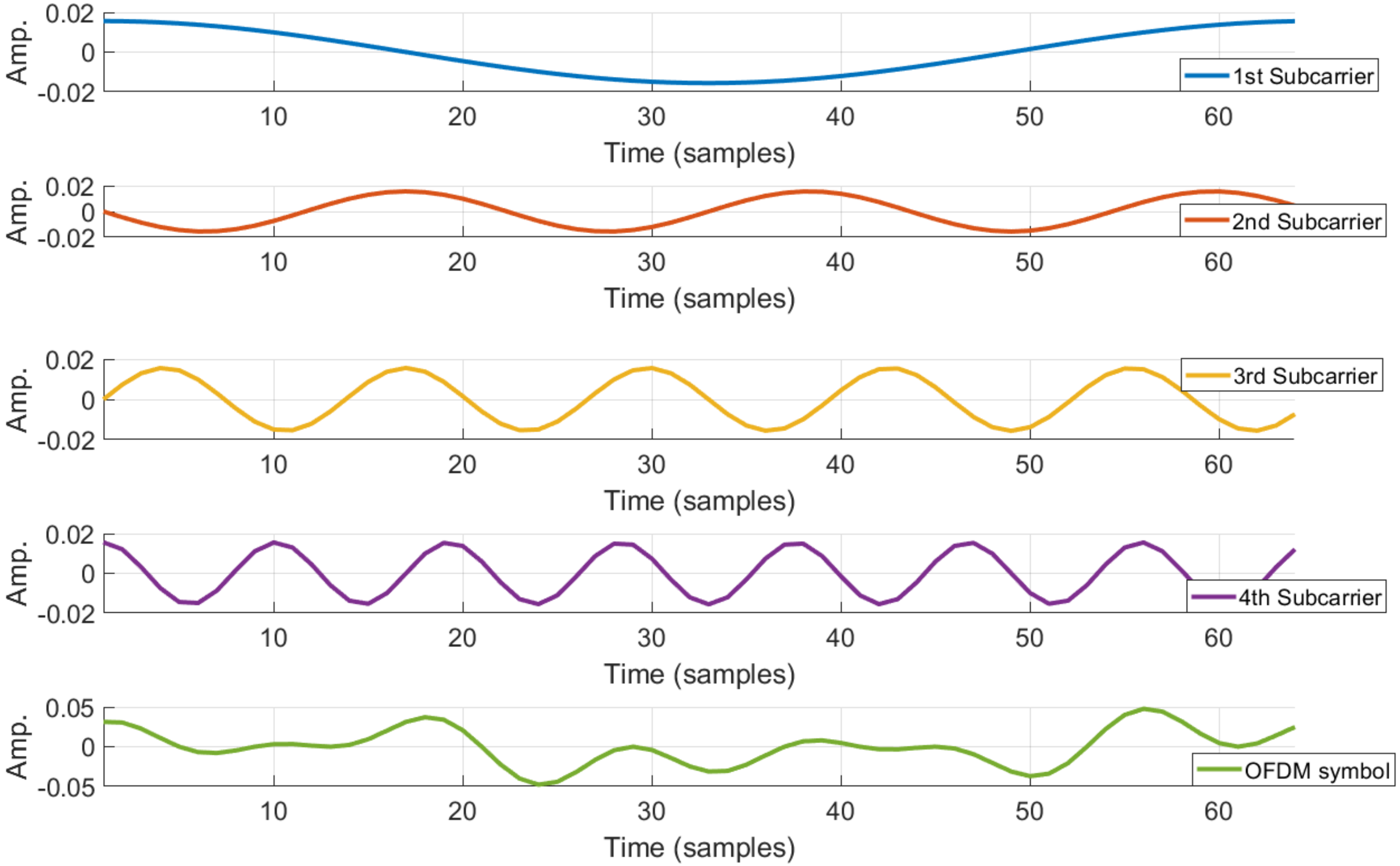
Time

Subcarriers are “Orthogonal”

- **Peaks** of each subcarrier coincide in frequency with zeros of other subcarriers
  - Carriers can be packed very densely with minimal interference
  - Requires very good control over frequencies

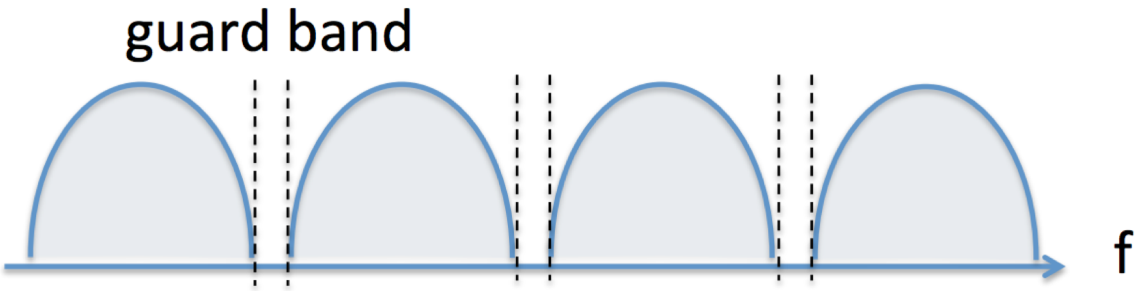


One OFDM symbol in time

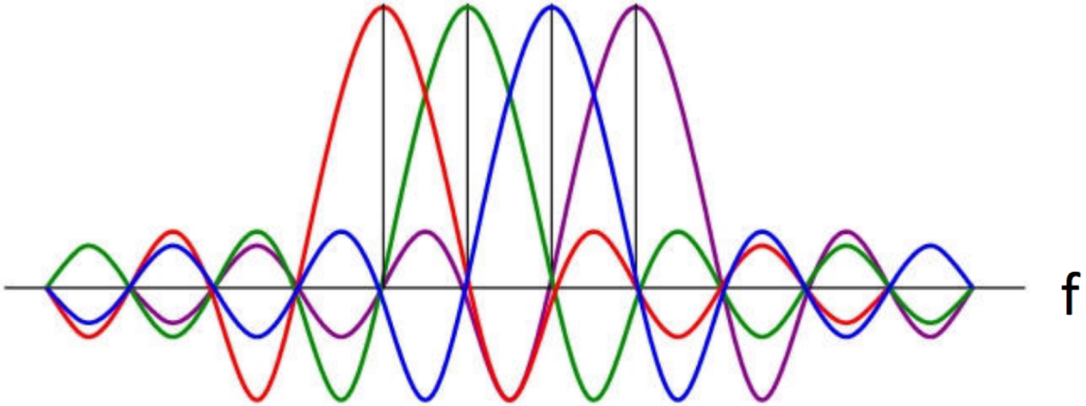


New **OFDM** *symbol time* T_s

Difference between FDM and OFDM



Frequency division multiplexing



Orthogonal sub-carriers in OFDM

Don’t need guard bands

Orthogonality of Subcarriers

Encode: frequency-domain samples $\xrightarrow{\text{IFFT}}$ time-domain sample

$$x(t)=\sum_{k=-N/2}^{N/2-1} X[k]e^{j2\pi kt/N}$$

Time-domain

Frequency-domain

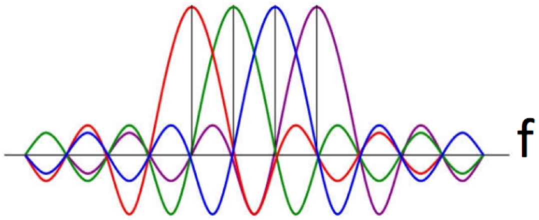
$$X[k]=\frac{1}{N}\sum_{t=N/2}^{N/2-1} x(t)e^{-j2\pi kt/N}$$

Decode: time-domain samples $\xrightarrow{\text{FFT}}$ frequency-domain sample

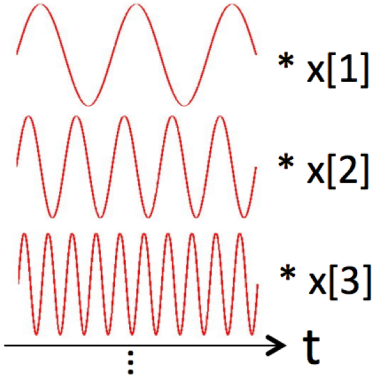
Orthogonality of any two bins : $\sum_{t=N/2}^{N/2-1} e^{-j2\pi kt/N}e^{-j2\pi pt/N}=0,\forall p\neq k$

OFDM: System Design

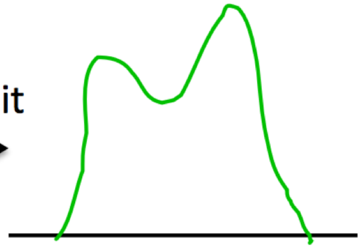
Transmitter:



IFFT



transmit



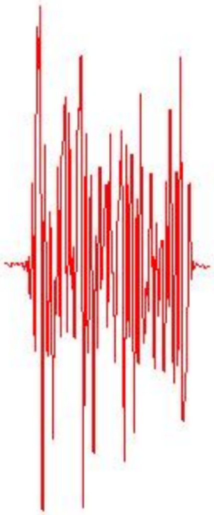
Data coded in frequency domain

Transformation to time domain:
each frequency is a sine wave
In time, all added up

Channel frequency
response

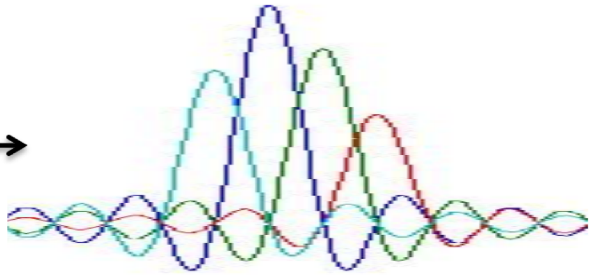
Receiver:

receive



Time domain signal

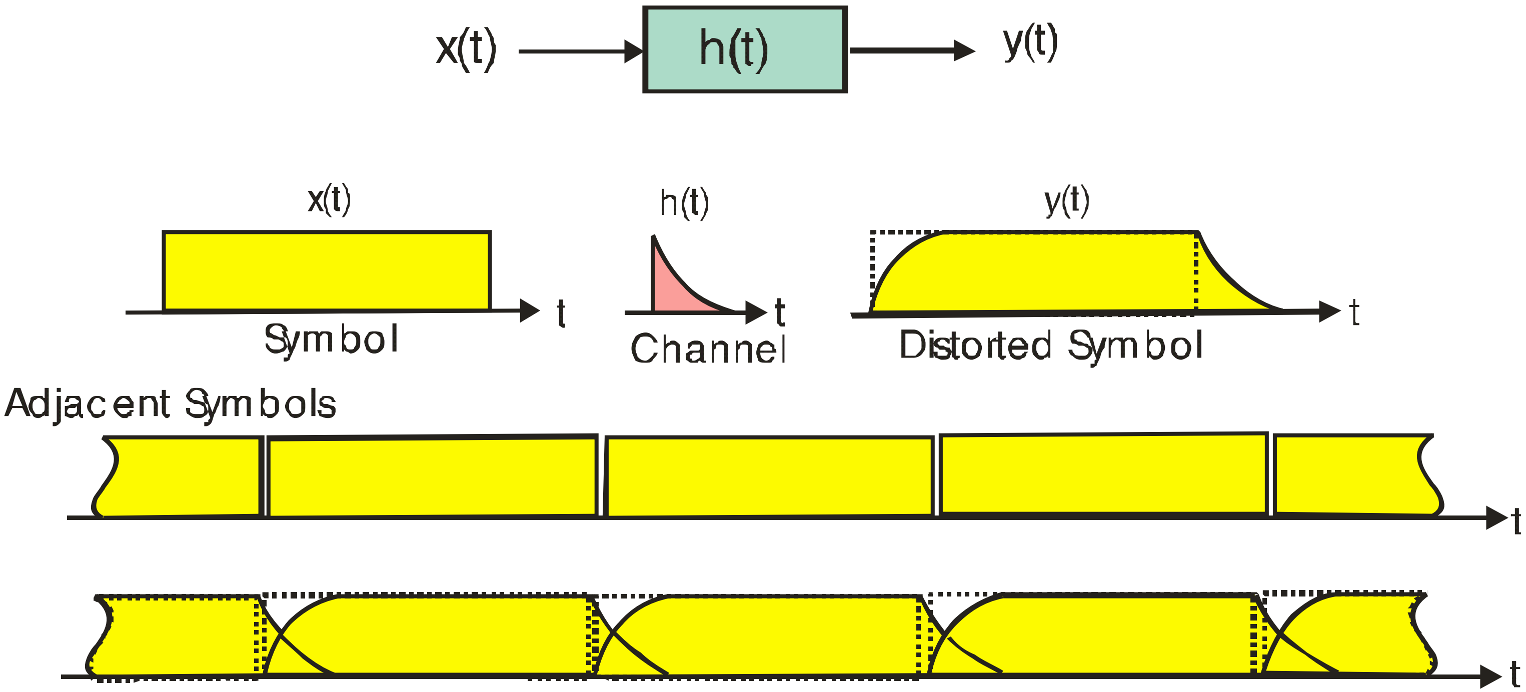
FFT



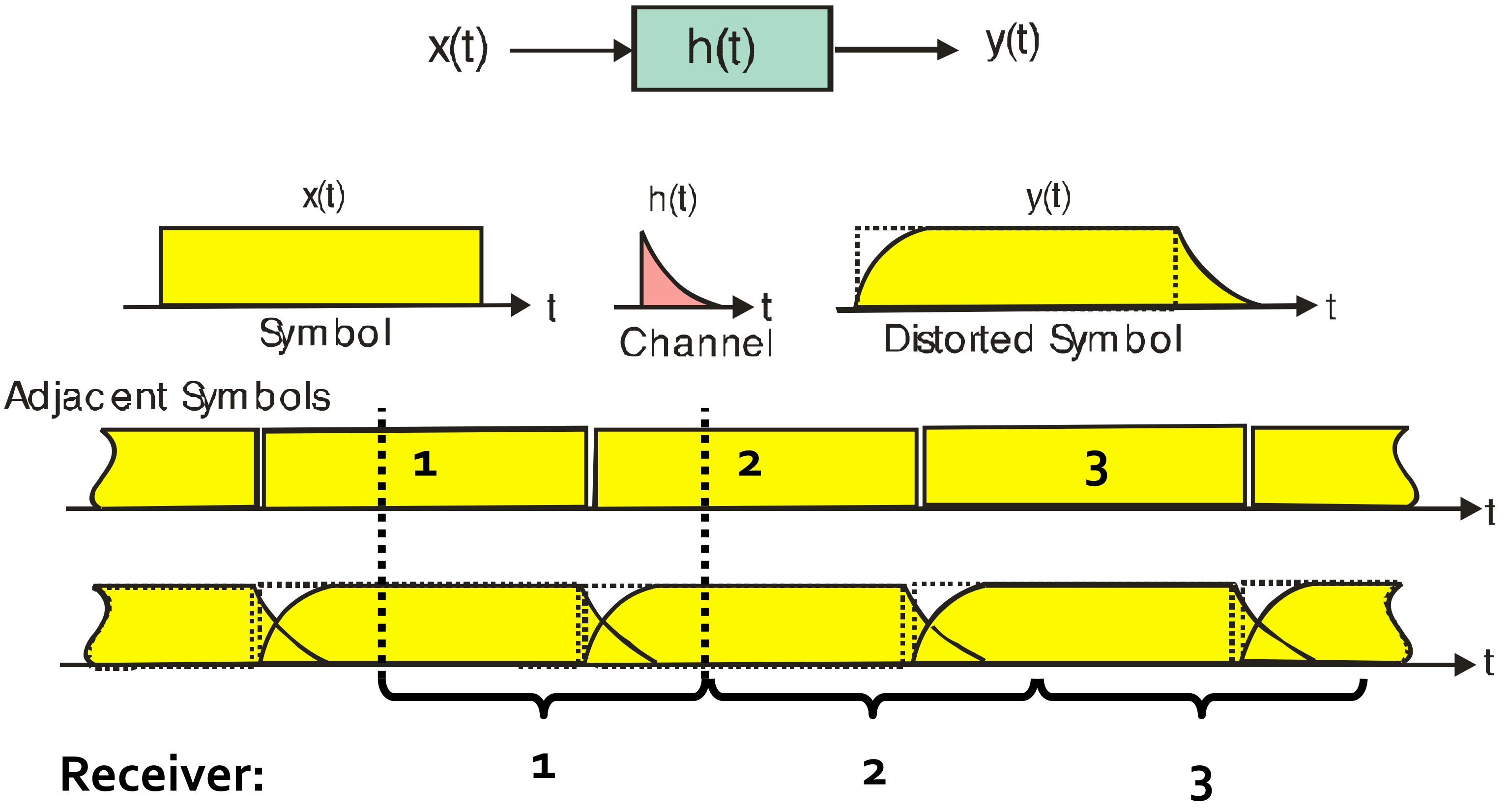
Frequency domain signal

Decode each subcarrier
separately

Problem: Inter-OFDM Symbol Interference

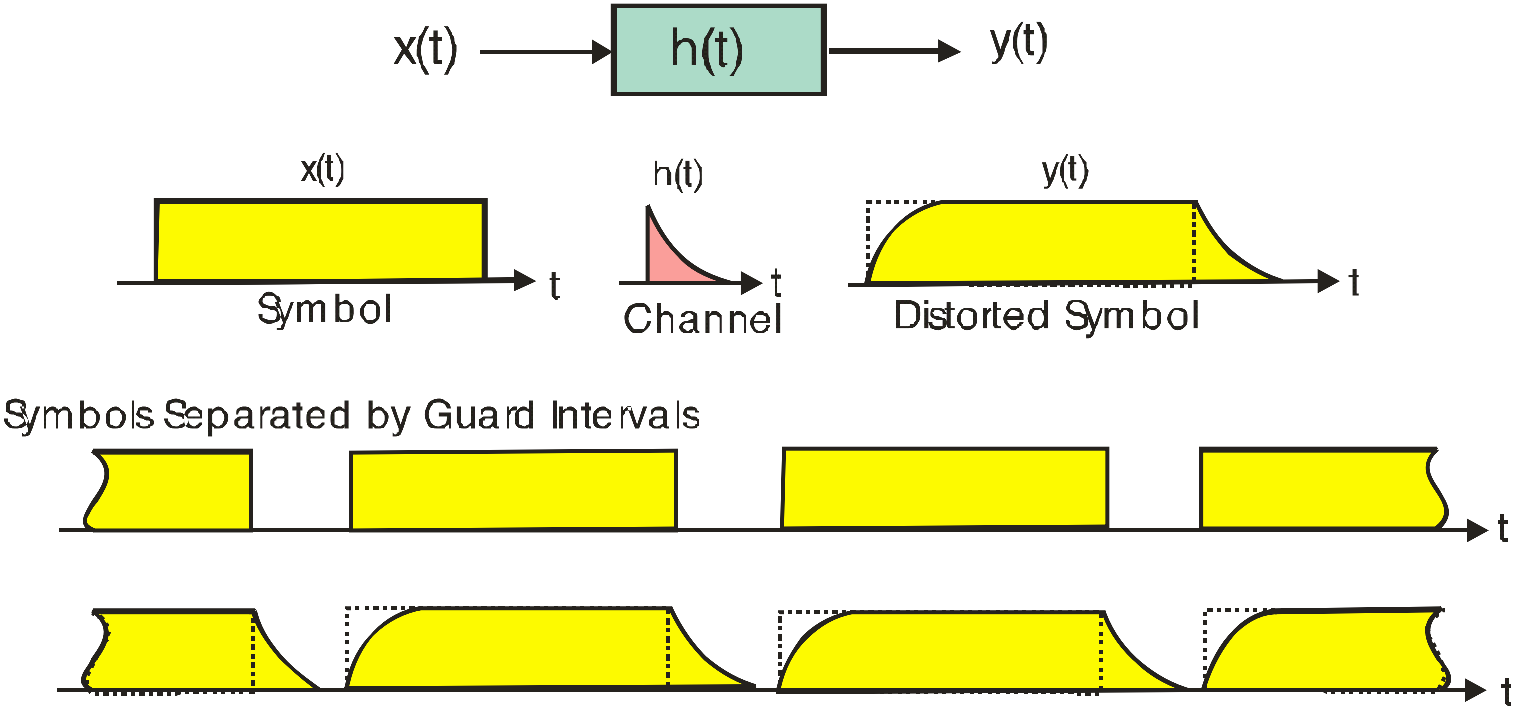


Problem: Receiver synchronization



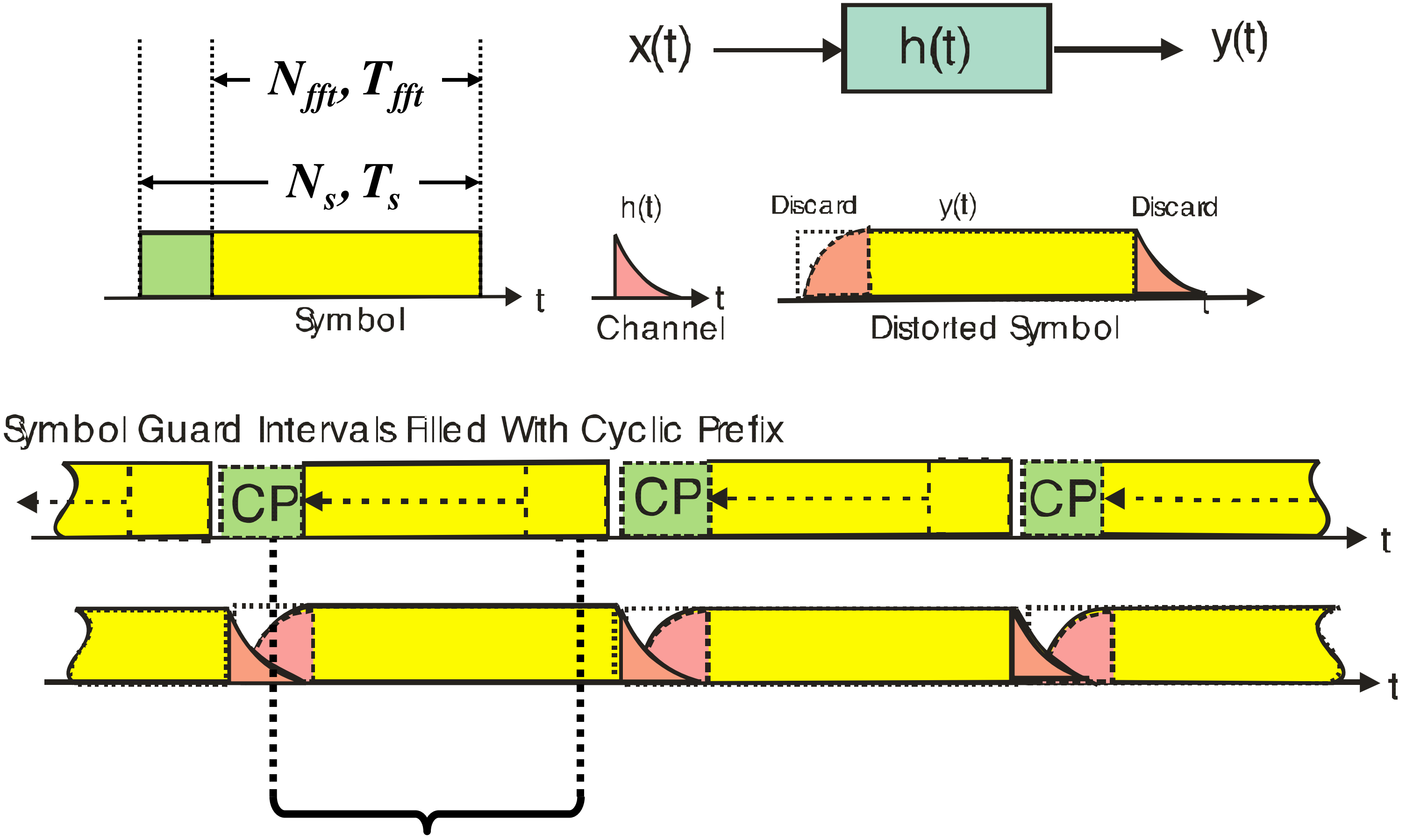
Receiver's view of Symbol 1 contains **actual Symbols 1 and 2**

Interference solution: Inter-symbol guard interval



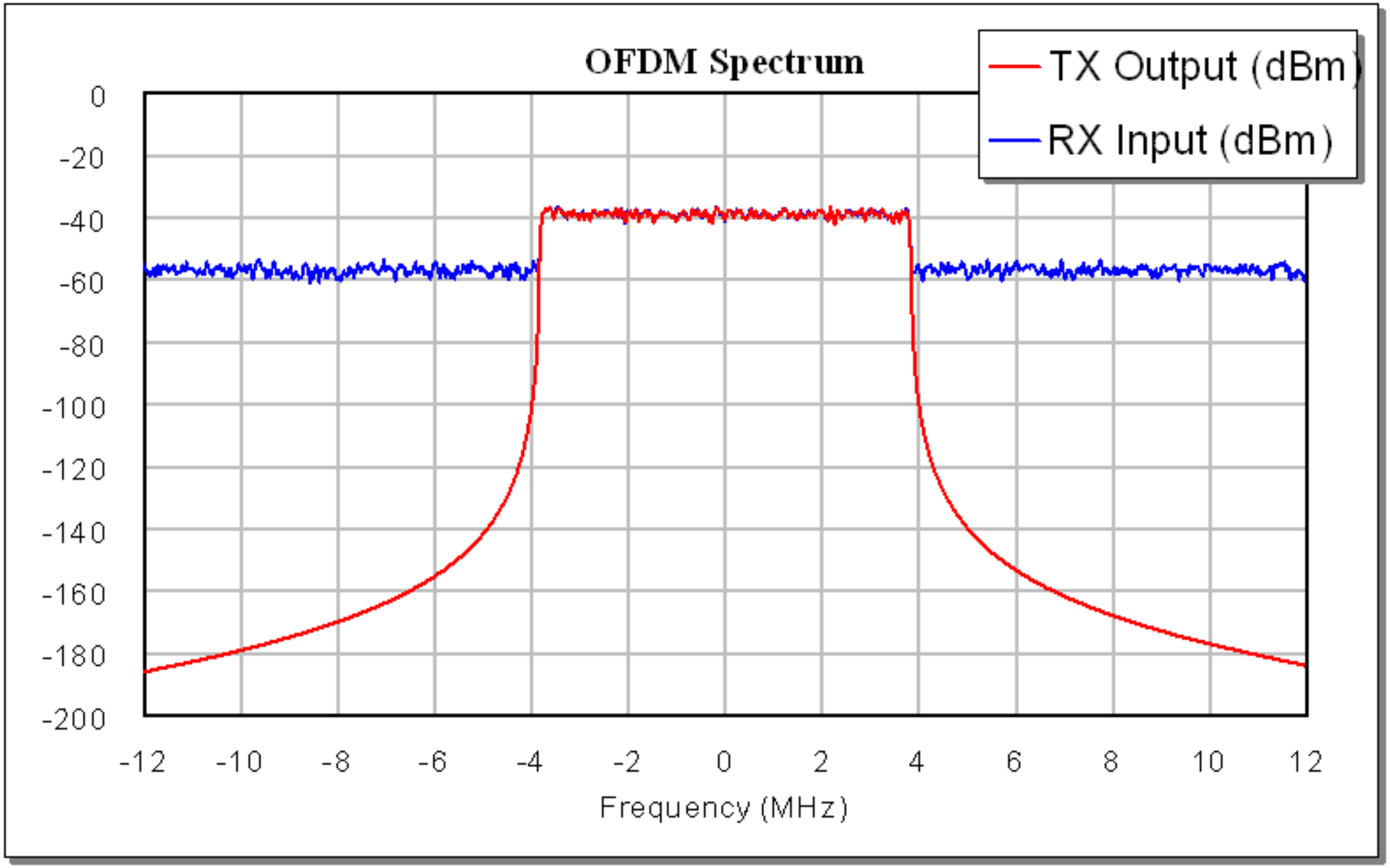
Guard interval between adjacent symbols mitigates adjacent symbol interference

Synchronization solution: Cyclic prefix



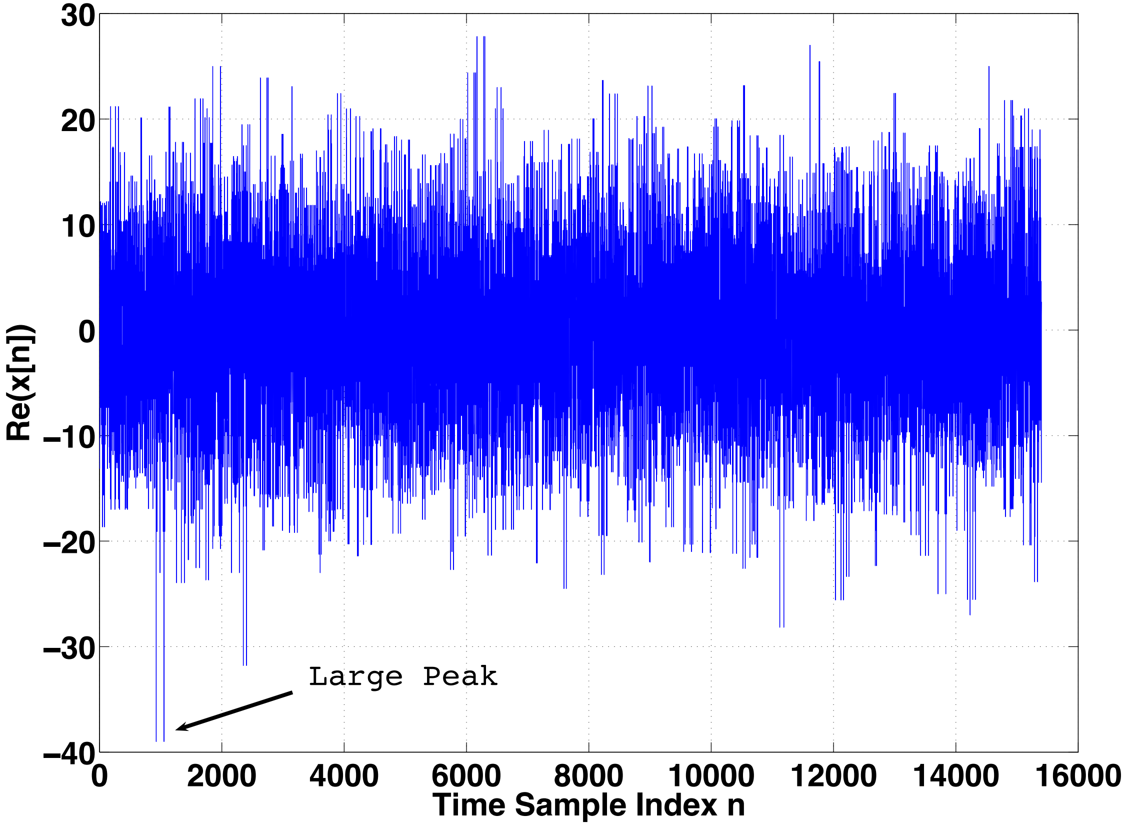
Receiver: **Symbol OK!**

OFDM signal: Frequency-Domain view



Uniform power in the frequency domain over the OFDM signal bandwidth

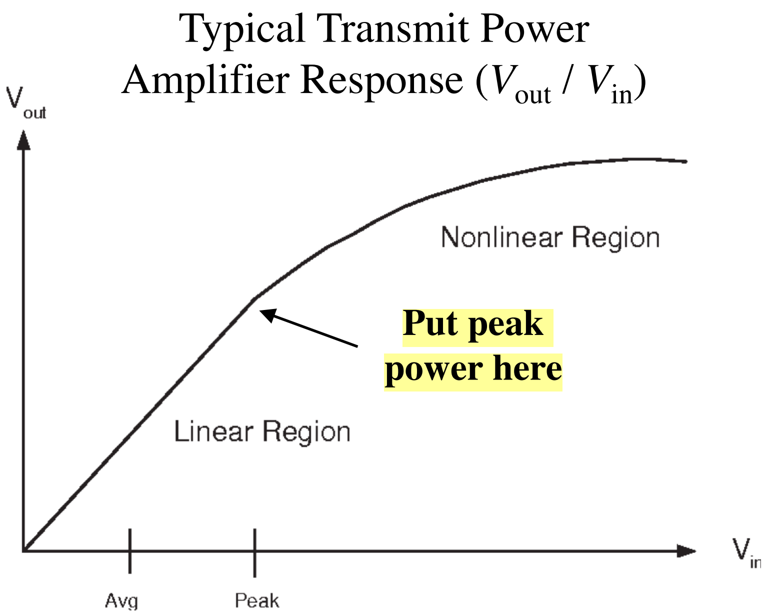
OFDM signal: Time Domain View



- Many low-frequency sinusoids in the time domain
- Occasionally **in time,** many will **all constructively interfere**
  - **Result:** High ratio of **peak power / average power**

Peak to Average Power & Transmit Amplifiers

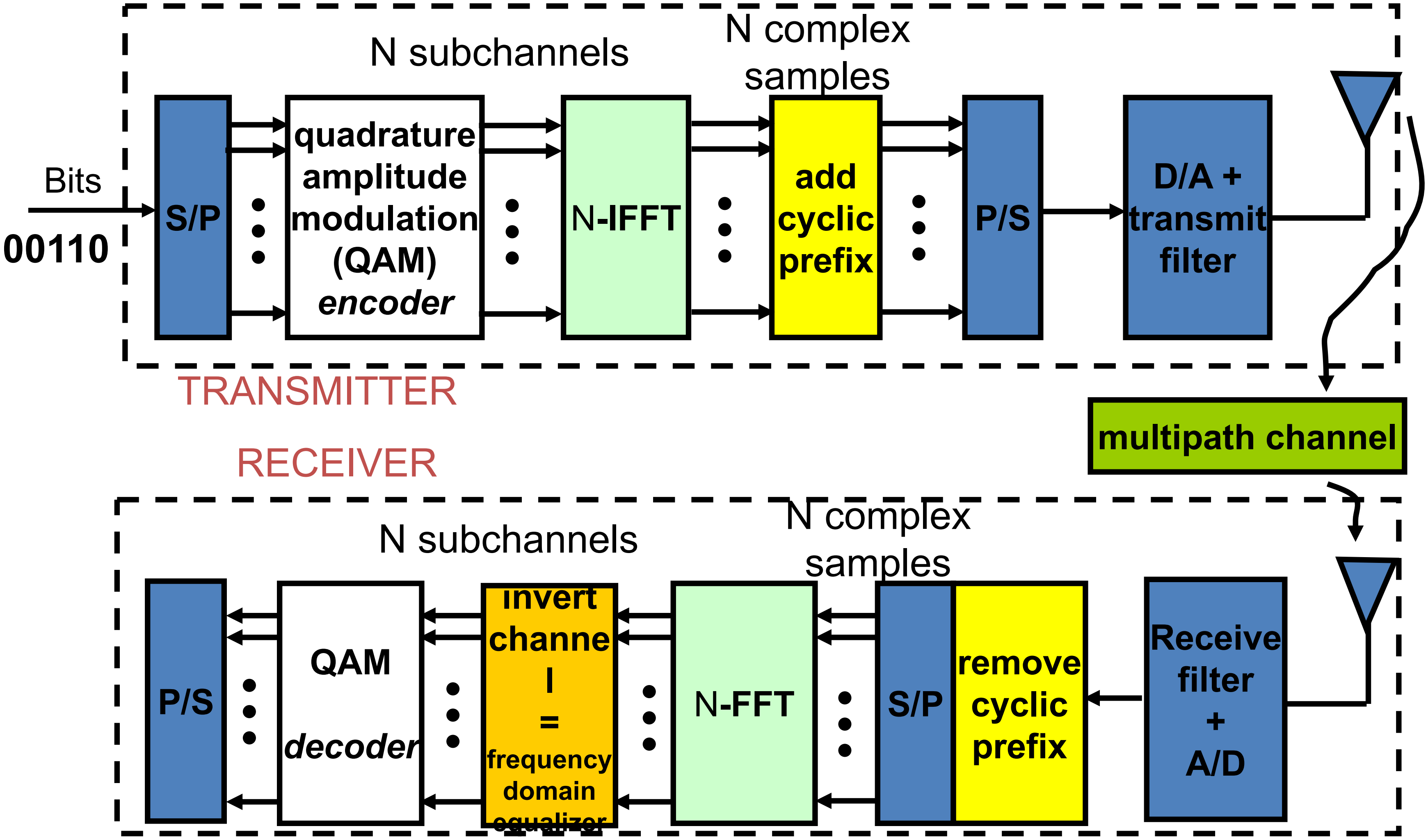
- ***Transmit power amplifier*** sits just before the transmit antenna
- Peak power in non-linear region causes signal distortion
  - So **lower input signal level** so that **peak input power falls in linear region**


Typical Transmit Power Amplifier Response (V_{out} / V_{in})

Put peak power here

- **High** peak to average power ratio (PAPR) →
  - **Low average** power level →
    - Signal mostly uses **fewer levels in discrete representation**, so **high quantization error** (another form of **distortion**)

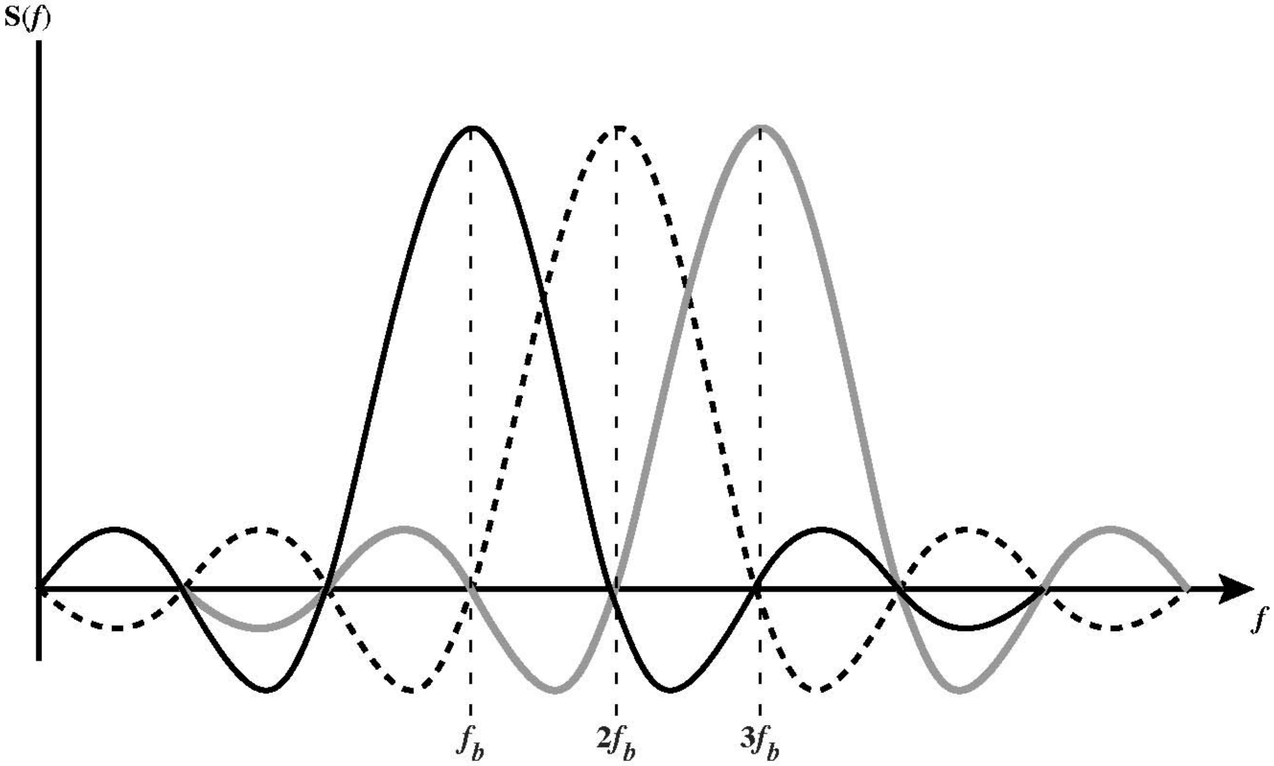
An OFDM Modem



Estimating the Channel

- Transmit known OFDM preamble symbol *x*
  - In frequency domain **on frequency *i*,** denote preamble X_i

- After FFT, hears frequency domain value Y_i



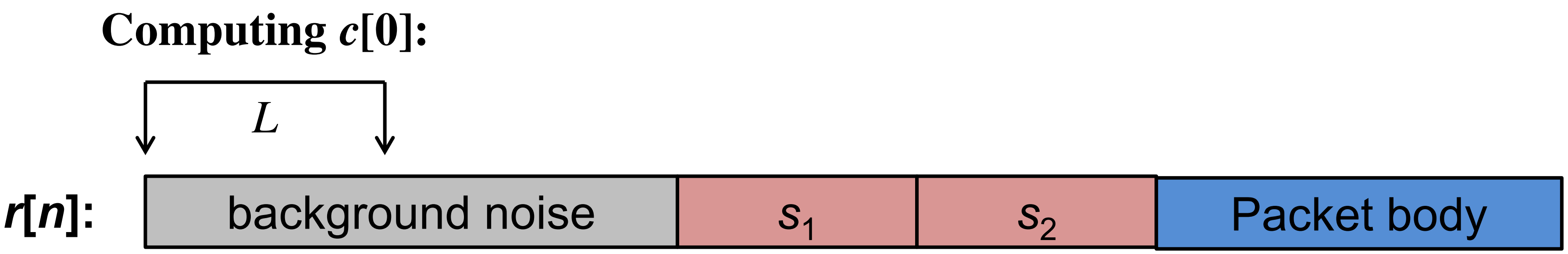
Packet detection

- OFDM uses **two identical, repeated symbols** s_1, s_2 in the preamble for packet detection:

- Receiver radio is **always listening, receiving samples**
  – Call this **received sample stream $r[n]$**

Searching for the preamble in noise

- Suppose each preamble symbol is of length *L*
- Receiver computes $\mathrm{c[n]} = \sum_{k=0}^{L-1} r[n+k]r^*[n+k+L]$

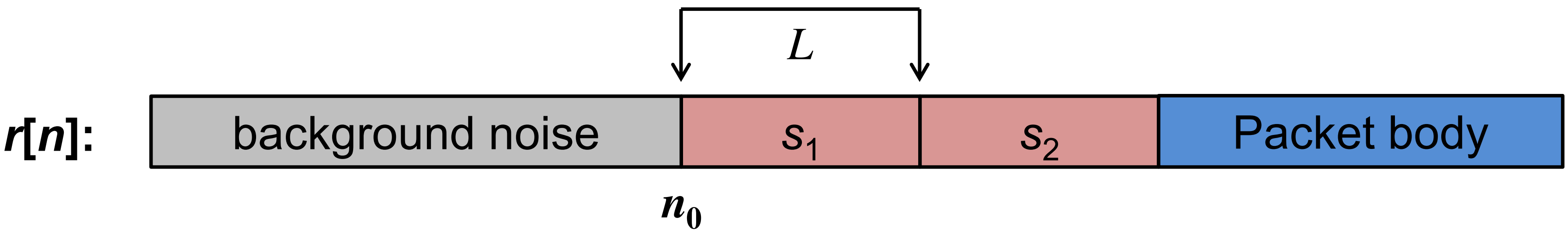


- **Angle of each term** in the sum is **random**
- **Sum** of complex numbers with **random angle ≈ 0**
  - **c[0] ≈ 0**

Search window encounters preamble

- Suppose preamble at position n_0
- Receiver computes $\mathrm{c[n]} = \sum_{k=0}^{L-1} r[n+k]r^*[n+k+L]$

Computing $c[n_0]$:



L

$r[n]$: background noise | s_1 | s_2 | Packet body

n_0

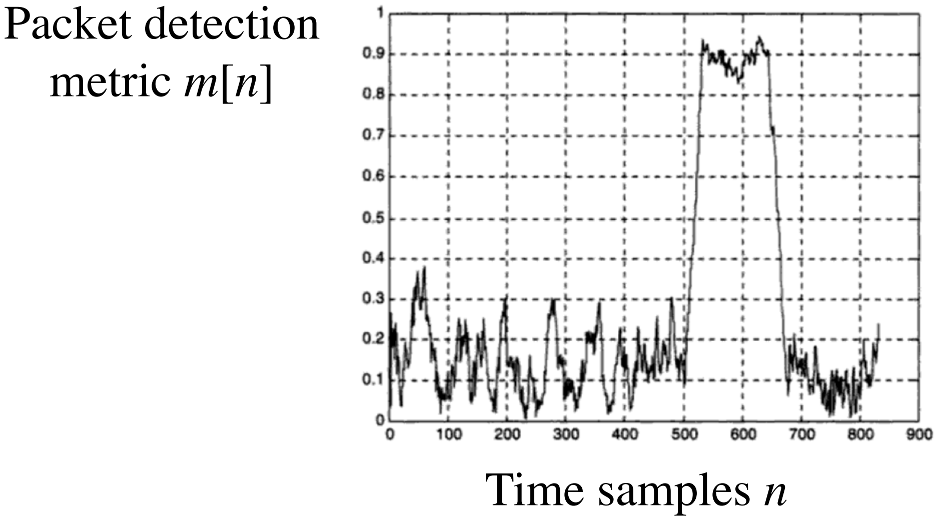
- **$\measuredangle(zz^*) = 0$, so angle of each term** in the sum is **≈ 0**
- **Sum** of complex numbers with **≈ 0 angle is large**
  - **$c[n_0]$ is large**

Schmidl-Cox Packet Detection

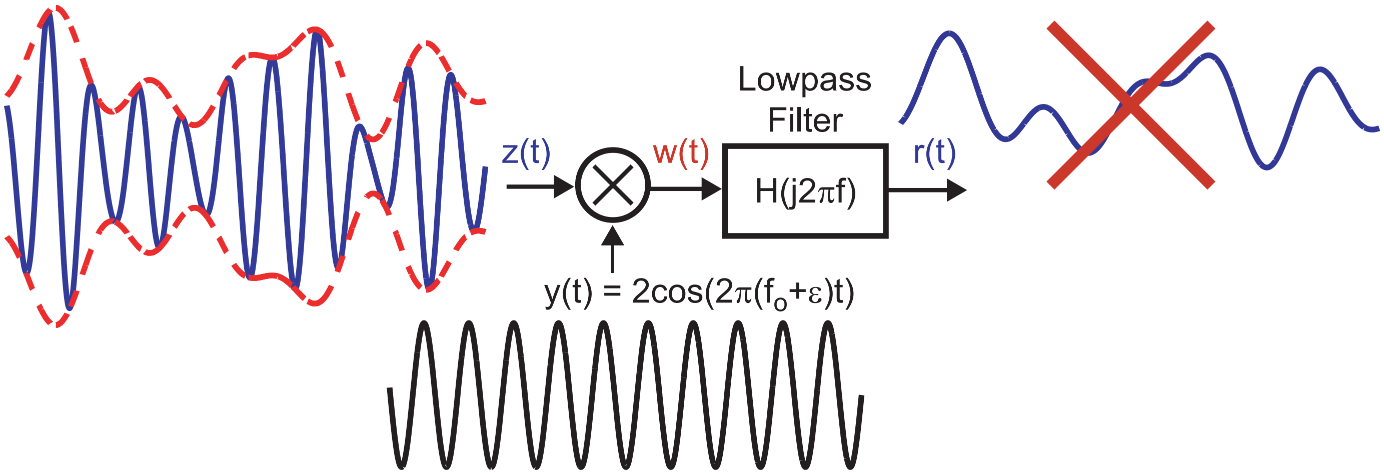
- $\mathrm{c[n]} = \sum_{k=0}^{L-1} r[n+k]r^*[n+k+L]$

- Normalize **power fluctuations in r[n]**, by measuring power:
  - $\mathrm{p[n]} = \sum_{k=0}^{L-1} |r[n+k]|^2$

- **Schmidl-Cox Packet Detection signal:** ***m*[*n*] =** *c*[*n*] / *p*[*n*]



A Closer Look at Carrier Frequency Offset



- **Limited precision** of frequency oscillators
- Up-convert baseband signal s_n to passband signal y_n:

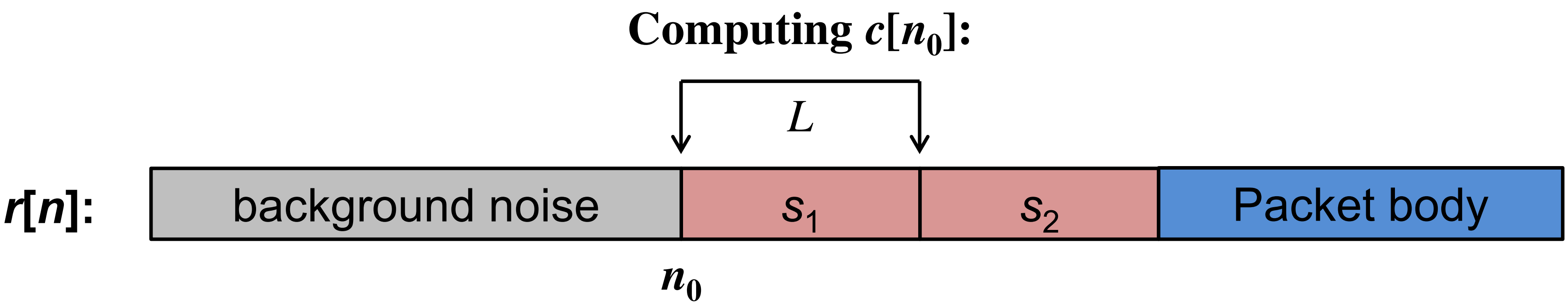
$$y_n = s_n e^{j2\pi f_{tx} n T_s}$$

- Down-convert passband signal y_n back to baseband:

$$\begin{aligned} r_n &= s_n e^{j2\pi f_{tx} n T_s} e^{j2\pi f_{rx} n T_s} \\ &= s_n e^{j2\pi \Delta f n T_s} \ \boldsymbol{(\Delta f = f_{rx} - f_{tx})} \end{aligned}$$

Estimating Carrier Frequency Offset

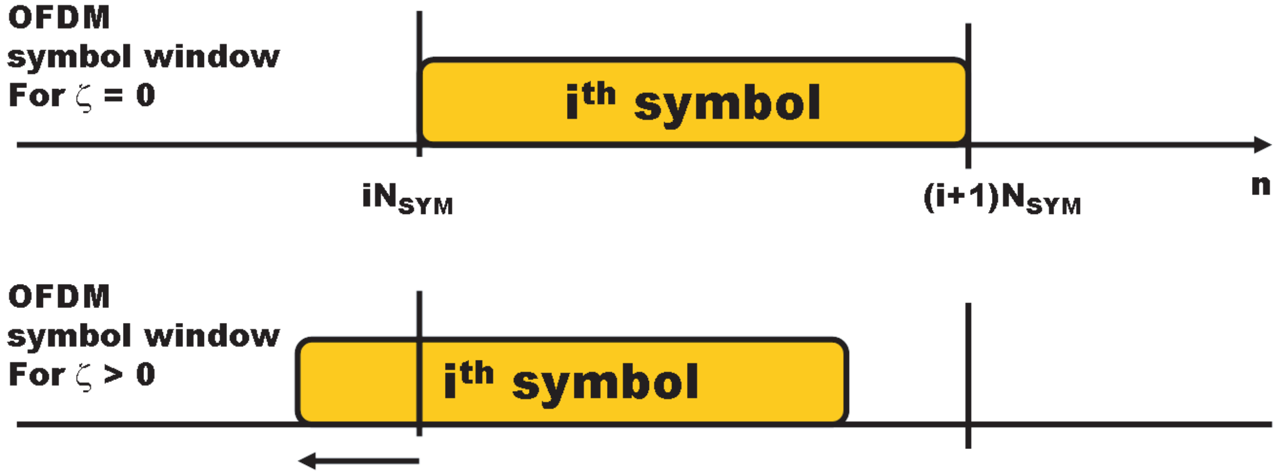
- Because of carrier frequency offset, $s_2 = s_1 e^{j2\pi\Delta f N T_s}$
  - $\mathrm{c}[n_0] = \sum_{k=0}^{L-1} r[n_0 + k] r^*[n_0 + k + L]$



- Consider the *k*th term in sum: $r[n_0 + k] r^*[n_0 + k] e^{j2\pi\Delta f N T_s}$
  - This is equal to $e^{j2\pi\Delta f N T_s} |r[n_0 + k]|^2$
    - So all terms have the **same angle** $2\pi\Delta f N T_s$

- **So, *carrier frequency offset estimator*** $\widehat{\Delta f} = \frac{\measuredangle \mathbf{c}[n_0]}{2\pi N T_s}$

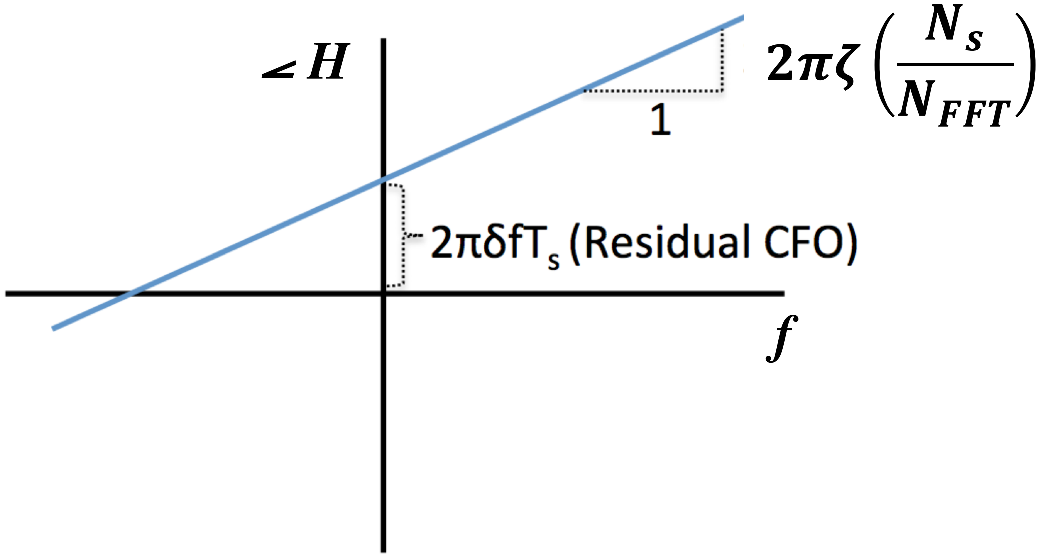
Sample Clock Offset



- The transmitter and receiver may sample the signal at slightly different rates, leading to a ***sample time offset ζ***

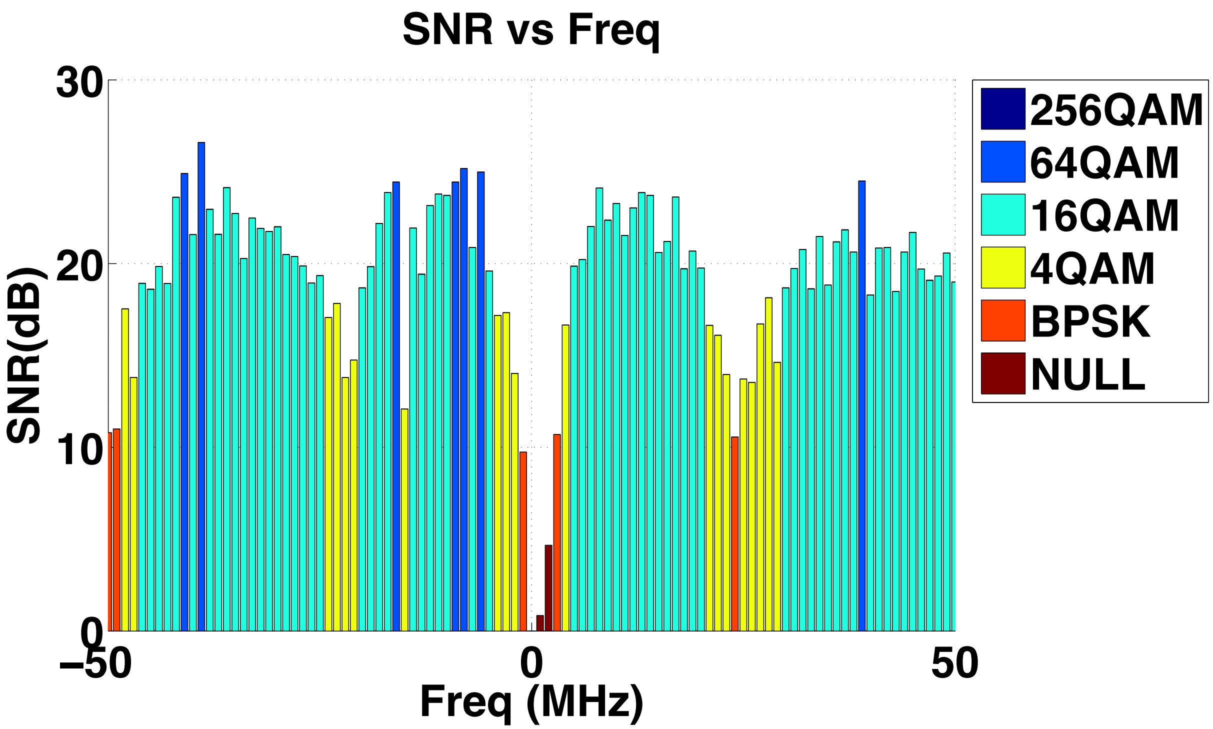
- All subcarriers experience the same sampling delay, but travel over different frequencies

Correcting Sample Clock Offset in the Frequency Domain



- Sample clock offset : slope
- Residual CFO: intersection with y-axis

Per-subcarrier Bit Rate Choice



Example: IEEE 802.11a, 802.11g

- OFDM with up to 48 subcarriers
  - Subcarrier spacing is 312.5 KHz
  - Subcarriers modulated: BPSK, QPSK, 16-QAM, or 64-QAM

- Uses a convolutional code at a rate of ½, 2/3, ¾ , or 5/6 to provide forward error correction

- Results in data rates of 6, 9, 12, 18, 24, 36, 48, and 54 MBps

- Cyclic prefix is 25% of a symbol time (16 vs 64)

Friday Precept:

Lab 4: Single-carrier transceiver on the HackRF hardware