Preliminaries: Radio Communication, Modulation, and Filtering

COS 463: Wireless Networks
Lecture 11

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[Parts adapted from I. Darwazeh, H. Balakrishnan, M. Perrot, C. Sodini , P. Steenkiste]

Today

- Radio fundamentals
  - **Radio Frequency (RF) Spectrum**
  - Energy and Noise

- Introduction to Modulation

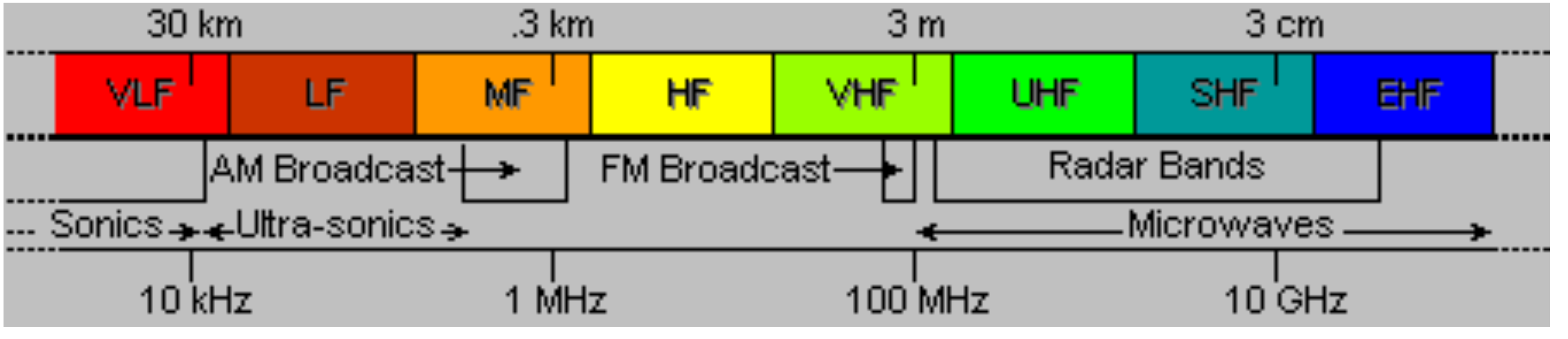
- Introduction to Filtering

Radio Frequency (RF)

- Electromagnetic signal that propagates through space
  - Transmitted at some **carrier frequency** f_c
  - Travels at the **speed of light** (c)

- **Wavelength** in air: $\lambda = c/f_c$

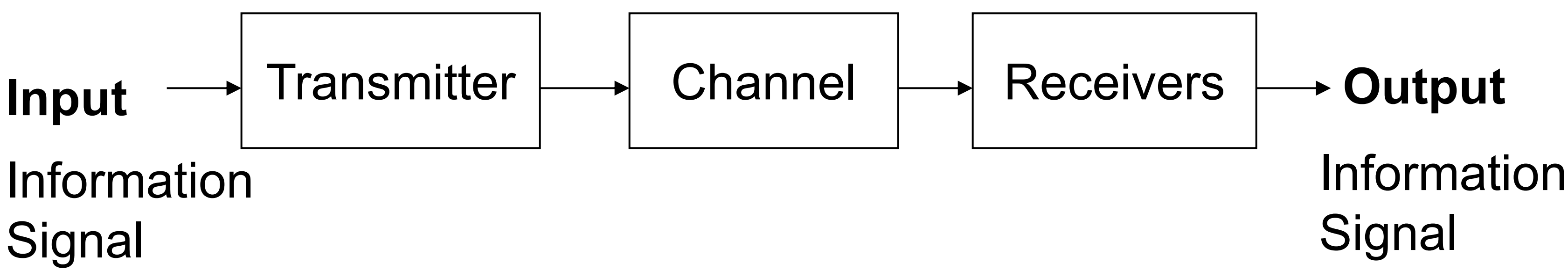
- f_c range: 3 KHz to >300 GHz (or, λ = 100 km to 1 mm)



Spectrum allocation in the US

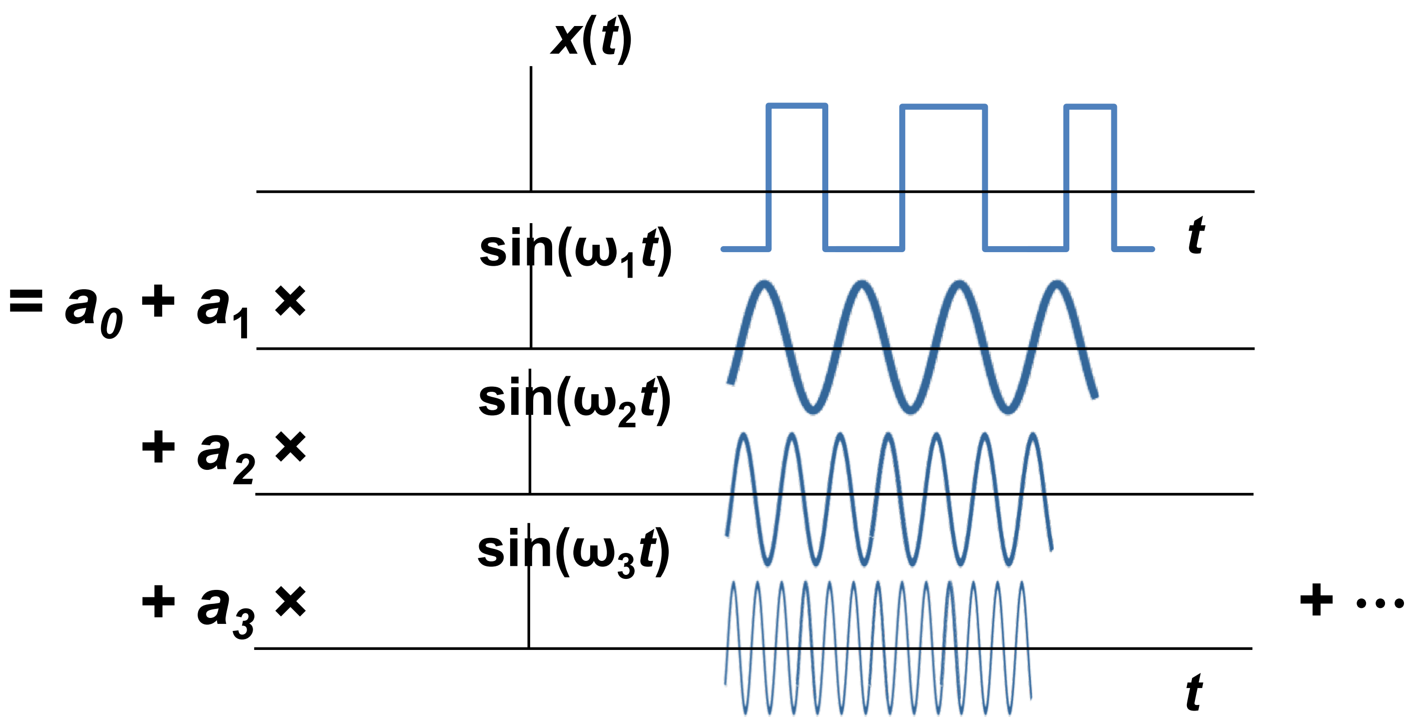
Information Transmission

- **Information** (voice, video, data etc.)
- Coded in **signals** (electromagnetic, optical, acoustic)
- Transmitted over a **channel** (physical medium such as free space, fiber, wire etc.)



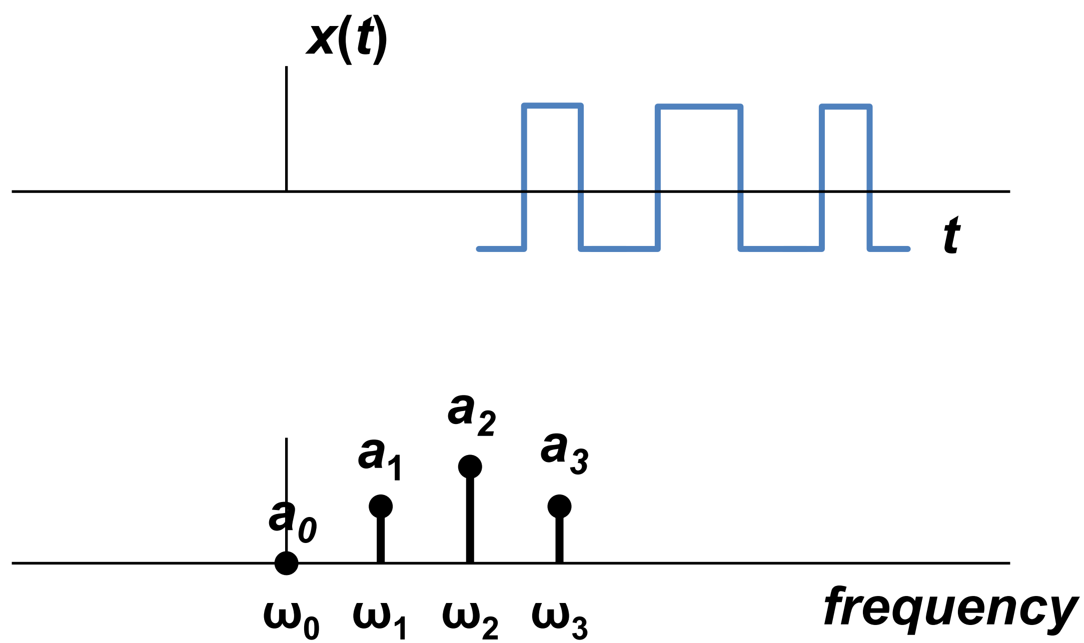
Key Tool: Fourier Series

- **Motivation:** $x(t)$ is our **information waveform**
- A periodic waveform $x(t)$ can be **represented** as a **sum of *weighted* (a_n) sinusoids of different *frequencies* ω_n**



Frequency Domain View

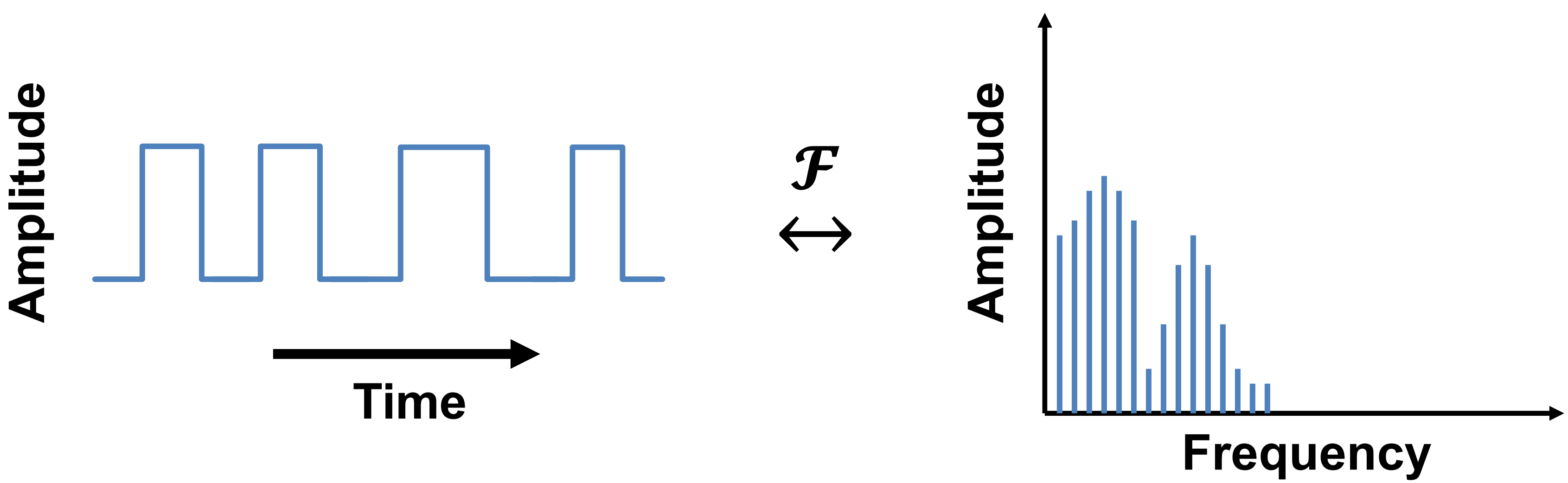
- **Plot** the weights a_n **versus** the frequencies ω_n:



Fourier Transform

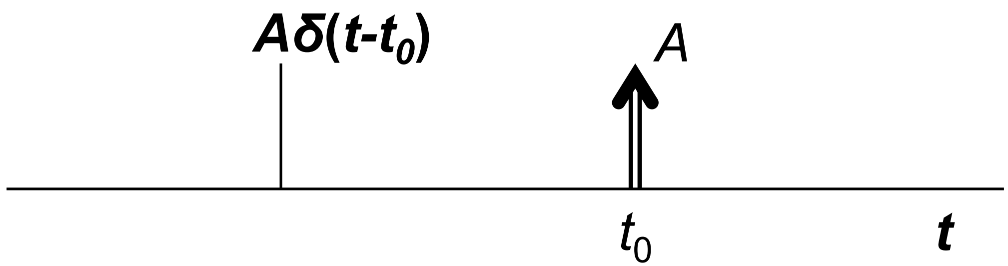
- **Fourier series** deals with **periodic signals**
- **Fourier transform** deals with **non-periodic signals**

- **Notation:** $x(t) \overset{\mathcal{F}}{\leftrightarrow} X(f)$



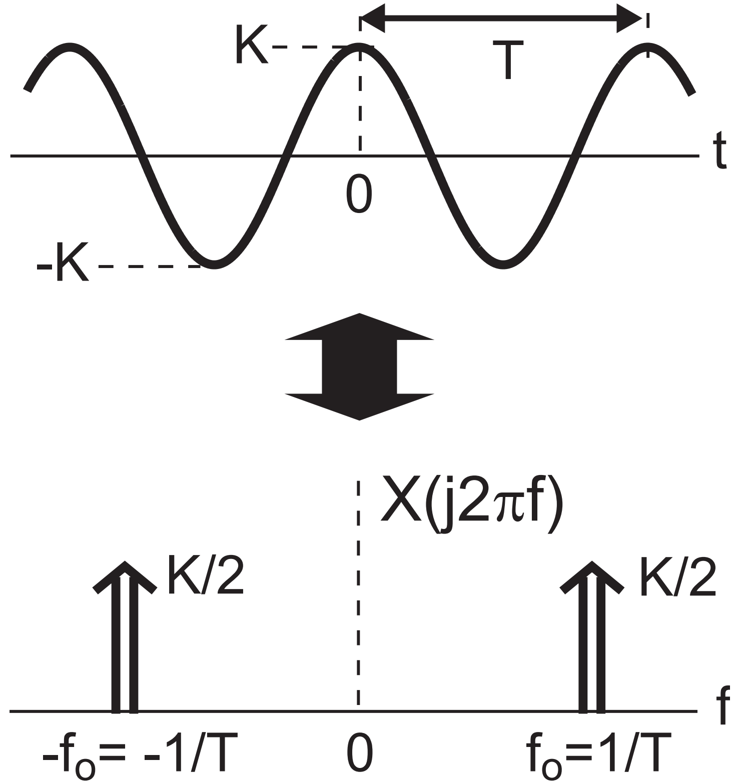
Key Tool: The Impulse Function

- An ***impulse*** of area A at time t_0 is denoted:



- Defined in terms of its **properties** when combined with other (information carrying) signals

Fourier Transform of cosine wave



Today

- Radio fundamentals
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  - Energy and Noise

- **Introduction to Modulation and Demodulation**

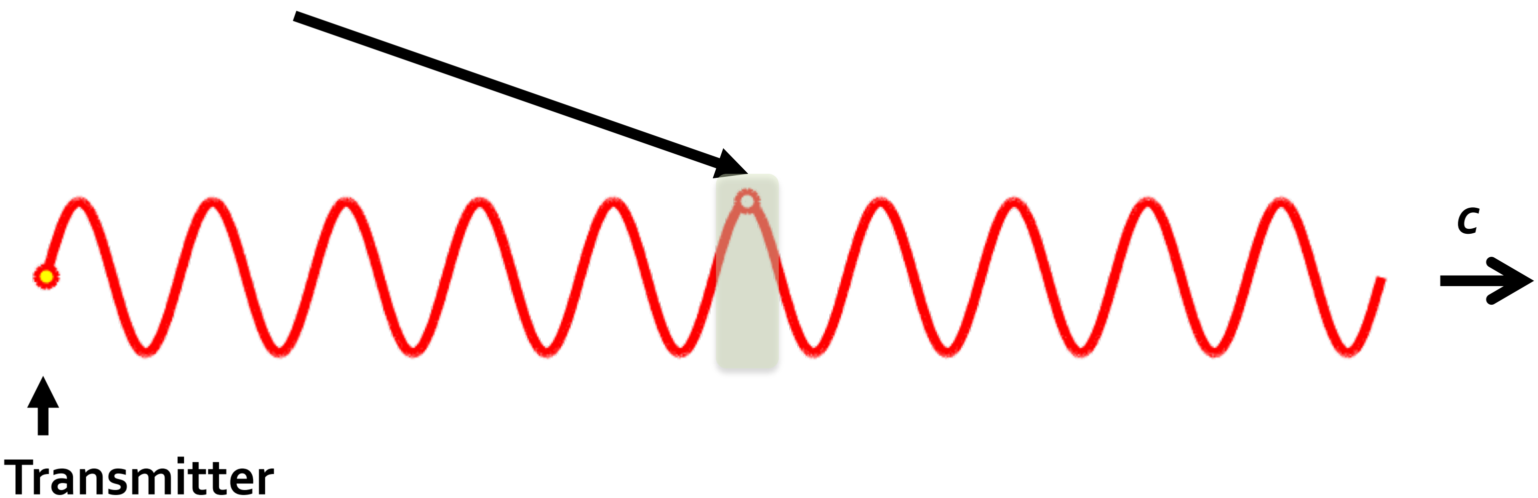
- Introduction to Filtering

Wireless Transmission

- **Physical medium** of wireless channel usually **free space**
- Radio wavelength ***inversely*** proportional to frequency (1 GHz → 30 cm, while 1 KHz → **30 km wavelength**)
- Since antenna and component size is related to wavelength we want to move the information signal to a higher frequency for smaller devices

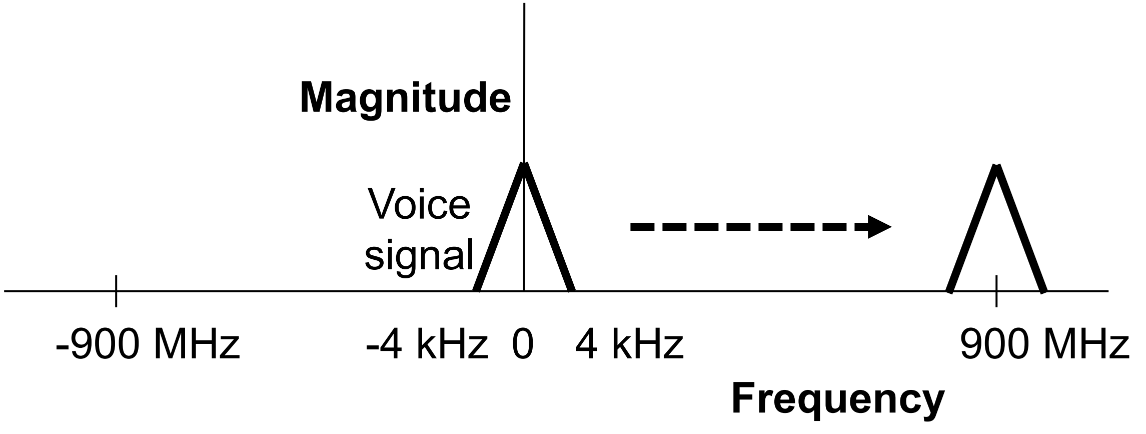
Sinusoidal *carrier signal*

- RF signal propagates away from transmitter at light speed *c*
- At an **instant in time:** signal "looks" sinusoidal **in space**
- At a **point in space**: signal **oscillates sinusoidally in time**



Goal of Modulation

Given an ***information signal***
Example: Voice signal with 4 KHz ***bandwidth***

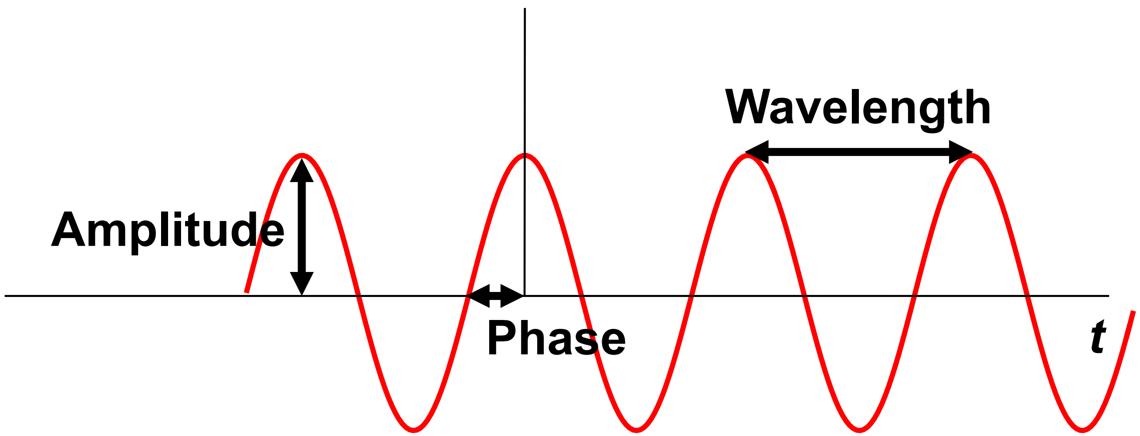


Shift information signal to the ***carrier frequency***
Example: 900 MHz carrier frequency

→ Information signal ***modulates*** (changes) the carrier signal

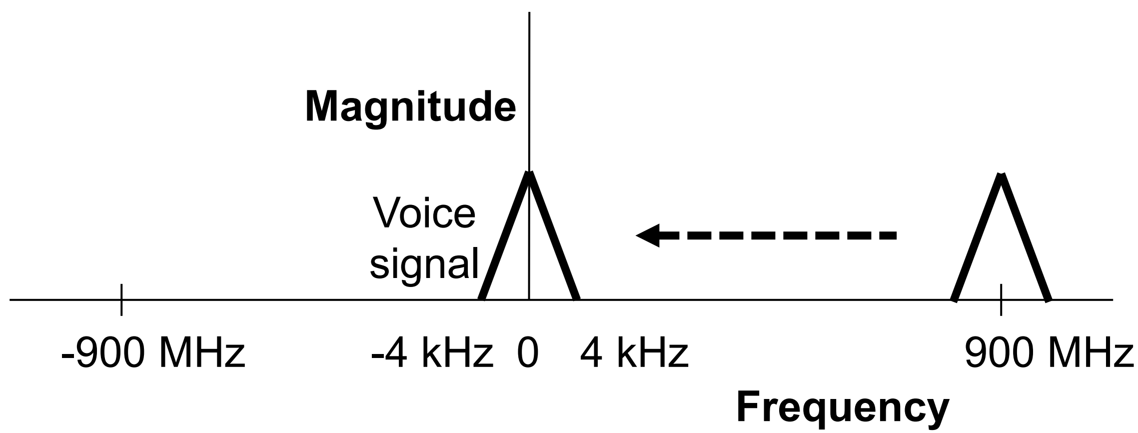
Carrier signal parameters

- The **information signal** modulates the **carrier's parameters:**



Goal of Demodulation

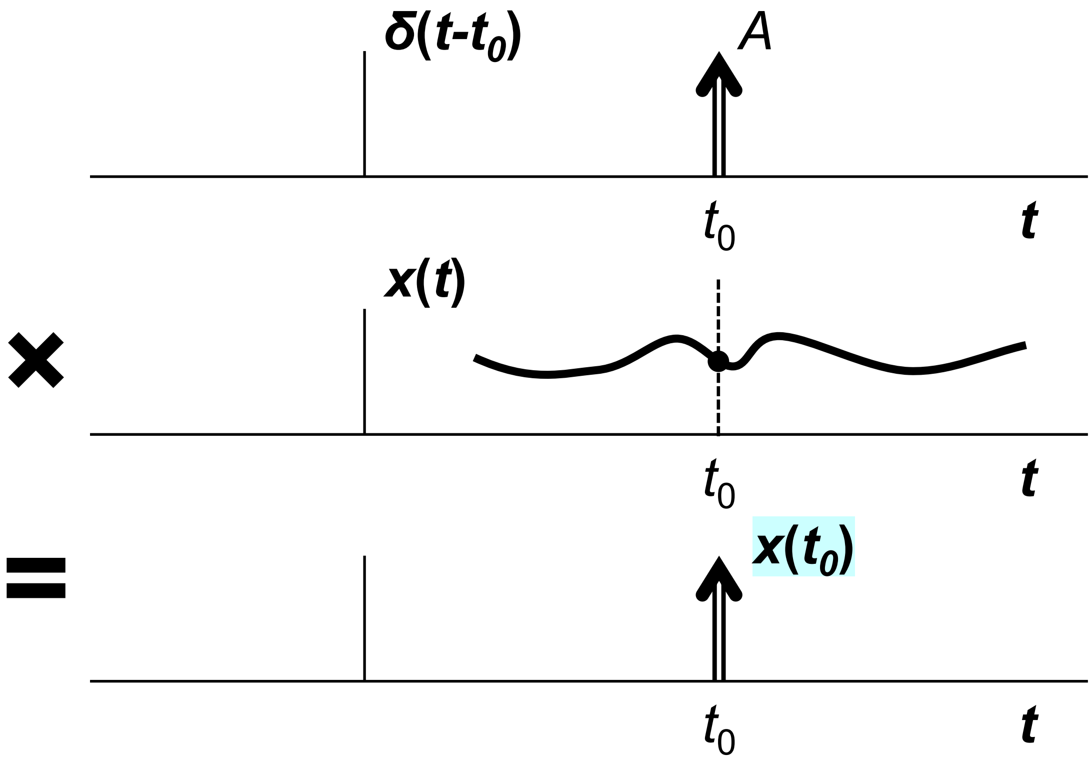
Given a transmitted signal,
Example: Voice signal centered at 900 MHz



Recover the **original signal** from the transmission.
Example: 4 KHz bandwidth voice signal centered at 0 Hz

Impulse: Sampling Property

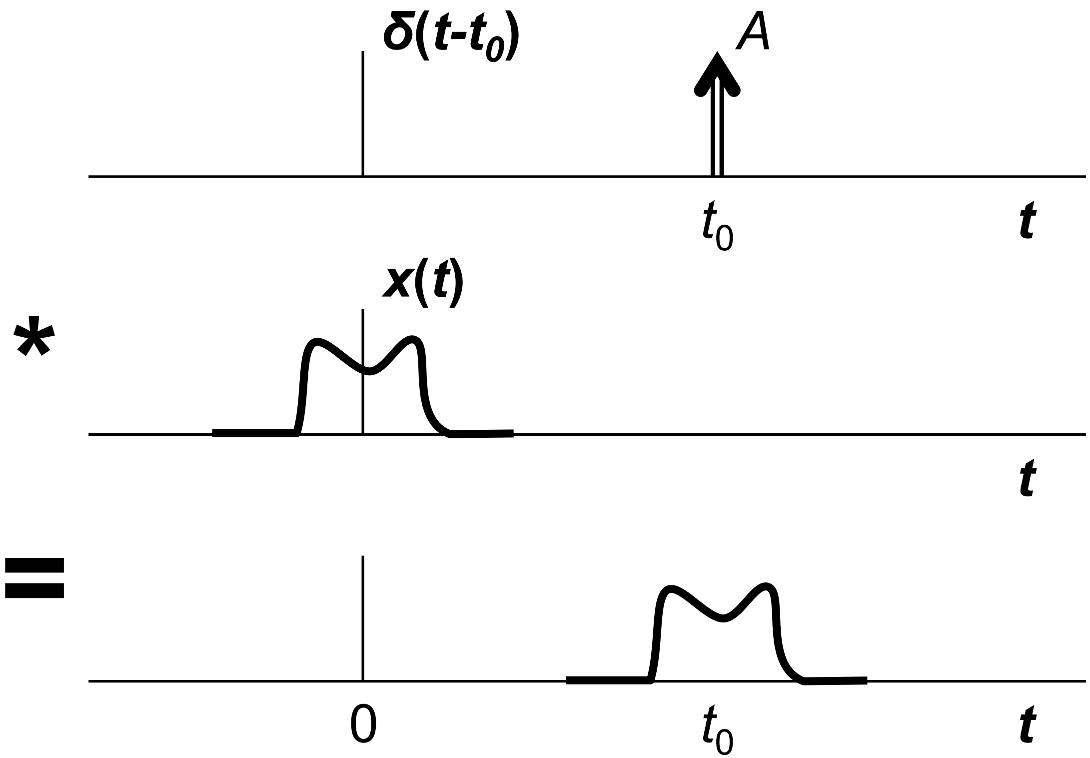
- **Multiplication** of a **function *x*(*t*)** with an impulse at time t_0:



- Results in **scaling** the impulse **by the value of *x*(*t*) at t_0**

Impulse: Convolution Property

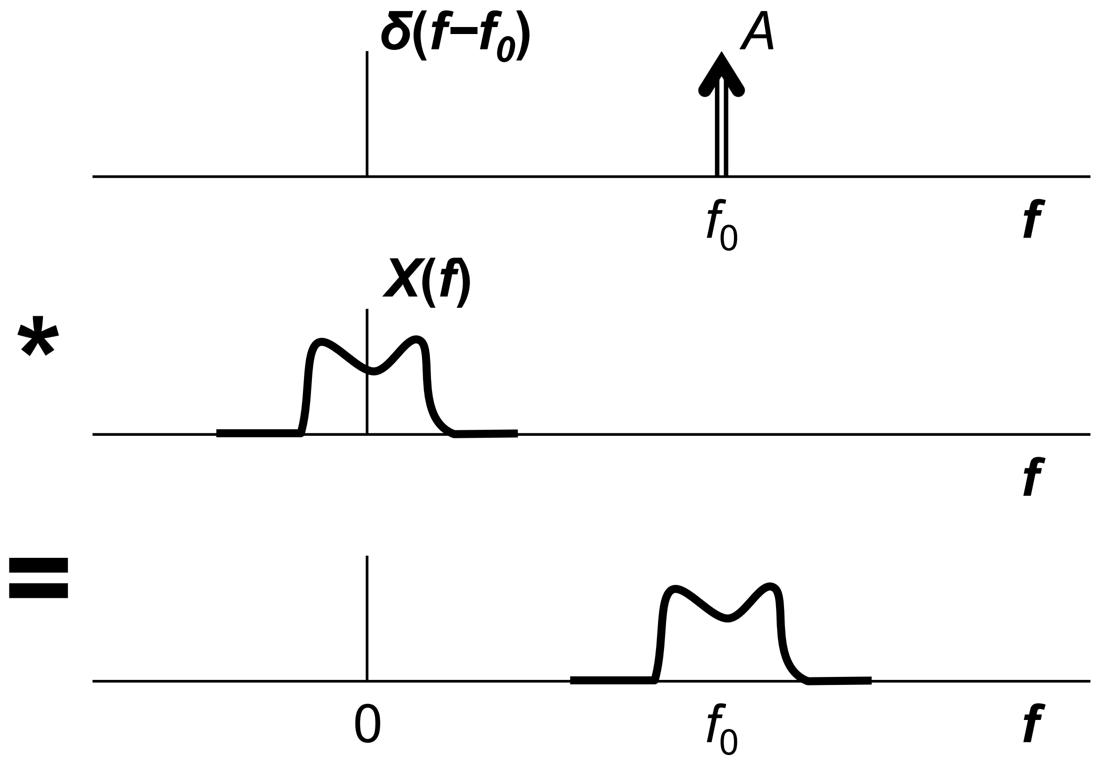
- Convolution of a **function $x(t)$** with an impulse at time t_0:



- Results in a **time shift of** $x(t)$ **by** t_0

Impulse: Convolution in Frequency

- Convolve **function *X*(*f*)** with impulse at frequency f_0:



- Results in a **frequency shift of** *X*(*f*) **by f_0**

Duality of Convolution and Multiplication

- **Multiplication** in time leads to **convolution** in frequency:

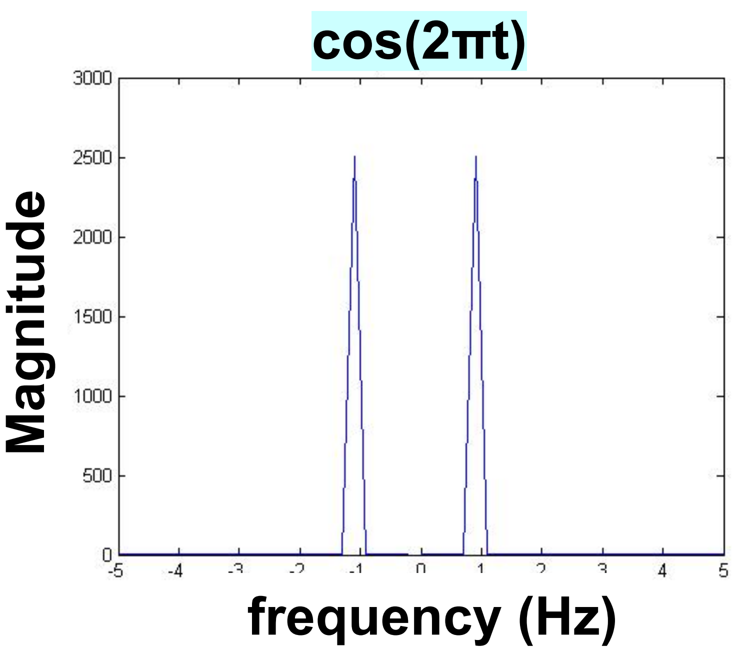
$$x(t)y(t) \overset{\mathcal{F}}{\leftrightarrow} X(f) * Y(f)$$

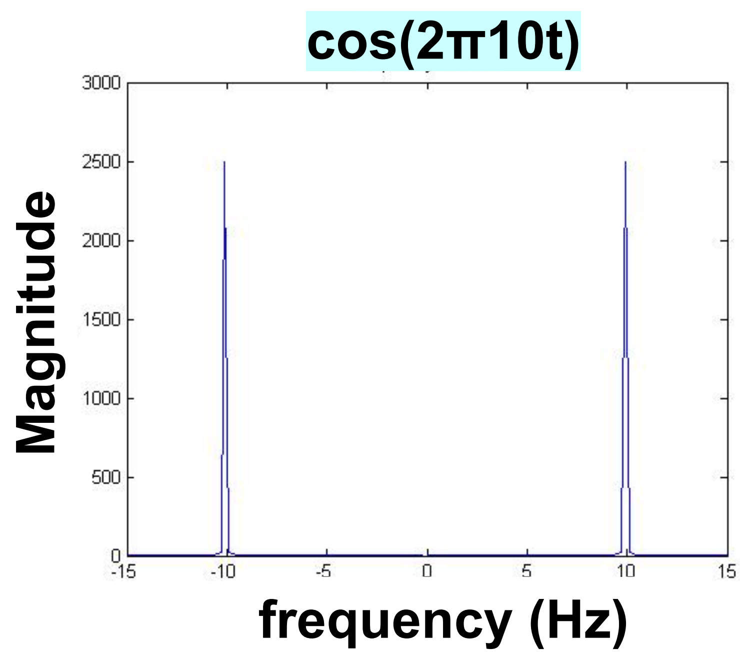
- **Convolution** in time leads to **multiplication** in frequency:

$$x(t) * y(t) \overset{\mathcal{F}}{\leftrightarrow} X(f)Y(f)$$

Principle of Modulation

- Given a cosine wave at frequency f_1 (1 Hz)
- ***Modulate*** it with carrier at frequency f_2 (10 Hz) by **multiplication**

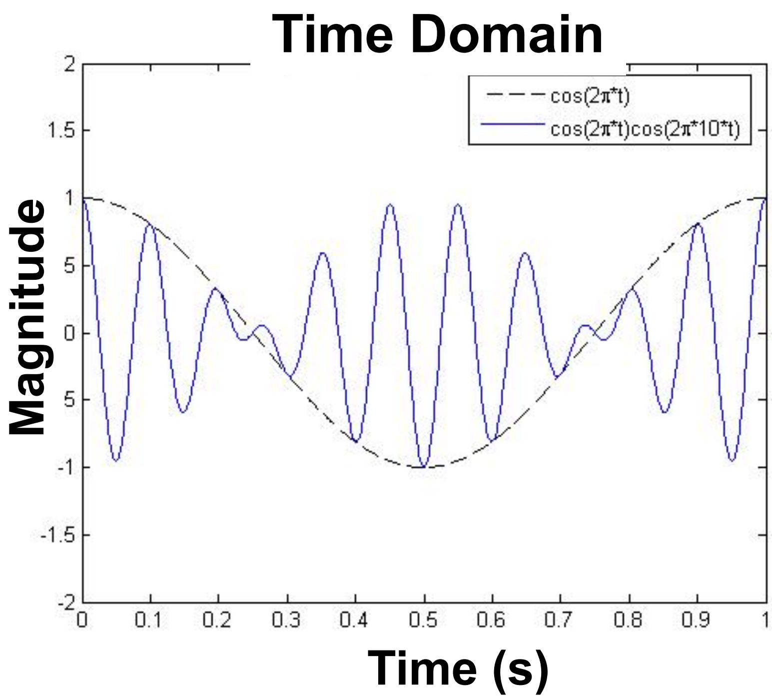


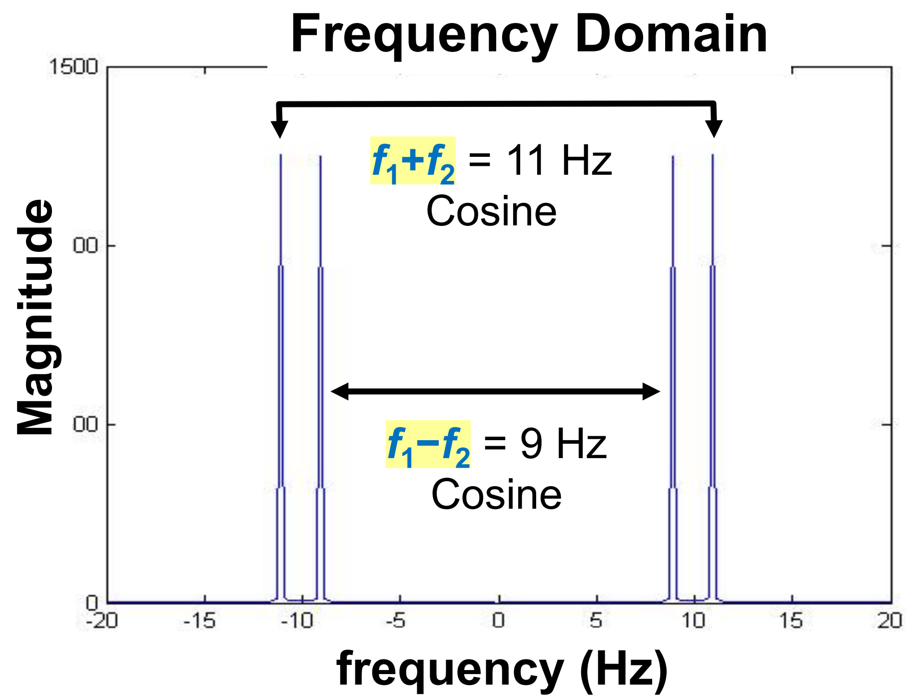


Modulate the two cosines: cos(2πt) × cos(2π10t)

Time Domain and Frequency Domain Views of Modulation

$$\cos(2\pi f_1 t) * \cos(2\pi f_2 t) = \frac{\cos\{2\pi(f_1 + f_2)t\} + \cos\{2\pi(f_1 - f_2)t\}}{2}$$





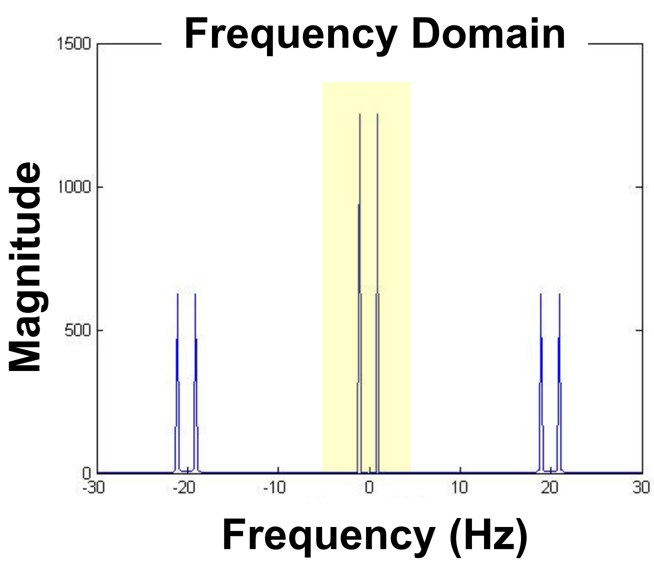
Principle of Demodulation

- Receiver multiplies the modulated signal containing $f_1 - f_2$ and $f_1 + f_2$ by **a copy of the carrier signal:** $cos(2\pi f_2 t)$

- Result contains **original signal** and higher frequency sinusoids:

$$\frac{1}{4}\cos(2\pi(f_1 - 2f_2)t) + \underbrace{\frac{1}{2}\cos(2\pi f_1 t)}_{\text{Original signal}} + \frac{1}{4}\cos(2\pi(f_1 + 2f_2)t)$$

Original signal



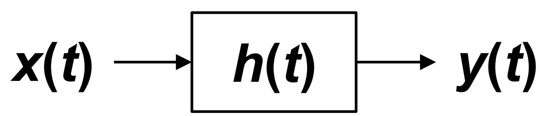
- How to remove the higher frequency sinusoids?

Today

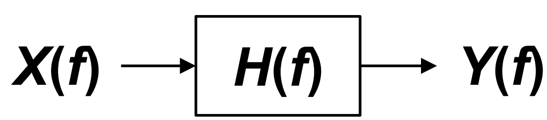
- Radio fundamentals
- Introduction to Modulation and Demodulation
- **Introduction to Filtering**

The Concept of Filtering

- With input, filter produces **output signal $y(t)$** by **convolution** with a **filter response $h(t)$**



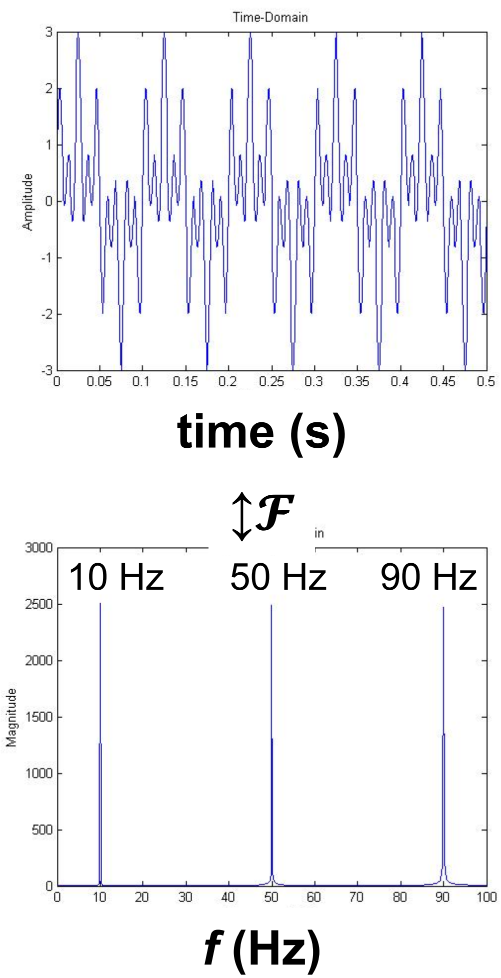
- So, in the frequency domain, the filter **multiplies** each input frequency f by $H(f) \overset{\mathcal{F}}{\leftrightarrow} h(t)$

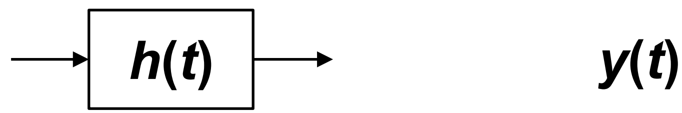


  – $Y(f) = X(f)H(f)$

Example Input Signal to Filter

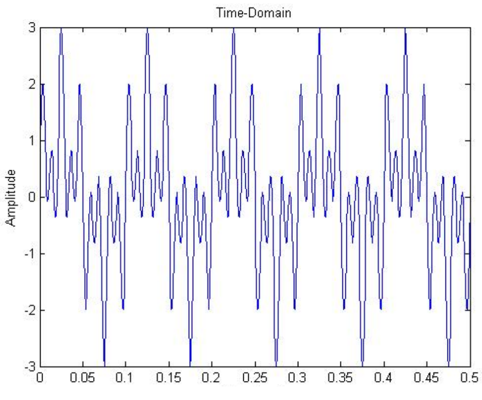
- **Input signal:** Sum of three sinusoids (10, 50, 90 Hz)



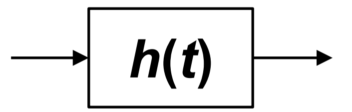


Low Pass Filter Example

- $H(f)$ = 1 below 20 Hz, approaches 0 above 20 Hz

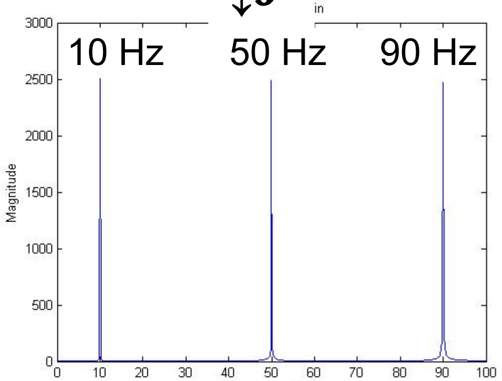


time (s)



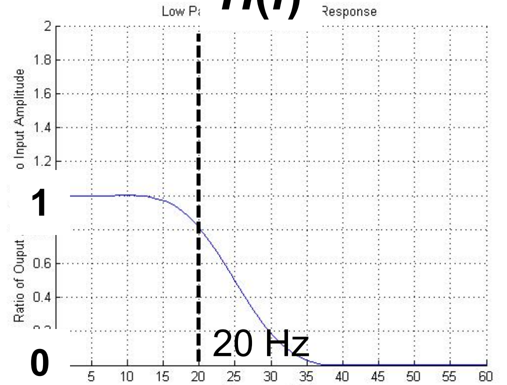
***y*(*t*)**

$\updownarrow \mathcal{F}$



***f* (Hz)**

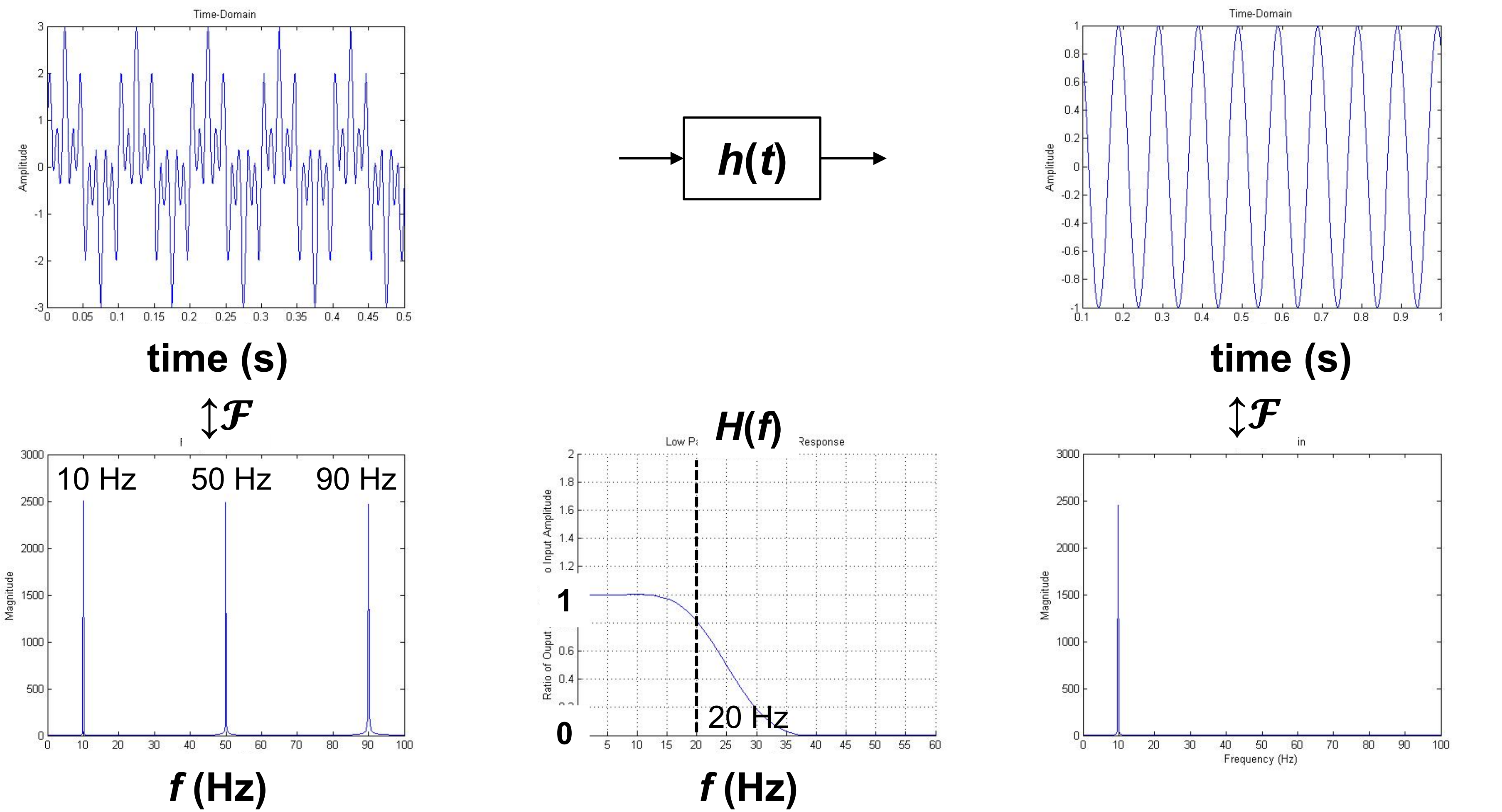
***H*(*f*)**



***f* (Hz)**

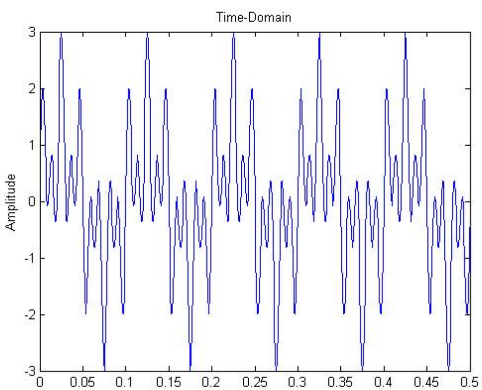
Low Pass Filter Output

- *H*(*f*) = 1 below 20 Hz, approaches 0 above 20 Hz

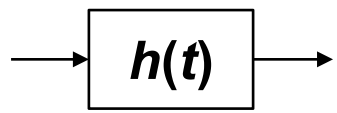


High Pass Filter Example

- $H(f)$ = 0 below 70 Hz, approaches 1 above 70 Hz

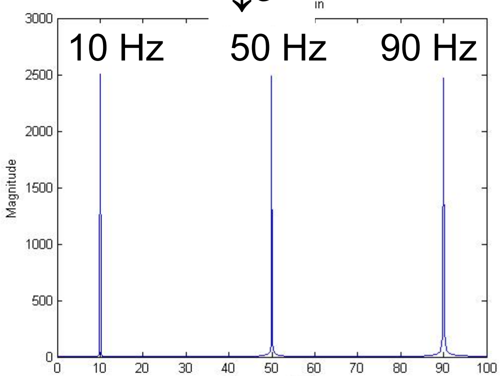


time (s)



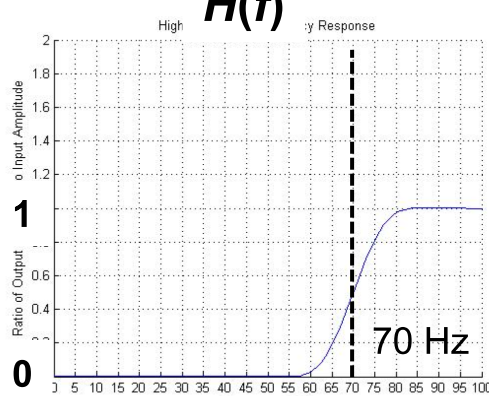
***y*(*t*)**

$\updownarrow \mathcal{F}$



***f* (Hz)**

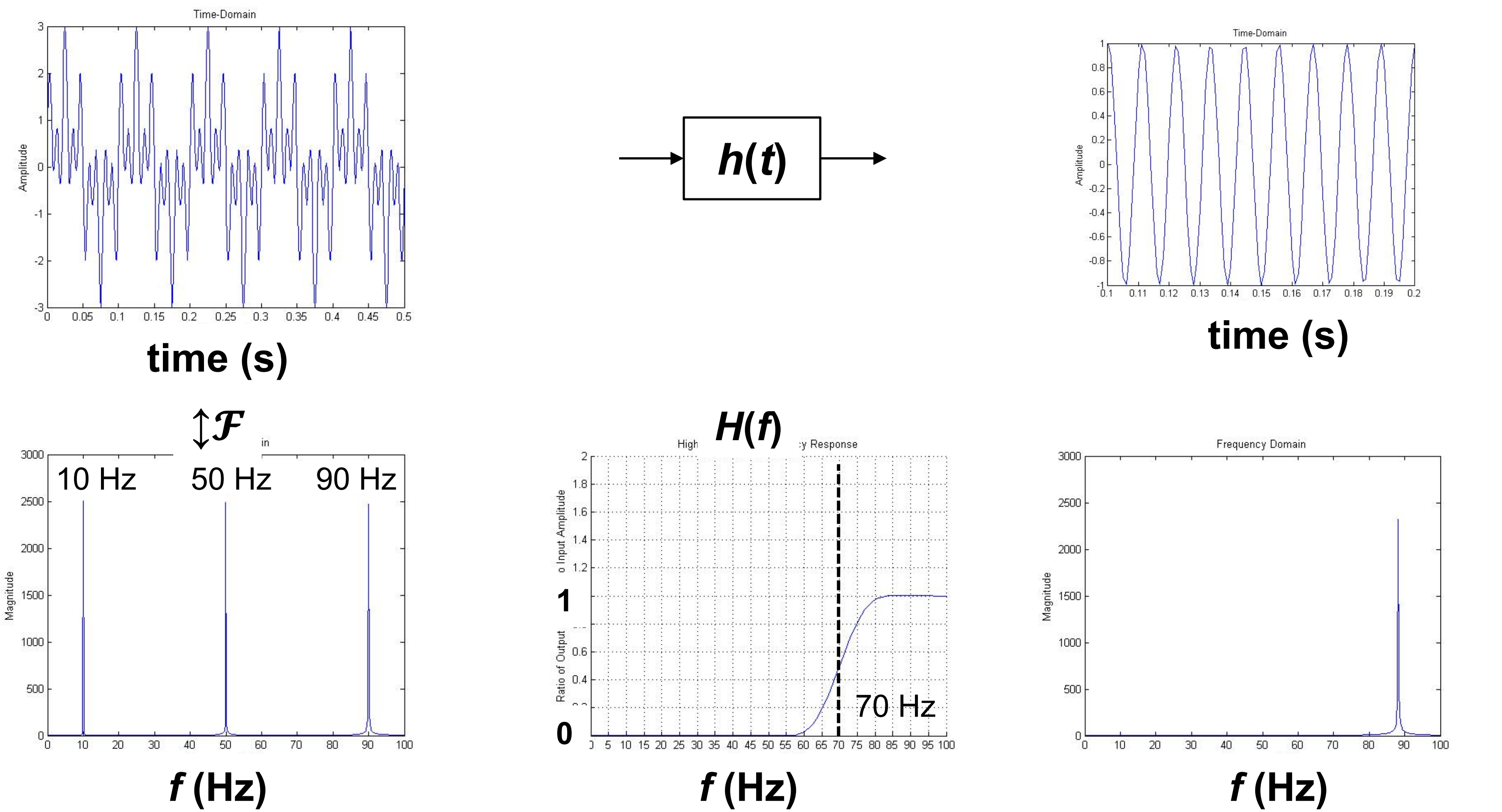
***H*(*f*)**



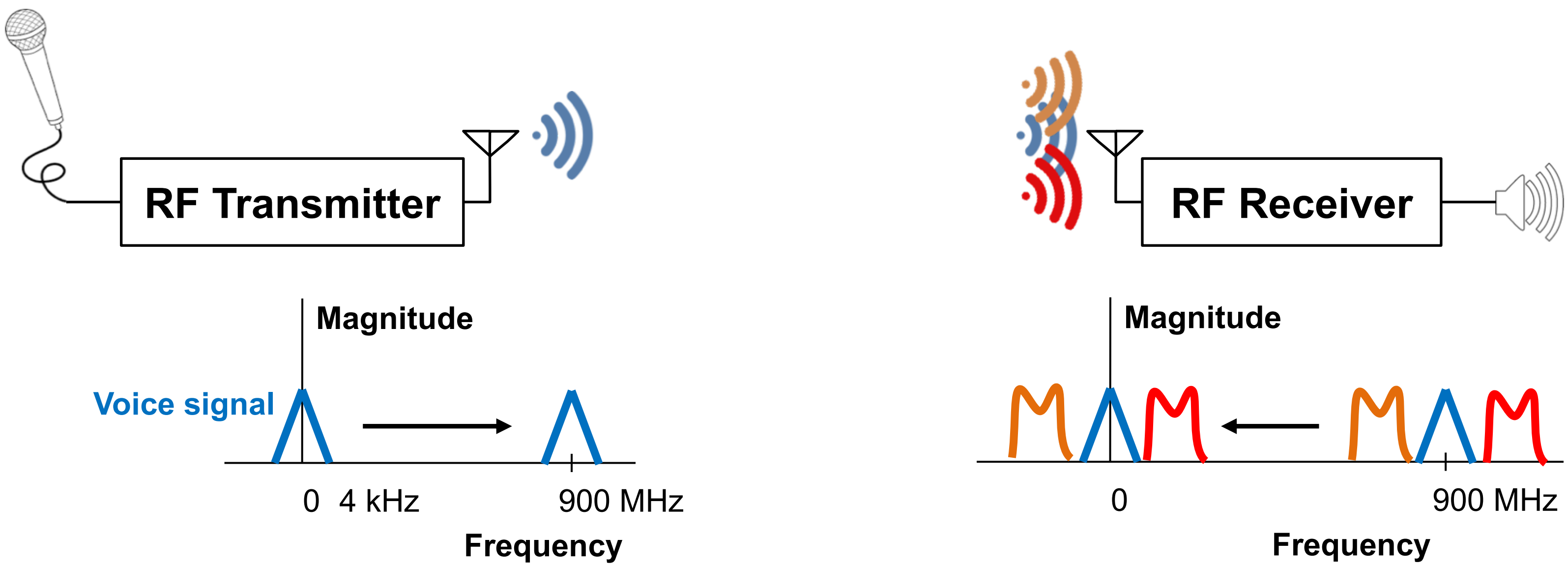
***f* (Hz)**

High Pass Filter Output

- $H(f)$ = 0 below 70 Hz, approaches 1 above 70 Hz



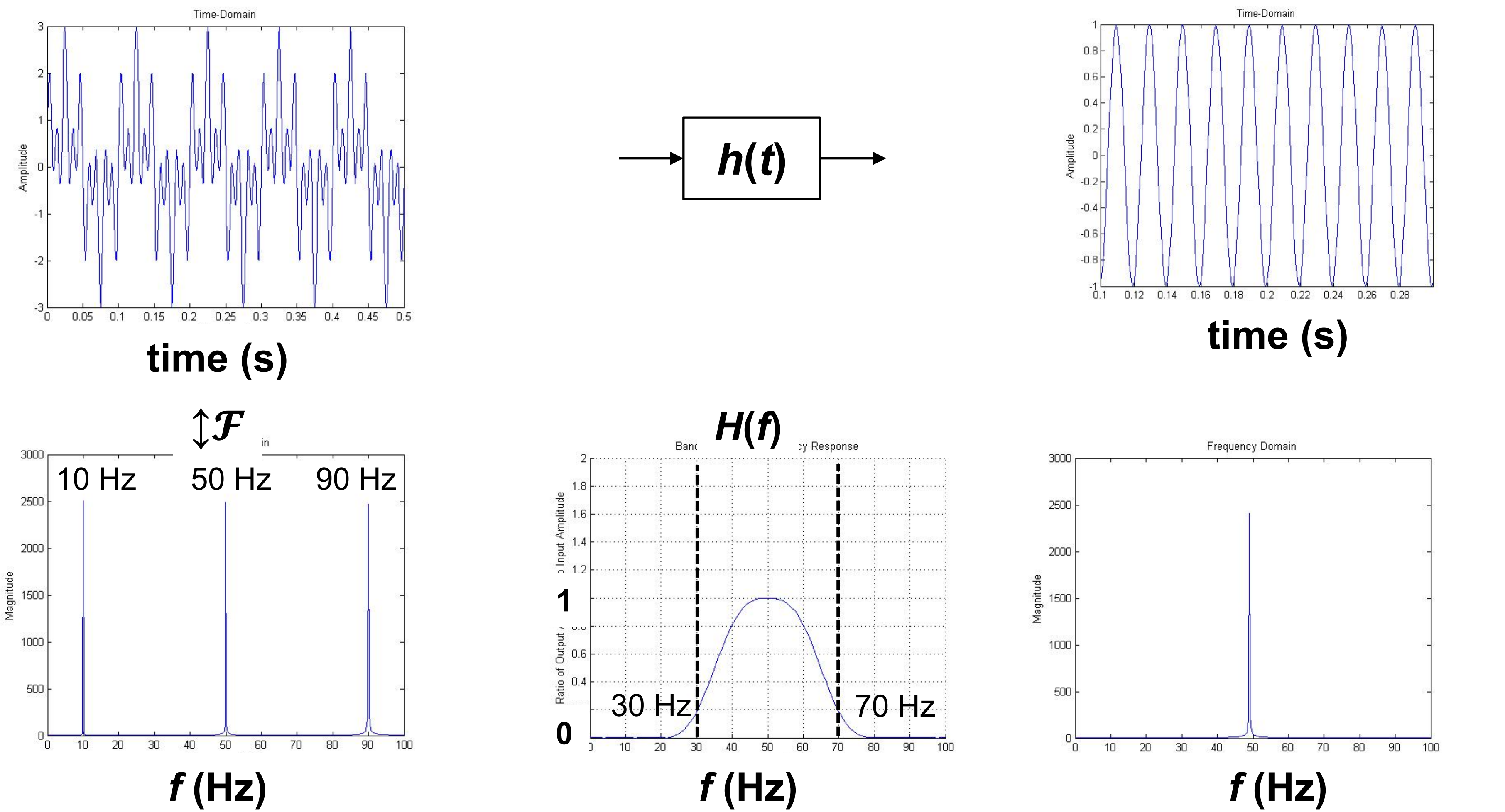
Bandpass Filter: Motivation



- Want to **receive** exclusively a certain **frequency band** of interest
  - In presence of other communication on **adjacent channels**

Bandpass Filter Output

- $H(f)$ = 0 below 30 Hz, above 70 Hz, approaches 1 elsewhere



Today

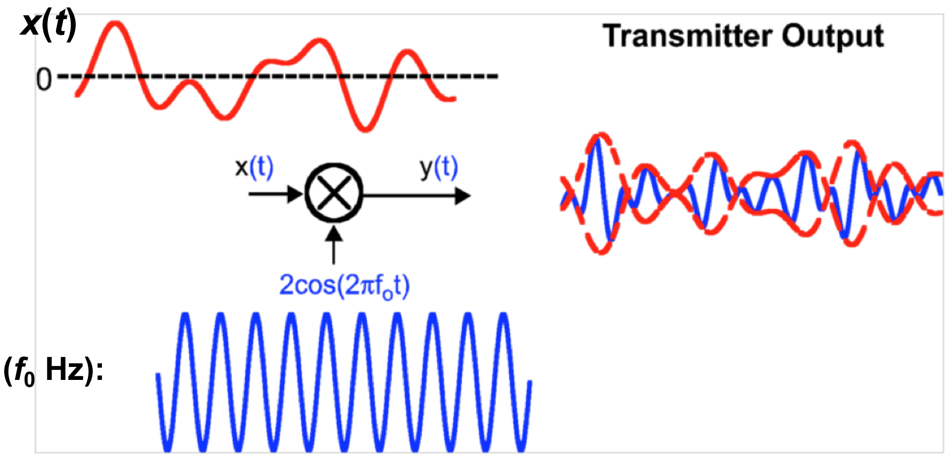
- Radio fundamentals

- **Introduction to Modulation and Demodulation**
  - **AM Radio**
  - Performance metrics

- Introduction to Filtering

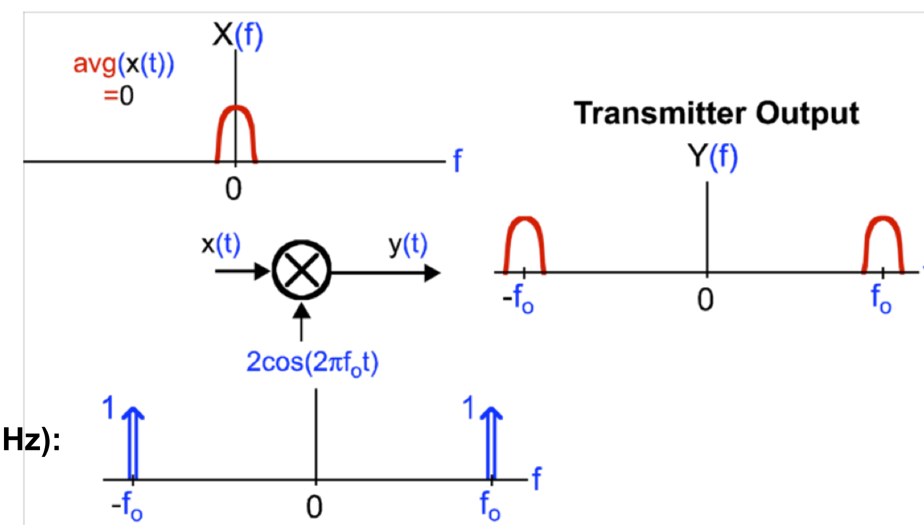
AM Radio Transmitter

Time domain view:



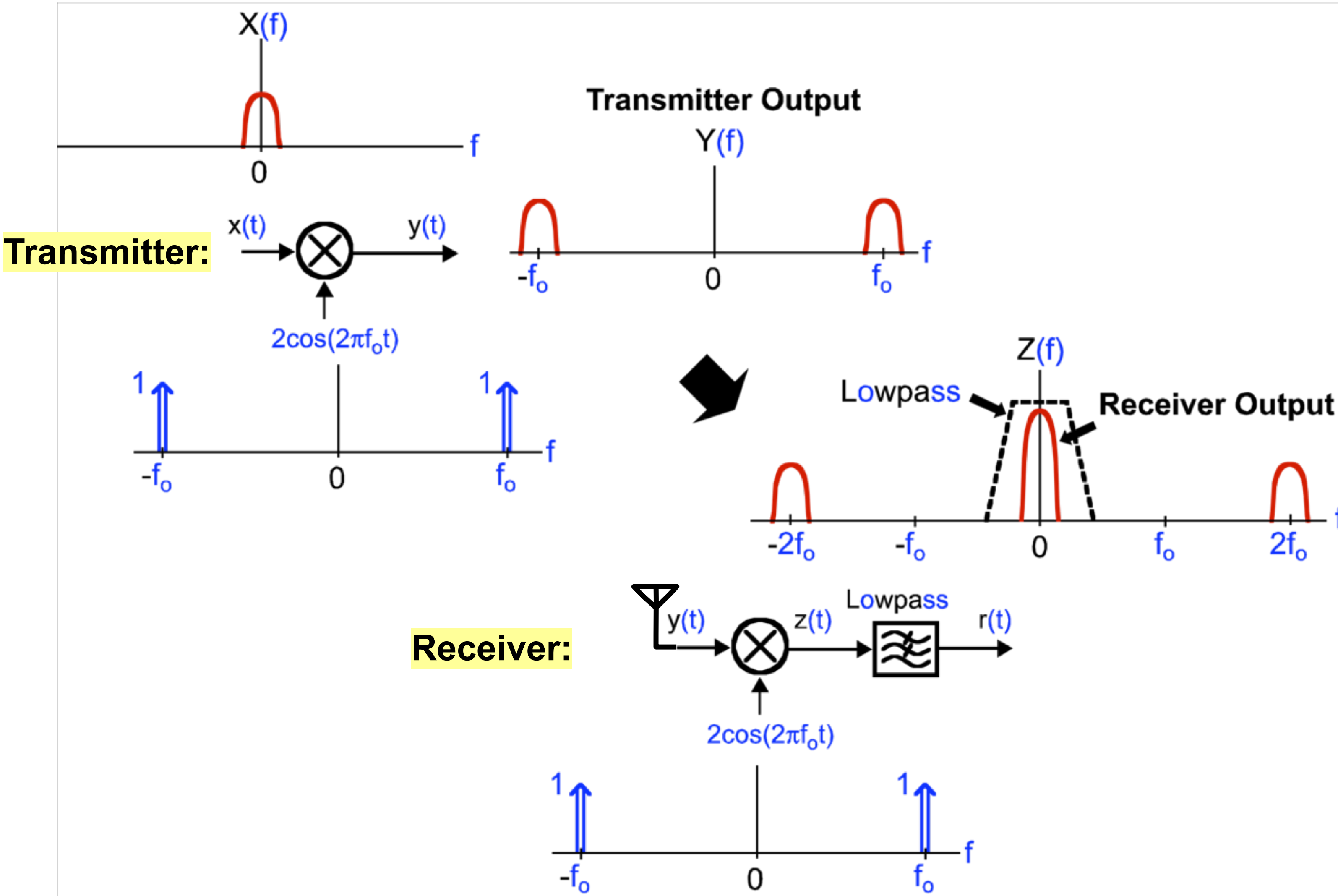
Carrier signal (f_0 Hz):

Frequency domain view:

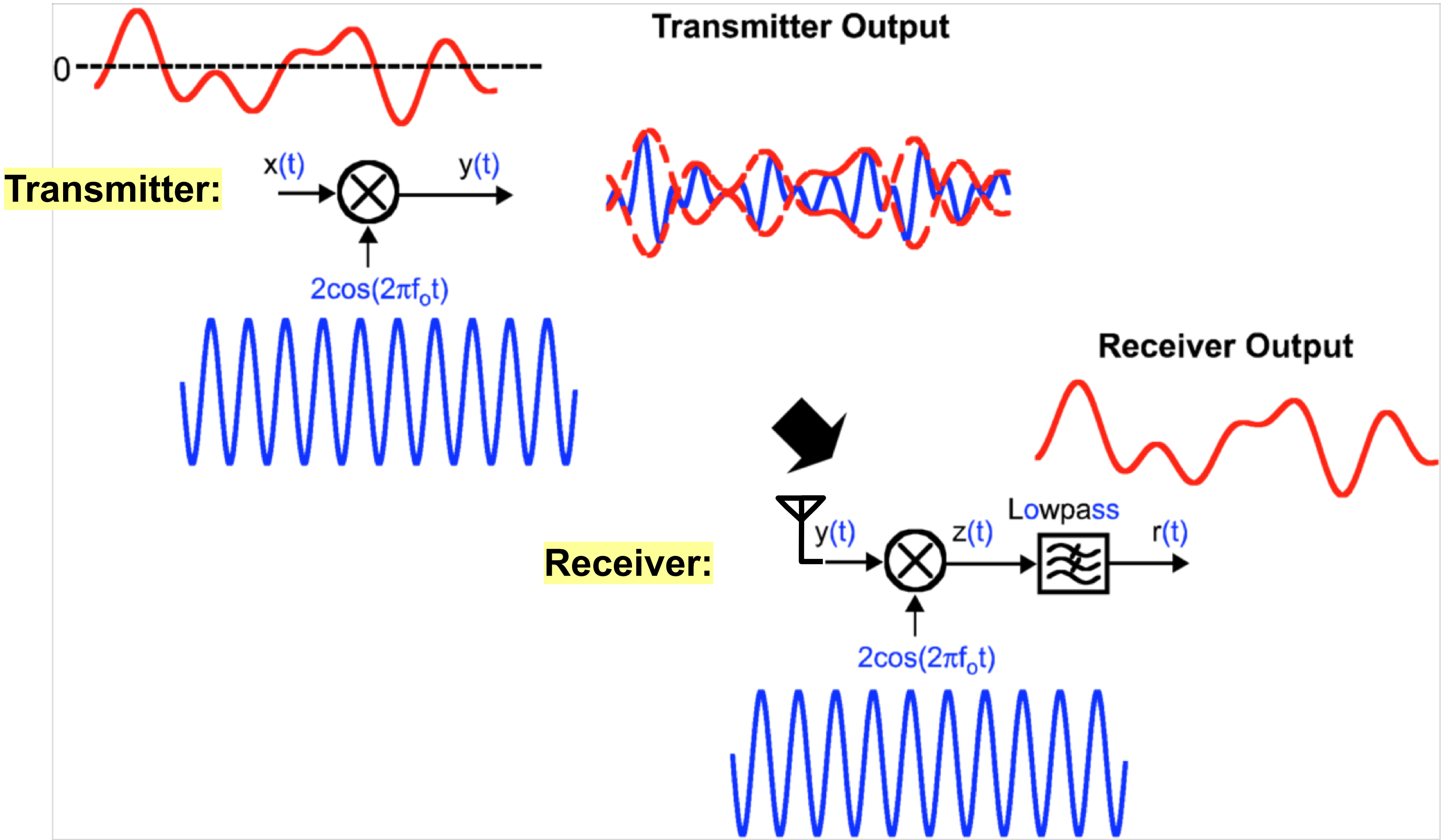


Carrier signal (f_0 Hz):

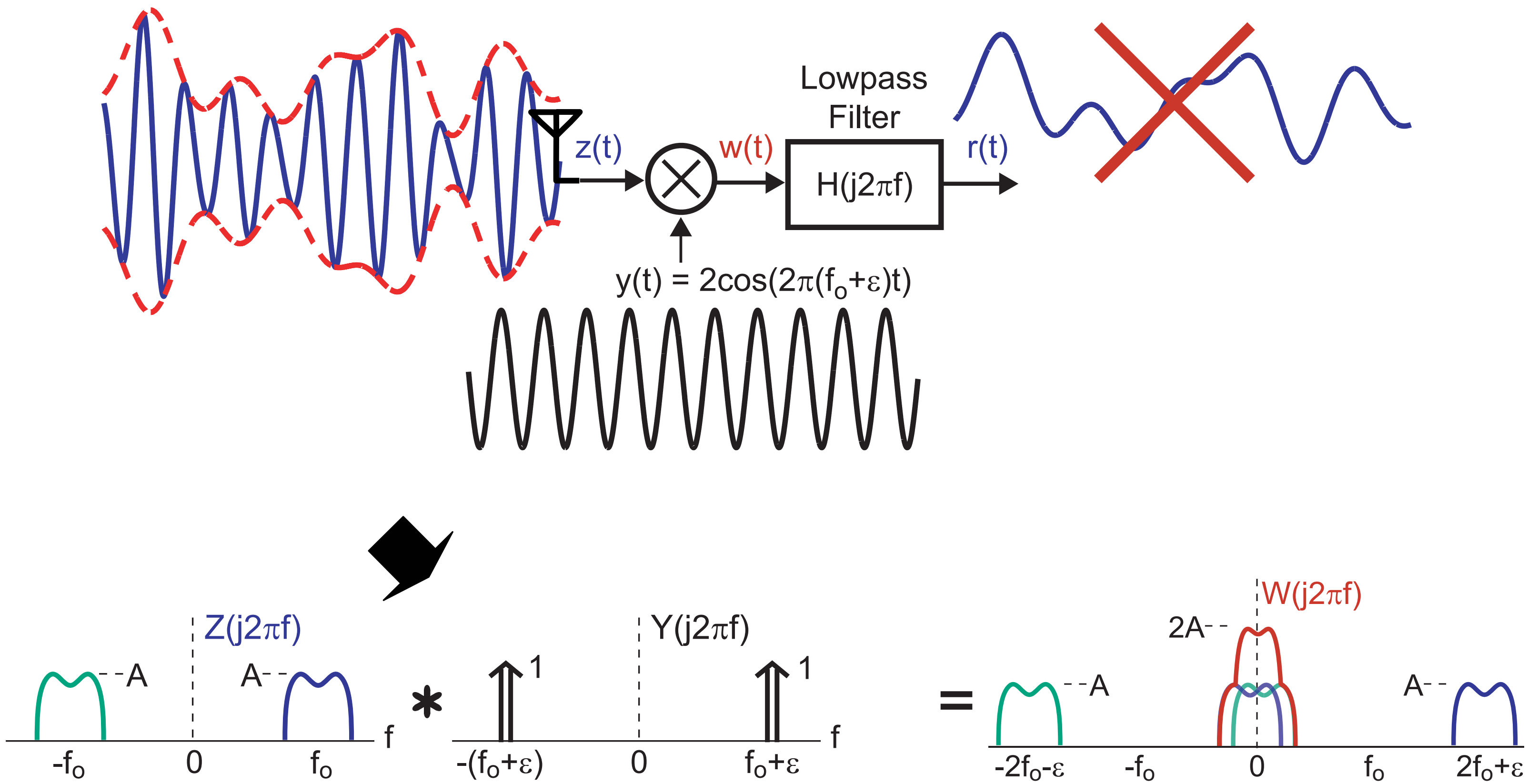
Demodulation: Frequency Domain View



Demodulation: Time Domain View



Impact of a frequency offset



Frequency offset ε at receiver **corrupts** **the output signal *r*(*t*)**

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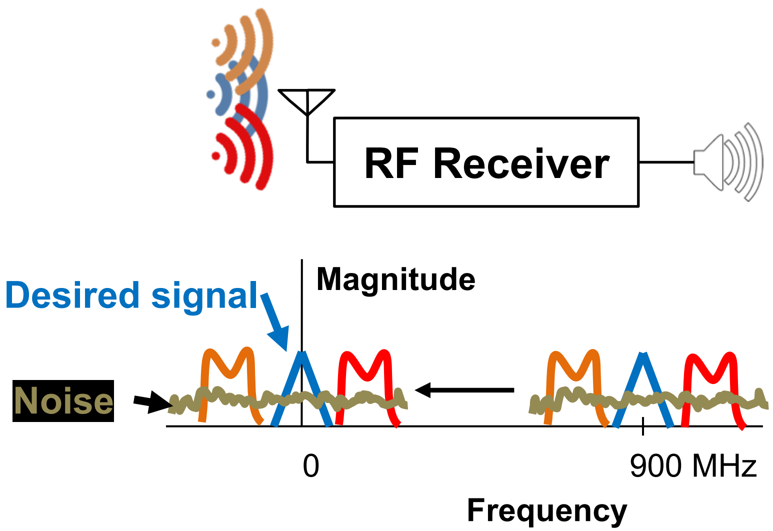
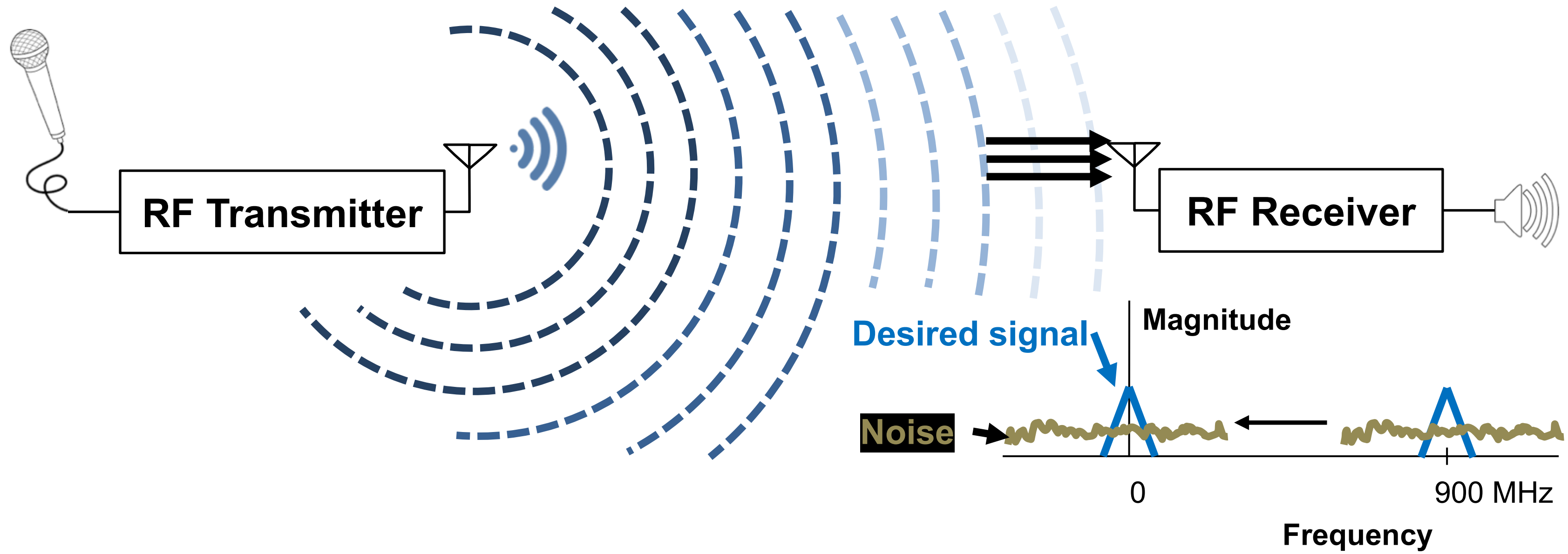
The Issue of Noise



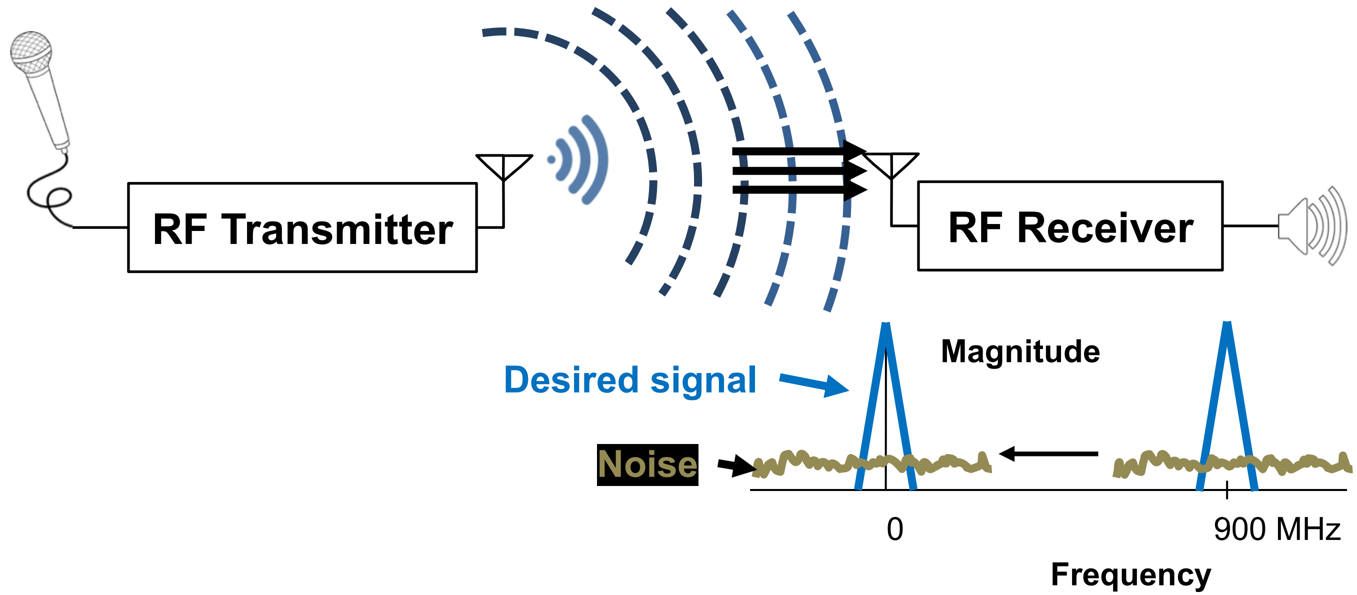
- ***Noise:*** **Unpredictable, corrupting** signal that **adds** to desired signal
  - For RF receiver, mostly comes from analog receiver amplifier circuitry
- **Undesired signals** also add to and corrupt desired signal

Energy Transfer in Wireless Communication



- Receiver antenna captures a **limited amount** of **desired signal's energy**
  - Depending on antenna size, distance, environment

Signal versus Noise



- Moving transmitter closer to receiver generally **increases desired signal energy**
- **Noise** from analog receiver circuitry **remains unchanged**
- **Next few lectures:** *How is system performance impacted?*

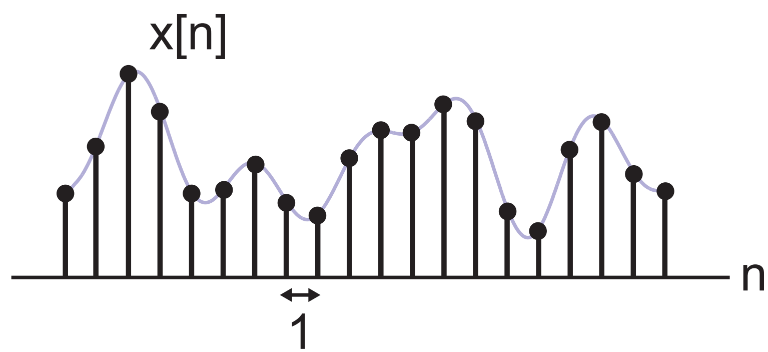
Today

- Radio fundamentals

- **Introduction to Modulation and Demodulation**
  - AM Radio
  - **Performance metrics**

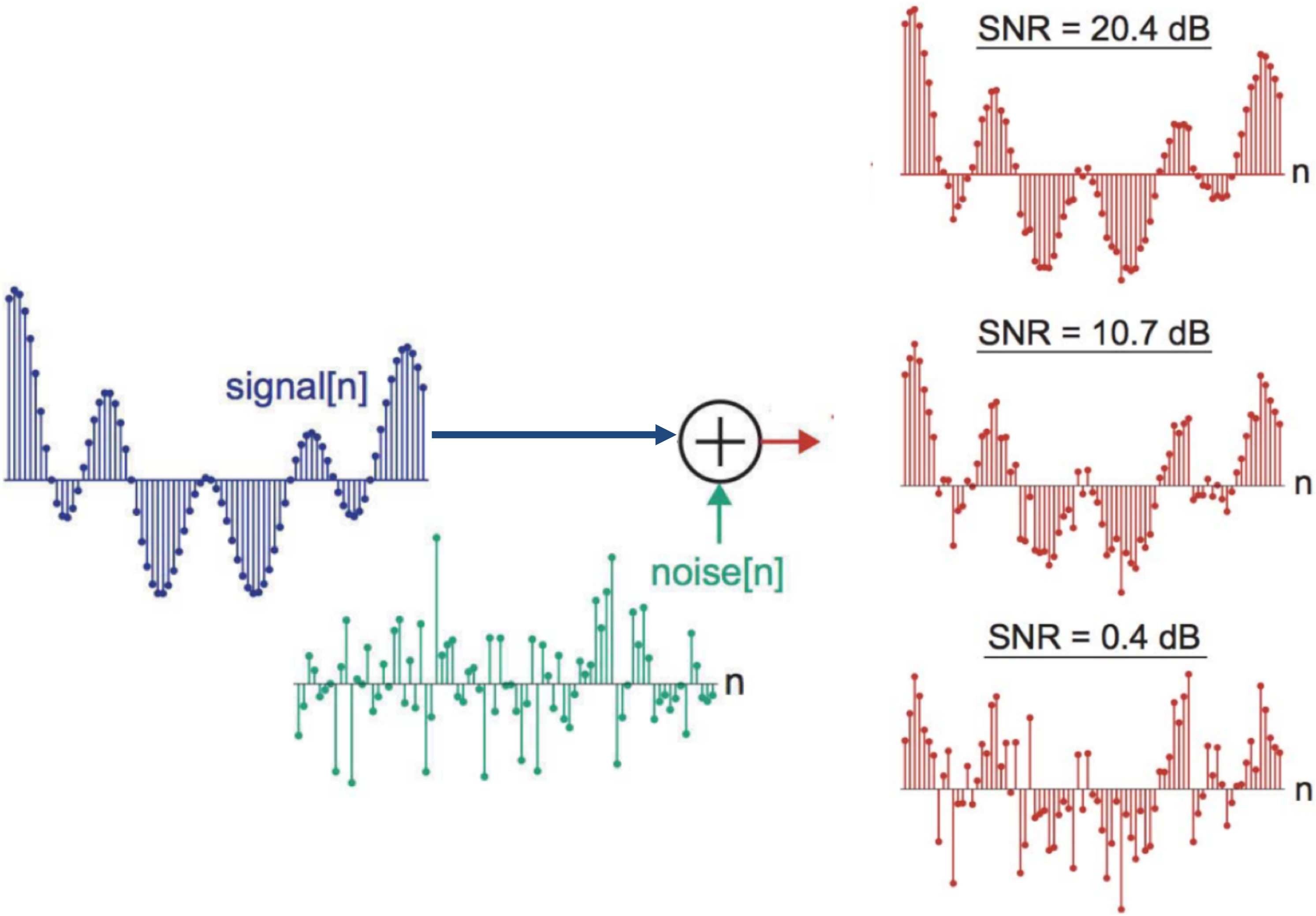
- Introduction to Filtering

Definition of Power, Energy



- Given a signal x[n]:
- ***Energy*** $E_x = \sum_{k=0}^{N-1}(x[k])^2$
- ***Power*** $P_x = \frac{1}{N}\sum_{k=0}^{N-1}(x[k])^2$

SNR Example



Signal to Noise Ratio (SNR)

- The Signal-to-Noise ratio (SNR) is useful in judging the impact of noise on system performance

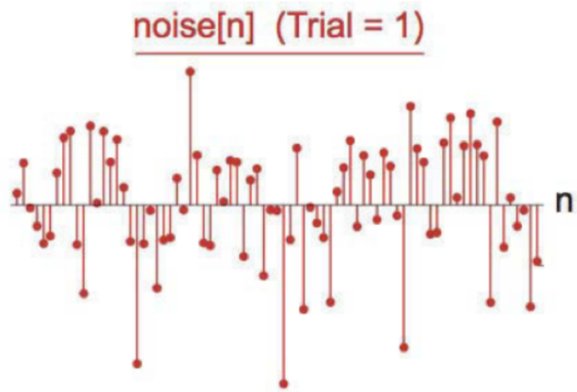
$$SNR = \frac{P_{signal}}{P_{noise}}$$

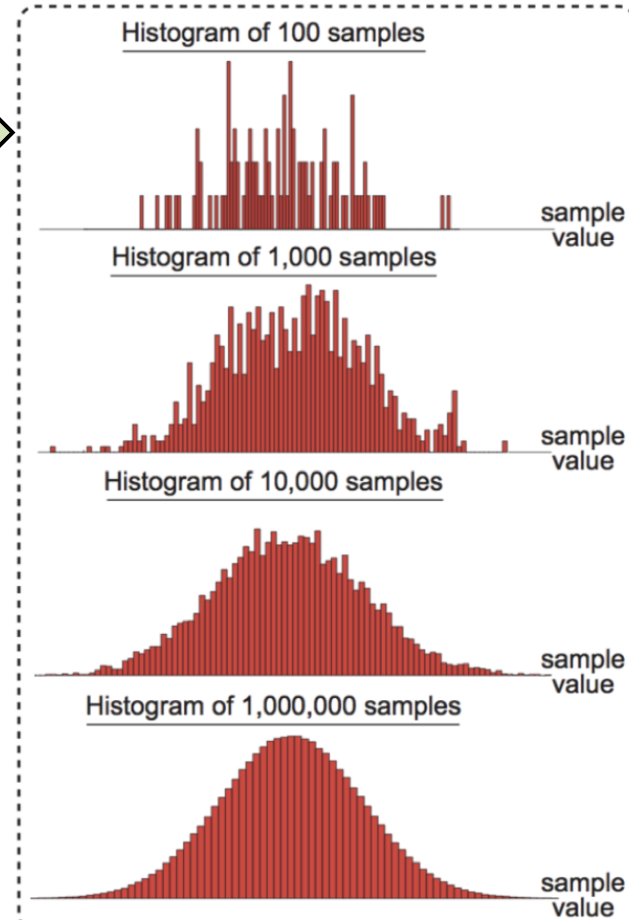
- SNR is often measured in decibels (dB):

$$SNR(dB) = 10log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

10logX	X
100	10000000000
90	1000000000
80	100000000
70	10000000
60	1000000
50	100000
40	10000
30	1000
20	100
10	10
0	1
-10	0.1
-20	0.01
-30	0.001
-40	0.0001
-50	0.000001
-60	0.0000001
-70	0.00000001
-80	0.000000001
-90	0.0000000001
-100	0.00000000001

Noise Characterization: From Samples to Histogram

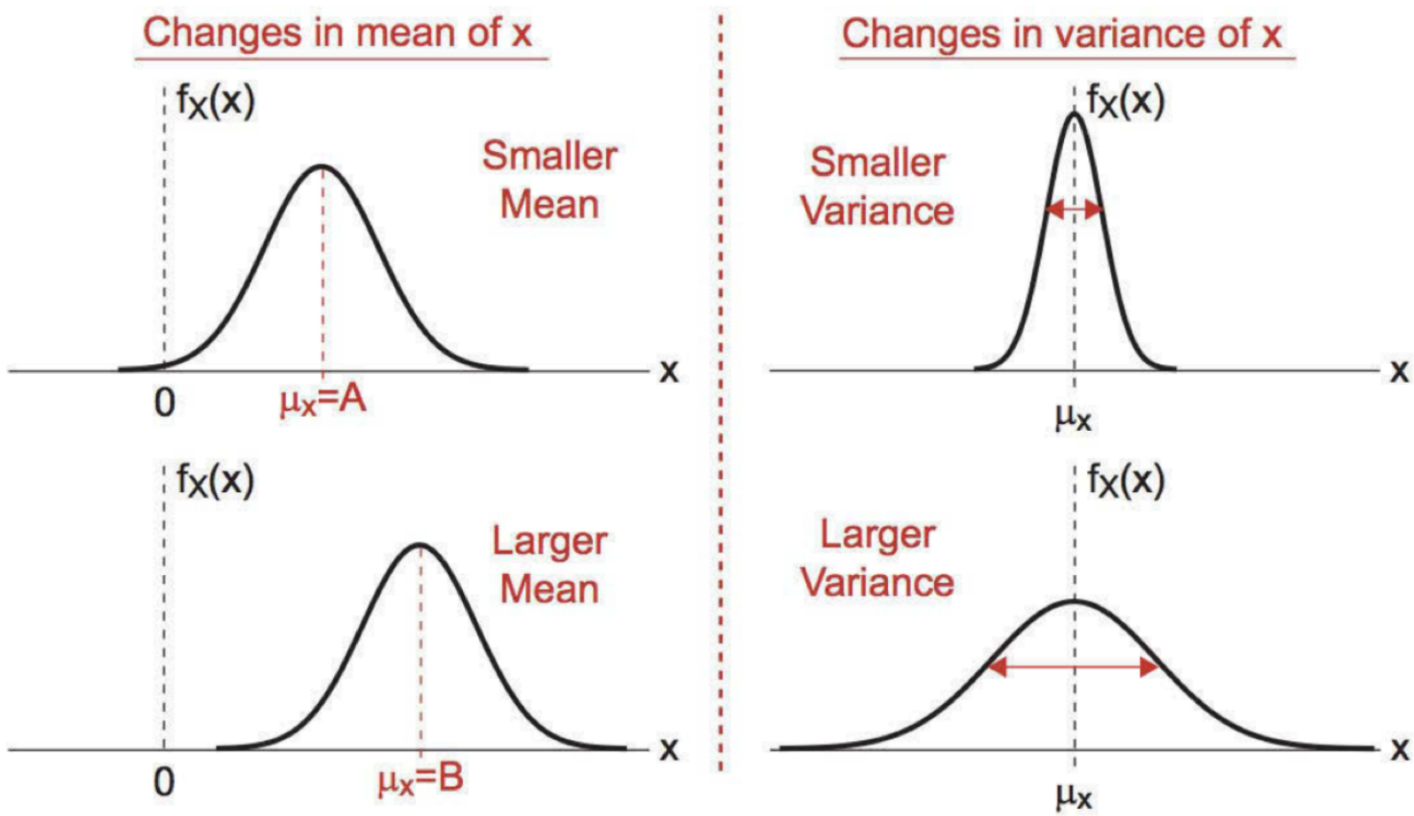




- **Experiment:** create histograms of sample values from signals of increasing lengths

- Typically converge to a shape calle d probability density function (PDF)

Visualizing Mean and Variance



Changes in mean shift the center of mass of PDF

Changes in variance narrow or broaden the PDF

Summary

- **Impulse function** is an important concept for frequency domain **"picture" analysis**
  - **Shifting, sampling properties** of impulse explain modulation and demodulation

- "Picture analysis" of modulation and filtering
  - **Modulation *shifts*** in frequency (convolution with impulses)
  - **Filtering *multiplies*** in frequency

Thursday:

In-Class Midterm (90 minutes)