Receiver Design and Performance; Shannon Capacity

COS 463: Wireless Networks
Lecture 13

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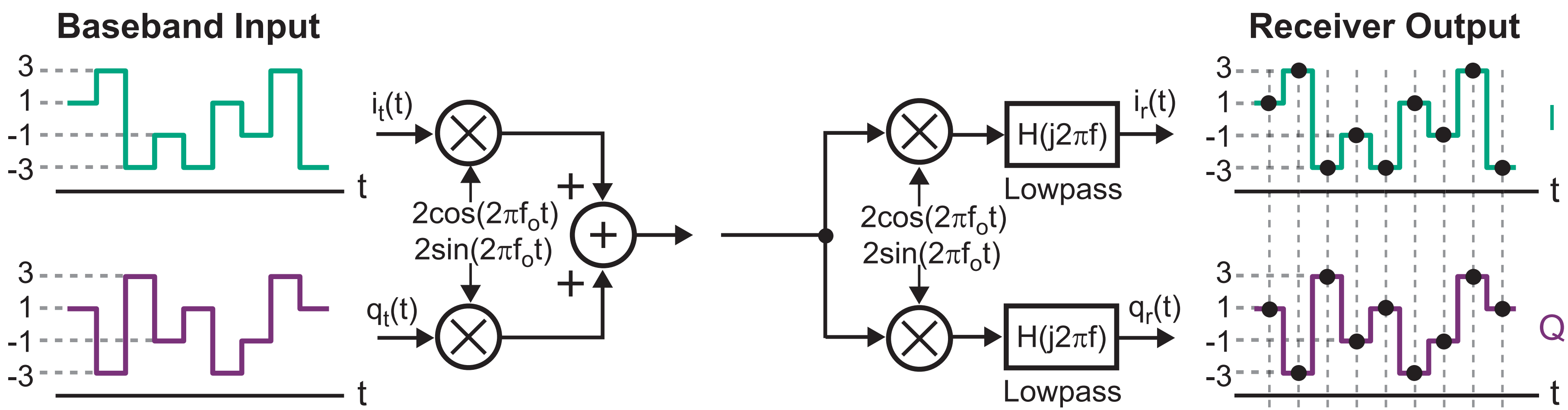
[Parts adapted from H. Balakrishnan, M. Perrott, C. Terman]

Today

1. **Receiver architecture**
   - **Tradeoffs between ISI and Noise**
   - Common filter design: Raised Cosine

2. Bit error rate and Shannon Capacity

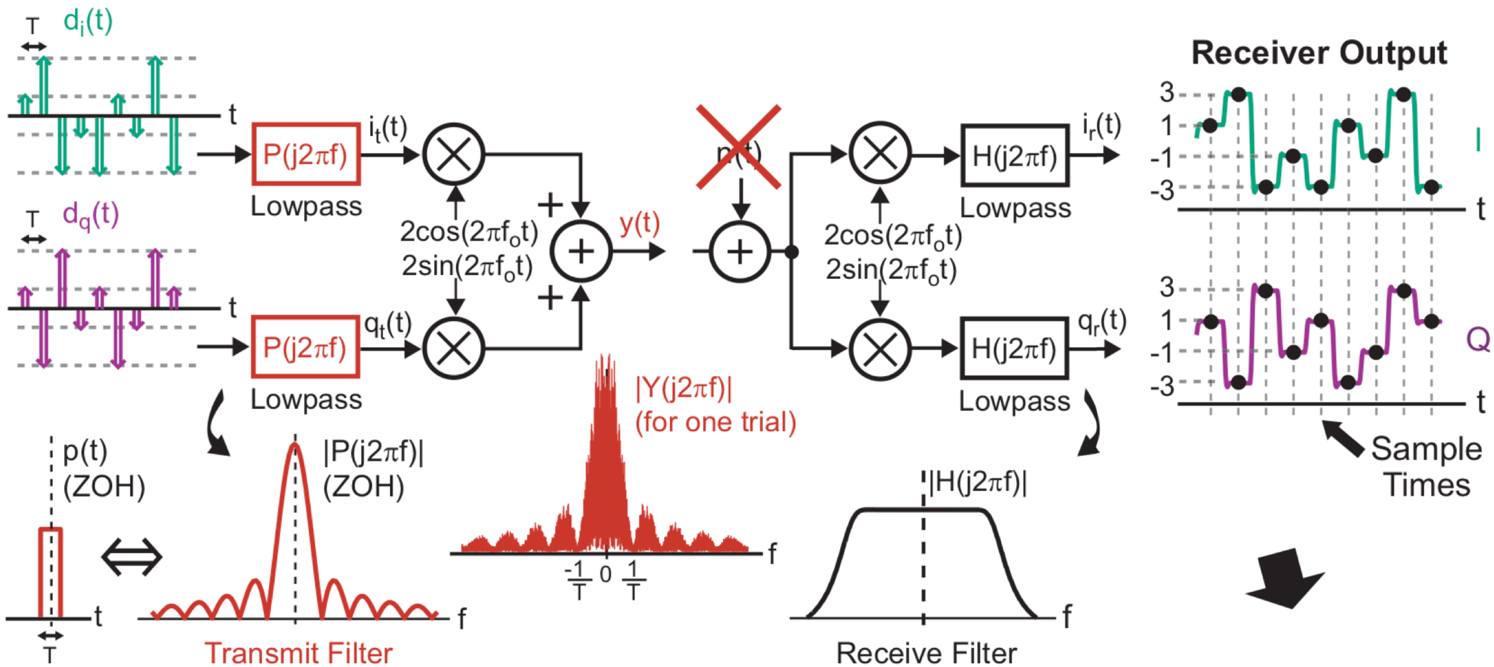
Review of Digital I/Q Modulation

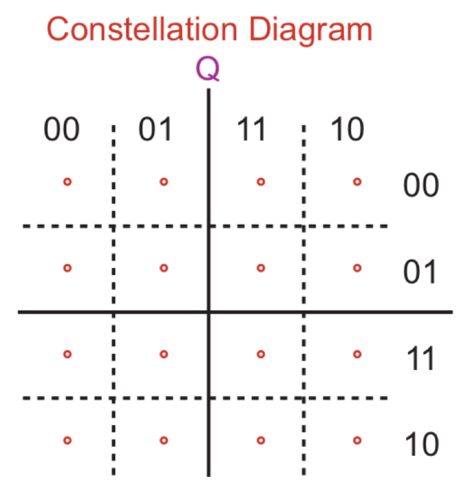


- Leverage **analog** communication channel to send **discrete-valued symbols**
  - *e.g.* send symbol from {-3,-1,1,3} on both I and Q channels every symbol period

- At receiver, **sample I/Q waveforms** every symbol period
  - **Associate each sampled I/Q value with symbol** from set, on both I and Q channels

Transmit and Receive Filters

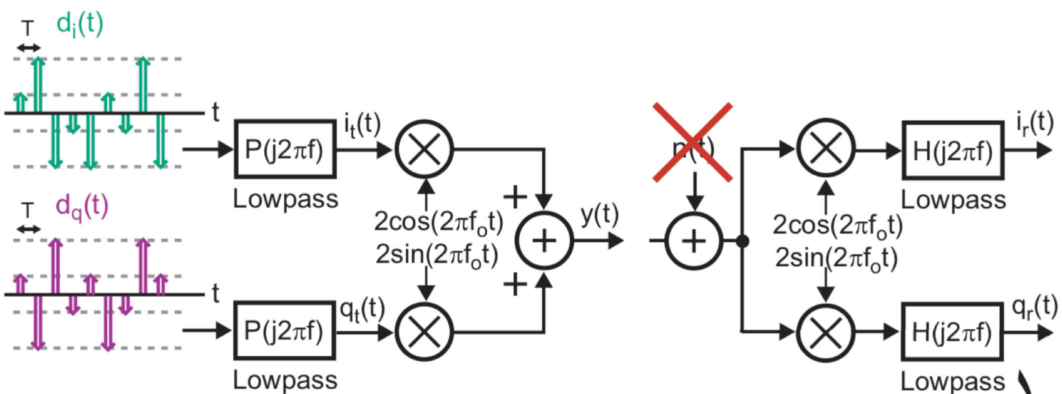


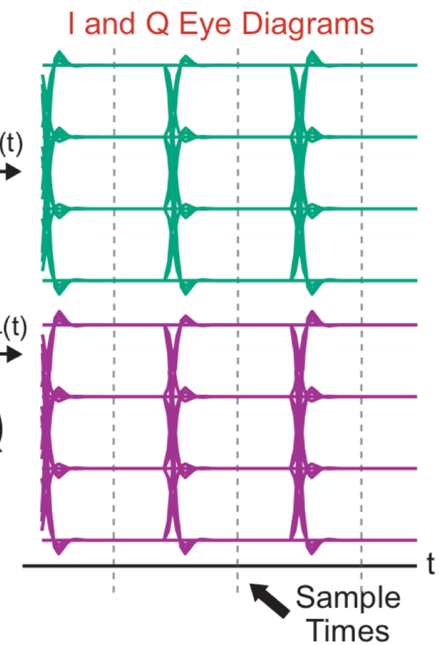


- **Last time:** Transmit filter
  - Tradeoff between transmitted bandwidth and intersymbol interference (ISI)

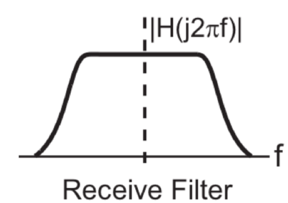
- **This time:** Receive filter (previously assumed very wide bandwidth so as not to influence ISI)

Tools for Examining ISI

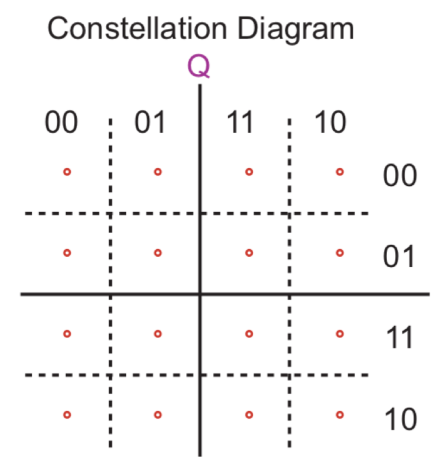




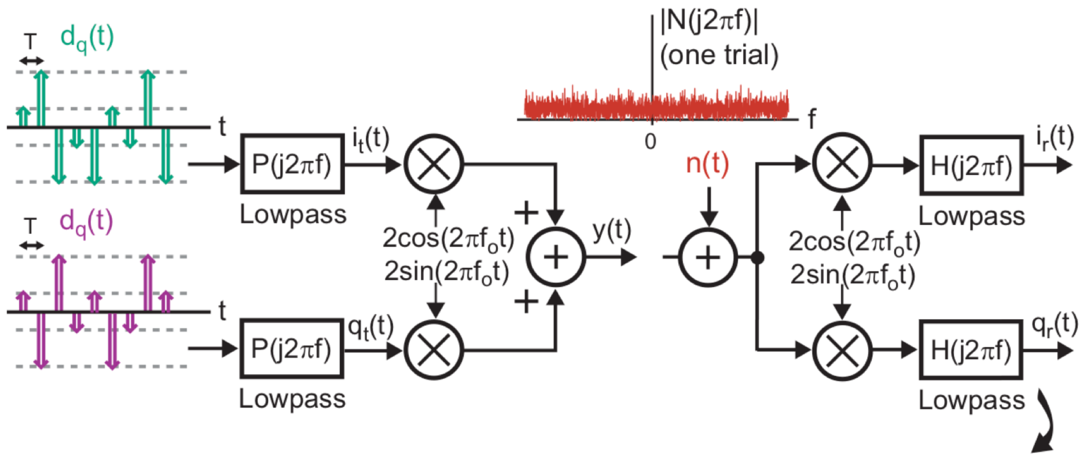
- **Eye Diagram**
  - For **transition behavior** between symbols
  - ISI causes **eye to close**

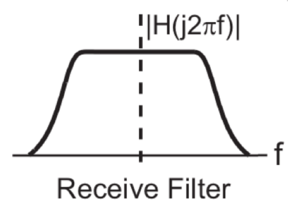


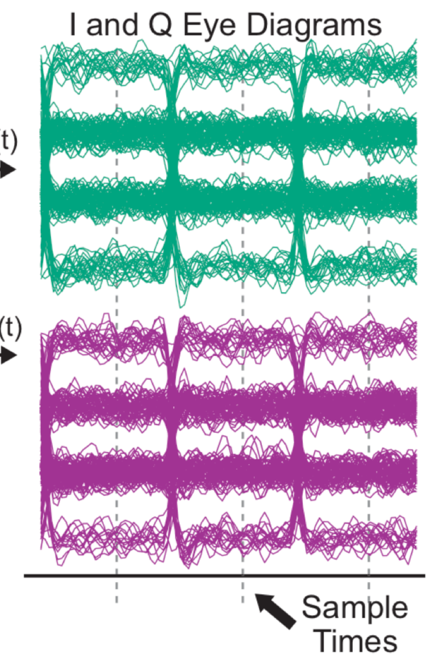
- **Constellation Diagram**
  - Shows aggregate placement of sampled I/Q values
  - **ISI spreads** the constellation points

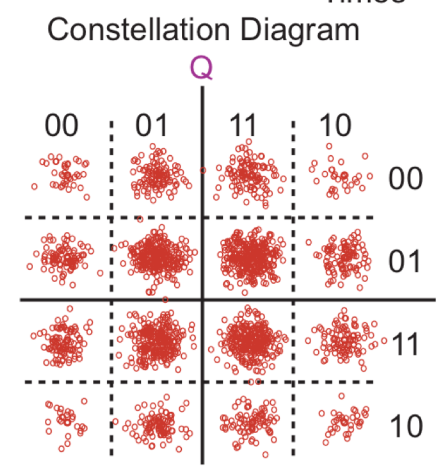


Impact of Receiver Noise





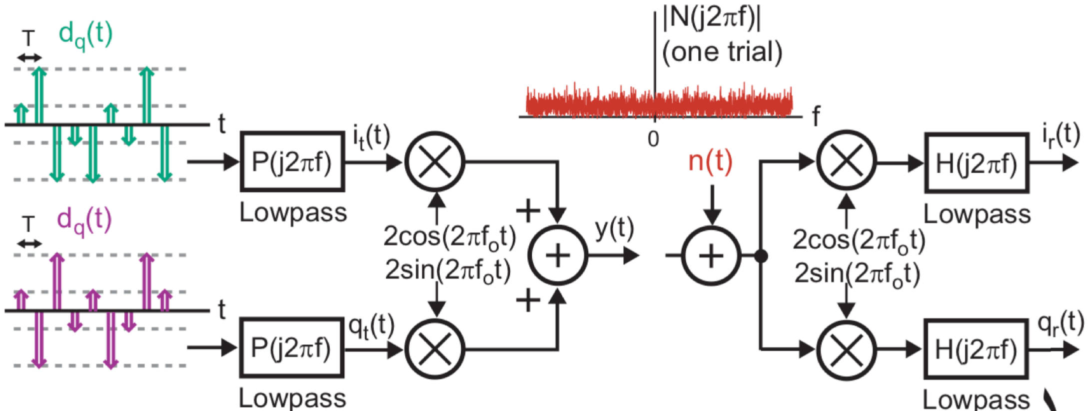


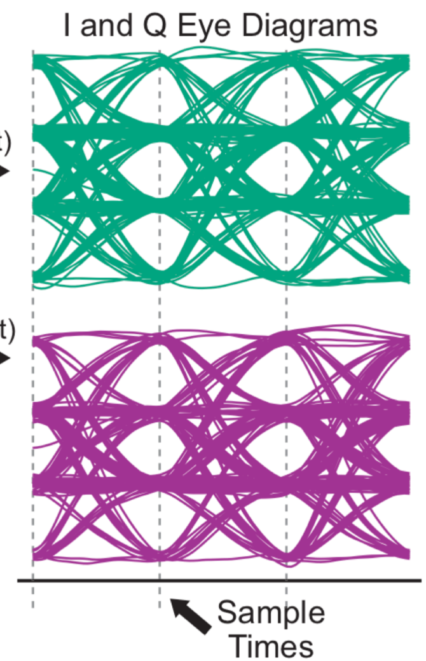


- **Receiver noise** adds to desired I/Q signals, causes **corruption**
  - Eye closes **further**
  - Constellation points spread

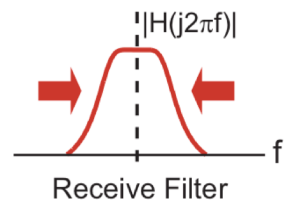
- **Key insight:** Lowering the receive filter bandwidth **improves the rejection** of background noise

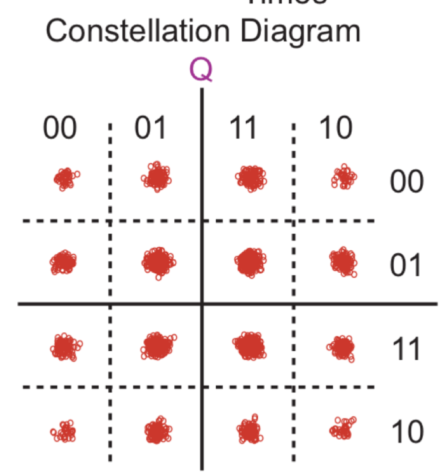
Impact of Lower Receiver Filter Bandwidth





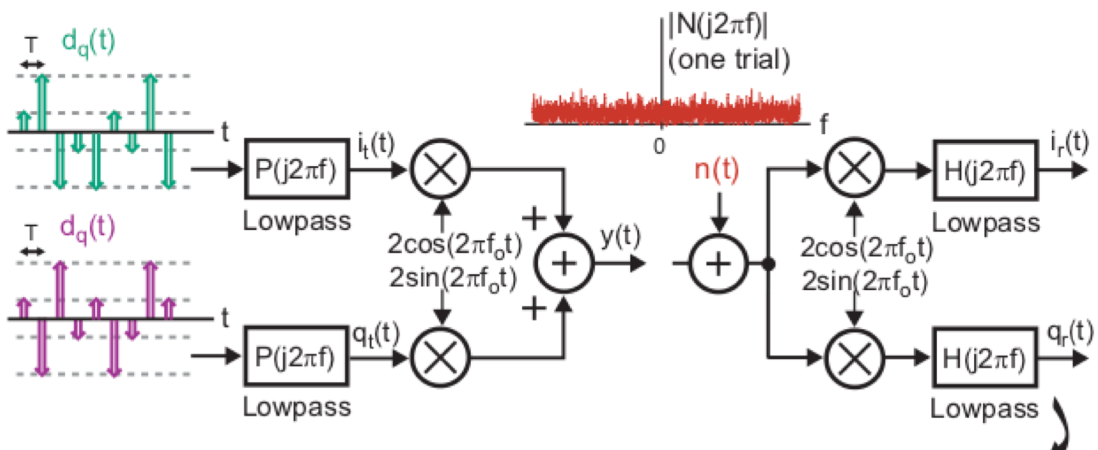
- **Primary noise source:** **Thermal noise** in receive circuits
  - Assume noise ***white*** (*i.e.*, flat frequency spectrum)

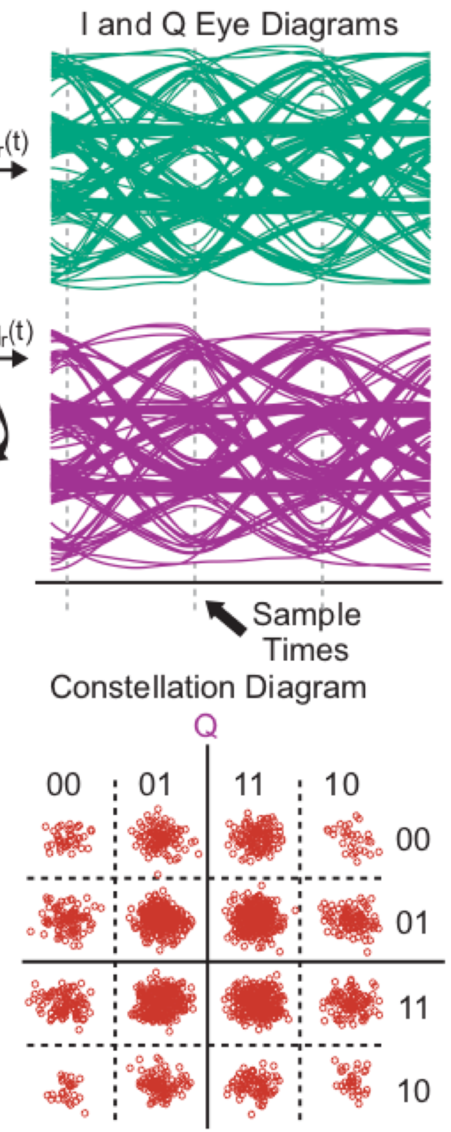




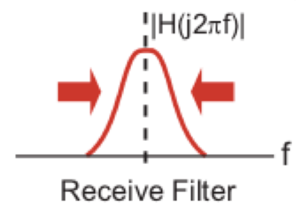
- Receive filter **only** passes noise within its **passband**
- How much can we **lower receive filter bandwidth?**

ISI Versus Noise



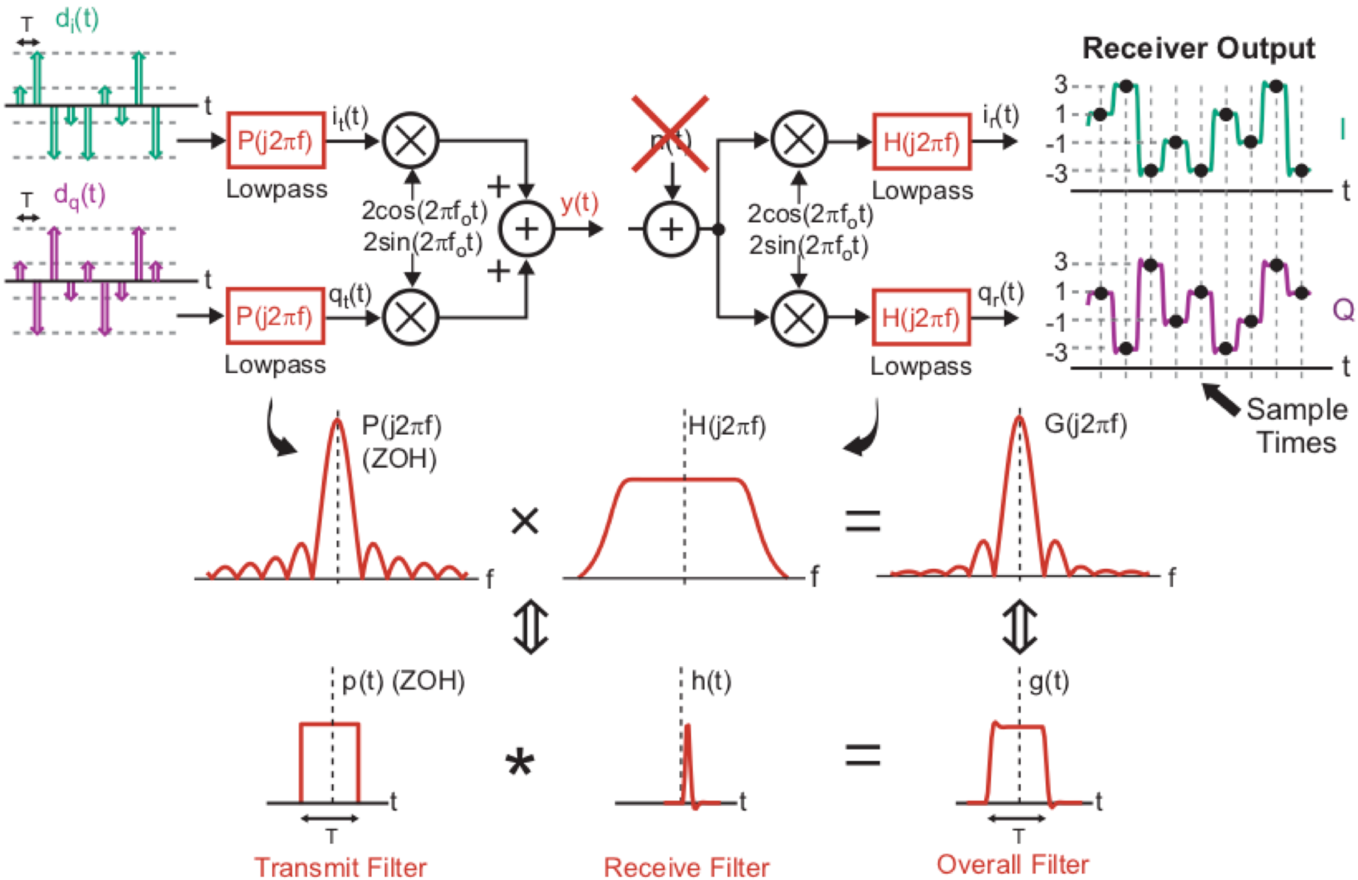


- **Lowering receive filter bandwidth** too much again **causes ISI to dominate**



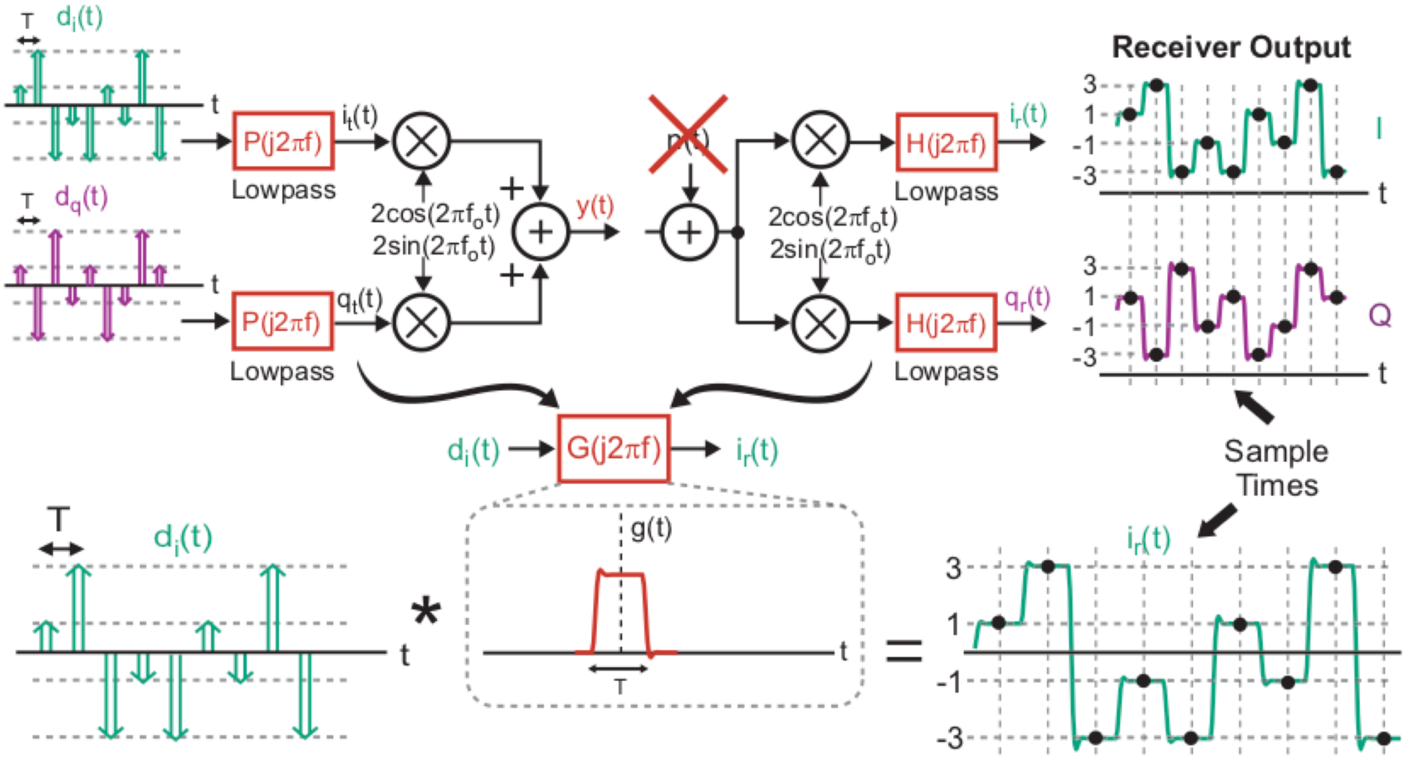
- Selection of receive filter bandwidth involves a **tradeoff between ISI, noise:**
  - Bandwidth too **high**: **High Noise**
  - Bandwidth too **low**: **High ISI**

Joint Transmit/Receive ISI Analysis



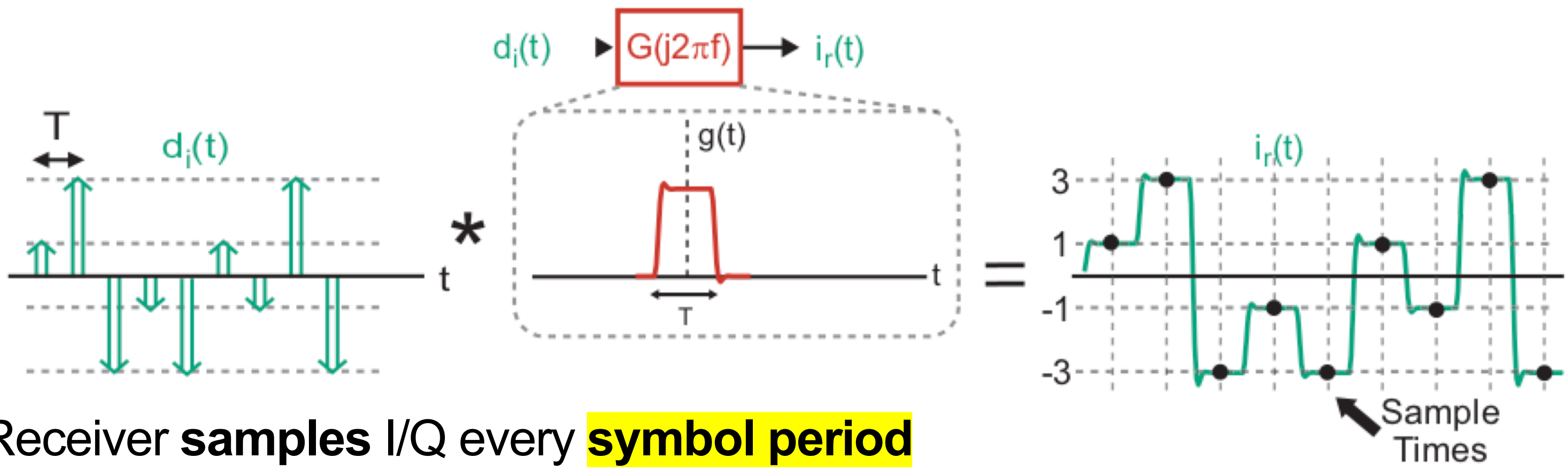
- **Both transmit and receive** filters influence **ISI**
  - Combined filter response: $G(2\pi jf) = P(2\pi jf)\, H(2\pi jf)$

Viewing Filtering in the Time Domain

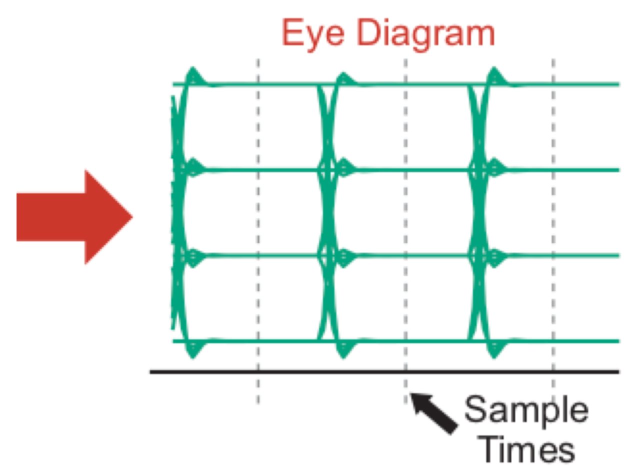


- **Filtering operation** corresponds to **convolution** in the time domain with impulse response
- Time domain view allows us to more clearly see **impact of overall filter on ISI**

Impulse Response and ISI: High Bandwidth

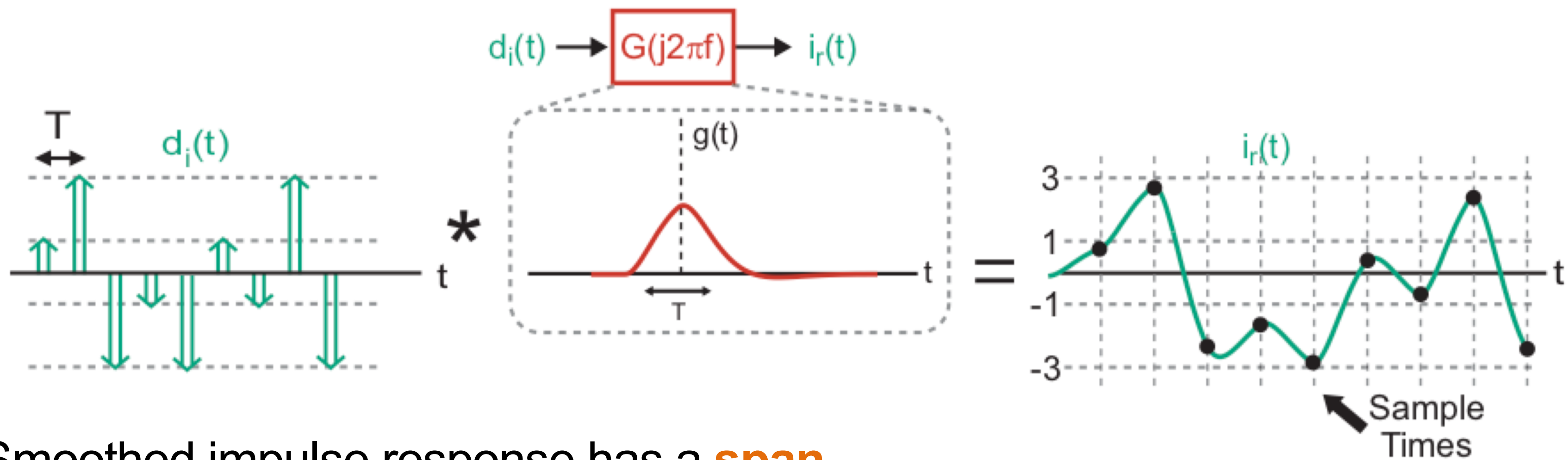


- Receiver **samples** I/Q every **symbol period**
  - Achieving zero ISI requires that **each symbol influence only one sample** at the combined filter output

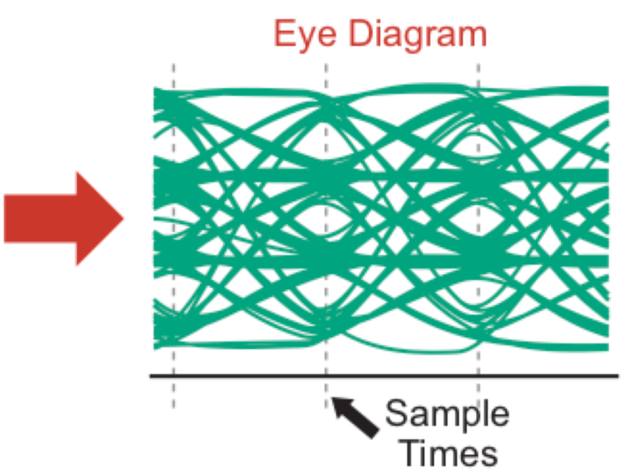


- **Issue: Want lower overall filter bandwidth** to reduce spectrum bandwidth and lower noise
  - But this causes **smoothing of *g*(*t*)**

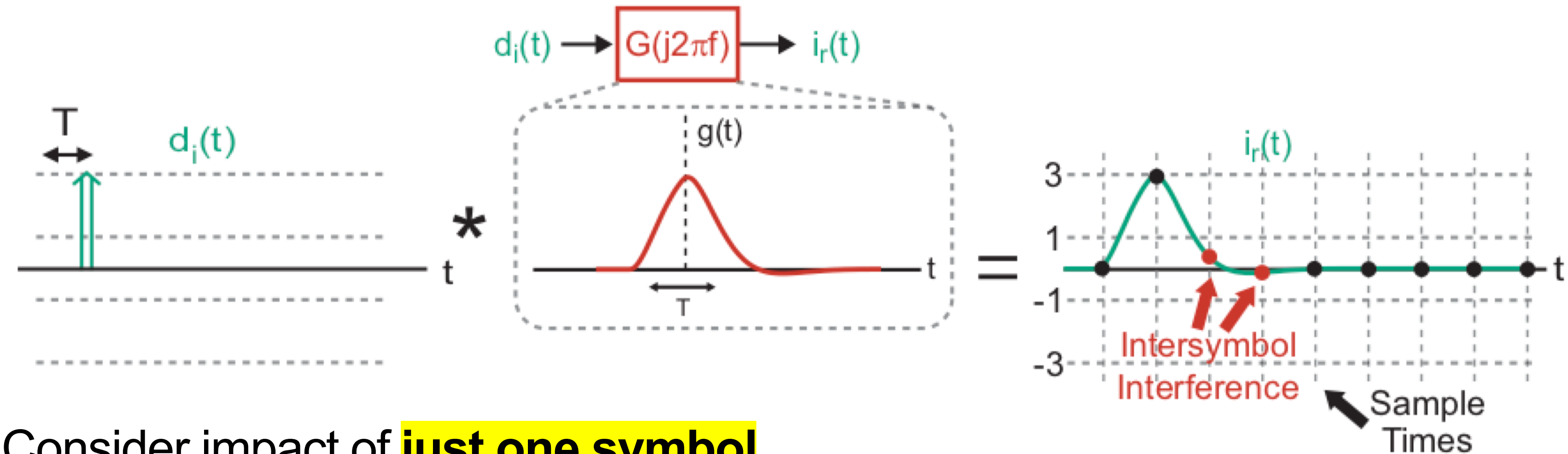
Impulse Response and ISI: Low Bandwidth



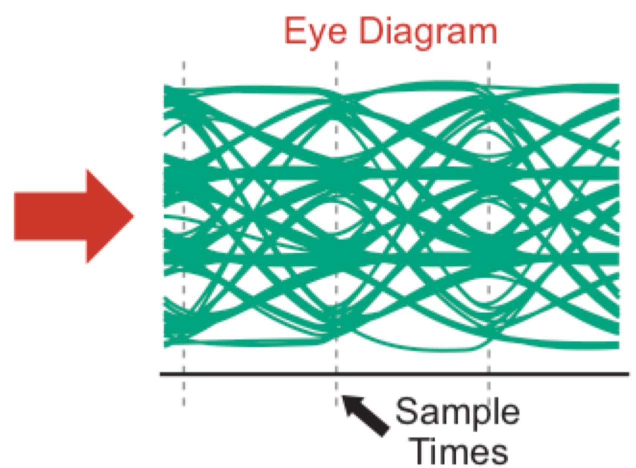
- Smoothed impulse response has a **span longer than one symbol period**
  - Convolution reveals that **each symbol impacts filter output** at **> 1 sample value**
    - **Inter-symbol interference** occurs



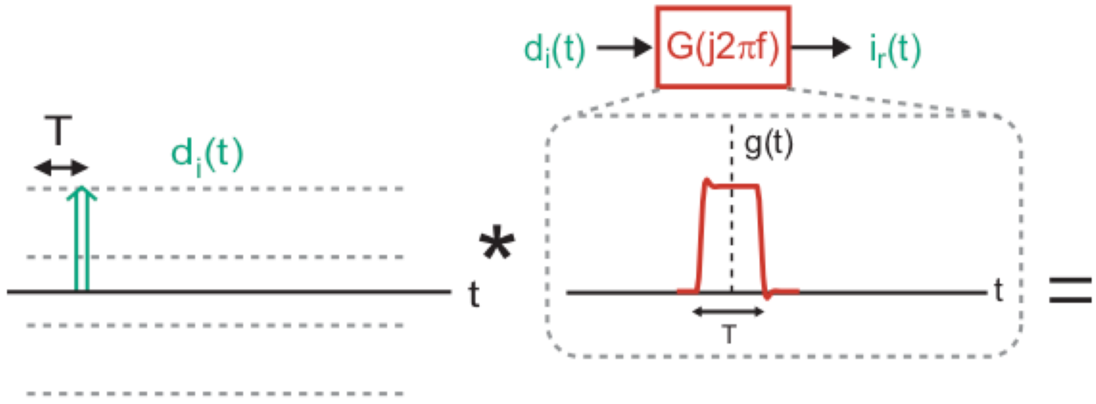
A More Direct View of the ISI Issue

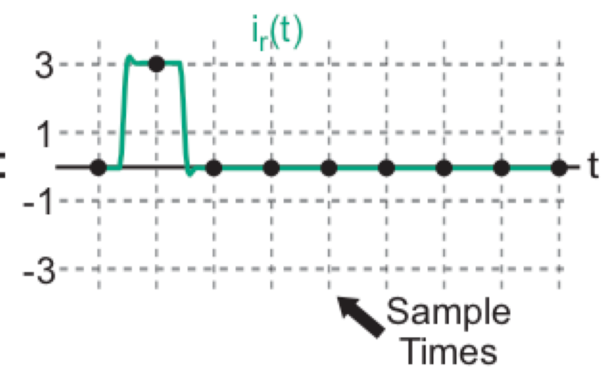


- Consider impact of **just one symbol**
  - Samples at filter output more **clearly show the impact** of the one symbol on **other sample values**



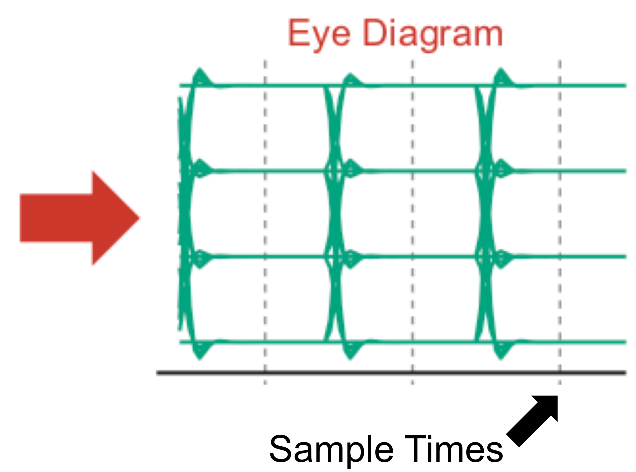
The Nyquist Criterion for Zero ISI



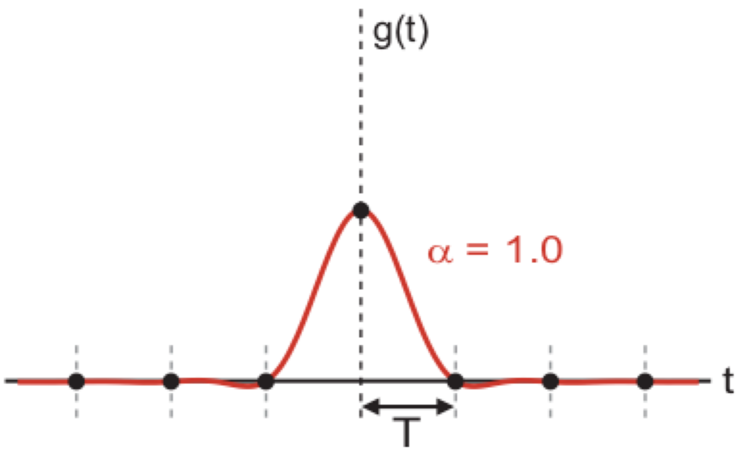


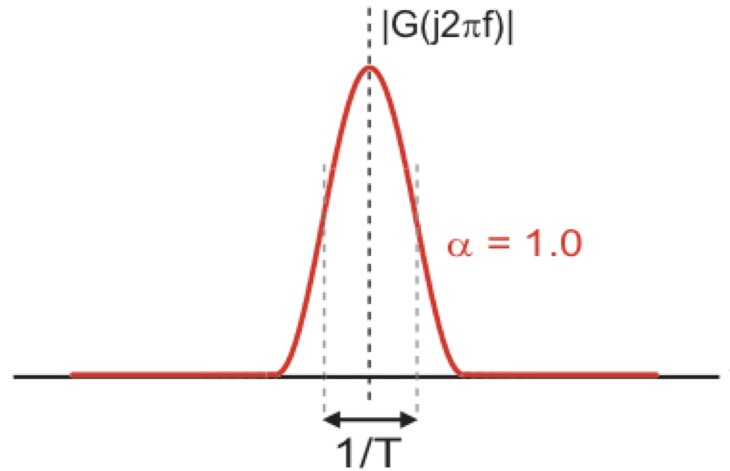
- Sample the impulse response of the overall filter at the symbol period
  - ***Nyquist <u>Criterion</u>:*** Resulting samples must have only one non-zero value to achieve zero ISI

- Can we design impulse response to span more than one symbol period and still meet the Nyquist Criterion for Zero ISI?



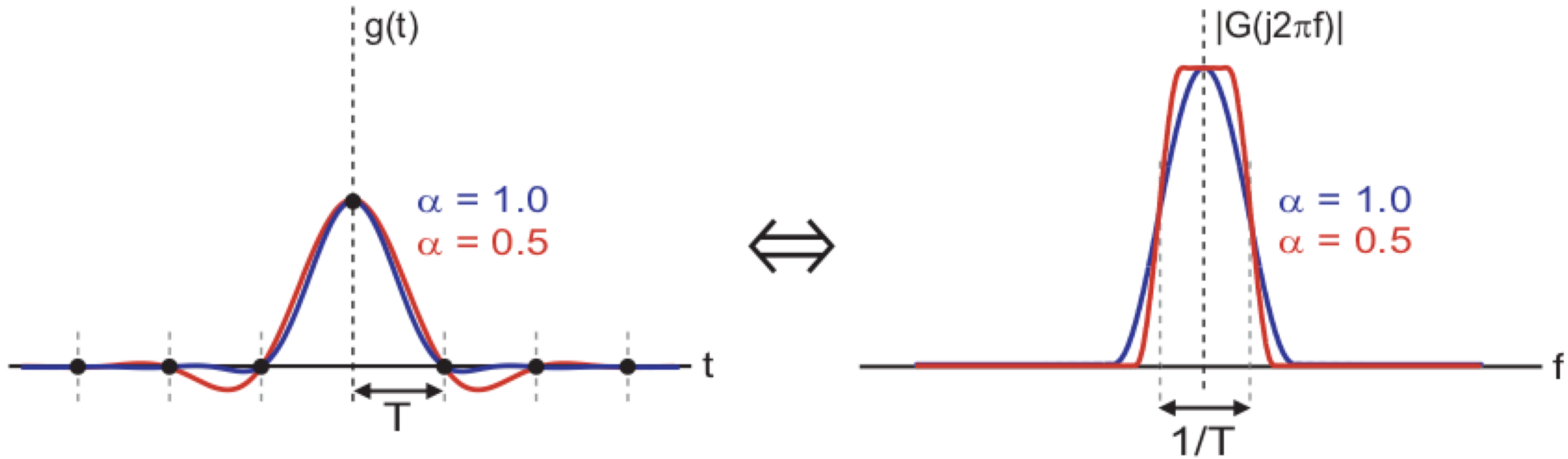
Raised Cosine Filter





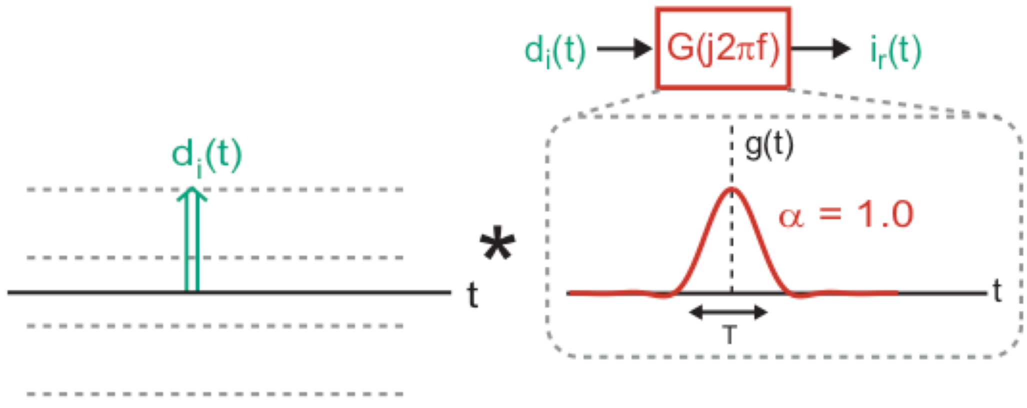
- Raised cosine filter achieves **low bandwidth and zero ISI**
  - Impulse response **spans more than one symbol,** but has **only one non-zero sample** value
  - Impulse response: $g(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\alpha \pi t/T)}{1-(2\alpha t/T)^2}$

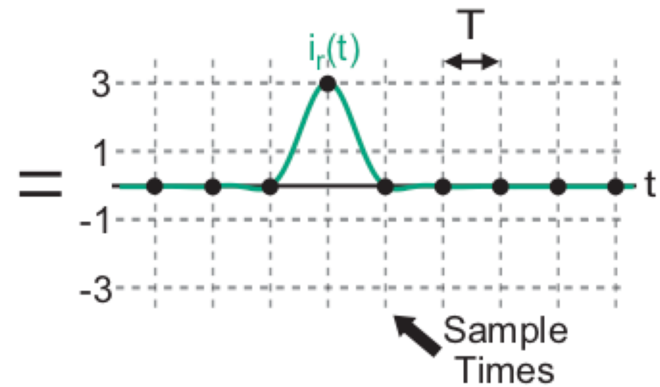
Raised Cosine Filter: Roll-off factor



- Parameter α (0 ≤ α ≤ 1) is referred to as the ***roll-off factor*** of the filter
  - Smaller values of α lead to:
    - Reduced filter bandwidth
    - Increased duration of the filter impulse response
- Regardless of α, the raised cosine filter **achieves zero ISI**

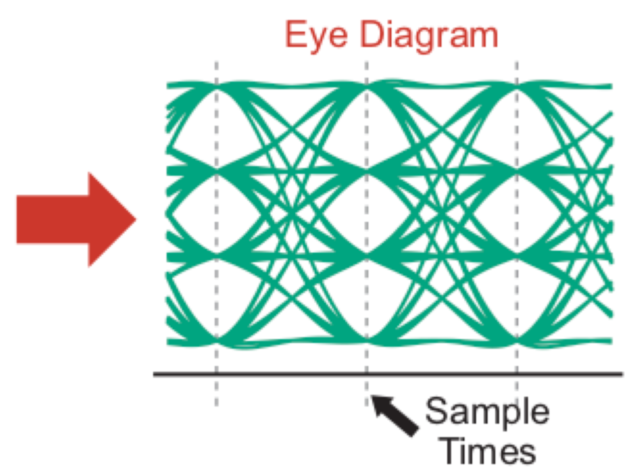
Impact of Large α on Eye Diagram



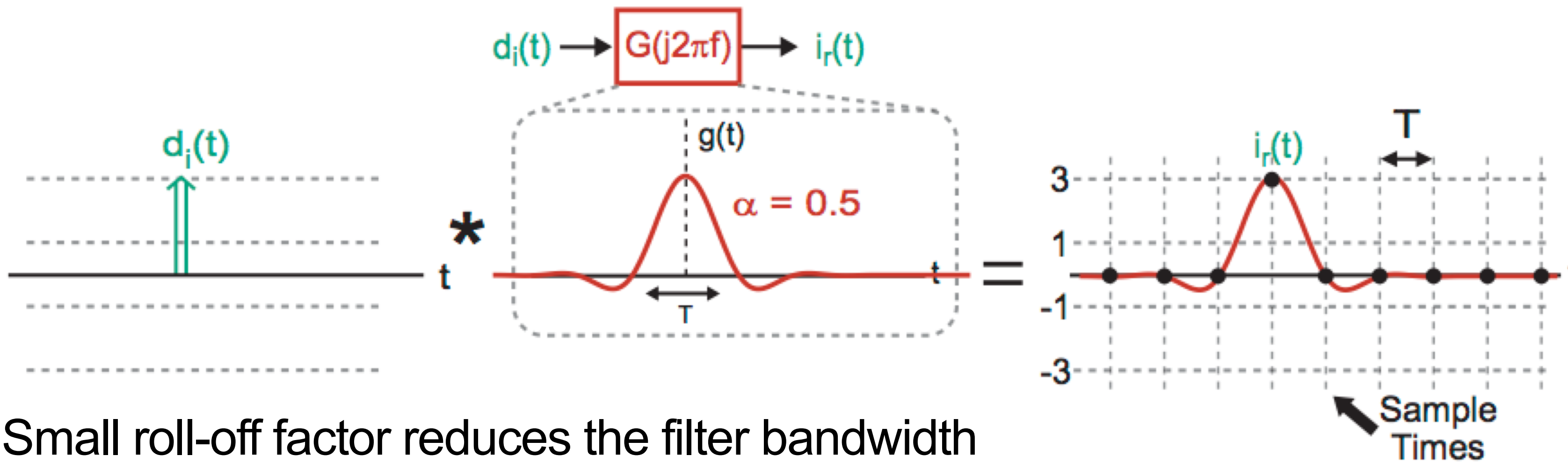


- **Large roll-off factor** leads to nice, **open** eye diagram

- **Key observation:** Achieving zero ISI **requires precise placement of sample times**
  - **Error** in placement of sample times leads to **substantial ISI**

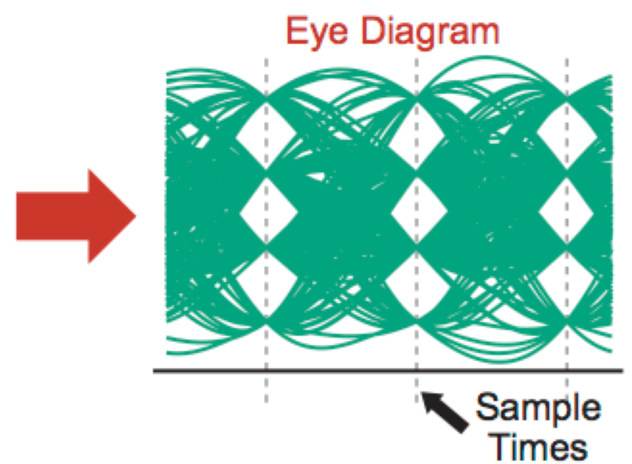


Impact of Small α on Eye Diagram

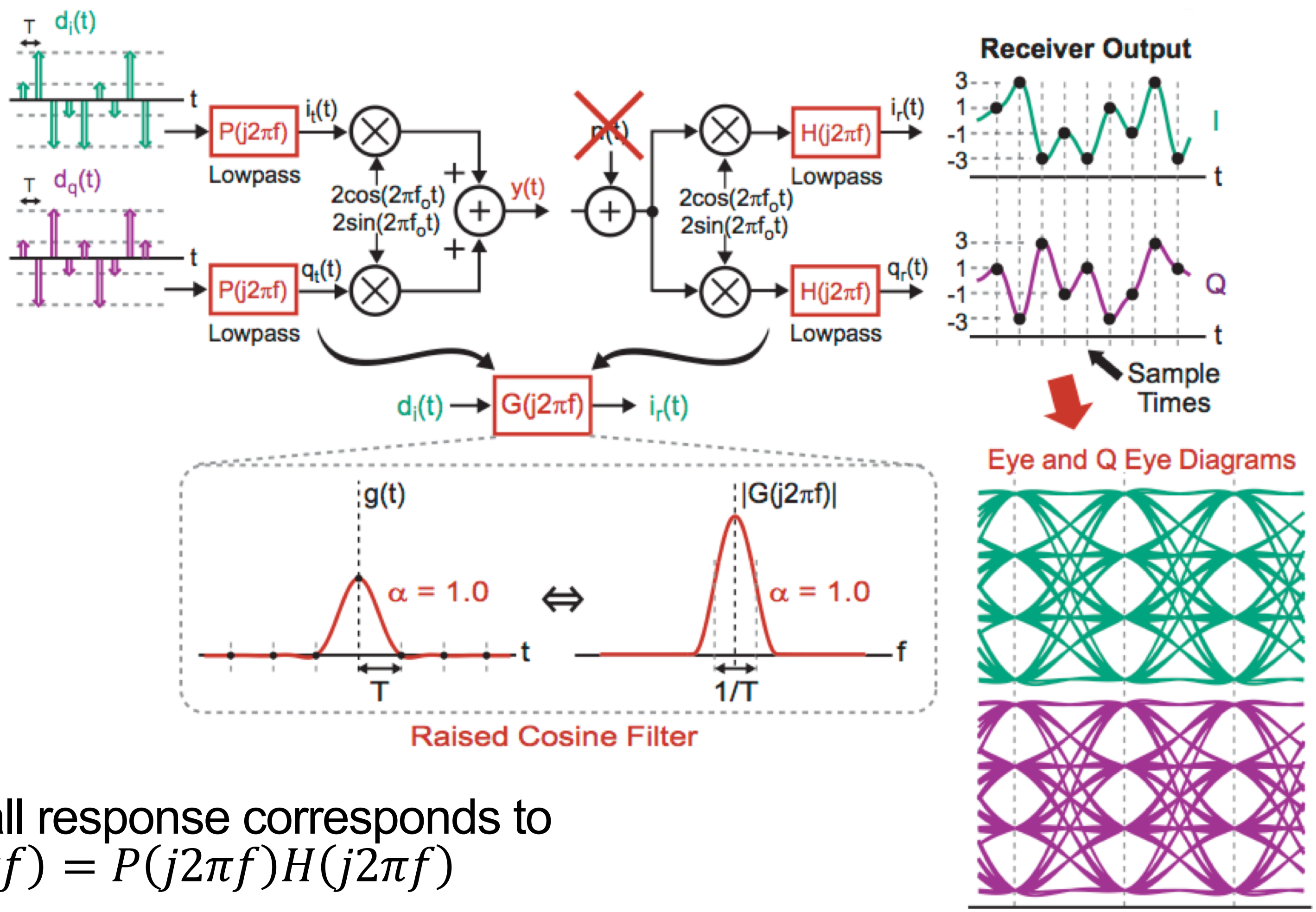


- Small roll-off factor reduces the filter bandwidth and still allows zero ISI to be achieved

- **Issue: Greater sensitivity** to sample time placement than for large α
  - Needs **greater receiver complexity** to ensure precise sample time placement

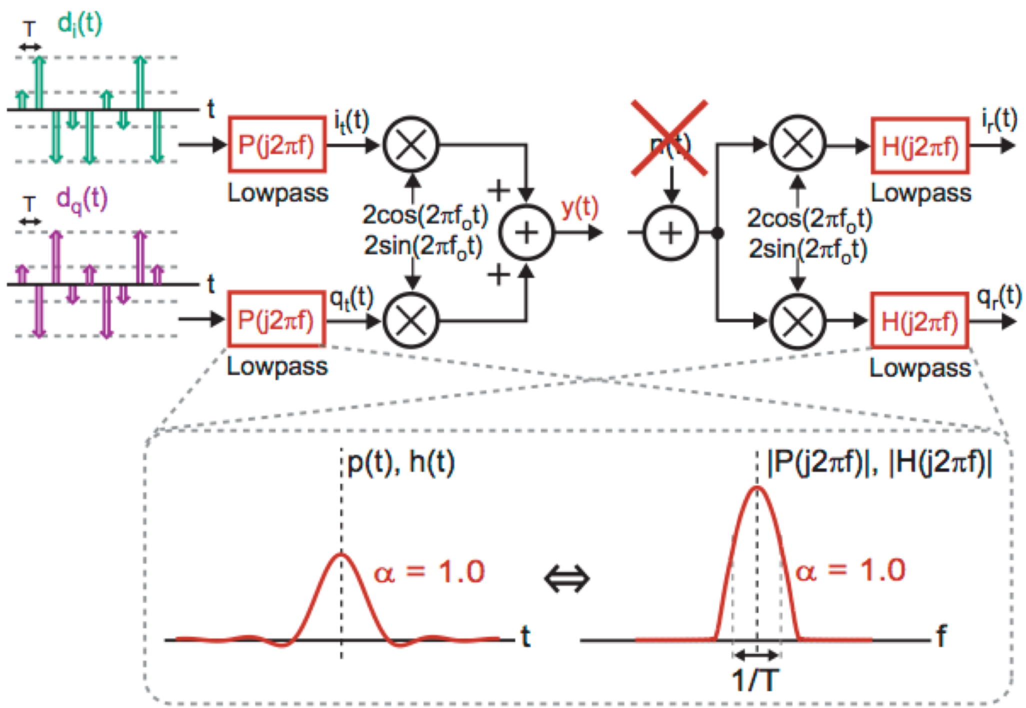


Transmitter and Receiver Filter Design

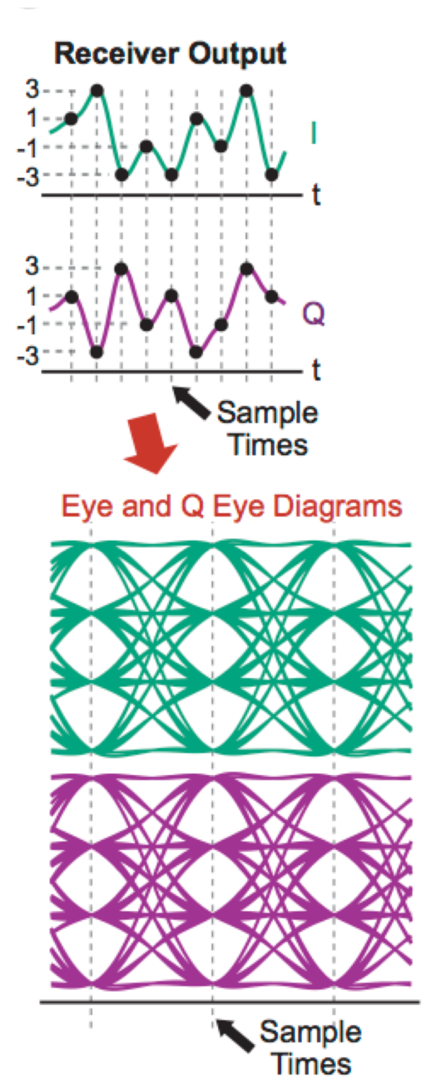


- Overall response corresponds to
  $G(j2\pi f) = P(j2\pi f)H(j2\pi f)$
  - How to choose P and H?

Matched Filter Design



Square-Root Raised Cosine Filter



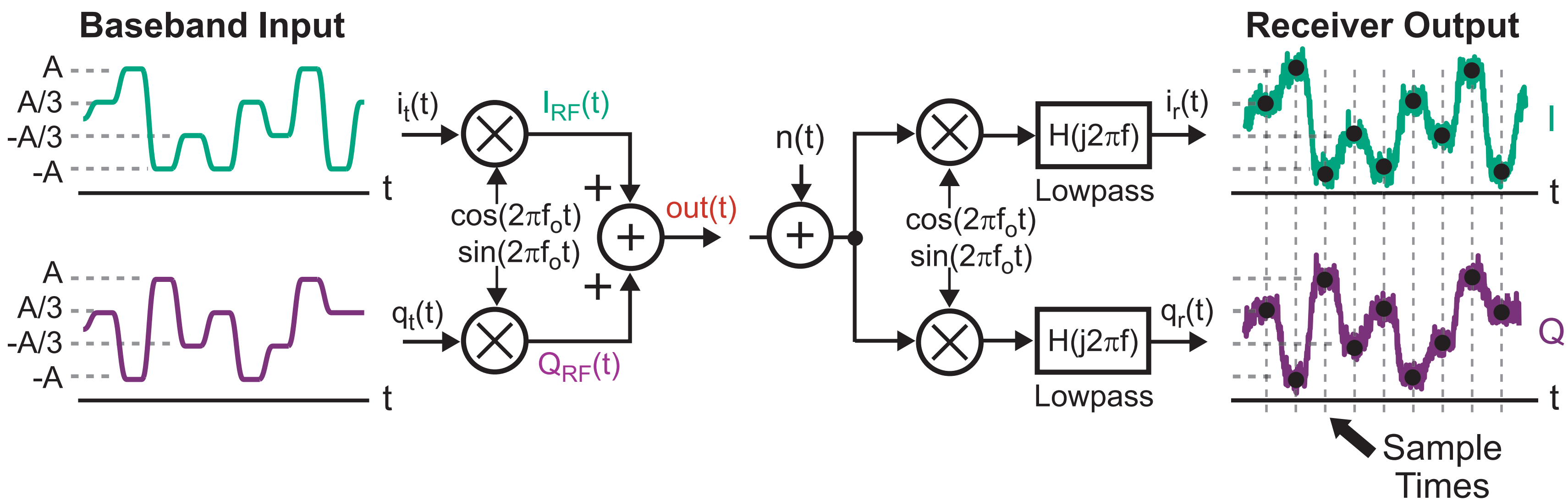
- Setting $P(j2\pi f) = H(j2\pi f)$ yields a ***matched filter*** design
  - Each filter chosen to be a **square-root** raised cosine filter

Today

1. Receiver architecture
   – Tradeoffs between ISI and Noise
   – Common filter design: Raised Cosine

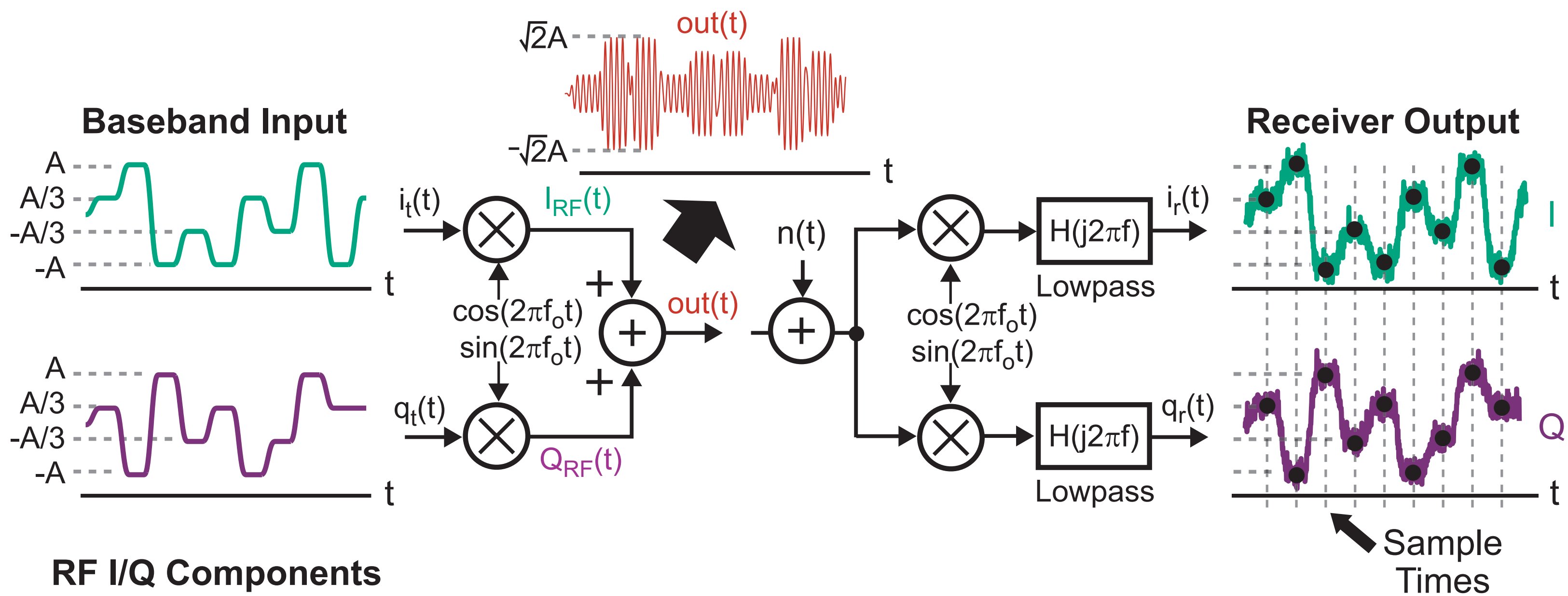
2. **Bit error rate and Shannon Capacity**

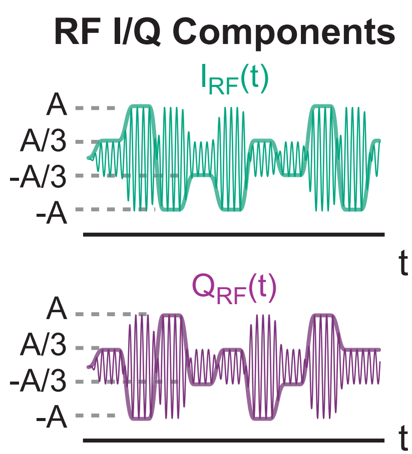
Review of Digital Modulation



- Transmitter sends discrete-value signals over analog communication channel
- Receiver samples recovered baseband signal
  - Noise and ISI corrupt received signal
- Key techniques:
  - Properly design transmit and receive filters for low ISI
  - Sample and slice received signals to detect symbols

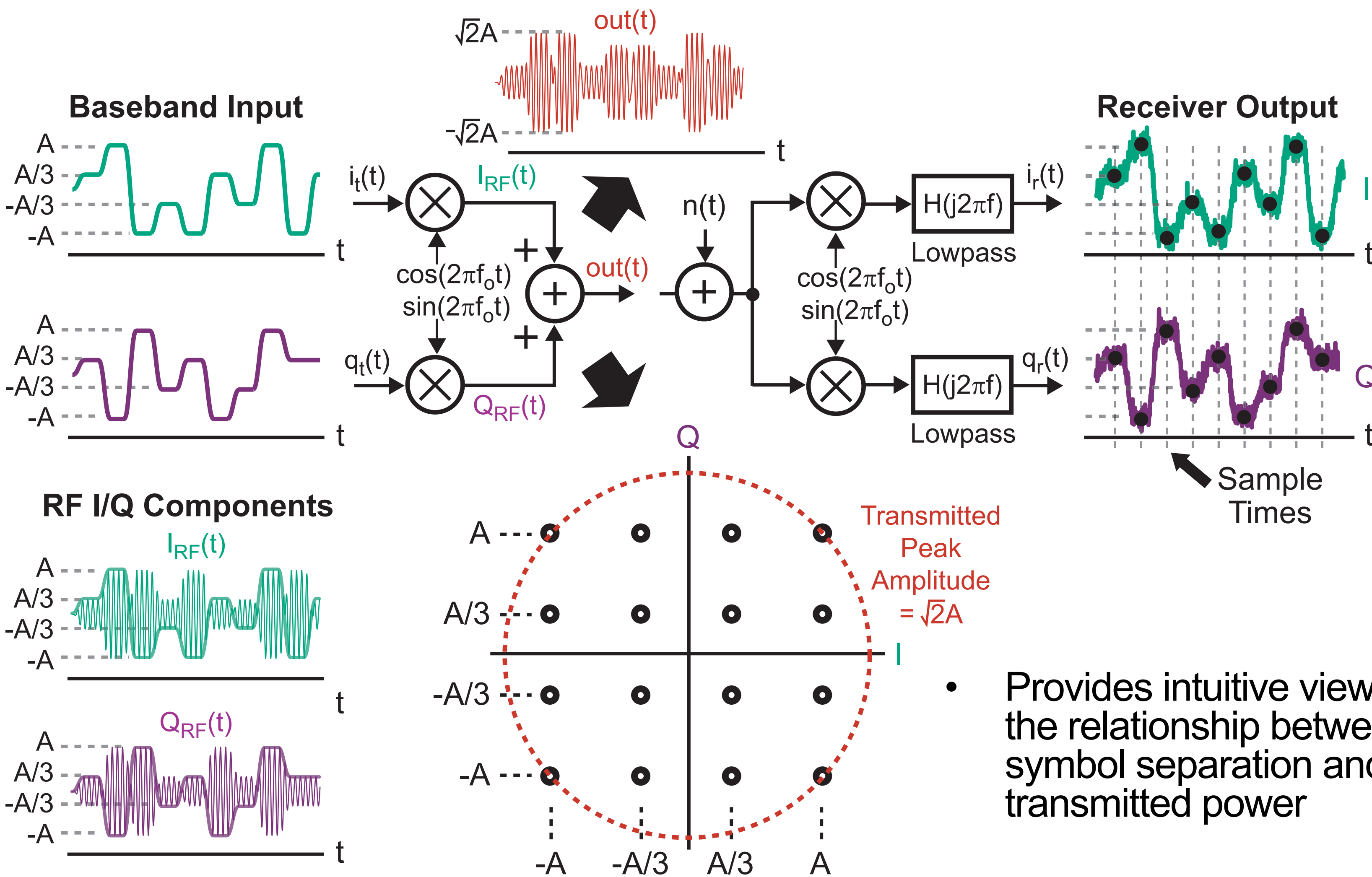
A Closer Look at the Transmitter





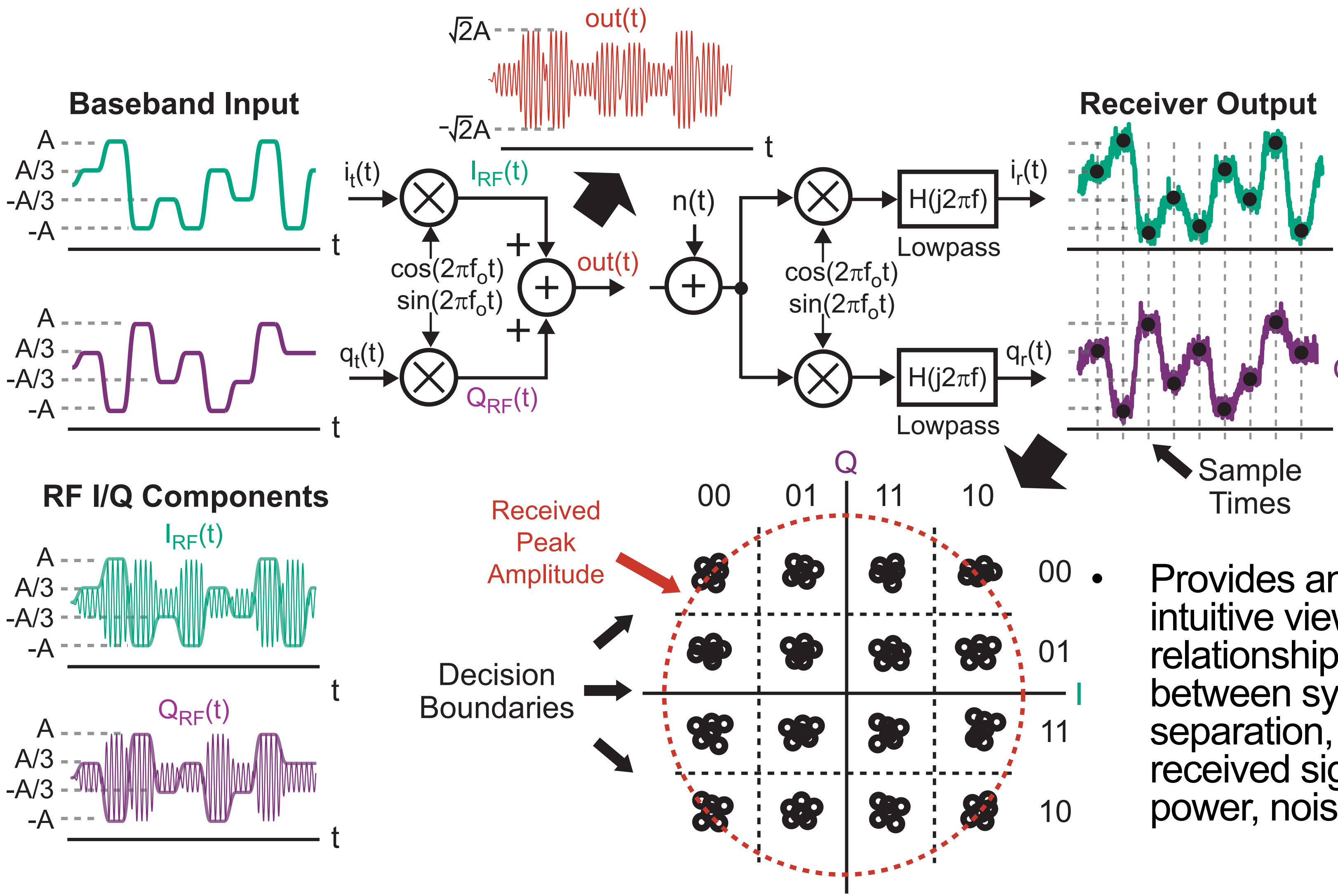
- Amplitude of I/Q transmit signals impact power of transmitted output
  - Output power limited within a given spectral band
  - Low output power desirable for portable applications (battery life)

A Constellation View of the Transmitter



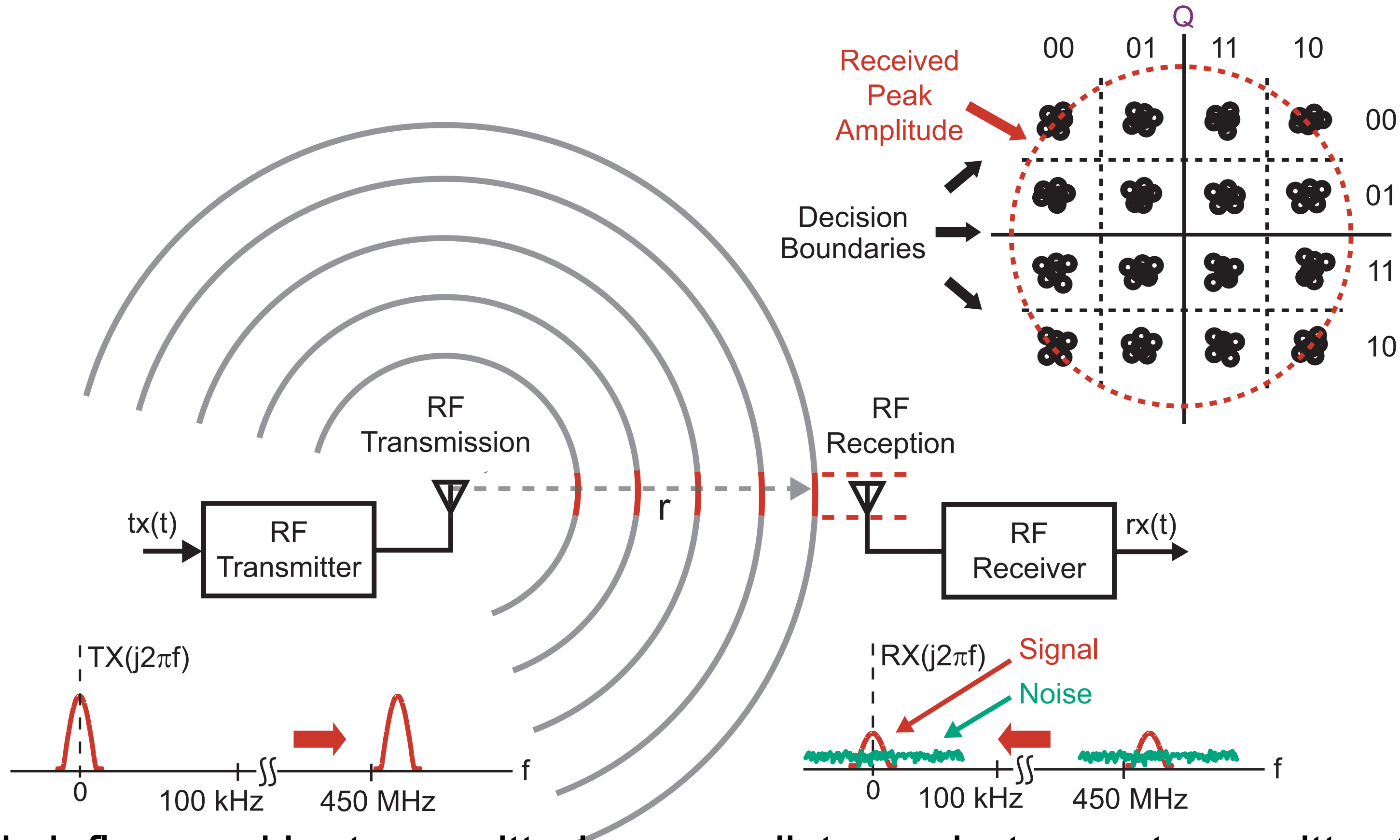
- Provides intuitive view of the relationship between symbol separation and transmitted power

A Constellation View of Receiver



- Provides an intuitive view of relationship between symbol separation, received signal power, noise

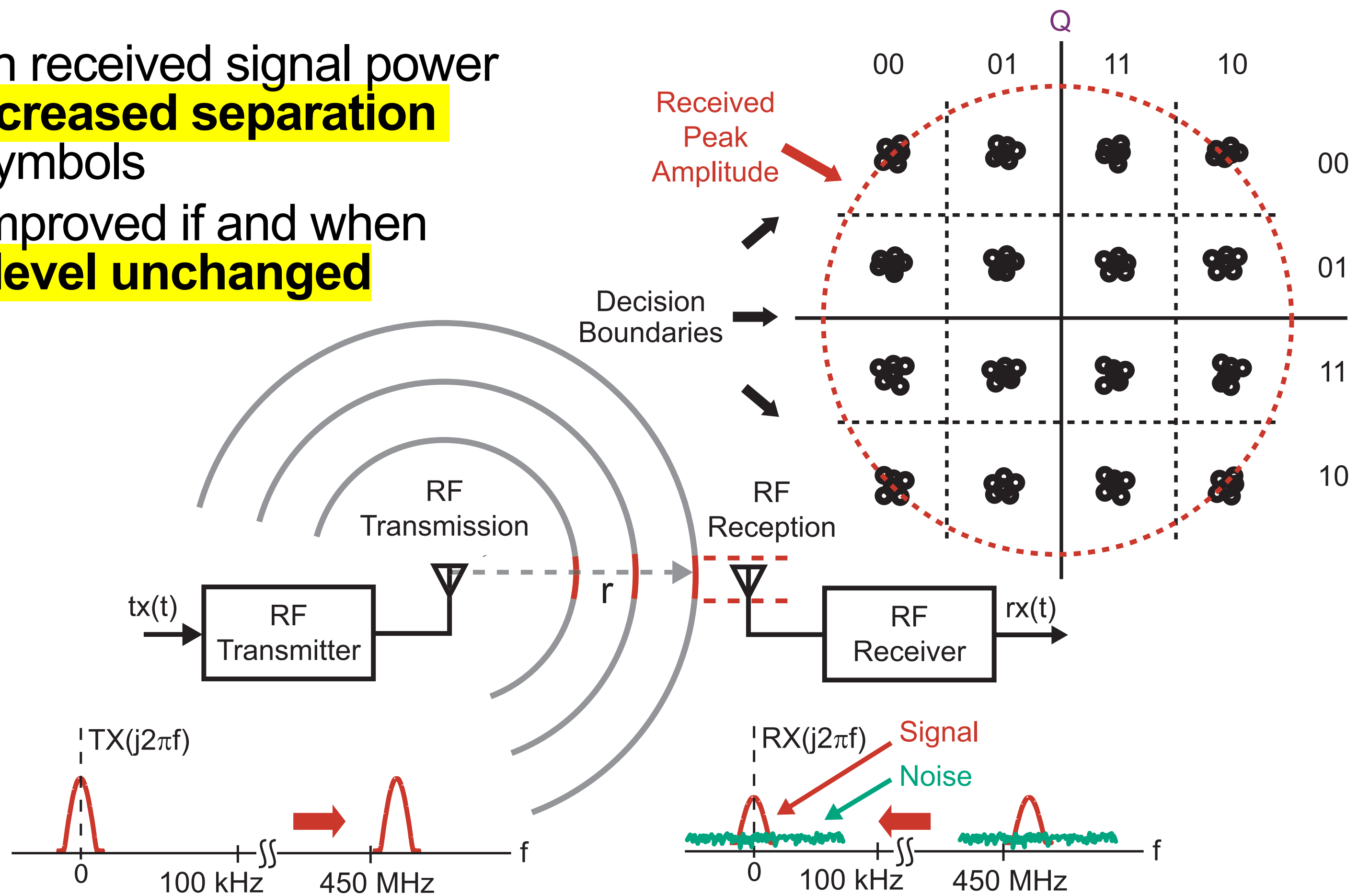
Impact of SNR on Receiver Constellation



- SNR is influenced by transmitted power, distance between transmitter & receiver, and background noise

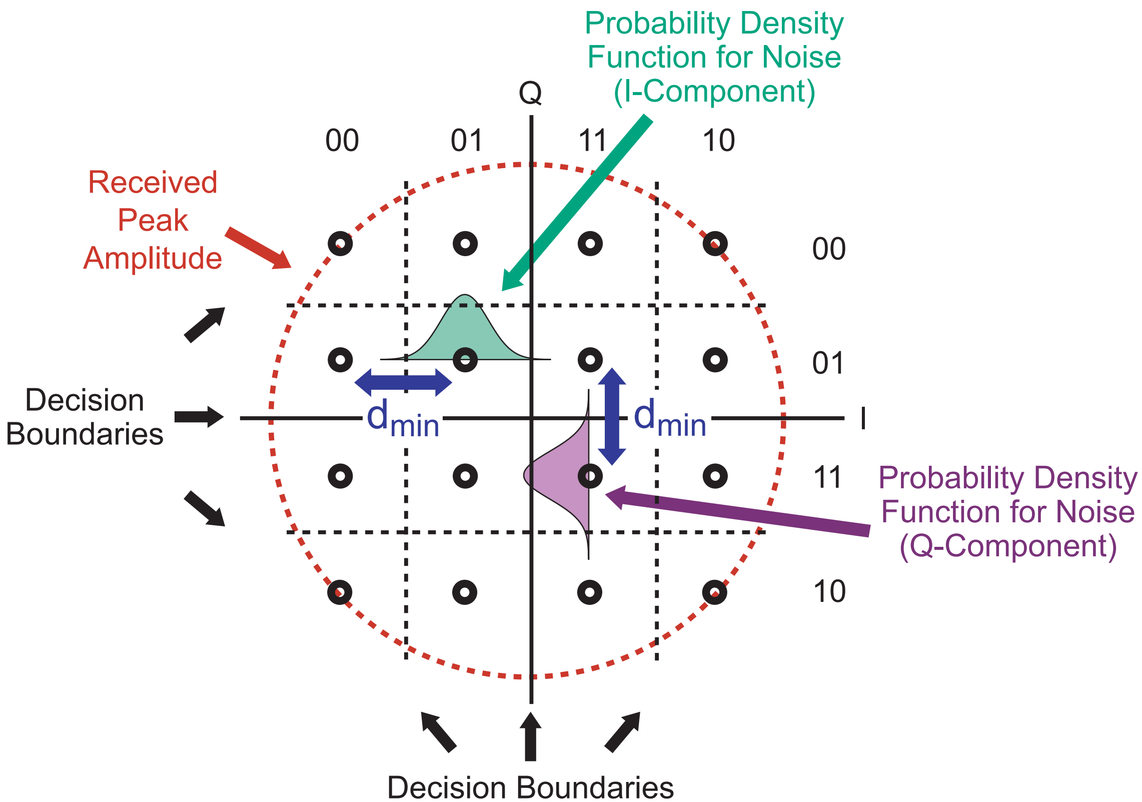
Impact of Increased signal on Constellation

- **Increase** in received signal power leads to **increased separation** between symbols
  - SNR improved if and when **noise level unchanged**



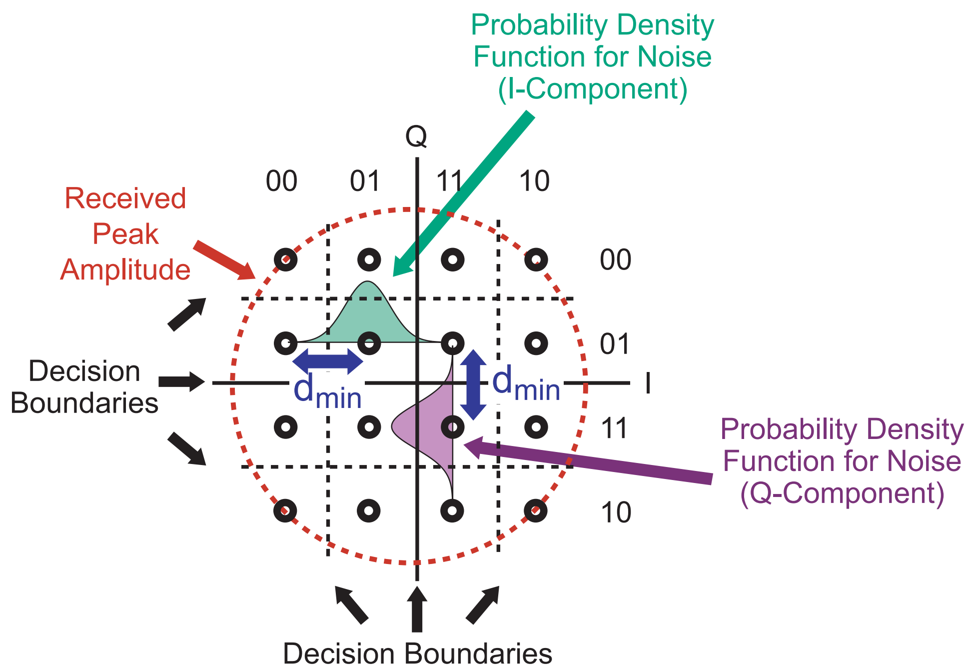
Quantifying the Impact of Noise

- Distribution of noise: zero-mean Gaussian distribution
  - Variance of noise determines the width of the Gaussian



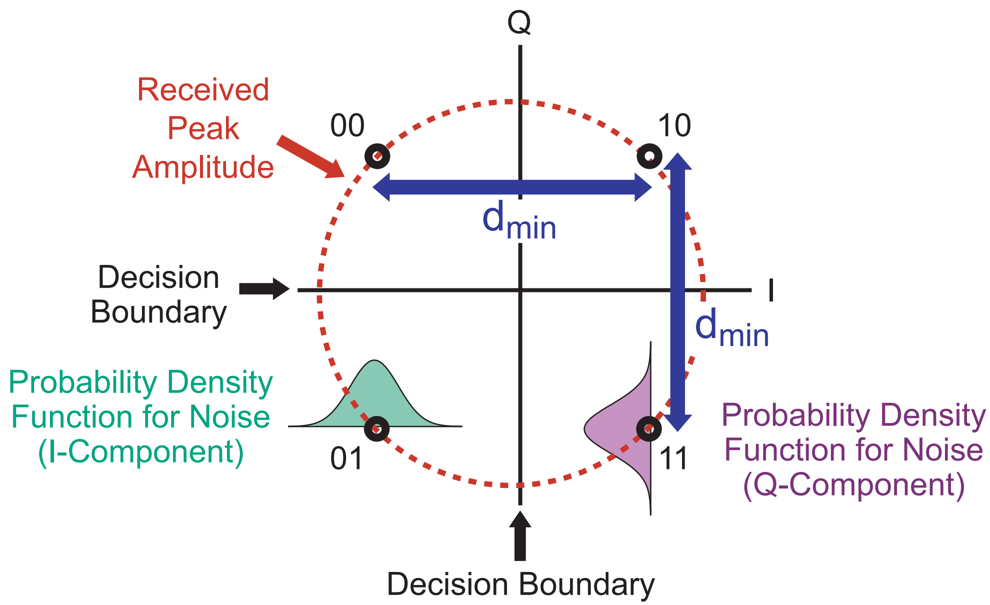
- Minimum separation between symbols: d_{min}
  - **Bit errors occur** when noise **moves a symbol by more than ½ d_{min}**

Impact of Reduced SNR



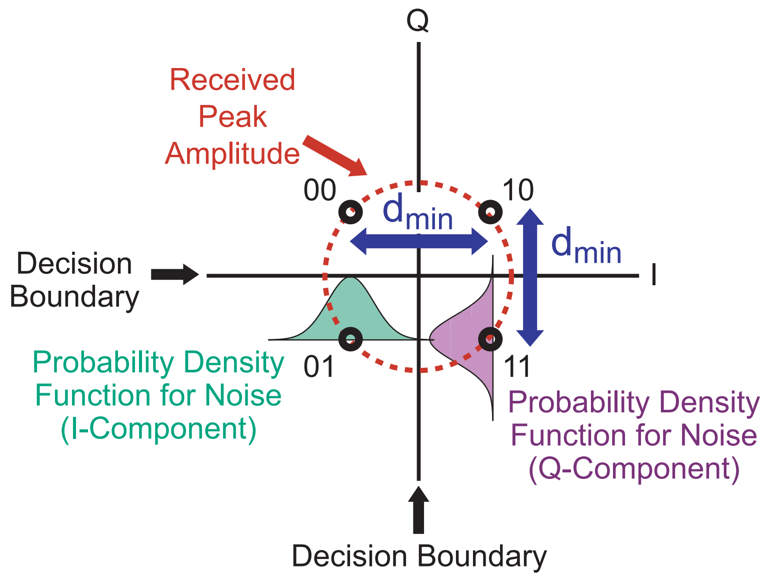
- Lower SNR leads to reduced value for d_{min}
- Leads to a higher bit error rate
  - Assuming noise variance unchanged
  - Assuming received signal power reduced

Impact of Constellation Size Reduction



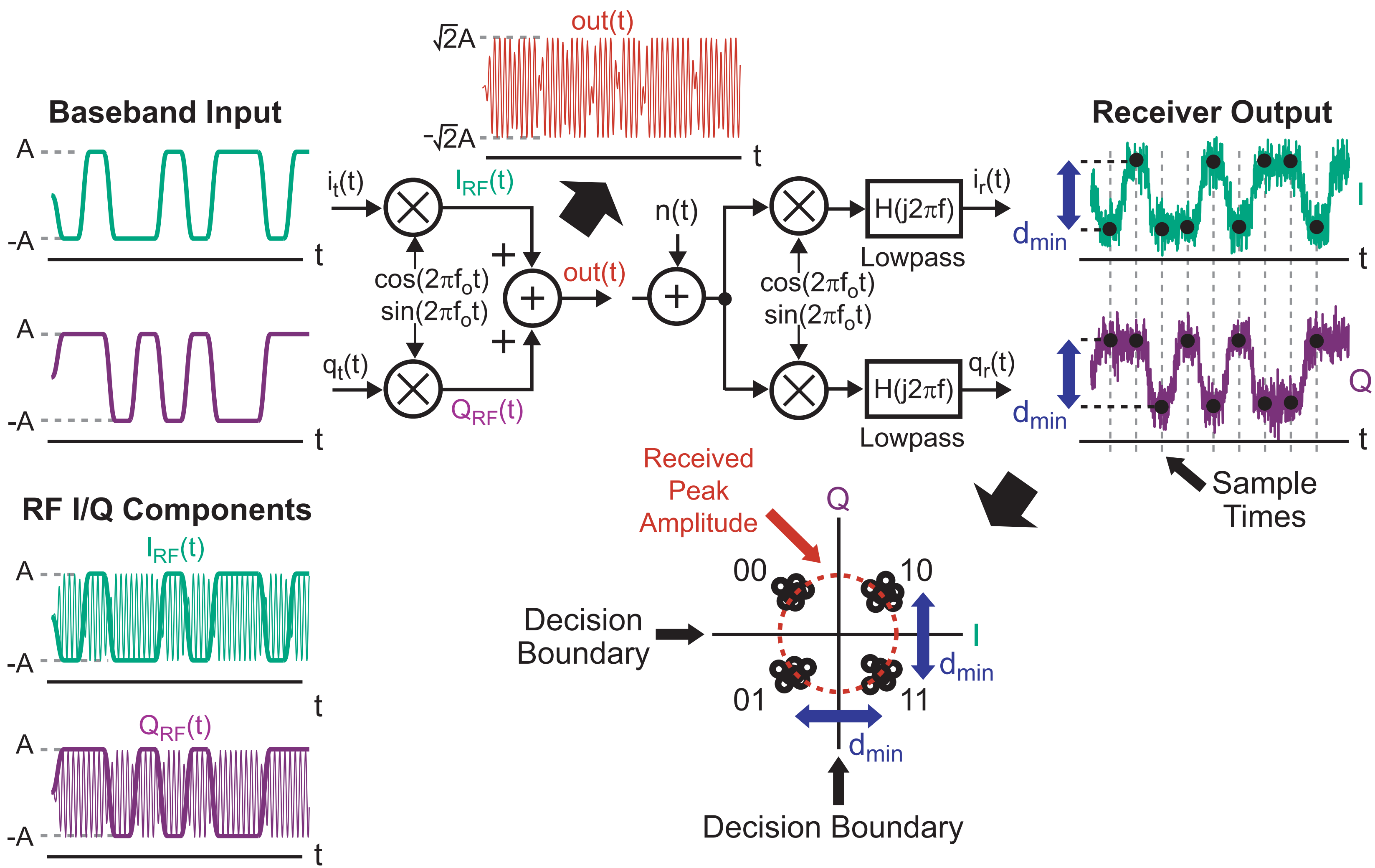
- Reducing the number of symbols leads to an increased value for dmin
- Leads to a lower bit error rate
  - Assuming signal power, noise variance constant

Can we Estimate Bit Error Rate?



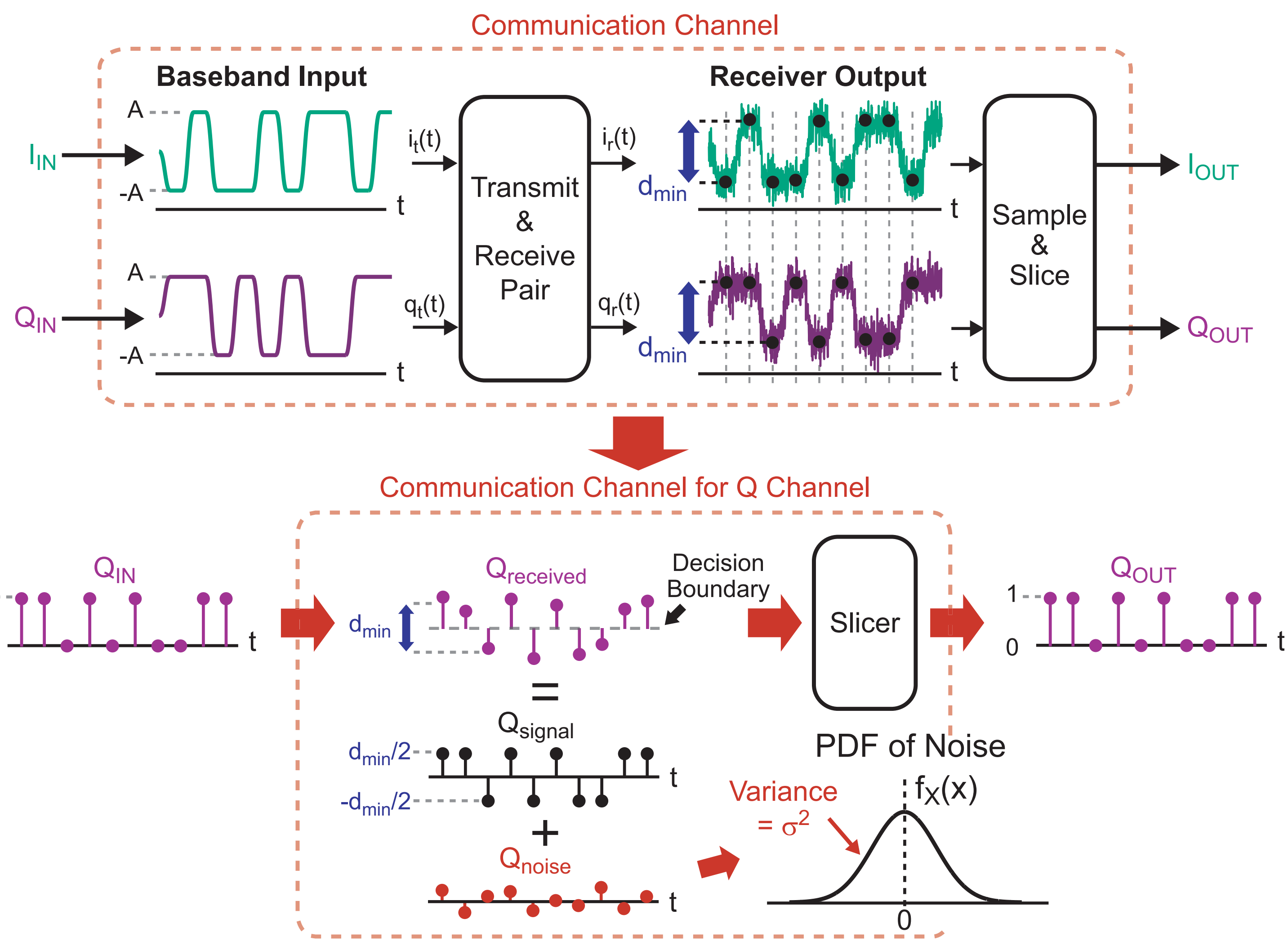
- Bit Error Rate depends on:
  - SNR (ratio of received signal power to noise variance)
  - Number of constellation points
    - Sets d_{min}, given a received signal power level

Let’s Start with a Detailed System View

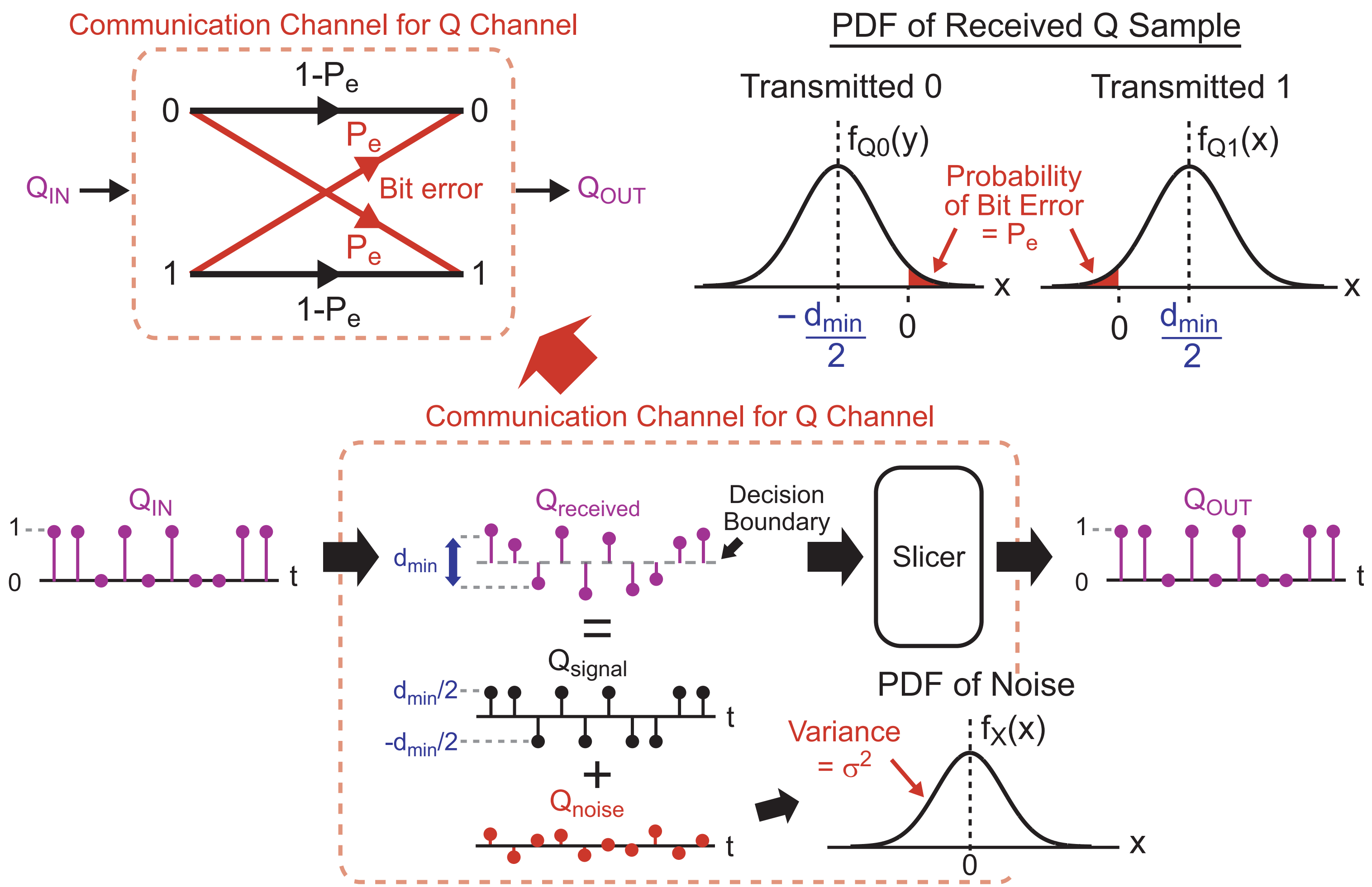


- Assumptions: No ISI, four-point constellation

A Closer Examination of Signal and Noise

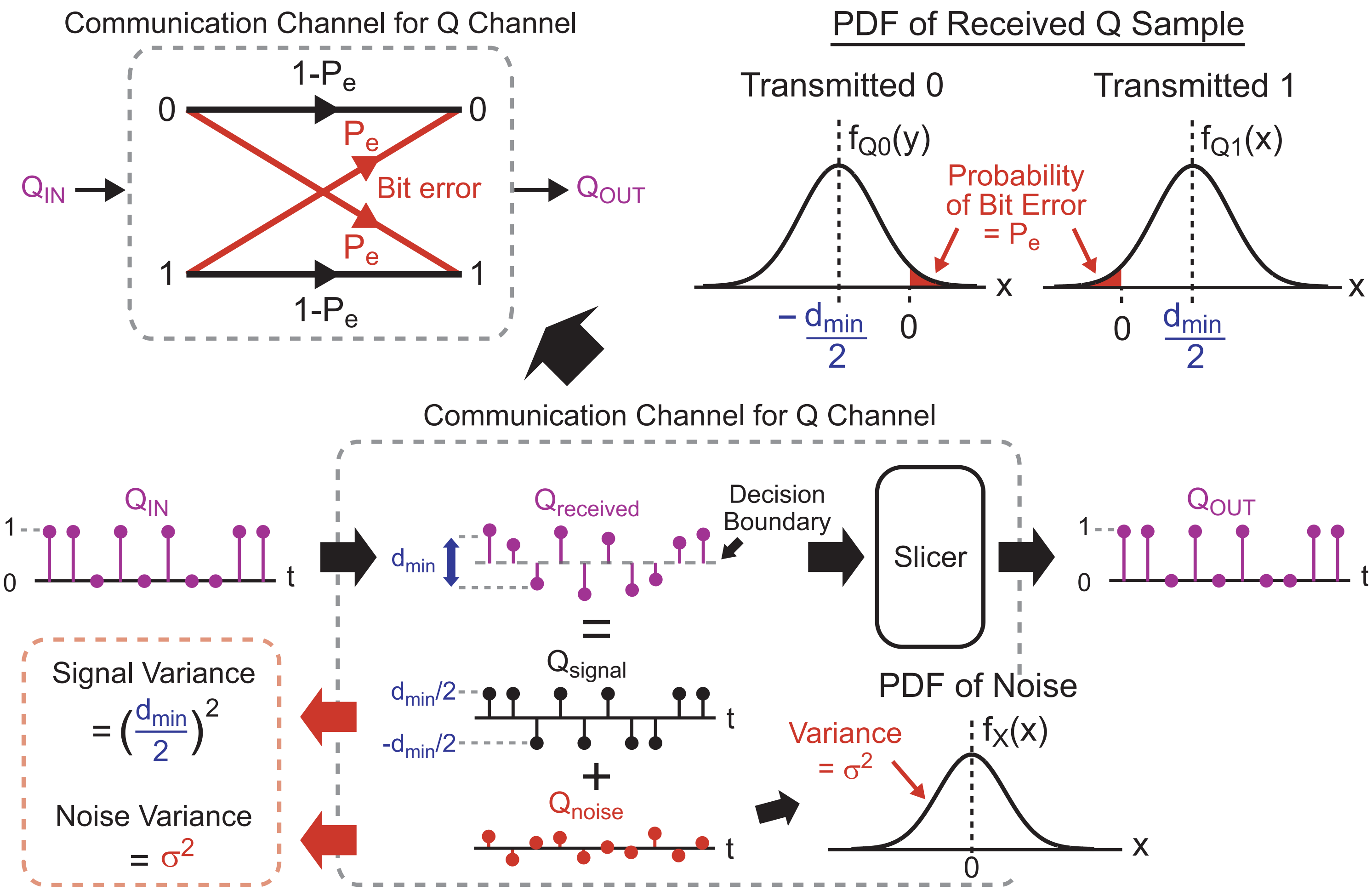


The Binary Symmetric Channel Model



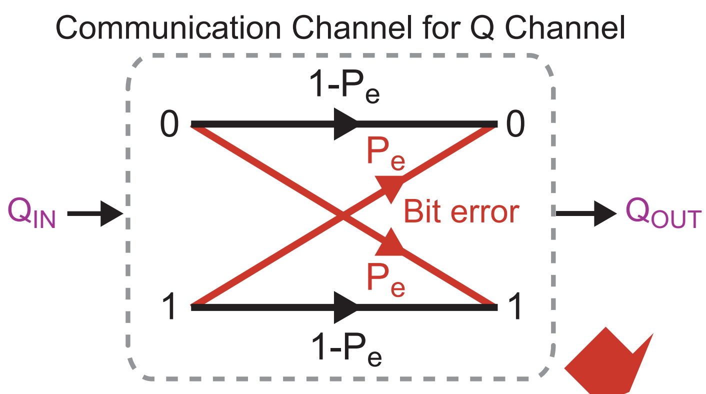
- Provides a binary signaling model of channel

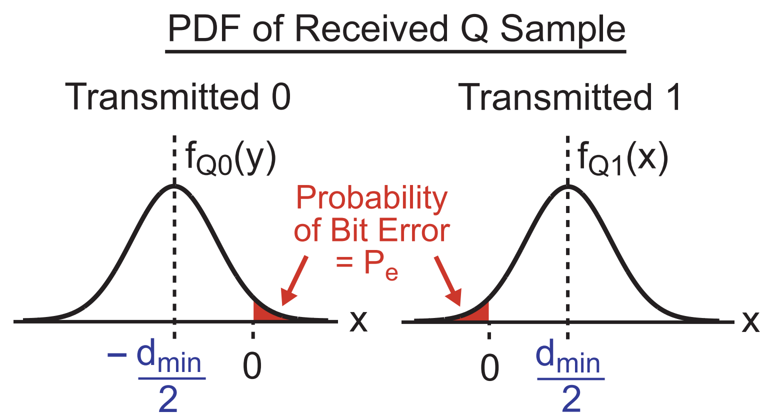
Computation of SNR

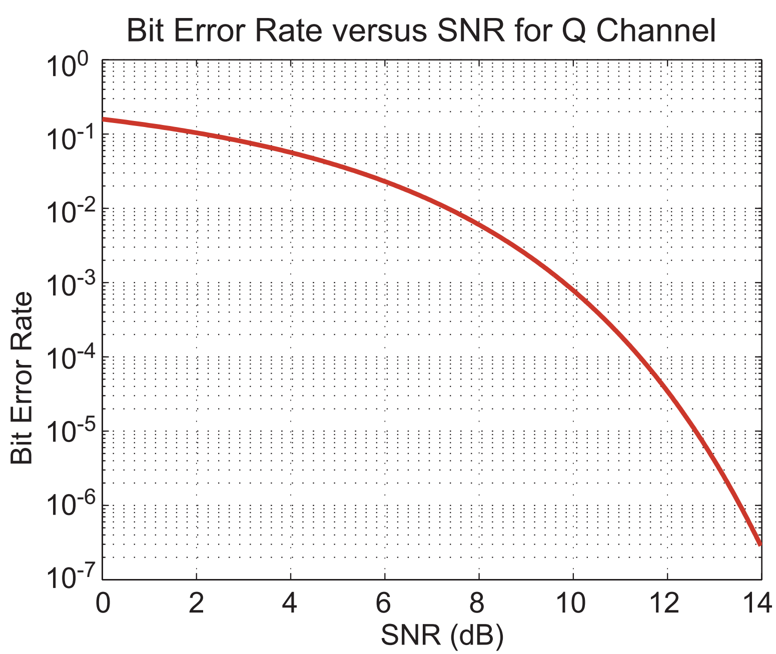


$$\Rightarrow \quad SNR(dB) = 10\log\left((d_{min}/2)^2/\sigma^2\right)$$

Resulting Bit Error Rate Versus SNR

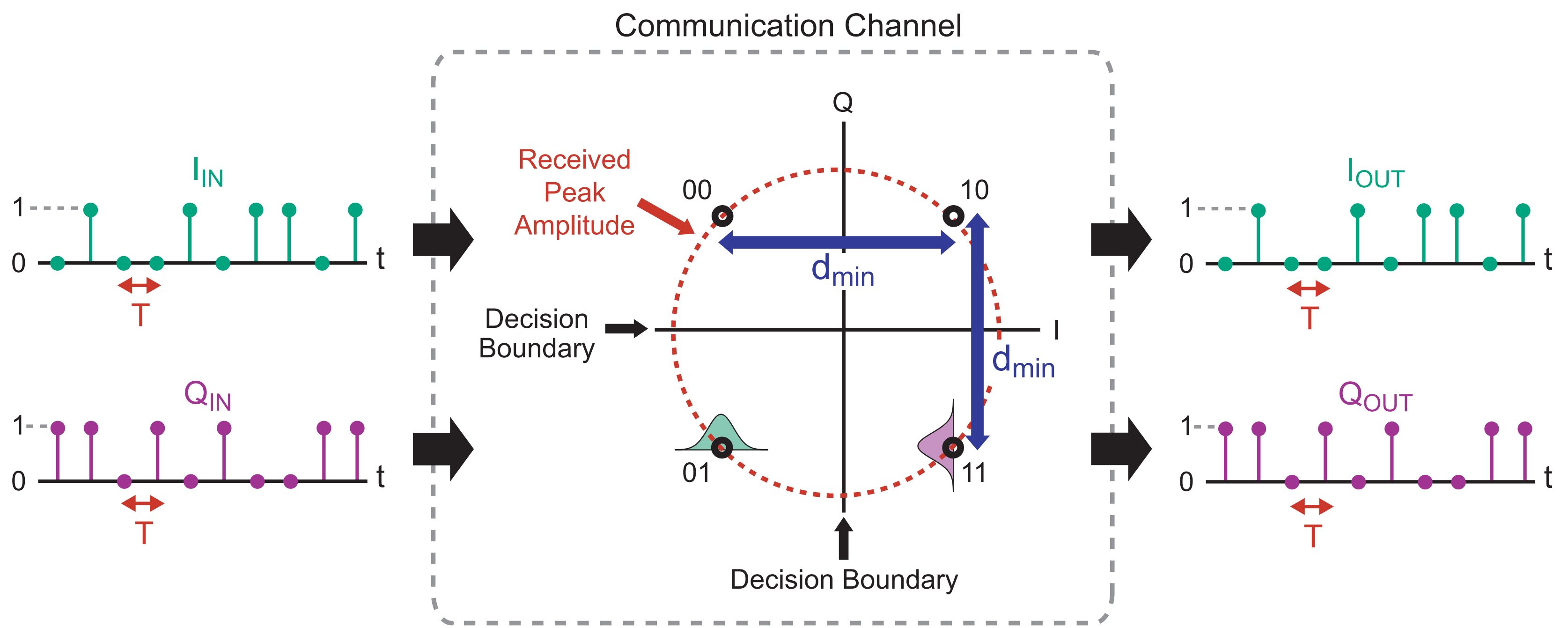






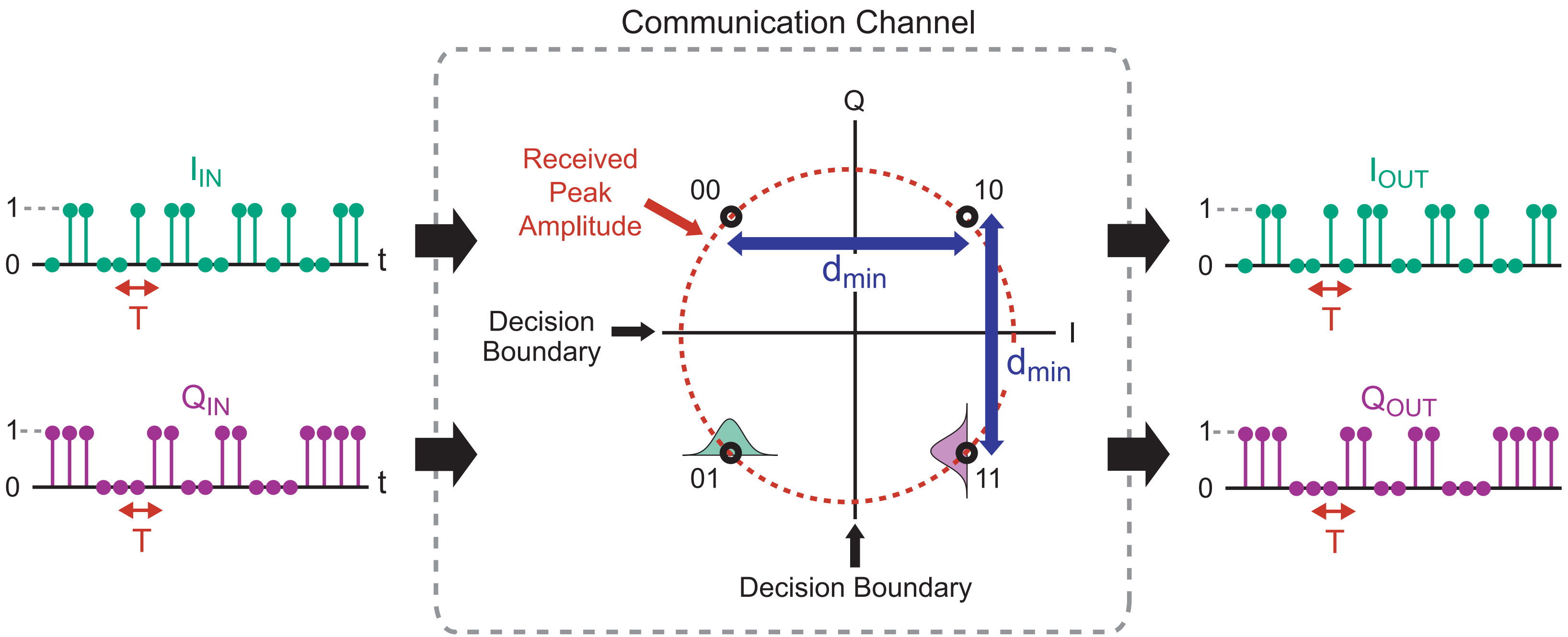
- Bit error rate P_e
- SNR (dB) = $10\log_{10}\left(\frac{\left(\frac{d_{min}}{2}\right)^2}{\sigma^2}\right)$
- Gaussian distribution of noise

Shannon Capacity



- In 1948, Claude Shannon proved that:
  - Digital communication can achieve **arbitrarily-low bit error rates if appropriate coding** methods are employed
  - The ***capacity,*** or **maximum rate** of a Gaussian channel with bandwidth *BW* **to support arbitrarily-low bit error rate communication** is:
    - $C = BW \log_2(1 + SNR)$ bits/second (SNR in **linear scale units**)

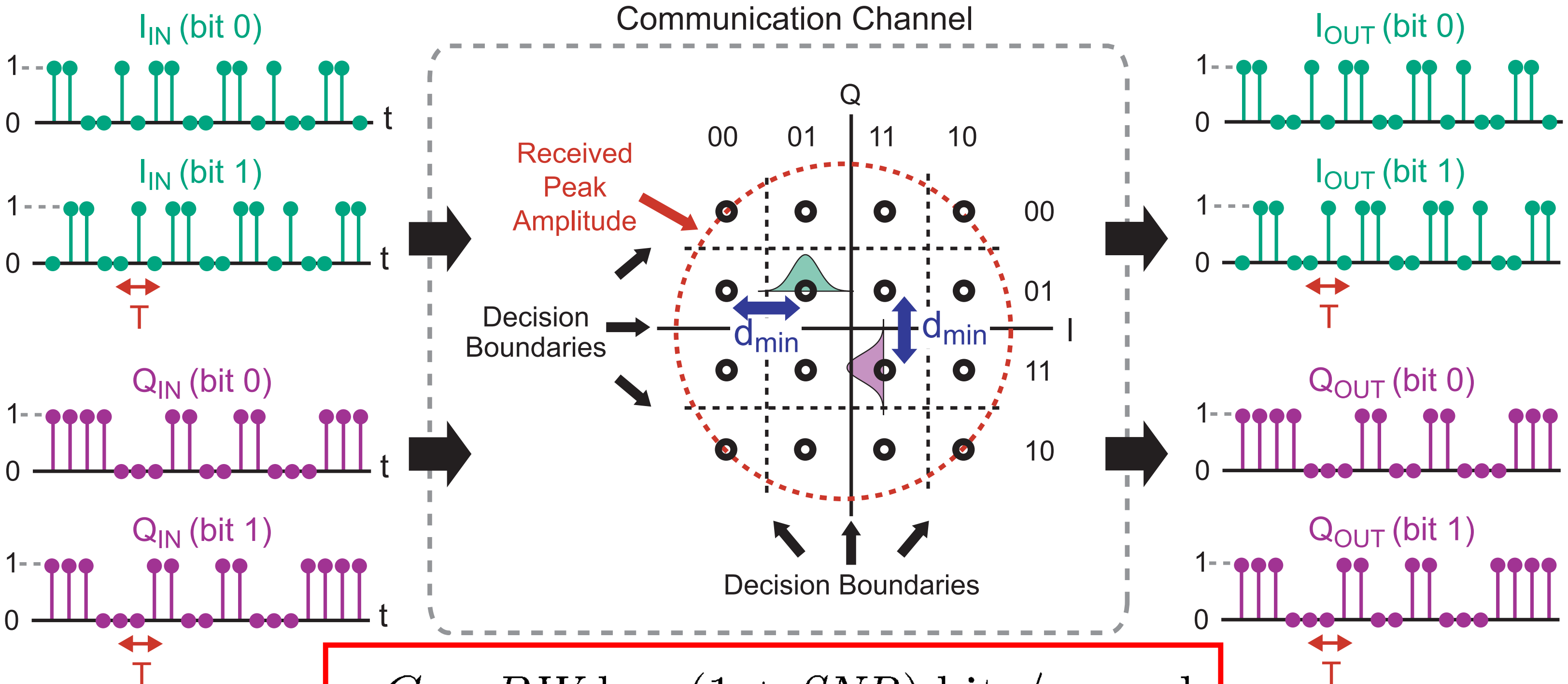
Impact of Channel Bandwidth on Capacity



$$C = BW \log_2(1 + SNR) \text{ bits/second}$$

- A doubling of bandwidth allows twice the number of bits to be sent in time T
  - Capacity (bits/second) increases linearly with bandwidth

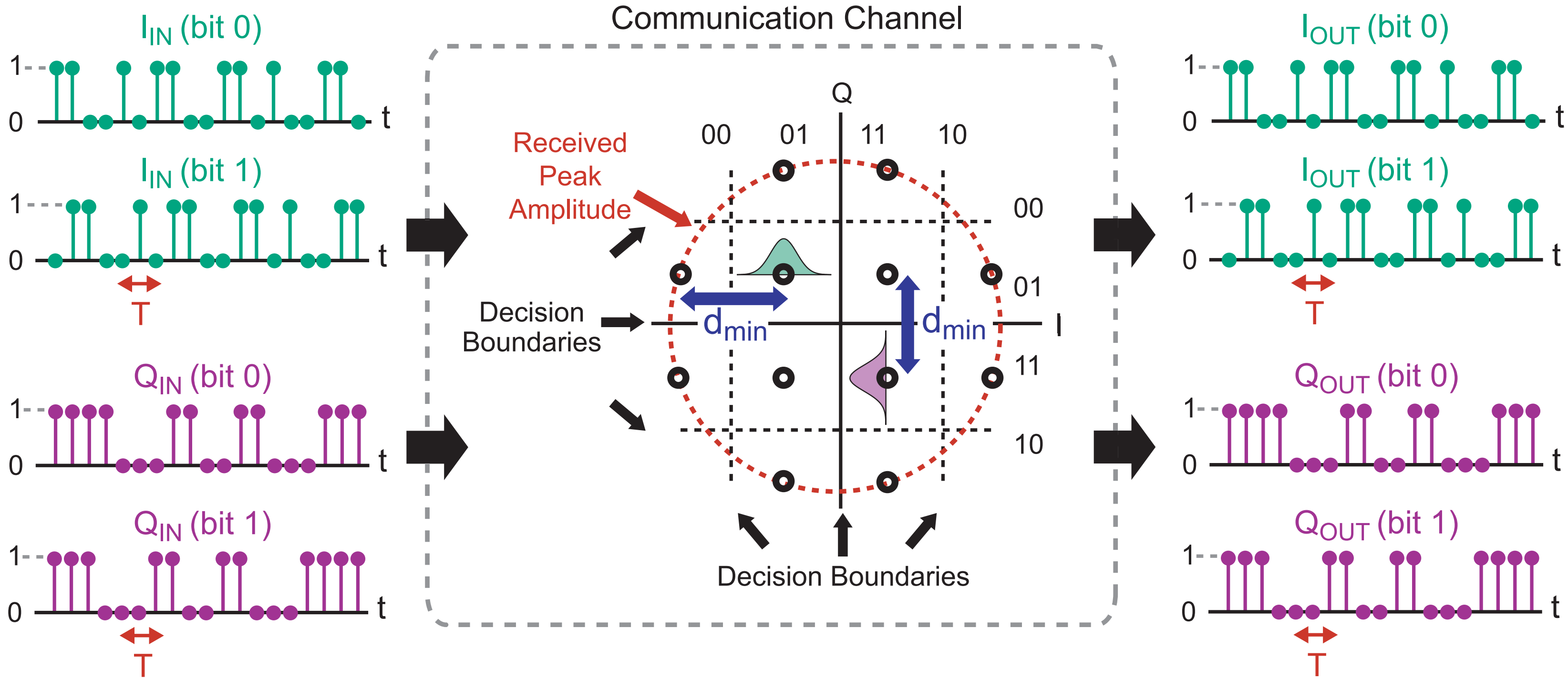
Impact of SNR on Capacity



$$C = BW \log_2(1 + SNR) \text{ bits/second}$$

- A higher SNR allows more bits to be sent per symbol
  - Adding n bits requires adding 2^n constellation points
    - Therefore leads to d_{min} being reduced by a factor of 2^n
  - **High SNR (>> 1):** Capacity **increases linearly** with **SNR** (dB, log scale)

Constellation Design (Symbol Packing)



- **Objective:** Design constellation to maximize d_{min} while packing as many points in as possible
  - Maximizing d_{min} achieves lowest uncoded error rate
  - Maximizing number of constellation points achieves highest uncoded data rate (bits/second)

Summary

- Constellation diagrams allow intuitive approach of quantifying uncoded bit error rate of a channel
  - Function of SNR and number of constellation points

- A digital communication channel can be viewed in terms of a binary signaling model
  - Focuses attention on **key issue of bit error rate**

- Coding theoretically allows **arbitrarily low bit error rate performance** of a practical digital communication link

Friday Precept:
Practical 802.11 PHY

Tuesday Topic:
The Wireless Channel