From AM Radio to Digital I/Q Modulation

COS 463: Wireless Networks
Lecture 12

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[Parts adapted from H. Balakrishnan, M. Perrott, C. Terman]

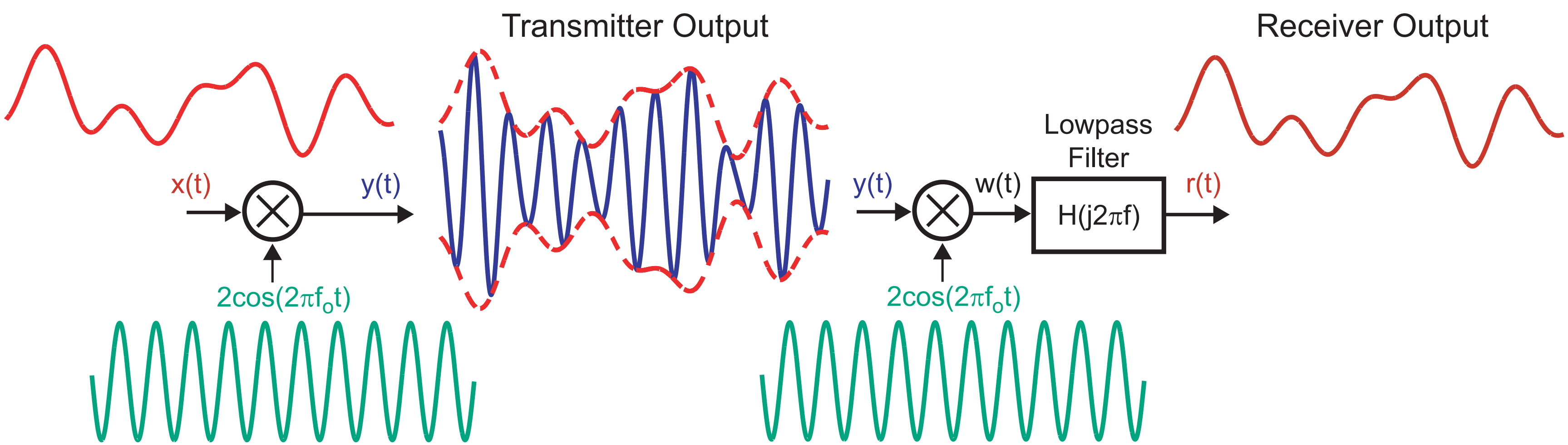
Roadmap

1. **Analog I/Q modulation**

2. Discrete-time processing of continuous signals
   – The Digital Abstraction
   – Quantization: Discretizing *values*
   – Sequences: Discretizing *time*

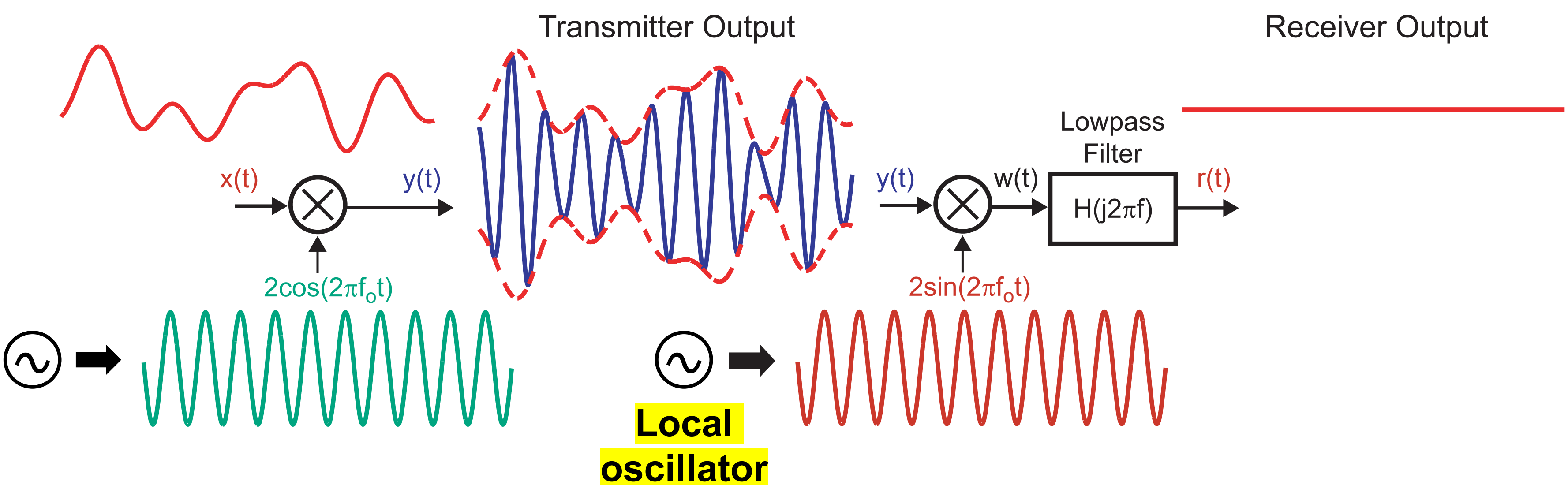
3. Digital I/Q Modulation

Review: AM Modulation & Demodulation



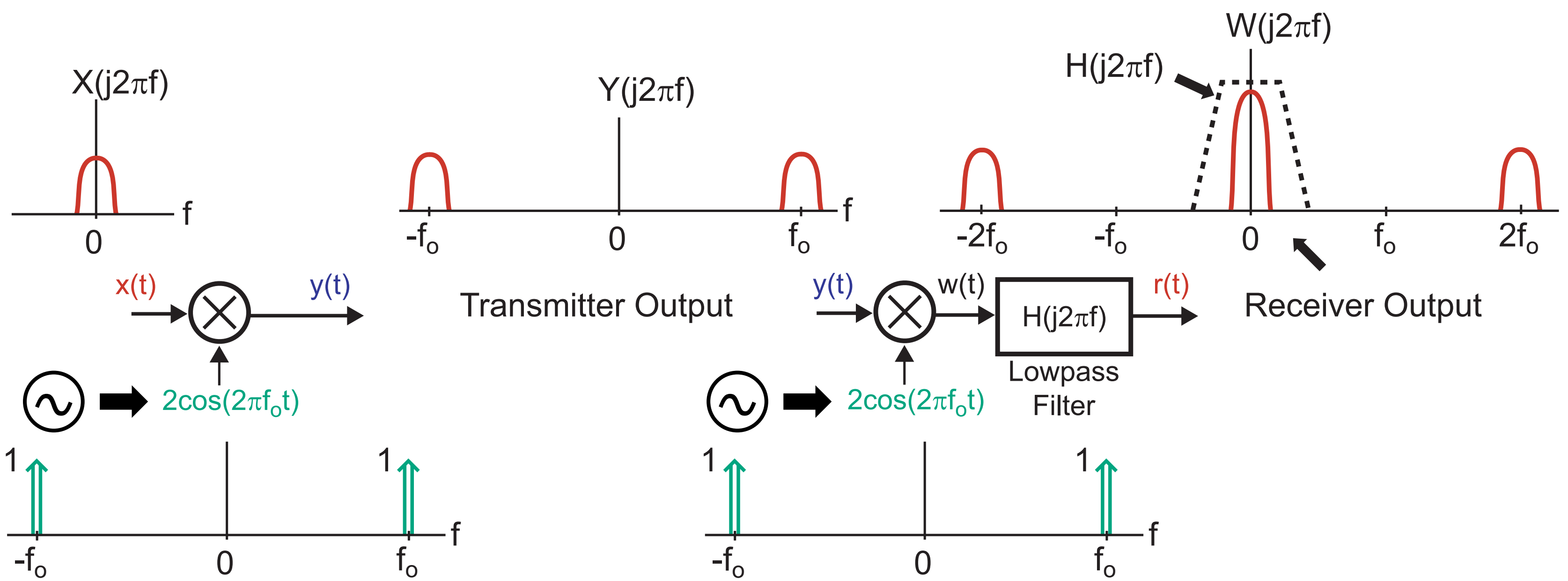
- **Multiplication** (*i.e.* ***mixing***) operation **shifts in frequency**
- **Low-pass filtering** passes **only the desired baseband signal** at the receiver

Impact of a 90º phase shift



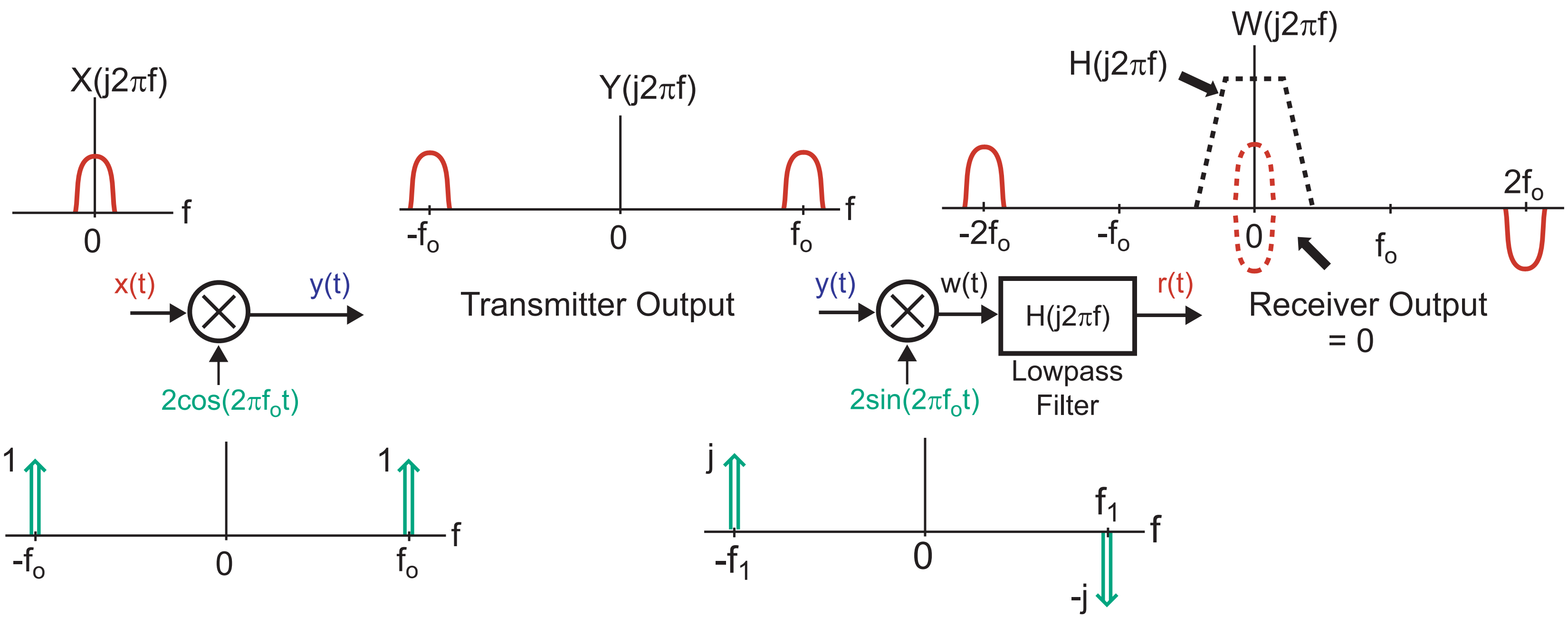
- **Receiver** uses **sine** instead of **cosine: no output**
  - $\sin(2\pi f_0 t)\cos(2\pi f_0 t) = ½ \sin(4\pi f_0 t)$
- Need to **synchronize phase** of transmitter and receiver ***local oscillators*** (***coherent*** demodulation)

Coherent Demodulation: Frequency-domain analysis



- Transmitter and receiver oscillators **phase-synchronized**
- Demodulated copies **add constructively at baseband**

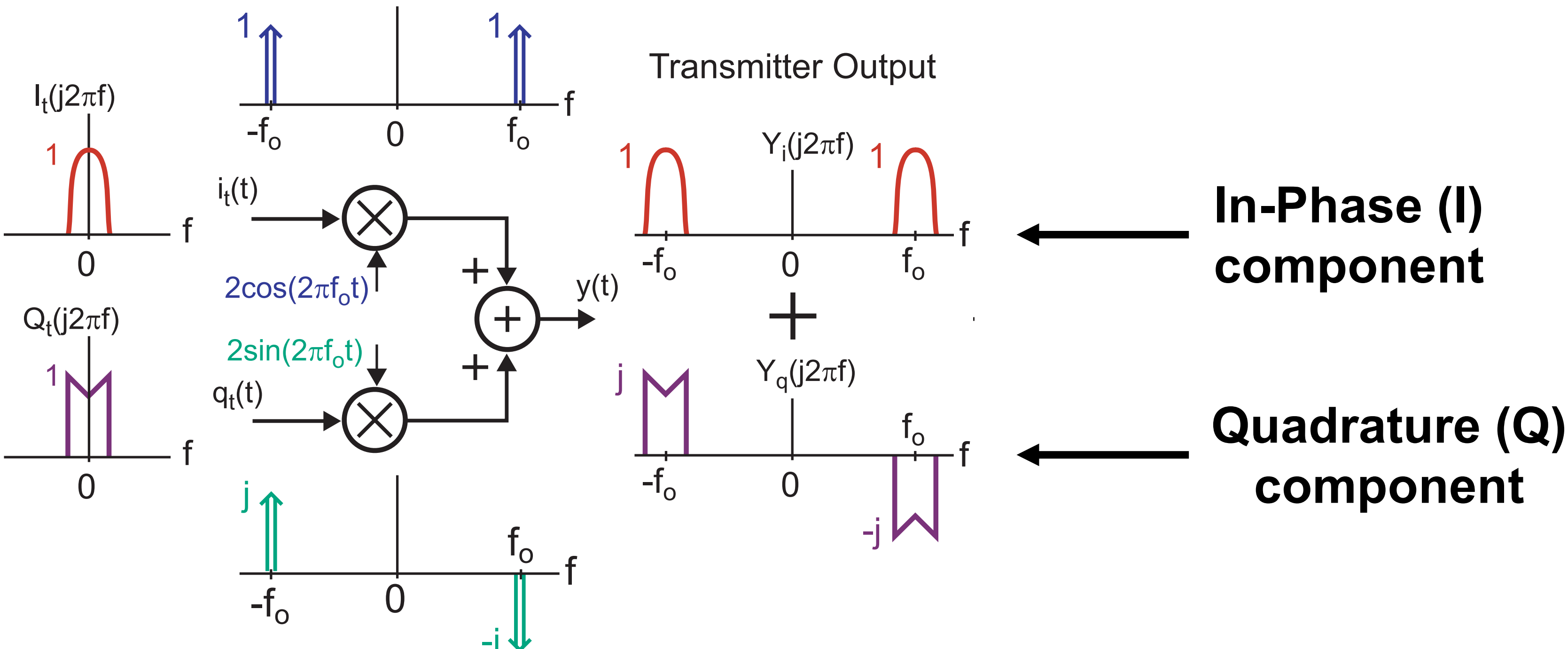
90º Phase Shift: Frequency-domain analysis



- Transmitter, receiver oscillators **are offset in phase by 90º**
- Demodulated copies **add destructively at baseband**
  - **Zero output from receiver**
- **Two separate channels!**

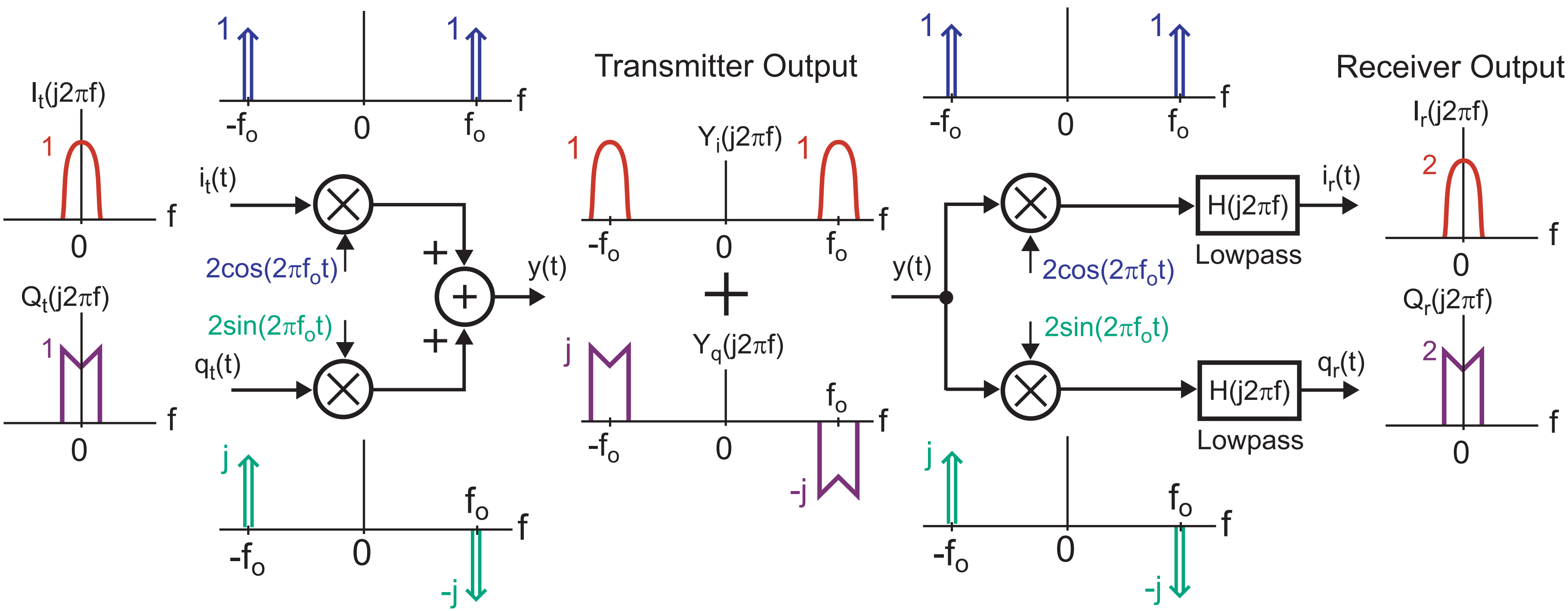
In-Phase/Quadrature Modulation

- Modulate each with a cosine & sine, **sum** the result



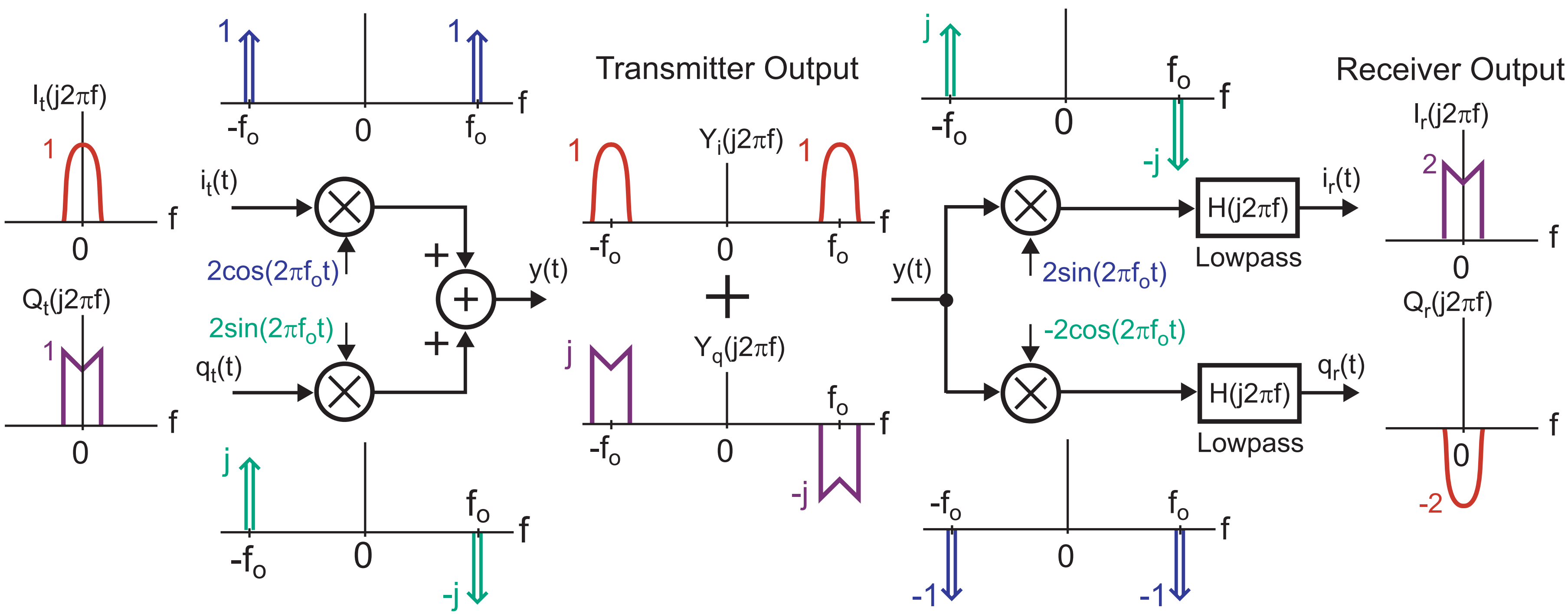
- I, Q signals occupy the **same frequency band**
  - One is **real (for cos)**, one is **imaginary (for sin)**

In-Phase/Quadrature Demodulation



- Demodulate with **both** a sine and a cosine
  - **Both I and Q channels are recovered!**

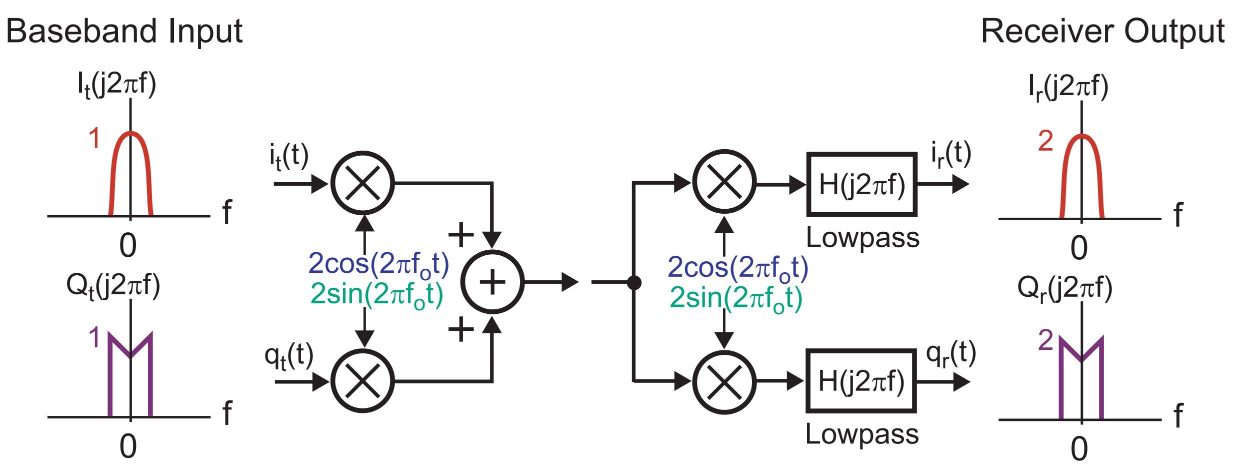
I/Q Demodulation: 90º Phase Shift



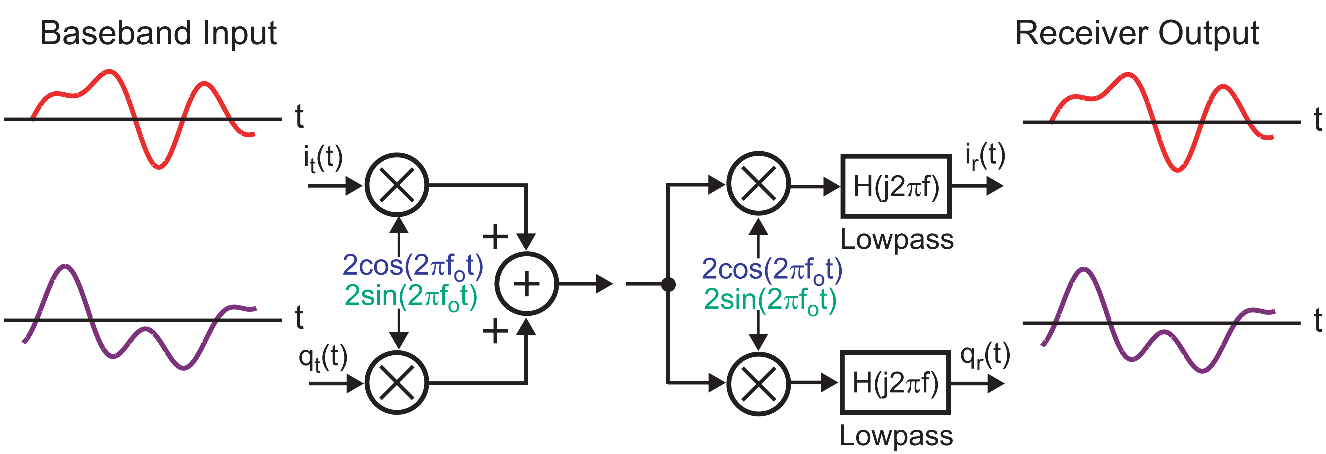
- I and Q channels get **swapped** at receiver
  - **Key observation:** No information is lost!

Summary of Analog I/Q modulation

- **Frequency domain view**



- **Time domain view**



I/Q modulation: Wrap-up

- I/Q modulation allows **twice the amount of information** to be sent **compared to basic AM**

- Impact of **phase offset** is to **swap I/Q**

- Impact of **frequency offset** is I/Q swapping (**small offset**)
  - Or **catastrophic corruption** (large offset) of received signal

Roadmap

1. Analog I/Q modulation

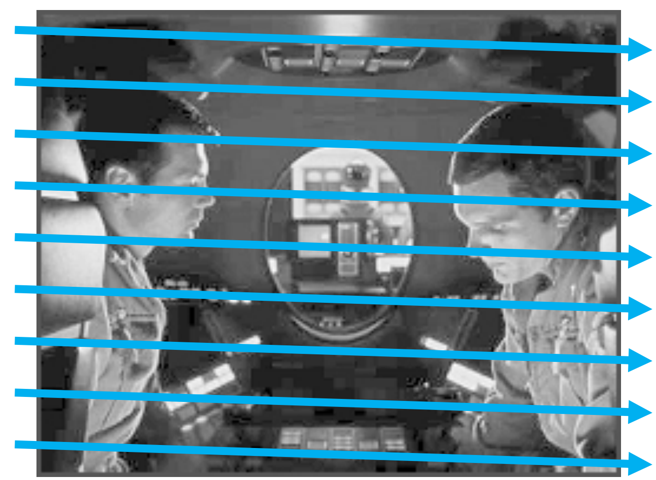
2. **Discrete-time processing of continuous signals**
   - **The Digital Abstraction**
   - Quantization: Discretizing *values*
   - Sequences: Discretizing *time*

3. Digital I/Q Modulation

Representing information with voltage

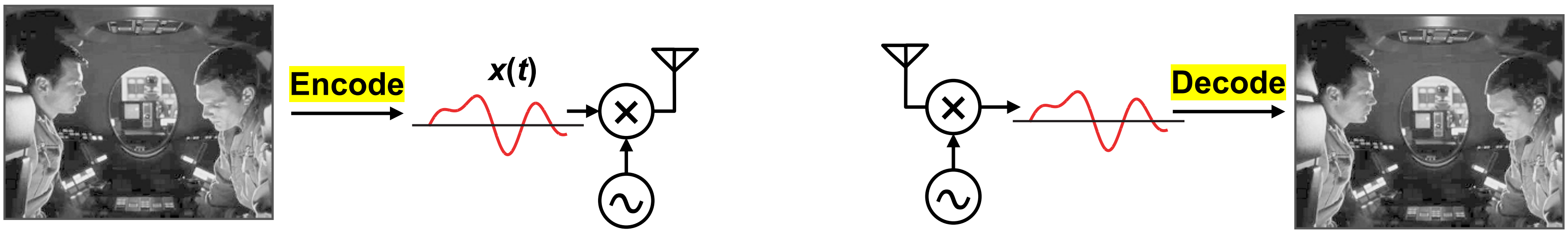
- **Encoding of pixel** at location (x, y) in a Black & White picture:
  - 0 Volts = Black
  - 1 Volt = White
  - 0.37 V = 37% Gray
  - etc...

- **Encoding of a picture:**
  - Scan points in prescribed order
  - Generate a continuous voltage waveform (**baseband signal**)

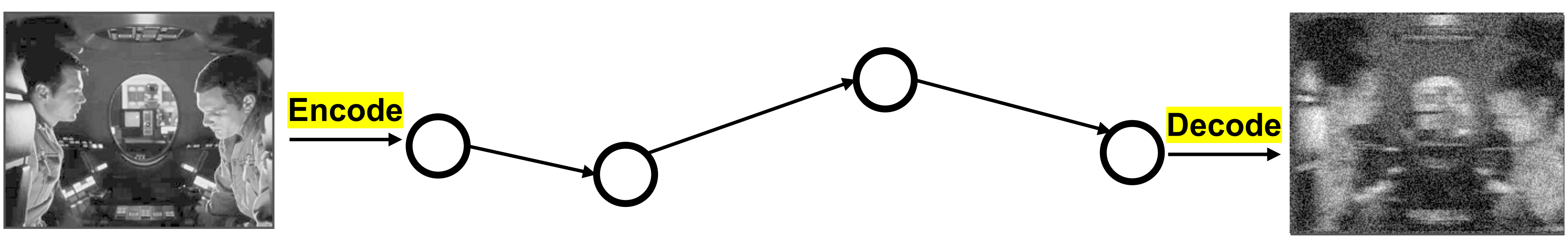


Let’s build a P2P television network

One hop:



Many hops:



Why did our network fail?

1. Transmitter doesn't work right
2. Receiver doesn't work right
3. Theory is imperfect
4. System architecture isn't right

- **Answer:** All of the above!

- Noise, transmitter/receiver imperfections **inevitable**, so must **design system to tolerate** some amount of error

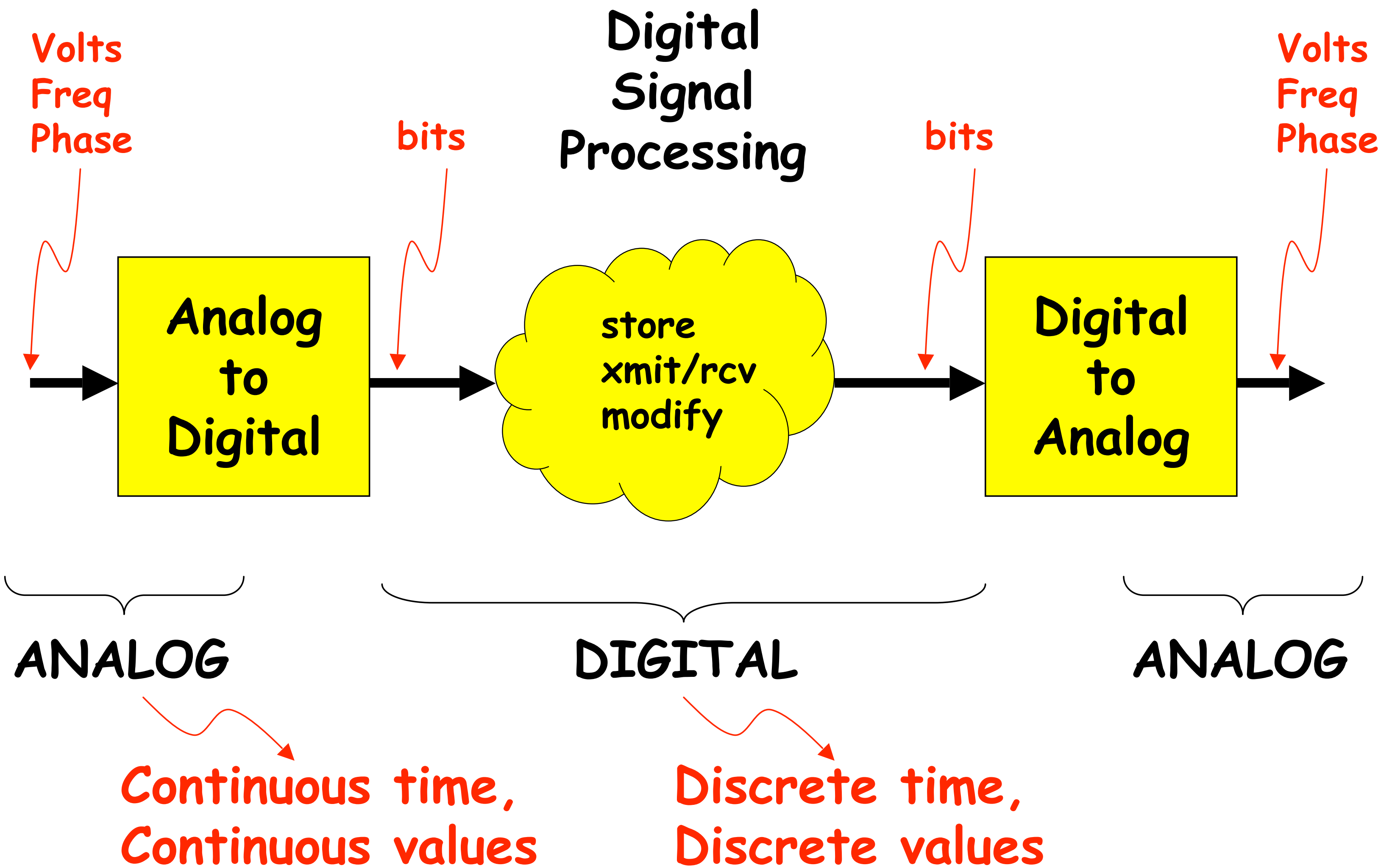
Analog communications issues

- **Problem:** It's hard to **distinguish legitimate analog waveforms from corrupted** ones
  - Every waveform is potentially legitimate!

- Small errors accumulate at each hop

- Endpoints can't help, so need to **eliminate errors at each hop**

Plan: *Mixed Signal* Architecture



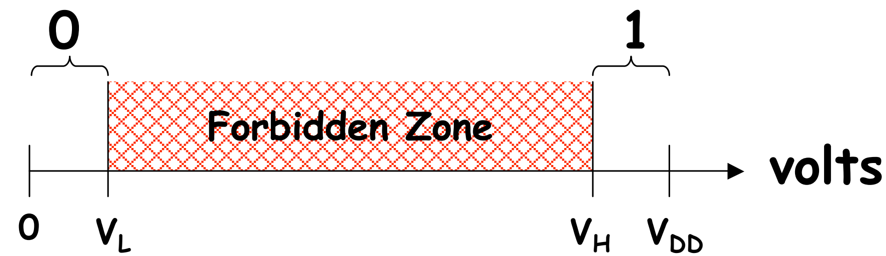
Discrete time, Discrete values

- Continuous-time, continuous-valued **waveform** is being converted into a discrete-valued **sequence**
  - **Only certain values** are ”allowed” to be used
- Questions yet to be answered:

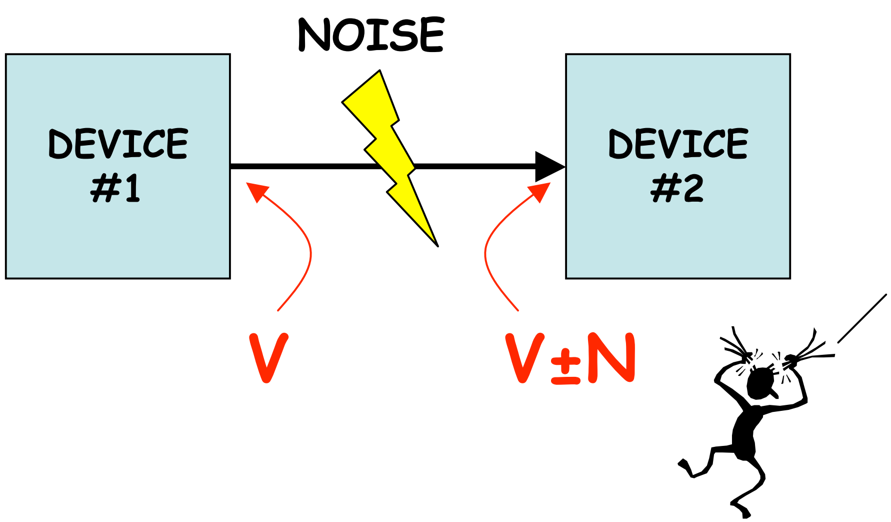
1. **How many discrete values** should we use?
2. **How rapidly in time** should we sample the waveform?

Digital signaling

- **First encoding attempt:**

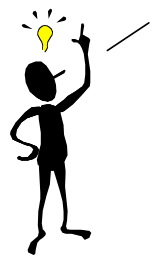


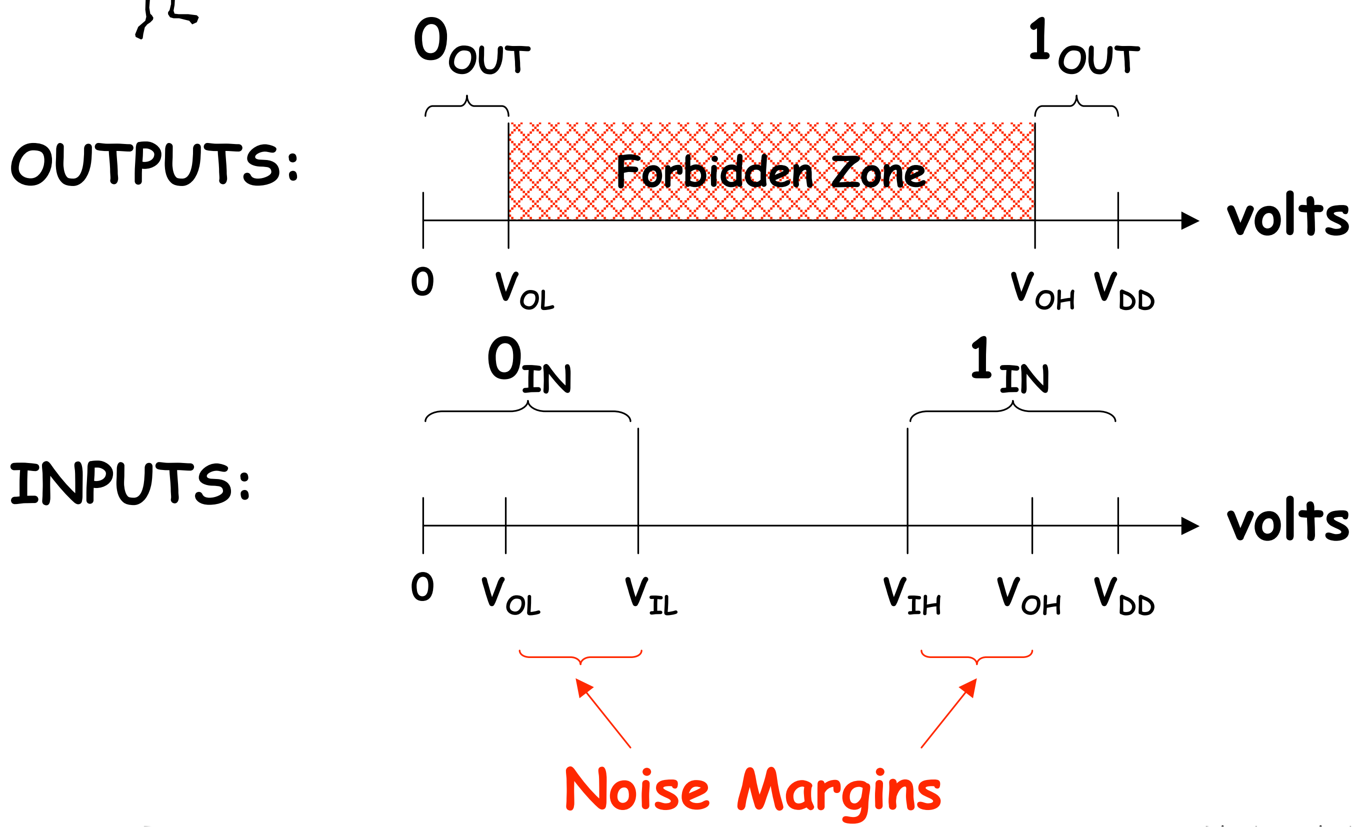
- But, what happens in the **presence of noise?**



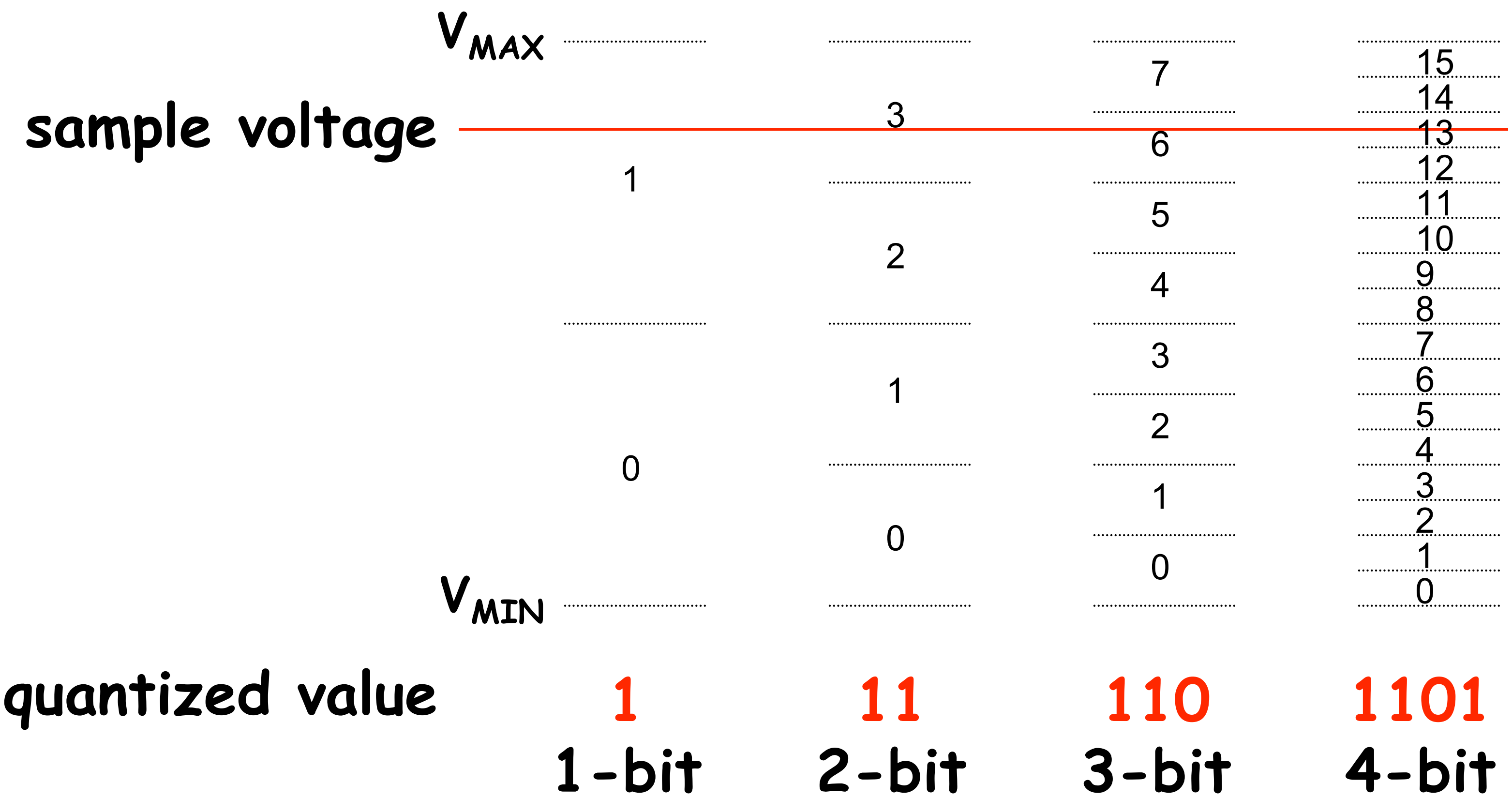
So an output voltage just below V_L might become an illegal input voltage in the forbidden zone!

Big Idea: Noise Margins

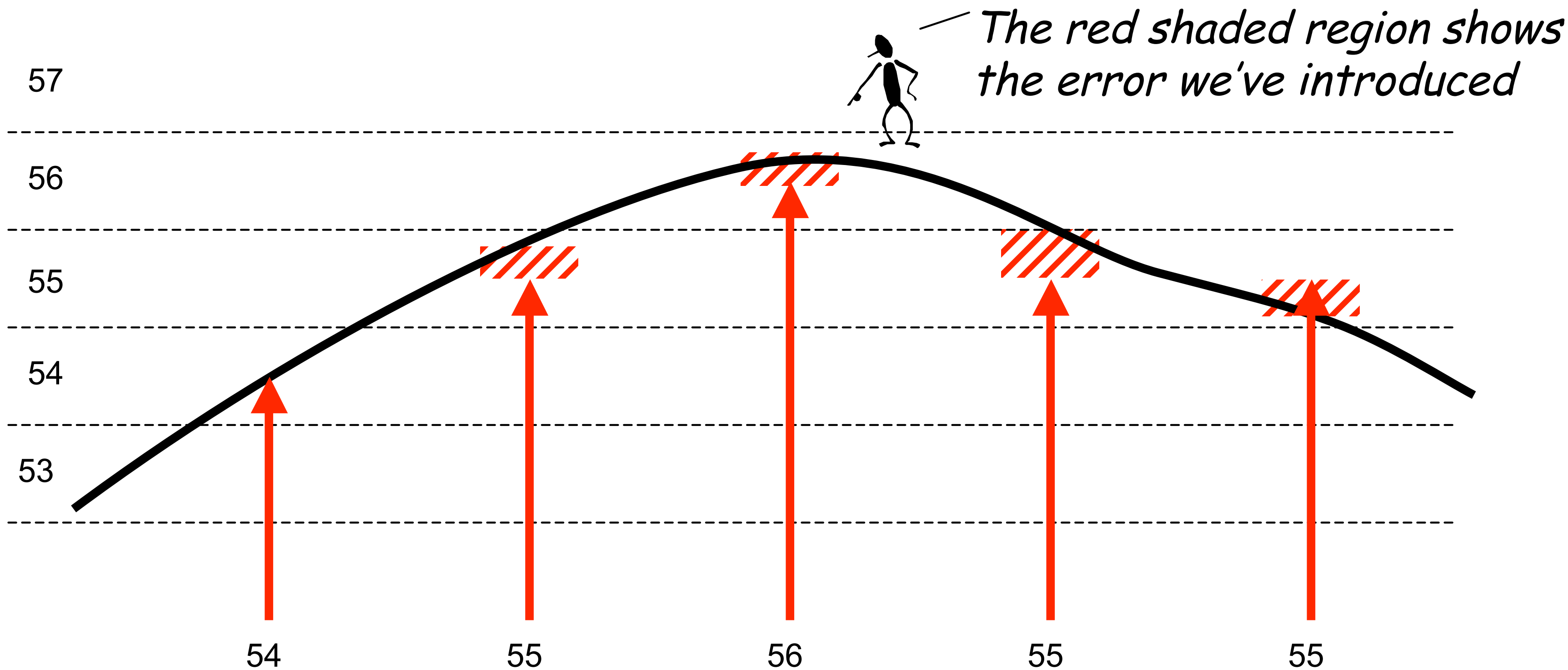




How many discrete values?



Quantization Error



- Introduce **at most ½ interval length** of quantization error

Roadmap

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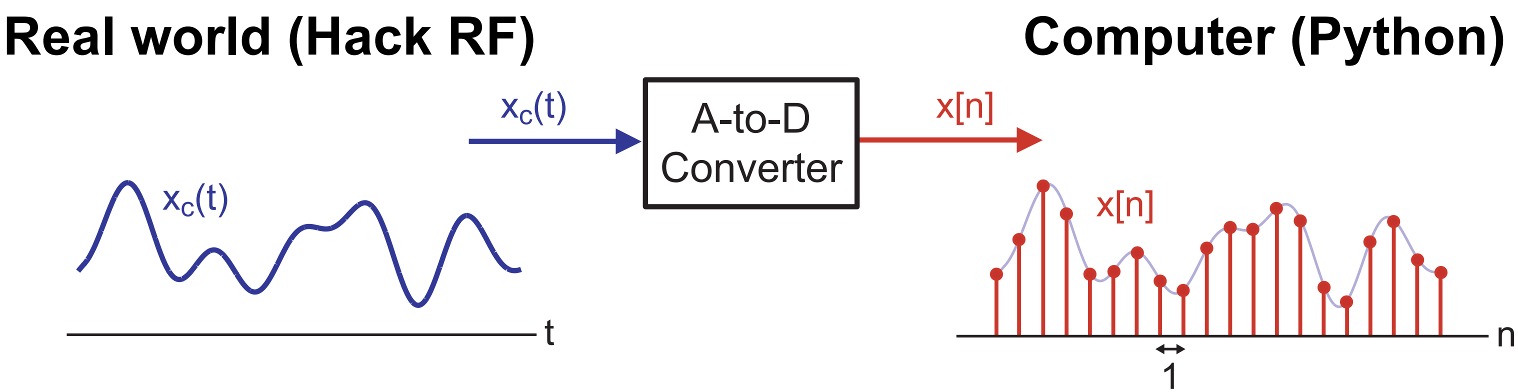
   - The Digital Abstraction
   - Quantization: Discretizing *values*
   - **Sequences: Discretizing *time***

3. Digital I/Q Modulation

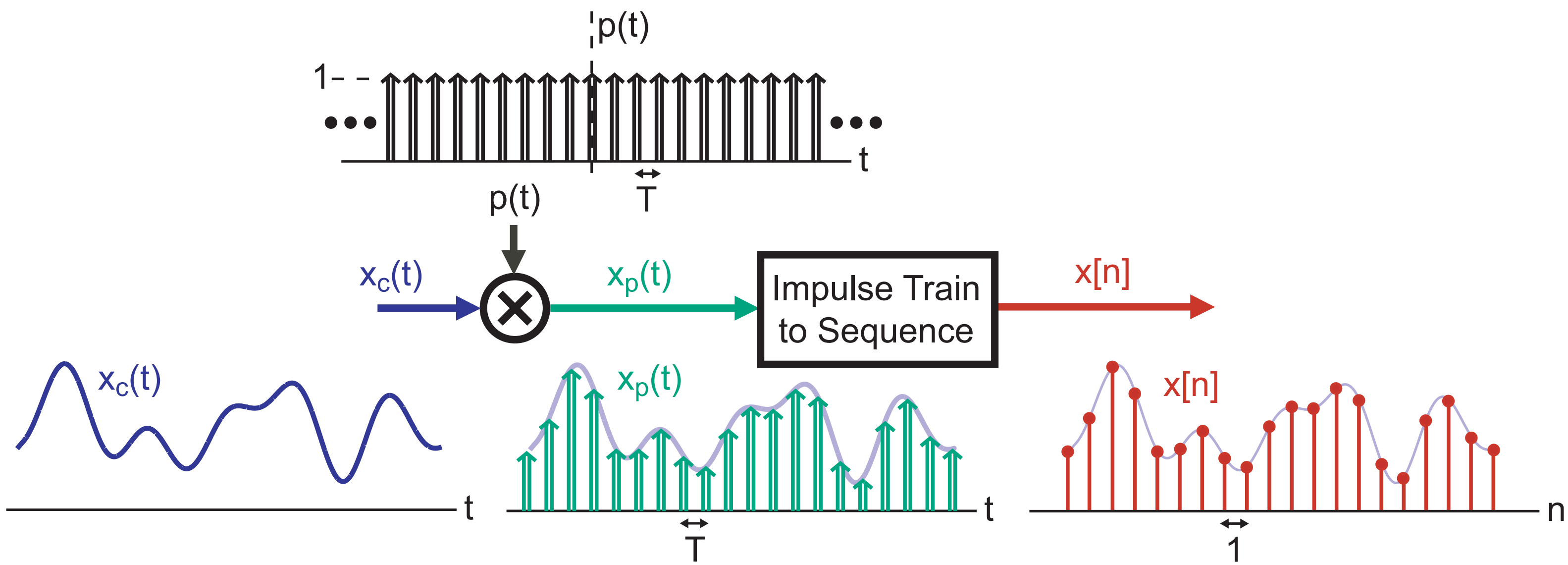
Need for Continuous to Discrete Conversion



- The boundary between analog and digital
  - Real world filled with continuous-time signals
  - Computers operate on **discrete-time sequences**

- Crossing the analog to digital boundary requires **conversion between the two**

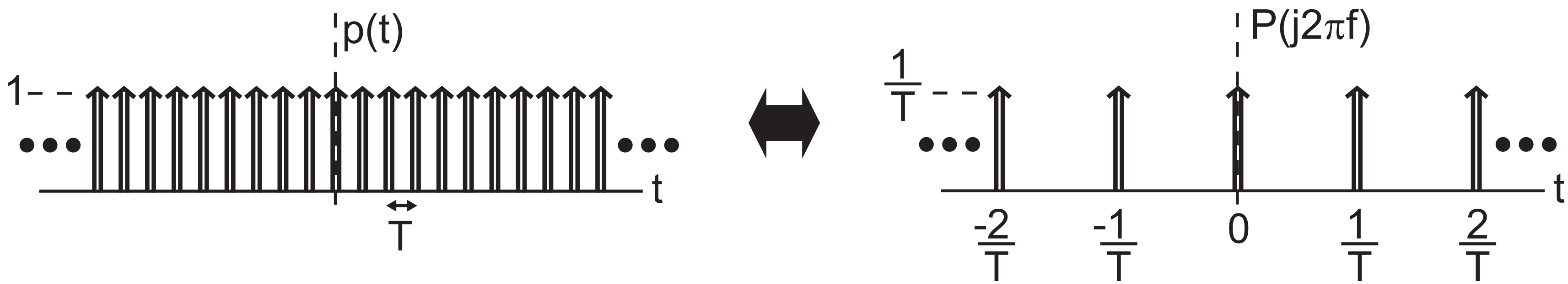
Modelling Continuous-to-Discrete Conversion



- Two-step process:

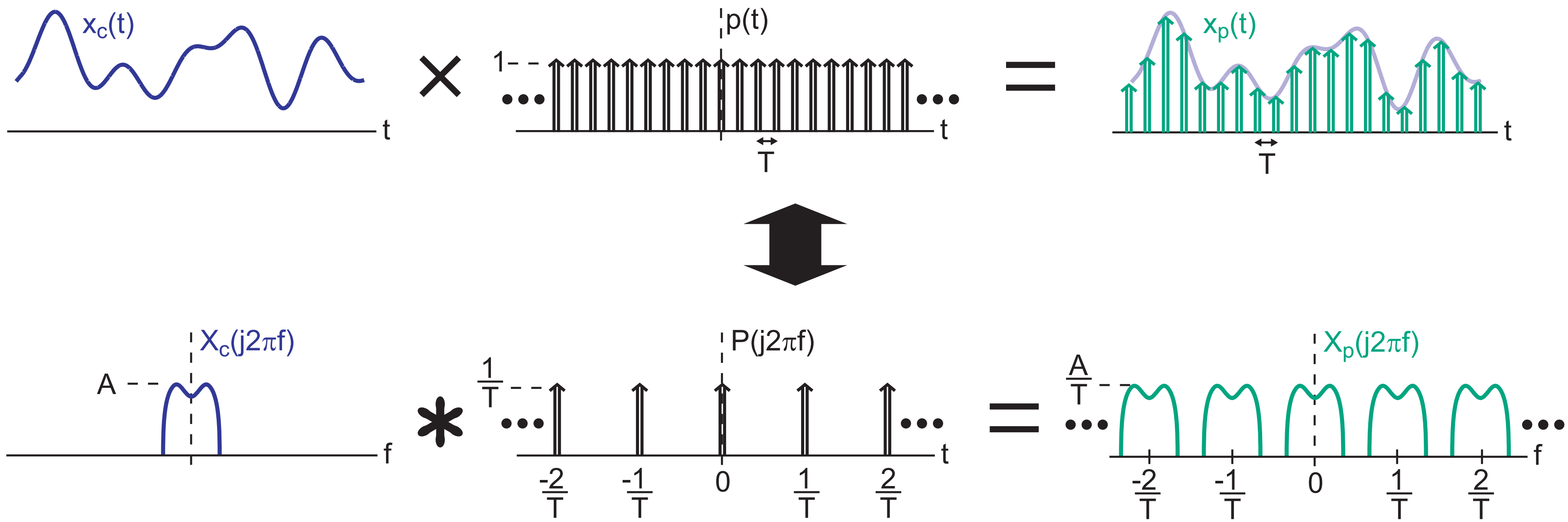
1. Sample continuous-time signal every ***T*** seconds
   – Model as multiplication of signal with impulse train
2. Create sequence from amplitude of scaled impulses
   – Model as rescaling of time axis (T → 1)

Fourier Transform of Impulse Train



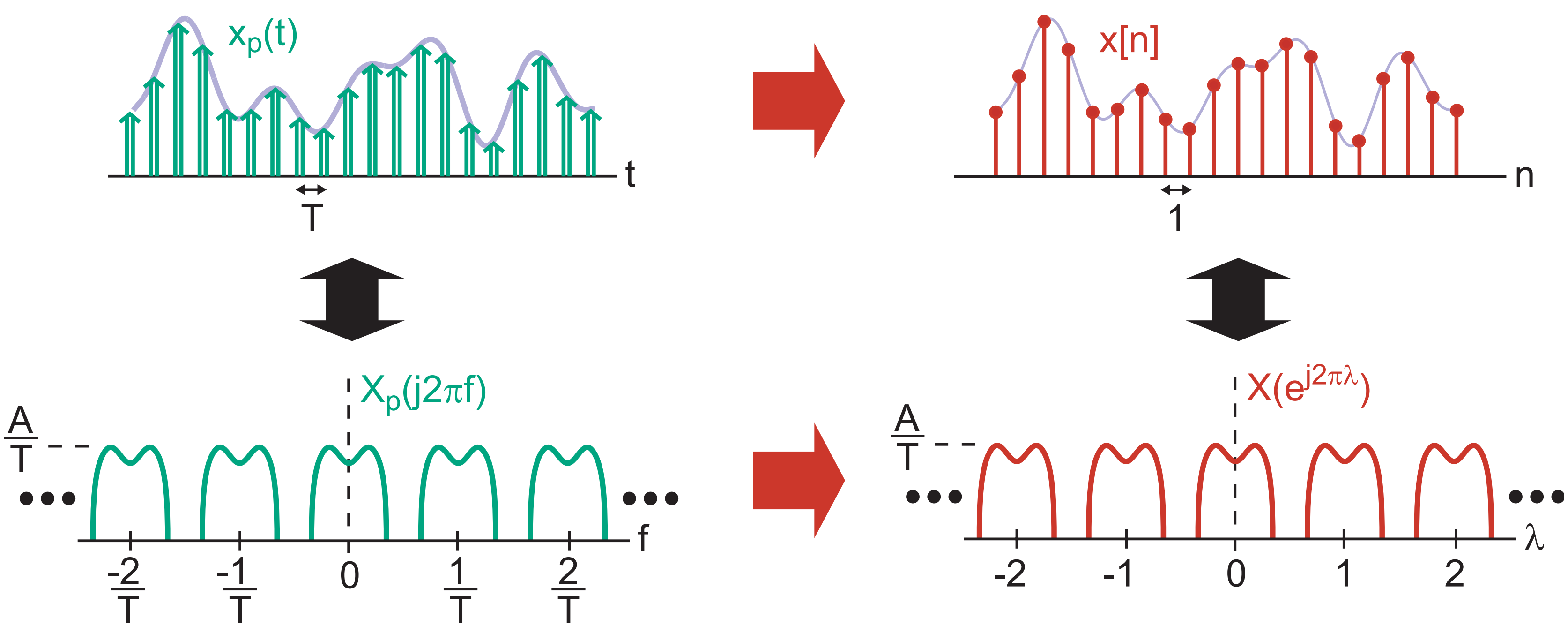
- **Impulse train in time** corresponds to **impulse train in frequency**
  – Spacing in time of **T seconds** corresponds to **1/T Hz** spacing in frequency

Frequency Domain view of Sampling



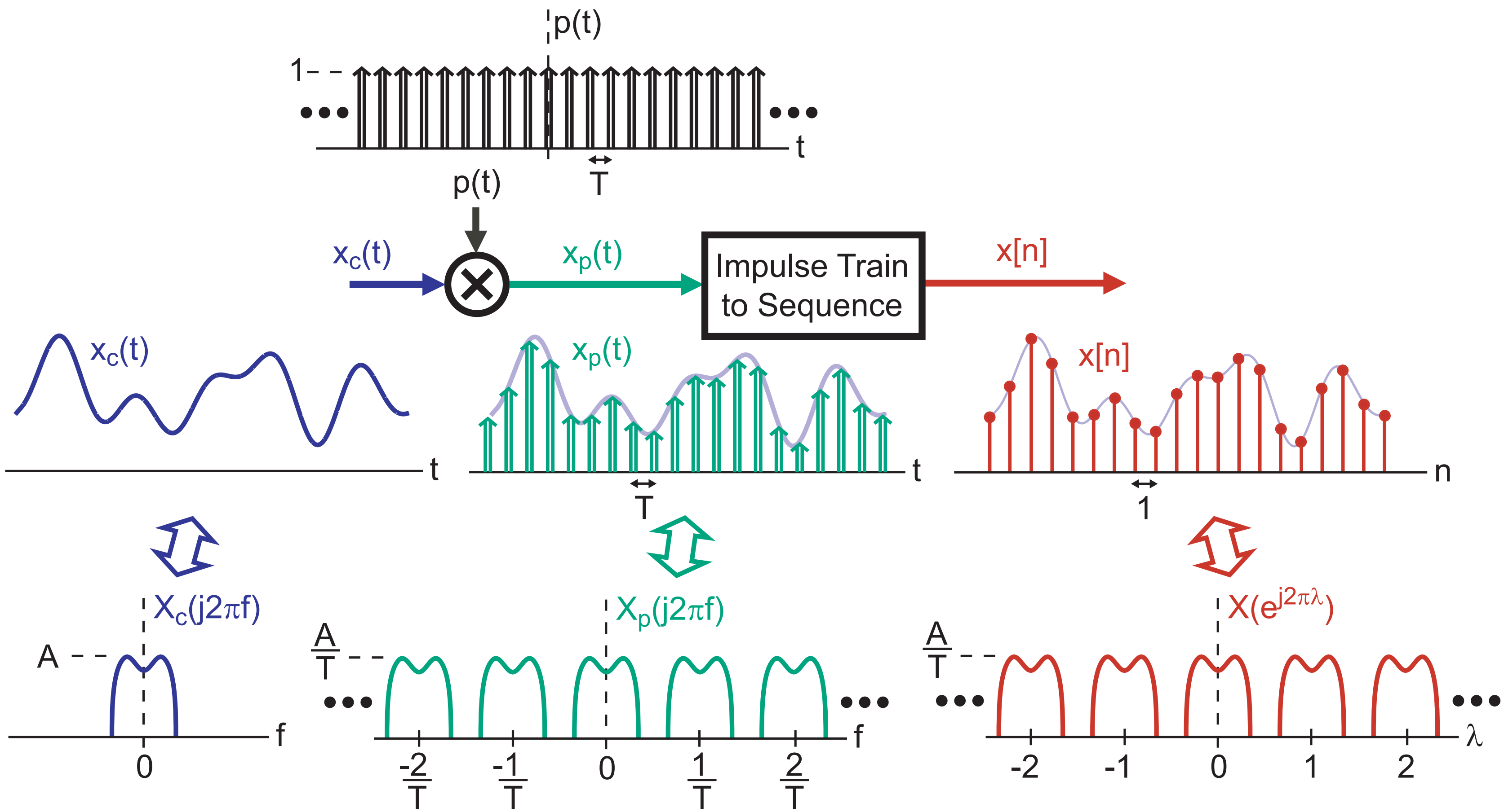
- Sampling in time leads to a **periodic Fourier Transform**, with period 1/T

Frequency Domain View of a Sequence



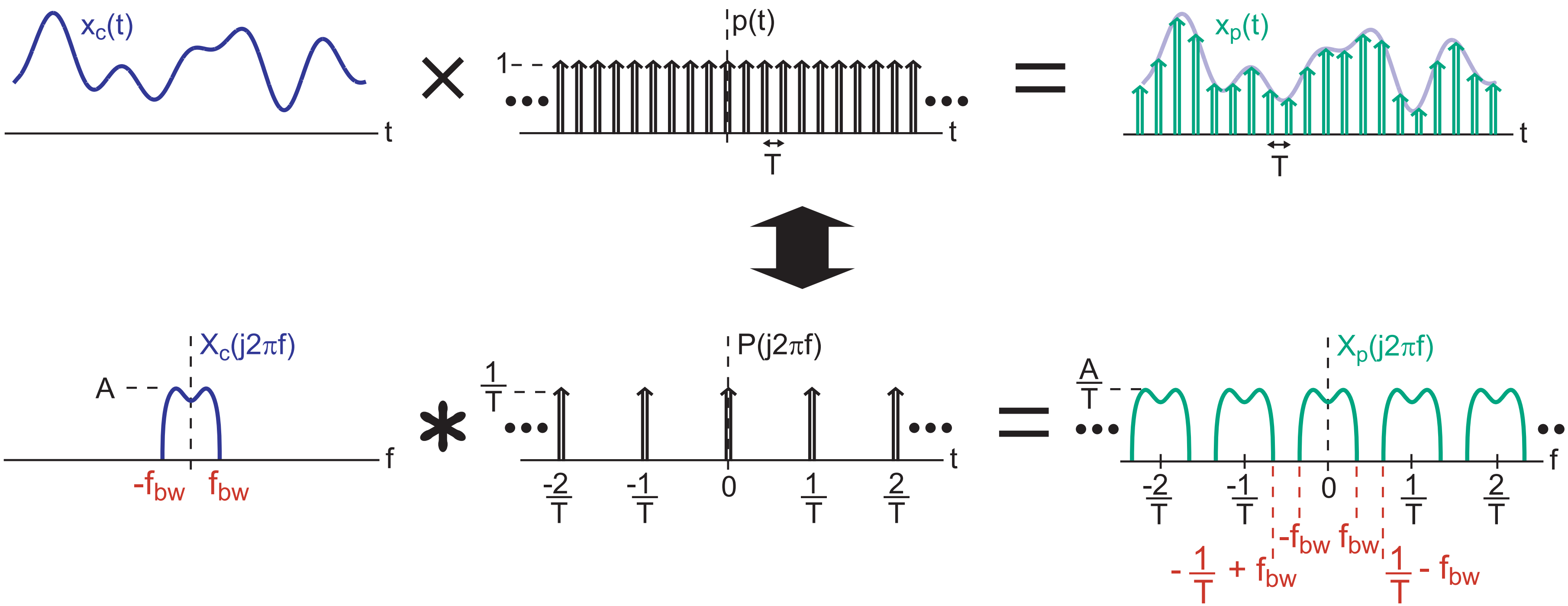
- Conversion to sequence **amounts to T = 1**
- Resulting Fourier Transform now periodic with period 1

Summary of Continuous-to-Discrete Conversion



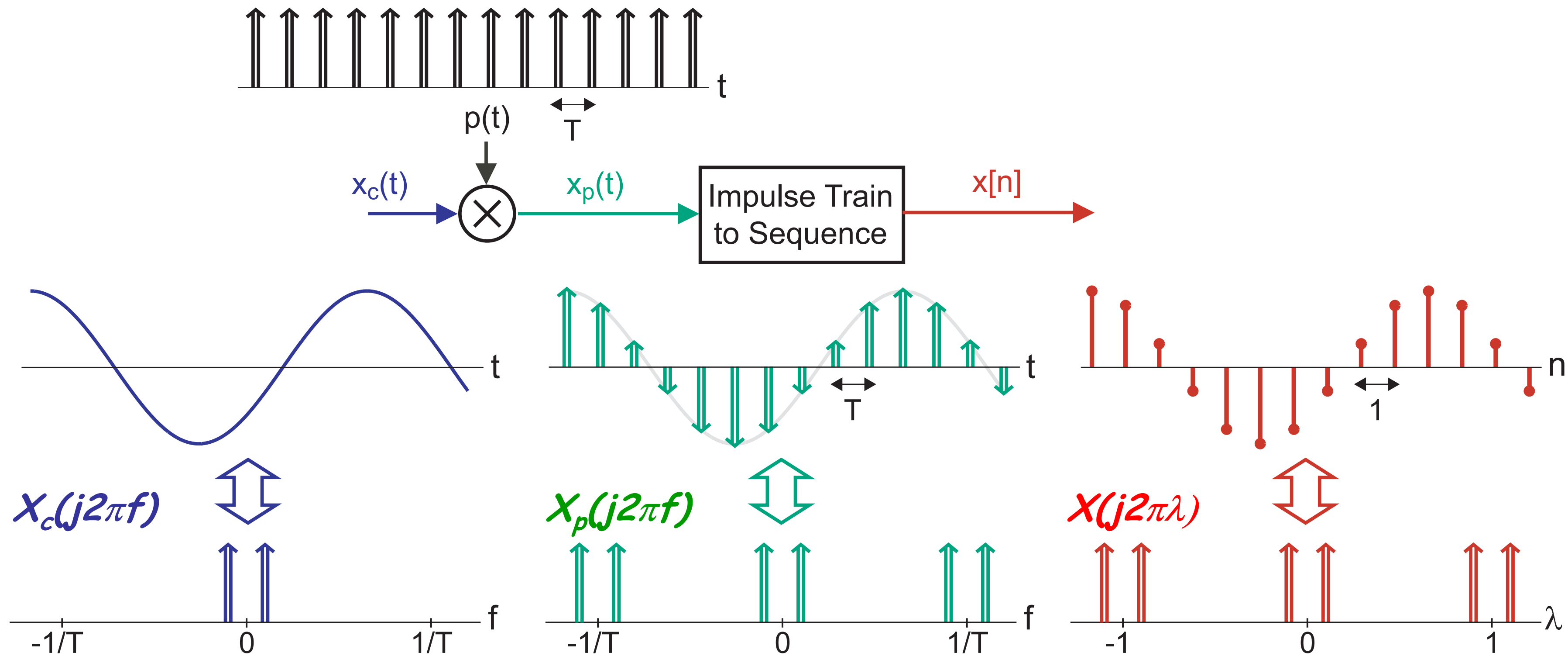
- Sampling leads to periodicity in frequency domain

The Sampling Theorem



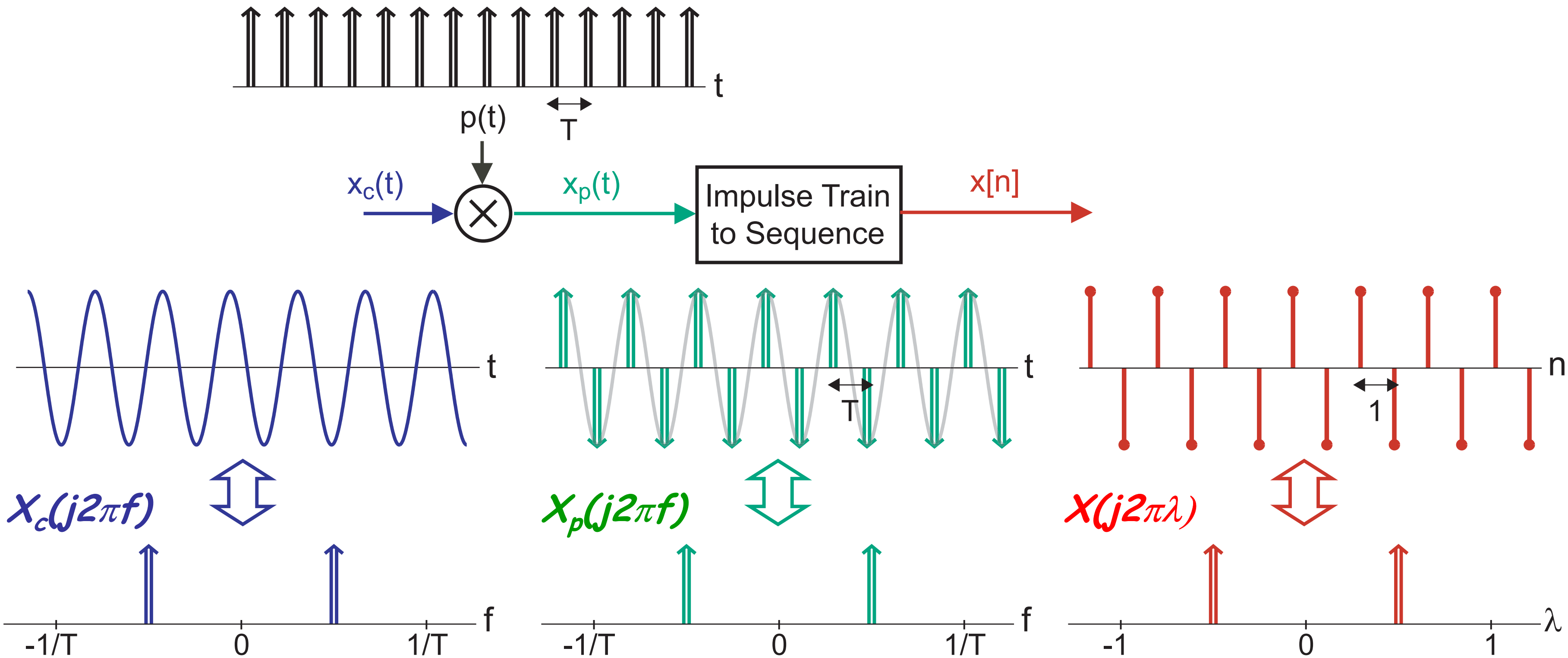
- **No overlap in frequency domain** (aliasing) when:
  - $^1/_T - f_{bw} \geq f_{bw}$ or $^1/_T \geq 2f_{bw}$
- We refer to the minimum 1/T that avoids aliasing as the ***Nyquist sampling frequency*** of a signal

Sine Wave Example: Sampling Above Nyquist Rate



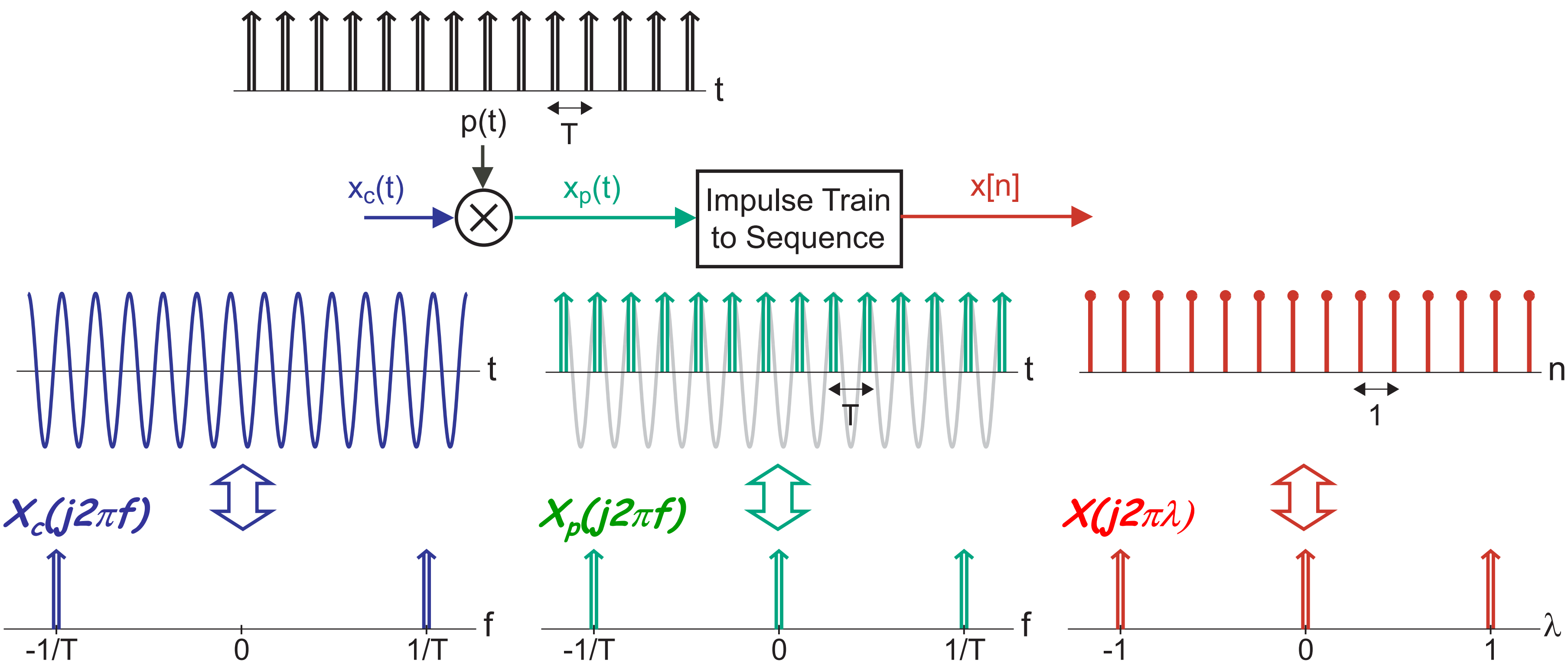
- **Time domain:** Resulting sequence **maintains same period** as input signal

Sine Wave Example: Sampling at Nyquist Rate



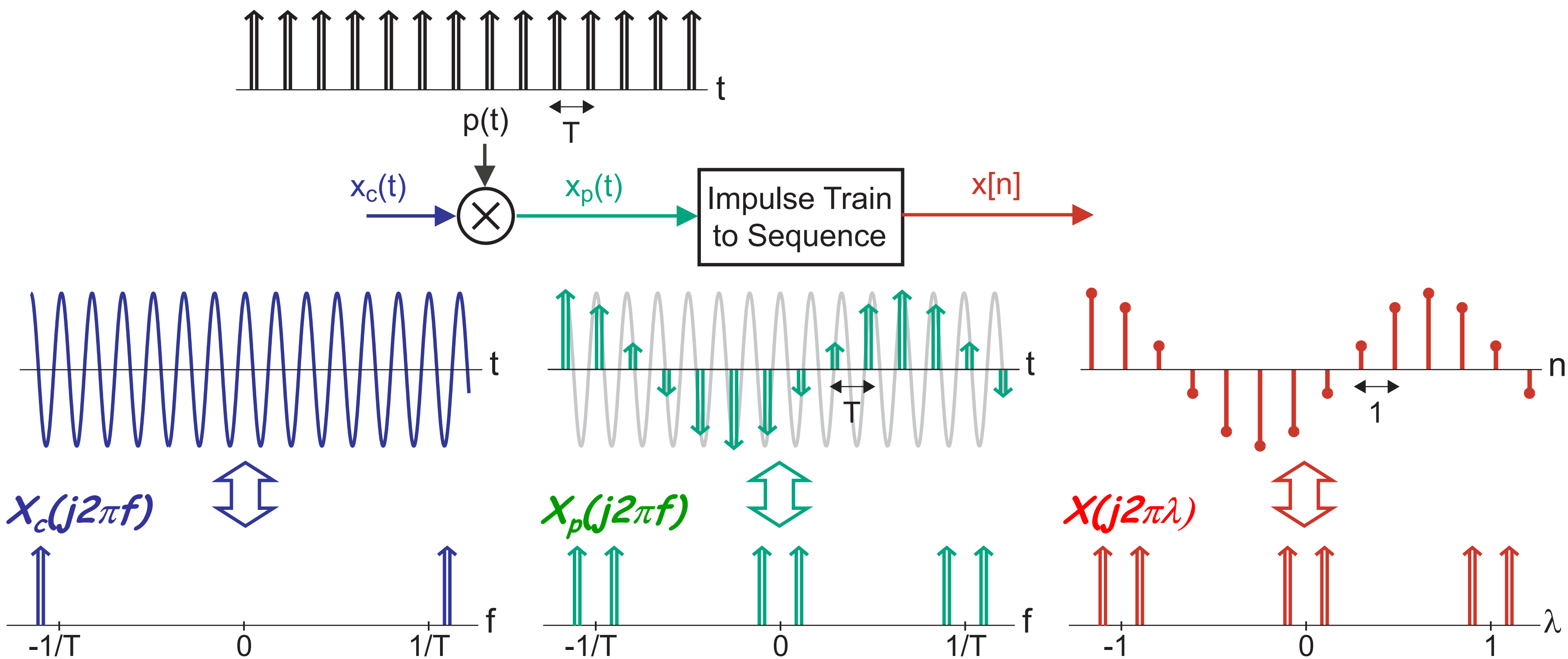
- **Time Domain:** Resulting sequence **maintains same period** as input signal

Sine Wave Example: Sampling at Half the Nyquist Rate



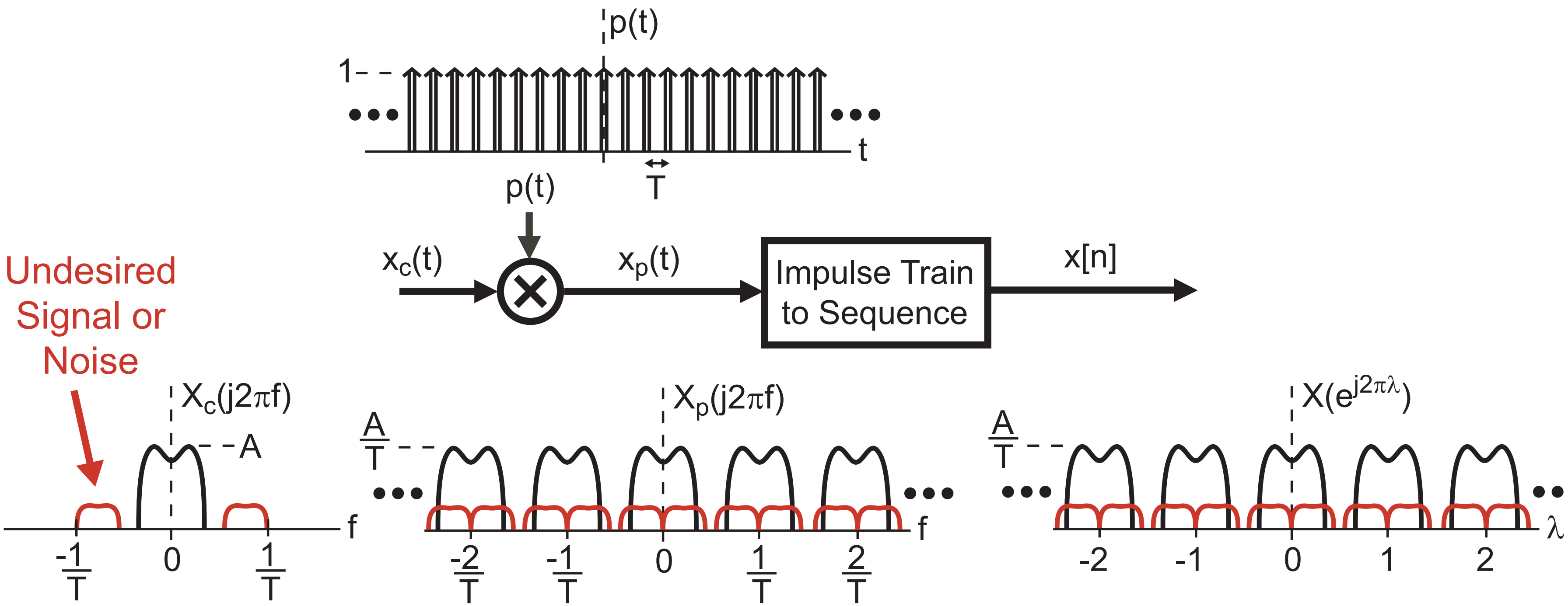
- Sequence now **appears as constant (zero frequency) signal**
- **Frequency domain: Aliasing to 0**

Sine Wave Example: Sampling Below the Nyquist Rate



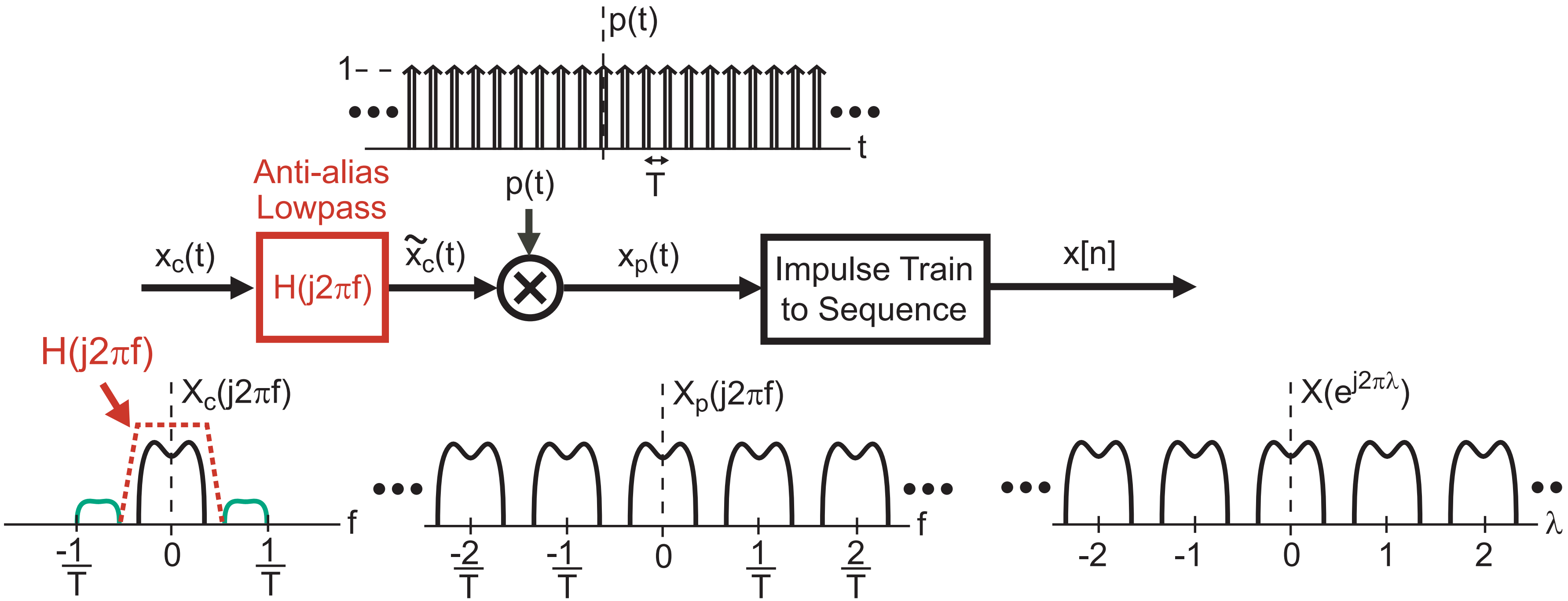
- Resulting sequence is now a **sine wave with a different period than the input**
- **Frequency domain: Aliasing** to a lower frequency

The Issue of High Frequency Noise



- Typically **set sample rate** to exceed **desired signal's** bandwidth
- Real systems can introduce **noise/other interference at high frequencies**
  - Sampling causes **noise to alias** into desired frequency band

Anti-Alias Filtering



- Practical A/D converters **include a continuous-time filter** before the sampling operation
  - Designed to **filter out noise and interference above 1/2T** in frequency, **prevent aliasing**

Summary: Advantages of Going Digital

- Allows **error correction** to be achieved
  - Less sensitivity to radio channel imperfections

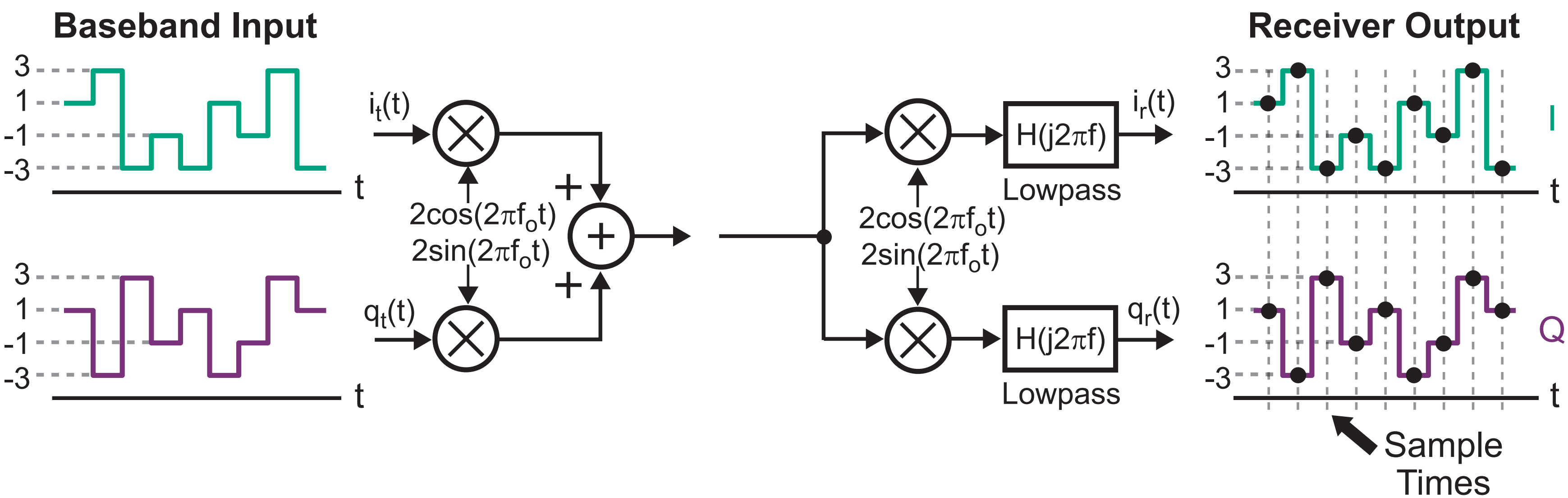
- **Enables compression** of information
  - More efficient use of the channel

- Supports a **wide variety of information** content
  - Voice, text, video can all be represented as digital bit streams

Roadmap

1. Analog I/Q modulation
2. Discrete-time processing of continuous signals
3. **Digital I/Q Modulation**

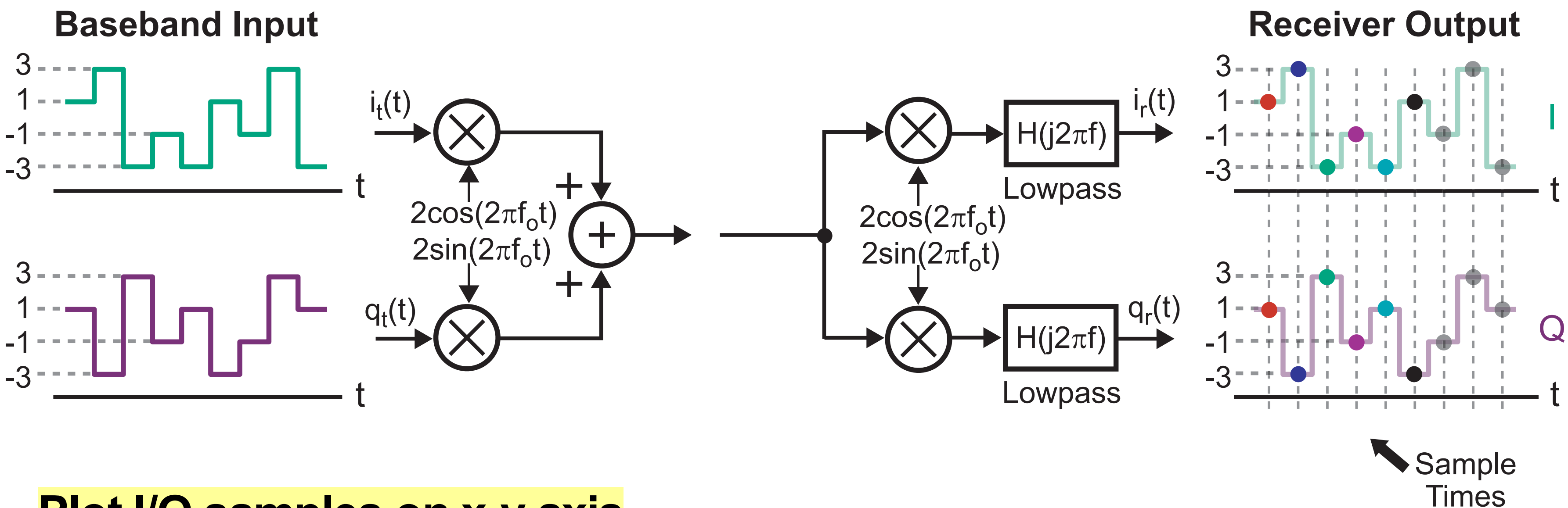
Digital I/Q modulation



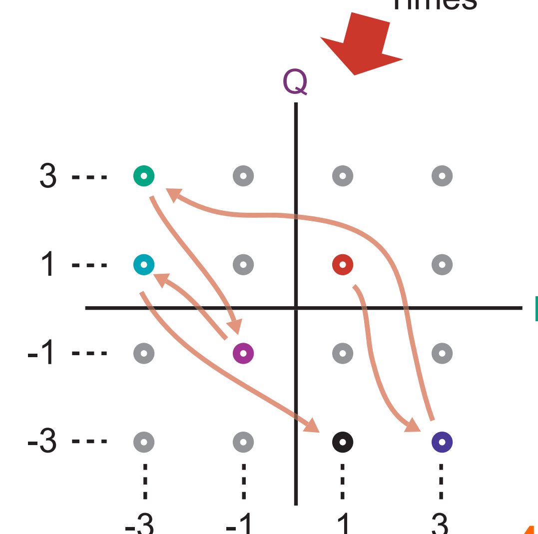
- Leverage analog communication channel to send **discrete-valued *symbols***
  - **Example:** Send symbol from set { -3, -1, 1, 3 } on both I and Q channels

- At receiver, **sample I/Q waveforms** every ***symbol period***
  - Associate each sampled I/Q value with symbols from same set on both I and Q channels

Constellation Diagrams

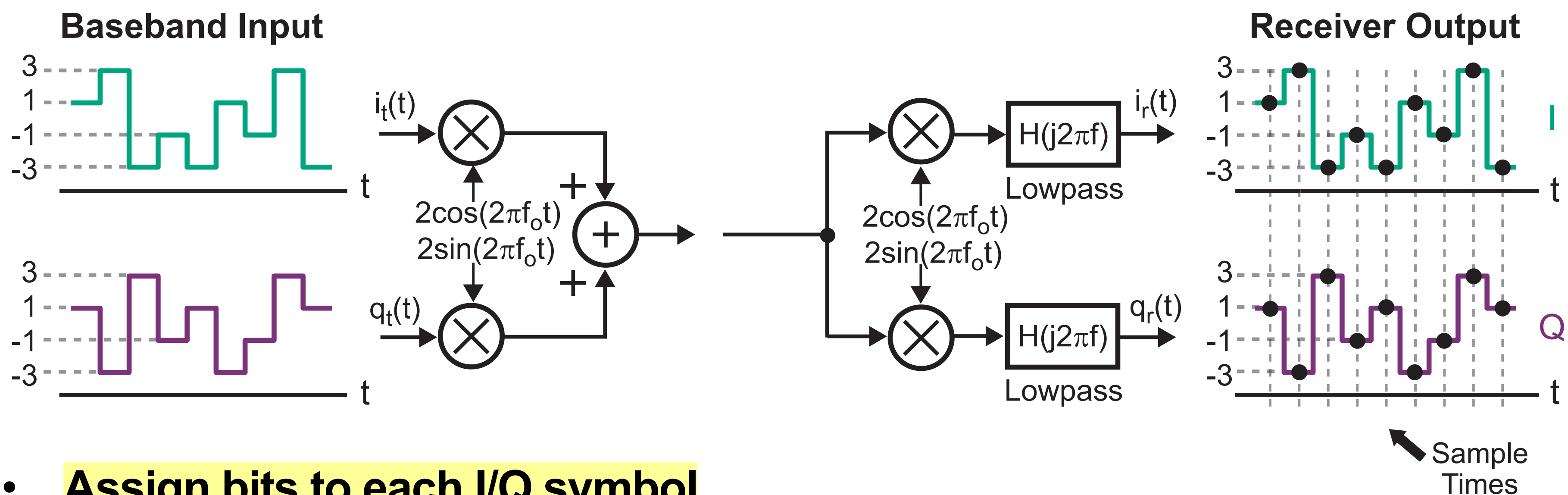


- **Plot I/Q samples on x-y axis**
  - As samples are plotted, constellation diagram eventually displays all possible symbol values

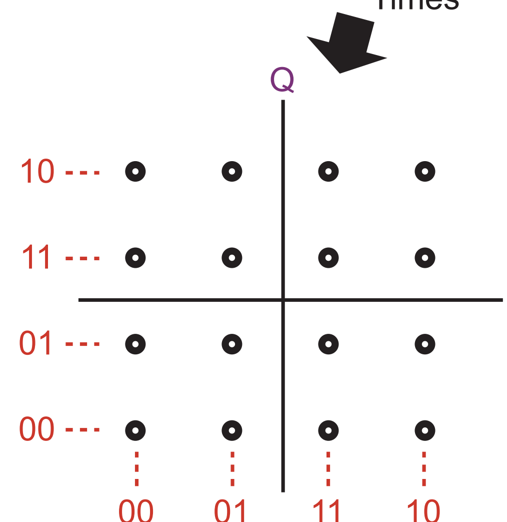


- Provides a sense of **how easy it is to distinguish** between different symbols

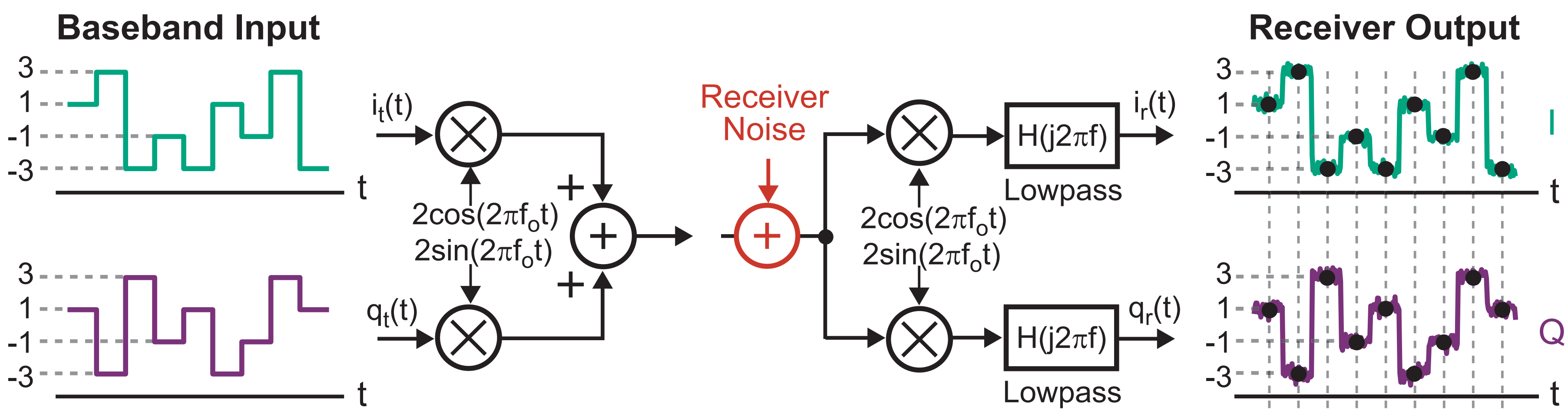
Sending Digital Bits



- **Assign bits to each I/Q symbol**
  - **Example:** I/Q = {1, 3} translates to bits 1110
  - ***Gray coding*** minimizes bit errors when symbol errors are made
    - **Example:** I/Q = {3, 3} translates to bits 1010 (one bit flip from {1, 3})

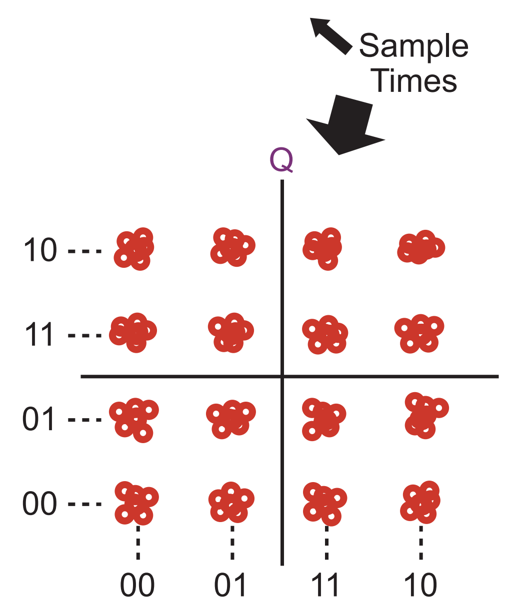


The Impact of Noise

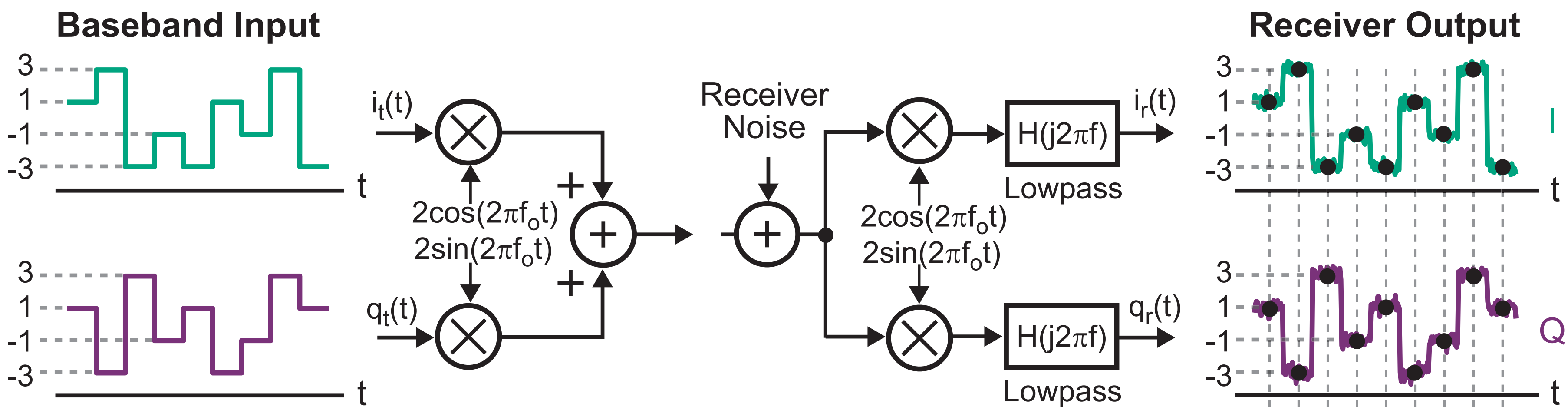


- **At receiver, noise perturbs** sampled I/Q values
  - Constellation points no longer consist of single points for each symbol

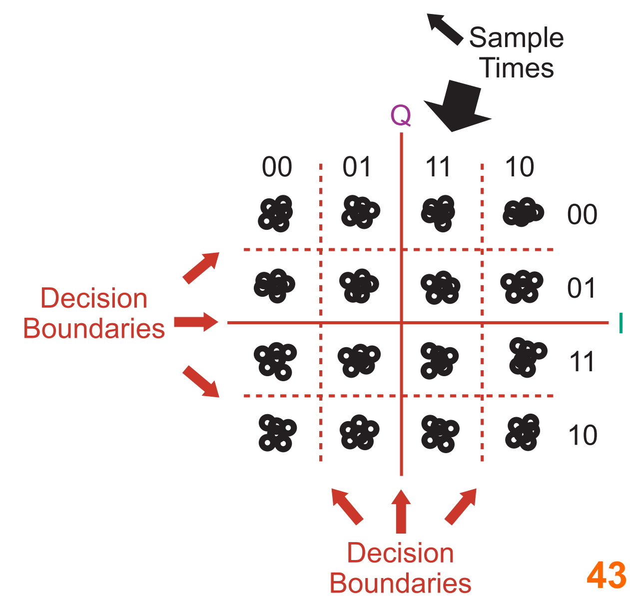
- **Issue:** What's the **best way of matching received I/Q samples** with their corresponding transmitted symbols?



Symbol Selection based on Slicing

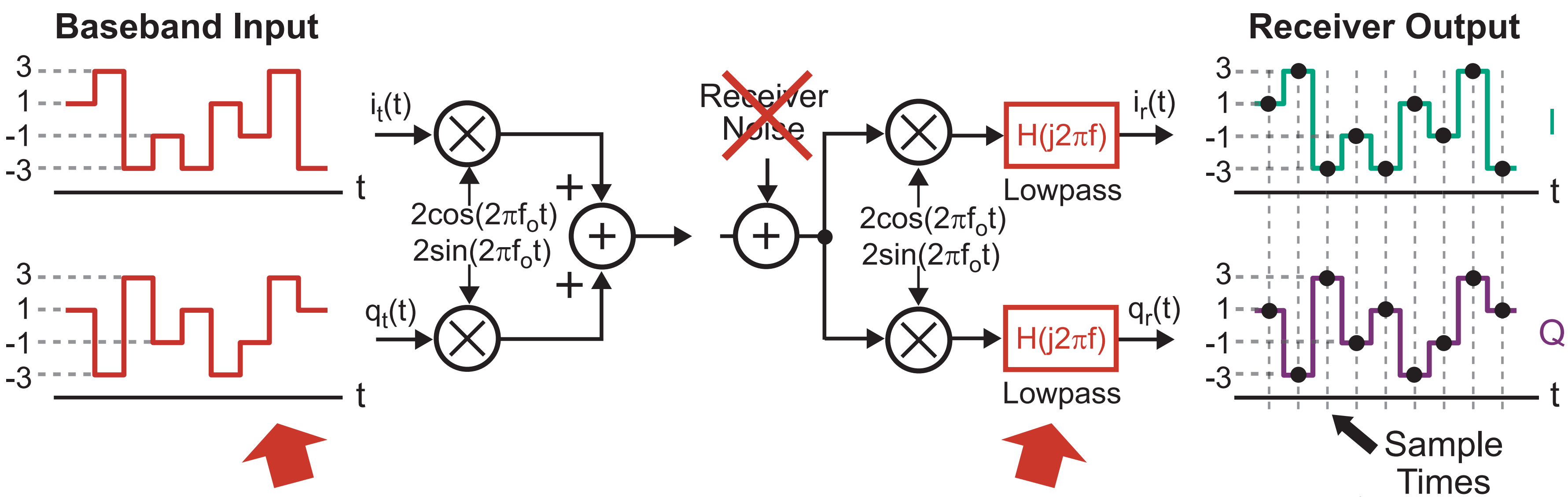


- Receiver matches I/Q samples to corresponding symbols based on ***decision regions*** comprised of decision boundaries
  - Choose decision boundaries to minimize symbol errors

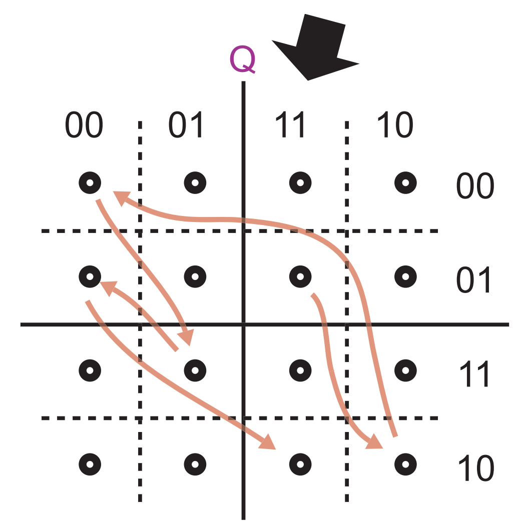


- Decision boundaries also called ***slicing levels***

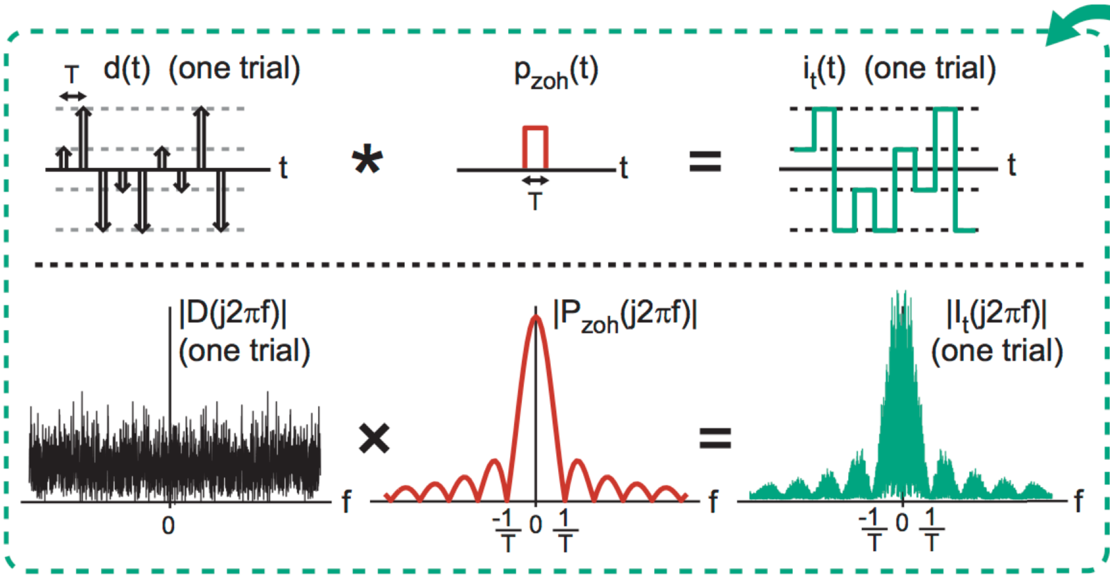
Transitioning Between Symbols

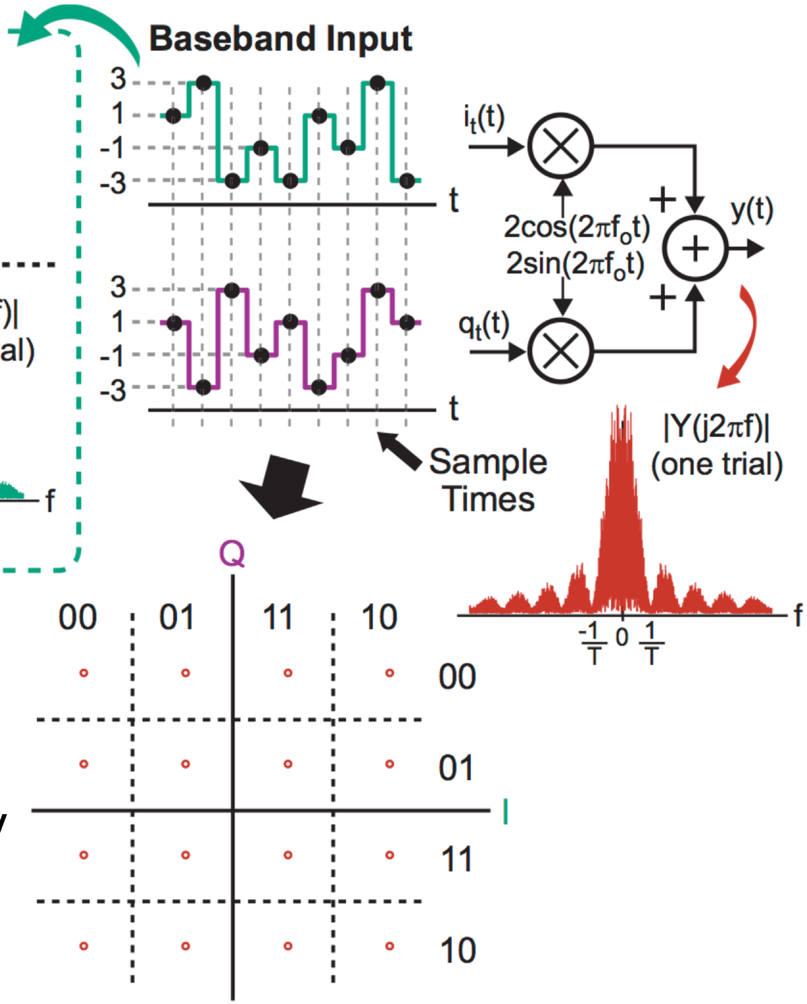


- Transition behavior influenced by transmit I/Q input waveforms **and** receive filter
  - **Today:** Focus on **what the transmitter does**
  - Ignore impact of noise for this analysis



Transitions and the Transmitted Spectrum

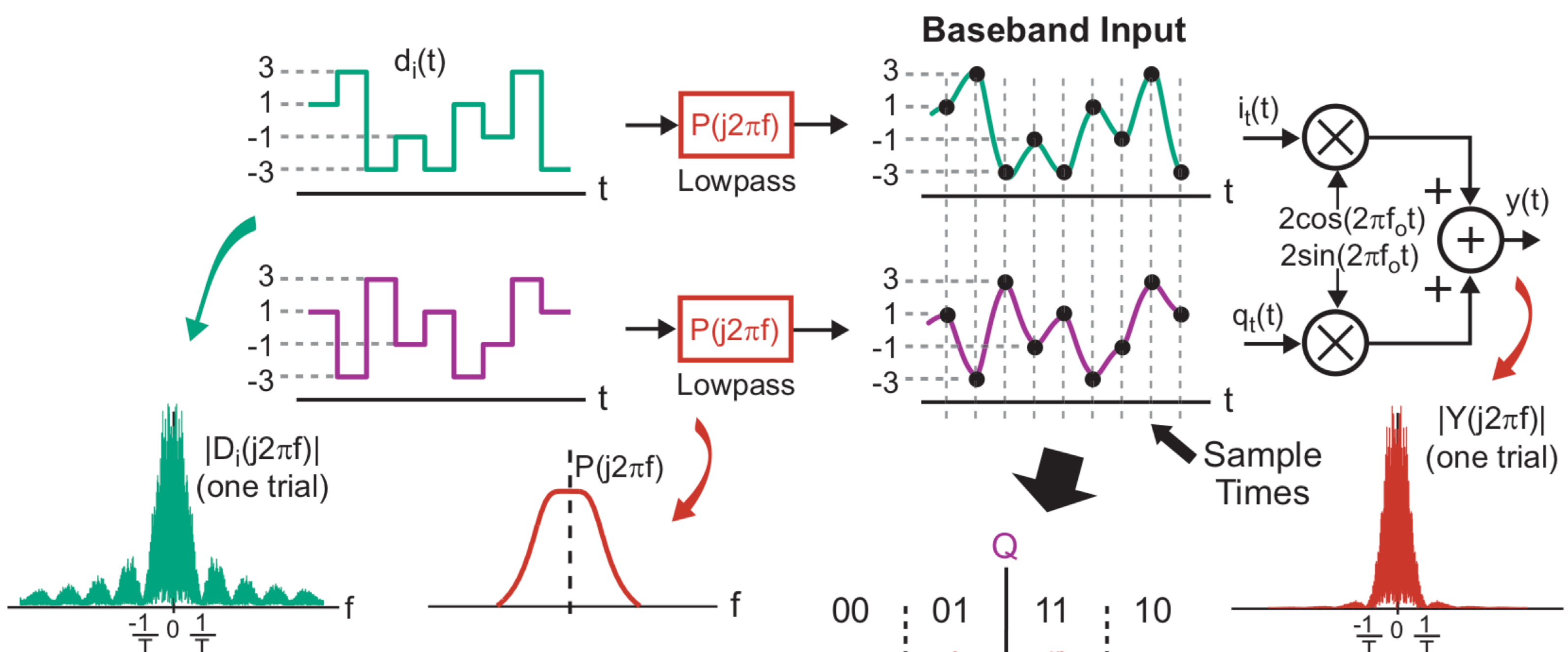


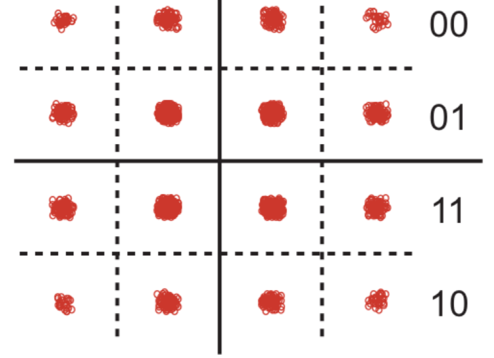


- Want transmitted spectrum **with minimal bandwidth**

- **Issue: Sharply changing I/Q** leads to very **wide bandwidth spectrum**

Impact of Transmit Filter

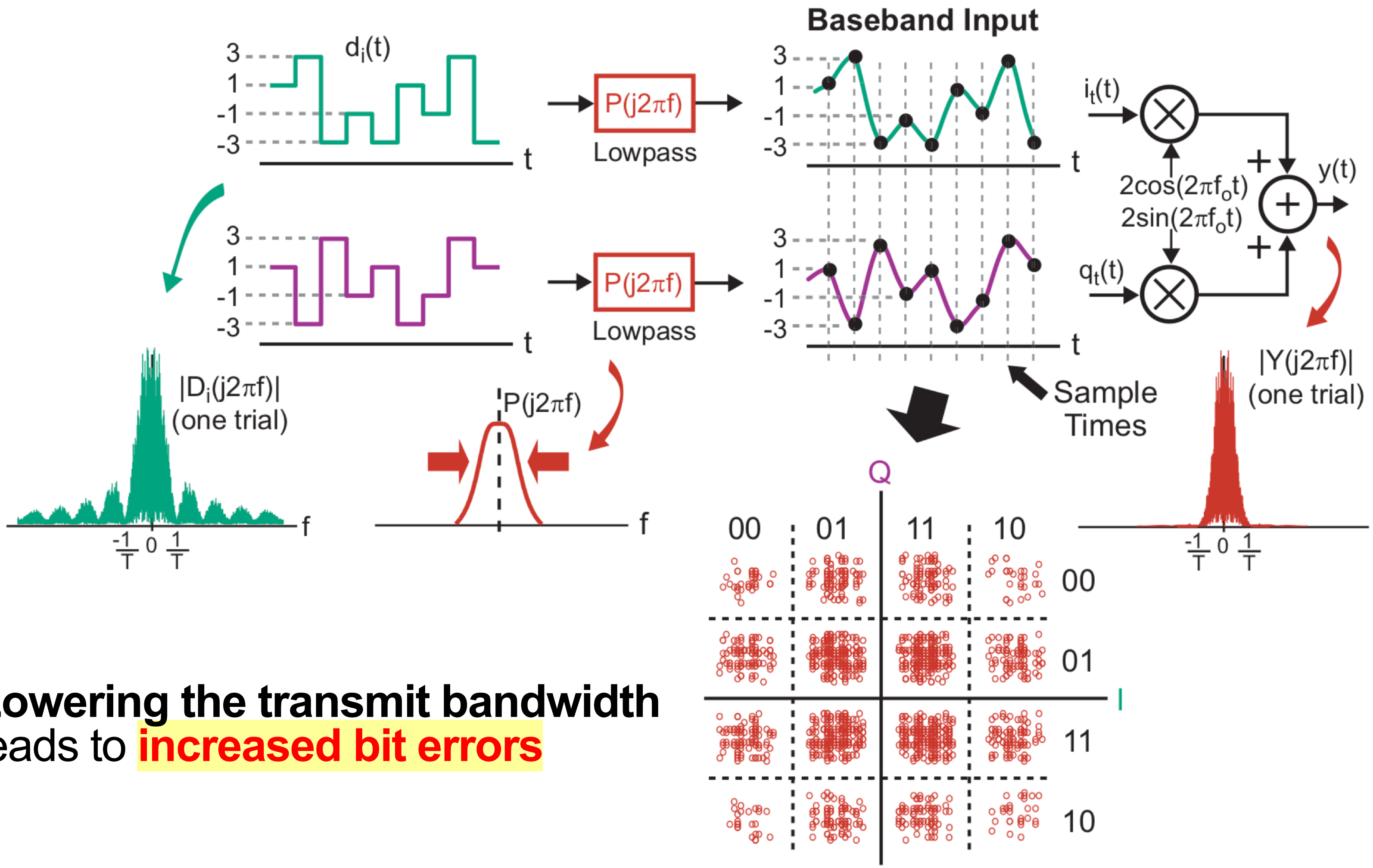




- Transmit filter **shapes the pulses**, **enables reduced bandwidth** for transmitted spectrum

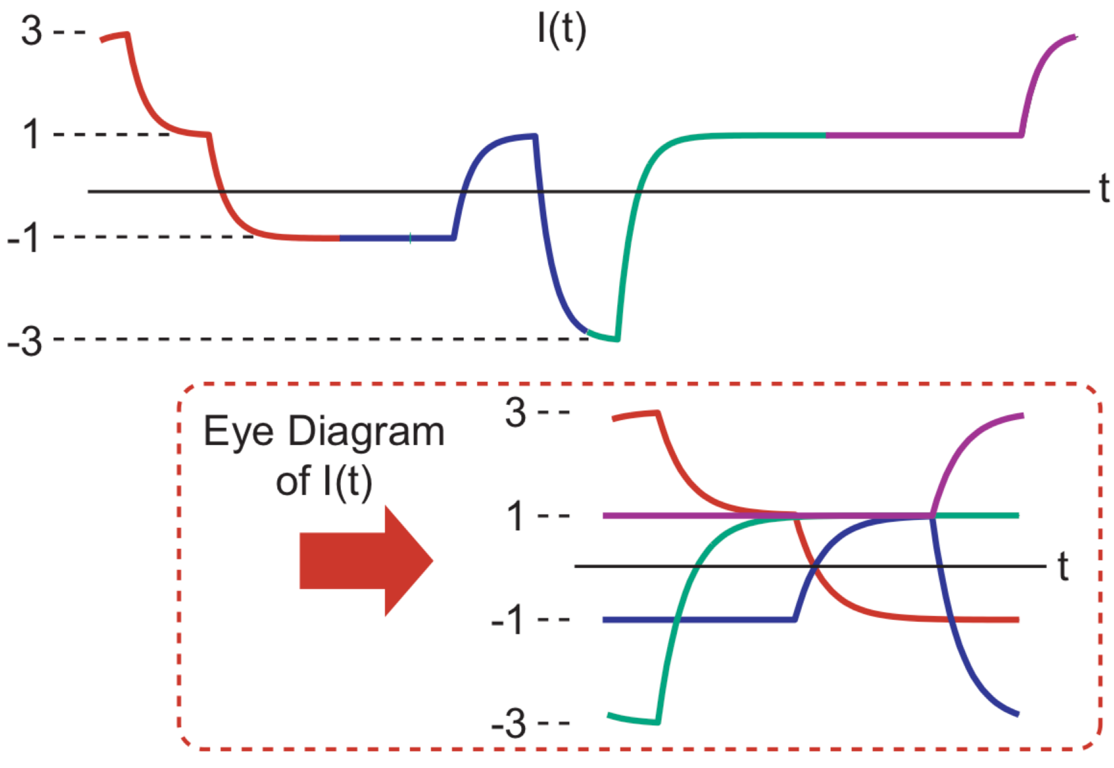
- **Next Issue:** Can lead to **bit errors**

Impact of Low Bandwidth Transmit Filter



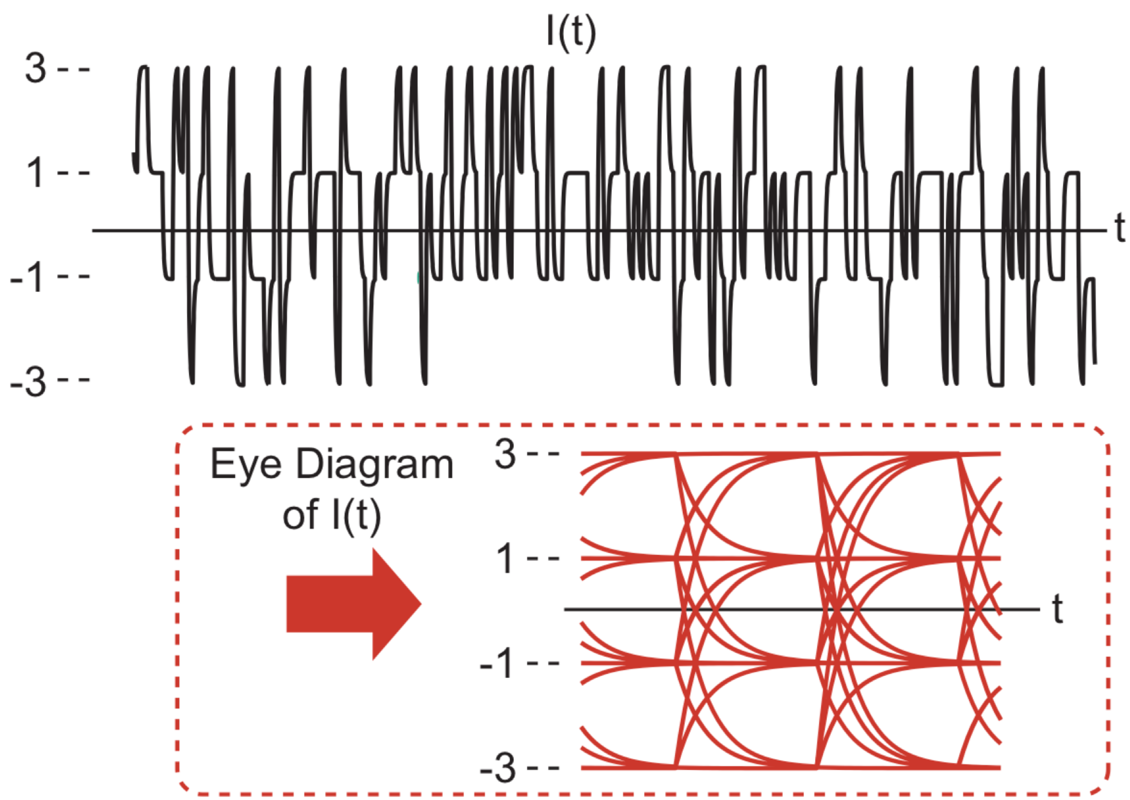
- **Lowering the transmit bandwidth** leads to **increased bit errors**

Eye Diagrams



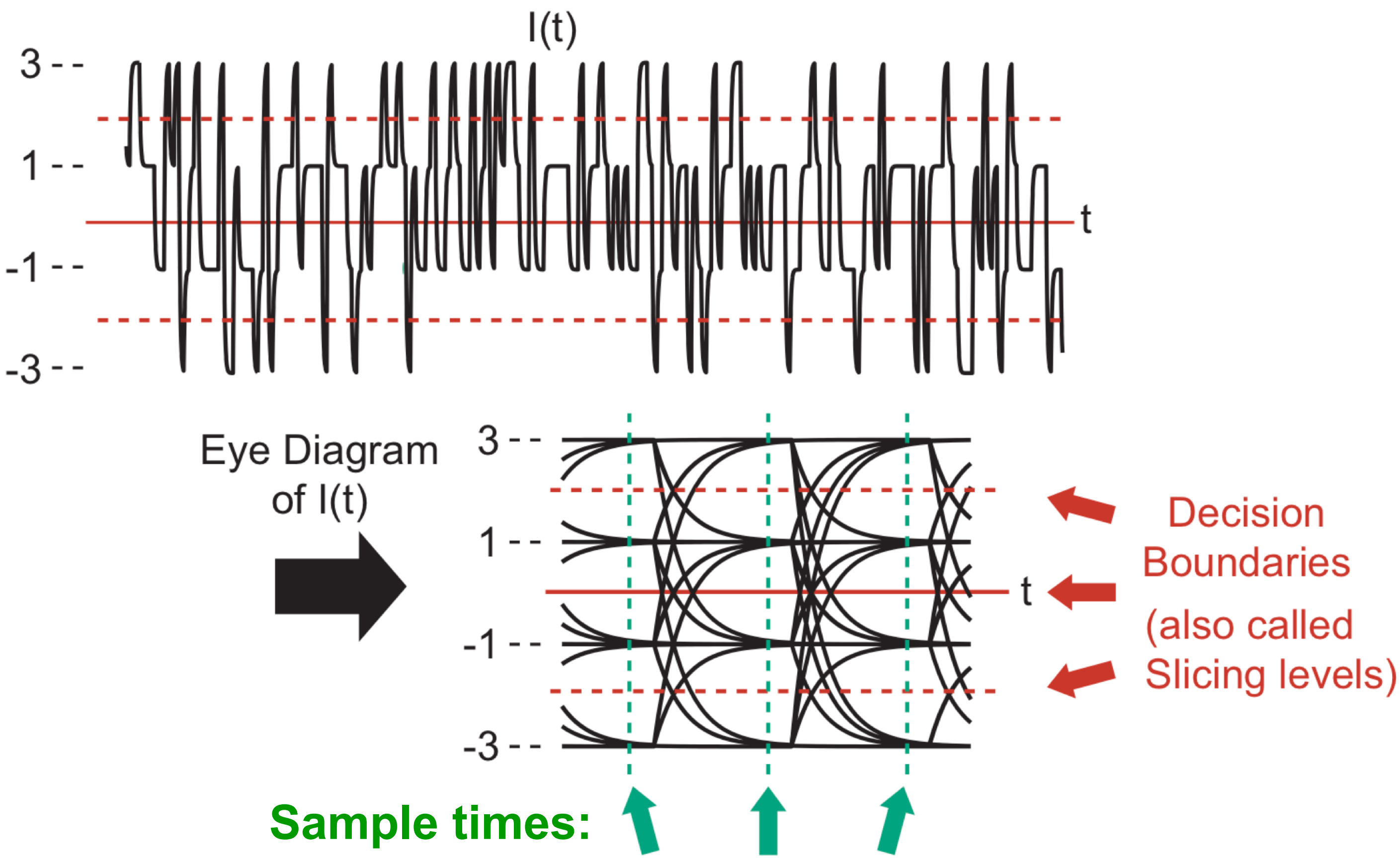
- ***Eye Diagram:*** **Wrap signal back onto itself** in **periodic time intervals,** retaining all "traces"

Looking at Many Symbols



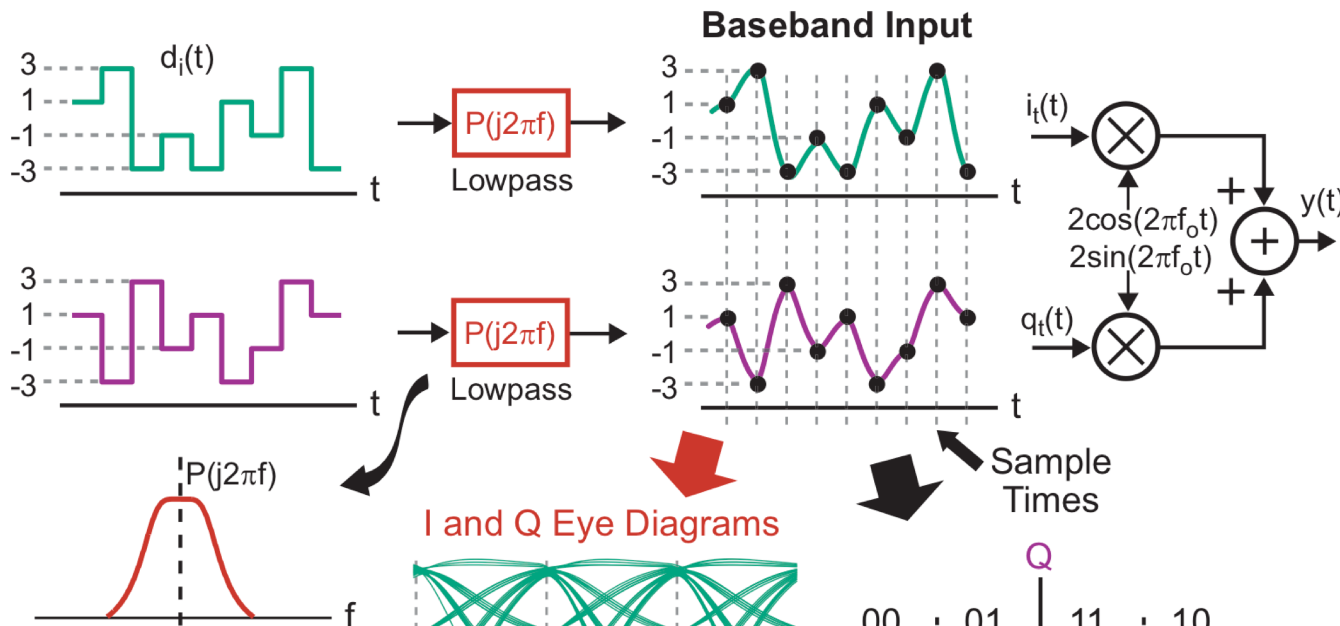
- Increasing the number of symbols eventually **reveals all possible symbol transition trajectories**
  - Intuitively **displays the impact of filtering** on transmitted signal

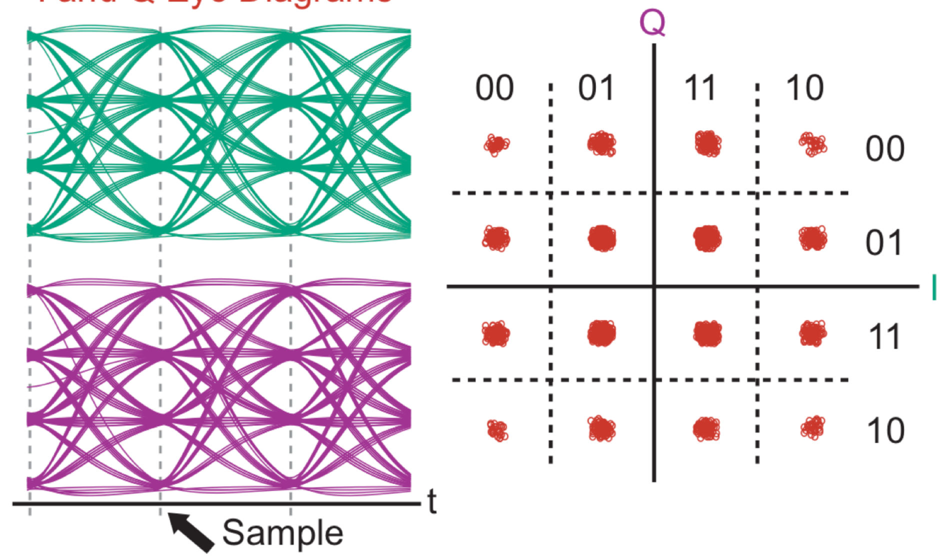
Assessing the Quality of an Eye Diagram



- Eye diagram **allows visual inspection of the impact of sample time and decision boundary** choices
  - **Large "eye opening"** implies less vulnerability to symbol errors

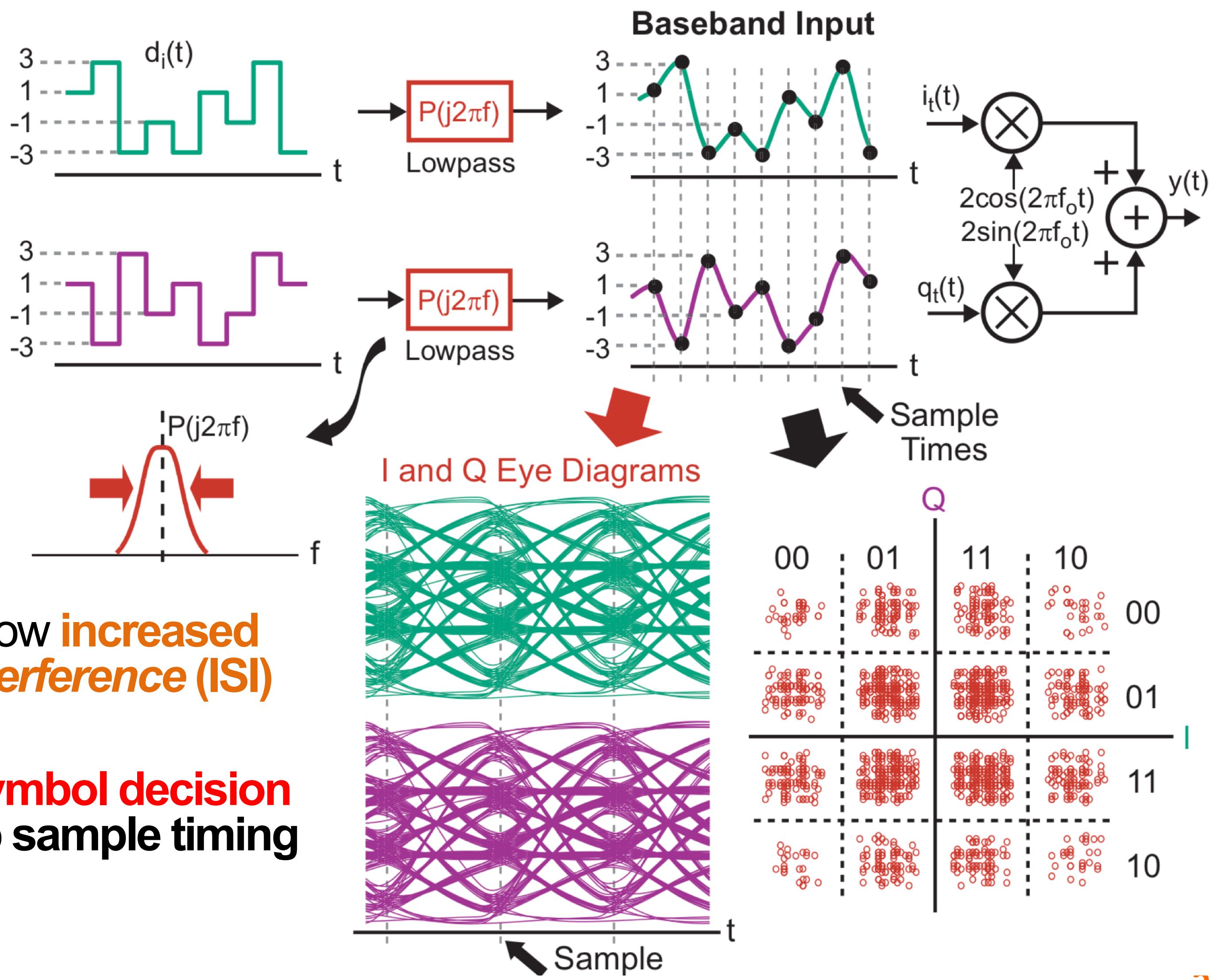
Relating Eye Diagrams to Constellation





- **Open eye diagrams** lead to **tight symbol groupings** in constellation
  - Assumes proper sample time placement

Impact of Low Transmit Bandwidth



- Eye diagrams show **increased *inter-symbol interference* (ISI)**
  - Also show **symbol decision sensitivity** to **sample timing placement**

Digital Modulation: Summary

- **Digital modulation:** Sends **discrete-valued symbols** through an **analog** communication channel
  - Receiver must **sample** I/Q signals at the appropriate time
  - Receiver matches I/Q sample values to corresponding symbols based on **decision regions**
  - **Constellation diagrams** are a convenient tool to see likelihood of bit errors being made

- **Choice of transmit filter: Tradeoff** between achieving a **minimal bandwidth** transmitted spectrum and **minimal ISI**
  - **Eye diagrams:** A convenient tool to see effects of ISI and sensitivity of bit errors to sample time choice

Thursday Topic:

Digital Communications II:
Receiver Processing, Performance

Friday Precept:

BER and Shannon's Limit