Receiver Design and Performance; Shannon Capacity

COS 463: Wireless Networks
Lecture 13

Kyle Jamieson

[Parts adapted from H. Balakrishnan, M. Perrott, C. Terman]
Today

1. Receiver architecture
   – Tradeoffs between ISI and Noise
   – Common filter design: Raised Cosine

2. Bit error rate and Shannon Capacity
Review of Digital I/Q Modulation

- Leverage **analog** communication channel to send **discrete-valued symbols**
  - e.g. send symbol from {-3,-1,1,3} on both I and Q channels every symbol period

- At receiver, **sample I/Q waveforms** every symbol period
  - **Associate each sampled I/Q value with symbol** from set, on both I and Q channels
Transmit and Receive Filters

• **Last time:** Transmit filter
  – Tradeoff between transmitted bandwidth and intersymbol interference (ISI)

• **This time:** Receive filter (previously assumed very wide bandwidth so as not to influence ISI)
Tools for Examining ISI

- **Eye Diagram**
  - For **transition behavior** between symbols
  - ISI causes **eye to close**

- **Constellation Diagram**
  - Shows aggregate placement of sampled I/Q values
  - **ISI spreads** the constellation points
Impact of Receiver Noise

- **Receiver noise** adds to desired I/Q signals, causes **corruption**
  - Eye closes **further**
  - Constellation points spread

- **Key insight:** Lowering the receive filter bandwidth improves the rejection of **background noise**
Impact of Lower Receiver Filter Bandwidth

- **Primary noise source:** Thermal noise in receive circuits
  - Assume noise *white* (*i.e.*, flat frequency spectrum)

- Receive filter **only** passes noise within its **passband**

- How much can we **lower receive filter bandwidth**?
ISI Versus Noise

- **Lowering receive filter bandwidth** too much again causes **ISI** to dominate

- Selection of receive filter bandwidth involves a **tradeoff between ISI, noise**:
  - Bandwidth too **high**: High Noise
  - Bandwidth too **low**: High ISI
Joint Transmit/Receive ISI Analysis

- Both transmit and receive filters influence ISI
  - Combined filter response: $G(2\pi f) = P(2\pi f) H(2\pi f)$
Viewing Filtering in the Time Domain

- **Filtering operation** corresponds to \textit{convolution} in the time domain with impulse response.

- Time domain view allows us to more clearly see \textit{impact of overall filter on ISI}
Impulse Response and ISI: High Bandwidth

- Receiver **samples** I/Q every **symbol period**
  - Achieving zero ISI requires that **each** symbol influence only **one sample** at the combined filter output

- **Issue:** **Want lower overall filter bandwidth** to reduce spectrum bandwidth and lower noise
  - But this causes **smoothing of** $g(t)$
Impulse Response and ISI: Low Bandwidth

- Smoothed impulse response has a **span longer than one symbol period**
  - Convolution reveals that **each symbol impacts filter output at > 1 sample value**

- **Inter-symbol interference occurs**
A More Direct View of the ISI Issue

- Consider impact of just one symbol
  - Samples at filter output more clearly show the impact of the one symbol on other sample values
The Nyquist Criterion for Zero ISI

- Sample the impulse response of the overall filter at the symbol period
  - **Nyquist Criterion**: Resulting samples must have only one non-zero value to achieve zero ISI

- Can we design impulse response to span more than one symbol period and still meet the Nyquist Criterion for Zero ISI?
Raised Cosine Filter

- Raised cosine filter achieves **low bandwidth and zero ISI**
  - Impulse response **spans more than one symbol**, but has **only one non-zero sample** value
  - Impulse response: \( g(t) = \frac{\sin(\pi t/T) \cos(\alpha \pi t/T)}{\pi t/T} \frac{1}{1-(2\alpha t/T)^2} \)
Parameter \( \alpha (0 \leq \alpha \leq 1) \) is referred to as the roll-off factor of the filter.

- Smaller values of \( \alpha \) lead to:
  - Reduced filter bandwidth
  - Increased duration of the filter impulse response

Regardless of \( \alpha \), the raised cosine filter achieves zero ISI.
Impact of Large $\alpha$ on Eye Diagram

- **Large roll-off factor** leads to nice, open eye diagram

- **Key observation:** Achieving zero ISI requires precise placement of sample times
  - Error in placement of sample times leads to substantial ISI
Impact of Small $\alpha$ on Eye Diagram

- Small roll-off factor reduces the filter bandwidth and still allows zero ISI to be achieved.

- **Issue:** Greater sensitivity to sample time placement than for large $\alpha$
  - Needs greater receiver complexity to ensure precise sample time placement.
Transmitter and Receiver Filter Design

- Overall response corresponds to
  \[ G(j2\pi f) = P(j2\pi f)H(j2\pi f) \]

  - How to choose P and H?
Matched Filter Design

- Setting $P(j2\pi f) = H(j2\pi f)$ yields a **matched filter** design
  - Each filter chosen to be a **square-root** raised cosine filter
1. Receiver architecture
   - Tradeoffs between ISI and Noise
   - Common filter design: Raised Cosine

2. Bit error rate and Shannon Capacity
Review of Digital Modulation

- Transmitter sends discrete-value signals over analog communication channel

- Receiver samples recovered baseband signal
  – Noise and ISI corrupt received signal

- Key techniques:
  – Properly design transmit and receive filters for low ISI
  – Sample and slice received signals to detect symbols
A Closer Look at the Transmitter

- Amplitude of I/Q transmit signals impact power of transmitted output
  - Output power limited within a given spectral band
  - Low output power desirable for portable applications (battery life)
A Constellation View of the Transmitter

Provides intuitive view of the relationship between symbol separation and transmitted power
A Constellation View of Receiver

- Provides an intuitive view of relationship between symbol separation, received signal power, noise.
Impact of SNR on Receiver Constellation

- SNR is influenced by transmitted power, distance between transmitter & receiver, and background noise
Impact of Increased signal on Constellation

- **Increase** in received signal power leads to **increased separation** between symbols
  - SNR improved if and when **noise level unchanged**

![Diagram showing RF Transmission and Reception with constellation diagram](image-url)

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<th>tx(t)</th>
<th>RF Transmitter</th>
<th>TX(j2πf)</th>
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*Figure showing increased signal power with improved separation between symbols.*
Quantifying the Impact of Noise

• Distribution of noise: zero-mean Gaussian distribution
  – Variance of noise determines the width of the Gaussian

• Minimum separation between symbols: $d_{\text{min}}$
  – Bit errors occur when noise moves a symbol by more than $\frac{1}{2} d_{\text{min}}$
Impact of Reduced SNR

- Lower SNR leads to reduced value for $d_{\text{min}}$
- Leads to a higher bit error rate
  - Assuming noise variance unchanged
  - Assuming received signal power reduced
Impact of Constellation Size Reduction

- Reducing the number of symbols leads to an increased value for $d_{\text{min}}$
- Leads to a lower bit error rate
  - Assuming signal power, noise variance constant
Can we Estimate Bit Error Rate?

- Bit Error Rate depends on:
  - SNR (ratio of received signal power to noise variance)
  - Number of constellation points
    - Sets $d_{\text{min}}$, given a received signal power level
Let’s Start with a Detailed System View

- Assumptions: No ISI, four-point constellation
A Closer Examination of Signal and Noise

Communication Channel

Baseband Input

Transmit & Receive Pair

Receiver Output

Sample & Slice

Communication Channel for Q Channel

PDF of Noise

\[ f_X(x) \]

Variance = \( \sigma^2 \)

Decision Boundary

Slicer

Q_{\text{signal}}

Q_{\text{noise}}

Q_{\text{received}}

Q_{\text{IN}}

Q_{\text{OUT}}

I_{\text{IN}}

I_{\text{OUT}}

Q_{\text{IN}}

Q_{\text{OUT}}

Decision Boundary

Slicer

PDF of Noise

\[ f_X(x) \]

Variance = \( \sigma^2 \)

Decision Boundary

Slicer

PDF of Noise

\[ f_X(x) \]

Variance = \( \sigma^2 \)
The Binary Symmetric Channel Model

- Provides a binary signaling model of channel
Computation of SNR

\[ \text{Signal Variance} = \left( \frac{d_{\text{min}}}{2} \right)^2 \]

\[ \text{Noise Variance} = \sigma^2 \]

\[ \Rightarrow \quad \text{SNR} (dB) = 10 \log \left( \left( \frac{d_{\text{min}}}{2} \right)^2 / \sigma^2 \right) \]
Resulting Bit Error Rate Versus SNR

Communication Channel for Q Channel

**PDF of Received Q Sample**

- Transmitted 0
  - $f_{Q_0}(y)$
  - Probability of Bit Error $= P_e$

- Transmitted 1
  - $f_{Q_1}(x)$

- Gaussian distribution of noise

- Bit error rate $P_e$

- SNR (dB) = $10 \log_{10} \left( \frac{d_{min}^2}{\sigma^2} \right)$

Bit Error Rate versus SNR for Q Channel
In 1948, Claude Shannon proved that:

– Digital communication can achieve **arbitrarily-low bit error rates** if **appropriate coding** methods are employed

– The **capacity**, or **maximum rate** of a Gaussian channel with bandwidth $BW$ to support **arbitrarily-low bit error rate communication** is:
  
  $$ C = BW \log_2 (1 + SNR) \text{ bits/second (SNR in linear scale units}) $$
Impact of Channel Bandwidth on Capacity

- A doubling of bandwidth allows twice the number of bits to be sent in time $T$
  - Capacity (bits/second) increases linearly with bandwidth

$$C = BW \log_2(1 + SNR) \text{ bits/second}$$
Impact of SNR on Capacity

- A higher SNR allows more bits to be sent per symbol
  - Adding $n$ bits requires adding $2^n$ constellation points
  - Therefore leads to $d_{\text{min}}$ being reduced by a factor of $2^n$
- High SNR ($>> 1$): Capacity increases linearly with SNR (dB, log scale)

$$C = BW \log_2(1 + SNR) \text{ bits/second}$$
Objective: Design constellation to maximize $d_{\min}$ while packing as many points in as possible
- Maximizing $d_{\min}$ achieves lowest uncoded error rate
- Maximizing number of constellation points achieves highest uncoded data rate (bits/second)
Summary

- Constellation diagrams allow intuitive approach of quantifying uncoded bit error rate of a channel
  - Function of SNR and number of constellation points

- A digital communication channel can be viewed in terms of a binary signaling model
  - Focuses attention on **key issue of bit error rate**

- Coding theoretically allows **arbitrarily low bit error rate performance** of a practical digital communication link
Friday Precept:
Practical 802.11 PHY

Tuesday Topic:
The Wireless Channel