3D Rendering Intro & Ray Casting

COS 426, Spring 2018
Princeton University
Syllabus

I. Image processing

II. Modeling

III. Rendering

IV. Animation

Rendering
(Michael Bostock, CS426, Fall99)

Image Processing
(Rusty Coleman, CS426, Fall99)

Modeling
(Dennis Zorin, CalTech)

Animation
(Angel, Plate 1)
What is 3D Rendering?

- Topics in computer graphics
  - Imaging = representing 2D images
  - Modeling = representing 3D objects
  - Rendering = constructing 2D images from 3D models
  - Animation = simulating changes over time
What is 3D Rendering?

- Construct image from 3D model
3D Rendering Scenario I

• Interactive
  ◦ Images generated in fraction of a second (e.g., 1/30) as user controls rendering parameters (e.g., camera)
    » Achieve highest quality possible in given time
    » Useful for visualization, games, etc.
3D Rendering Scenario II

• Offline
  ◦ One image generated with as much quality as possible for a particular set of rendering parameters
    » Take as much time as is needed (minutes, hours…)
    » Photorealism: movies, cut scenes, etc.
3D Rendering Issues

- What issues must be addressed by a 3D rendering system?
3D Rendering Issues

• What issues must be addressed by a 3D rendering system?
  ○ Camera
  ○ Visible surface determination
  ○ Lights
  ○ Reflectance
  ○ Shadows
  ○ Indirect illumination
  ○ Sampling
  ○ etc.
3D Rendering Issues

• What issues must be addressed by a 3D rendering system?
  ○ Camera
  ○ Visible surface determination
  ○ Lights
  ○ Reflectance
  ○ Shadows
  ○ Indirect illumination
  ○ Sampling
  ○ etc.
Pinhole Camera Parameters

- **Position**
  - Eye position \((p_x, p_y, p_z)\)

- **Orientation**
  - View direction \((d_x, d_y, d_z)\) or “look at” point
  - Up direction \((u_x, u_y, u_z)\)

- **Coverage**
  - Field of view \((\text{fov}_x, \text{fov}_y)\)

- **Resolution**
  - \(x\) and \(y\)
3D Rendering Issues

• What issues must be addressed by a 3D rendering system?
  ○ Camera
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  ○ Shadows
  ○ Indirect illumination
  ○ Sampling
  ○ etc.
Visible Surface Determination

- The color of each pixel on the view plane depends on the radiance ("amount of light") emanating from visible surfaces.

How find visible surfaces?
A Characterization of Ten Hidden-Surface Algorithms

ACM Comput. Surv. 6, 1 (March 1974)
3D Rendering Issues

• What issues must be addressed by a 3D rendering system?
  ◦ Camera
  ◦ Visible surface determination
  ◦ Lights
  ◦ Reflectance
  ◦ Shadows
  ◦ Indirect illumination
  ◦ Sampling
  ◦ etc.
Lighting Simulation

[Diagram showing light sources (L1, L2), a viewer, and the normal vector (N) on a surface (V)].
3D Rendering Issues

• What issues must be addressed by a 3D rendering system?
  ○ Camera
  ○ Visible surface determination
  ○ Lights
  ○ Reflectance
  ○ Shadows
  ○ Indirect illumination
  ○ Sampling
  ○ etc.
Shadows

- Occlusions from light sources
  - Soft shadows with area light source
3D Rendering Issues

• What issues must be addressed by a 3D rendering system?
  ◦ Camera
  ◦ Visible surface determination
  ◦ Lights
  ◦ Reflectance
  ◦ Shadows
  ◦ Indirect illumination
  ◦ Sampling
  ◦ etc.
Indirect Illumination

Jensen

Henrik Wann Jensen
3D Rendering Issues

• What issues must be addressed by a 3D rendering system?
  ◦ Camera
  ◦ Visible surface determination
  ◦ Shadows
  ◦ Reflectance
  ◦ Indirect illumination
  ◦ Sampling
  ◦ etc.
Sampling

• Scene can be sampled with any ray
  ◦ Rendering is a problem in sampling and reconstruction
Rendering Method I: Ray Casting
Ray Casting

- The color of each pixel on the view plane depends on the radiance emanating along rays from visible surfaces in scene
Scene

- Scene has:
  - Scene graph with surface primitives
  - Set of lights
  - Camera

```c
struct R3Scene {
    R3Node *root;
    vector<R3Light *> lights;
    R3Camera camera;
    R3Box bbox;
    R3Rgb background;
    R3Rgb ambient;
}:
```
Scene Graph

- Scene graph is hierarchy of nodes, each with:
  - Bounding box (in node’s coordinate system)
  - Transformation (4x4 matrix)
  - Shape (mesh, sphere, … or null)
  - Material (more on this later)
Scene Graph

• Simple scene graph implementation:

```c
struct R3Node {
    struct R3Node *parent;
    vector<struct R3Node *> children;
    R3Shape *shape;
    R3Matrix transformation;
    R3Material *material;
    R3Box bbox;
};
```

```c
struct R3Shape {
    R3ShapeType type;
    R3Box *box;
    R3Sphere *sphere;
    R3Cylinder *cylinder;
    R3Cone *cone;
    R3Mesh *mesh;
};
```
Ray Casting

- For each sample (pixel) ...
  - Construct ray from eye position through view plane
  - Compute radiance leaving first point of intersection between ray and scene
Ray Casting

• Simple implementation:

```c
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene, &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```
Ray Casting

• Simple implementation:

```c
R2Image *RayCast(R3Scene *scene, int width, int height)
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    R2Image *image = new R2Image(width, height);
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            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene, &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```
Constructing Ray Through a Pixel

Ray: $P = P_0 + tV$
Constructing Ray Through a Pixel

• 2D Example

\[ \Theta = \text{frustum half-angle} \]
\[ d = \text{distance to view plane} \]

\[ \text{right} = \text{towards} \times \text{up} \]

\[ P_1 = P_0 + d \times \text{towards} - d \times \tan(\Theta) \times \text{right} \]
\[ P_2 = P_0 + d \times \text{towards} + d \times \tan(\Theta) \times \text{right} \]

\[ P = P_1 + ((i + 0.5) / \text{width}) \times (P_2 - P_1) \]
\[ V = (P - P_0) / \|P - P_0\| \]
\[ (d \text{ cancels out...}) \]

Ray: \[ P = P_0 + tV \]
Ray Casting

- Simple implementation:

```c
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene, &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```
Ray Casting

- Simple implementation:

```c
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray)
{
    R3Intersection intersection = ComputeIntersection(scene, ray);
    return ComputeRadiance(scene, ray, intersection);
}
```

```c
struct R3Intersection {
    bool hit;
    R3Node *node;
    R3Point position;
    R3Vector normal;
    double t;
};
```
Ray Casting

- Simple implementation:

```cpp
R3Rrgb ComputeRadiance(R3Scene *scene, R3Ray *ray)
{
    R3Intersection intersection = ComputeIntersection(scene, ray);
    return ComputeRadiance(scene, ray, intersection);
}
```

```cpp
struct R3Intersection {
    bool hit;
    R3Node *node;
    R3Point position;
    R3Vector normal;
    double t;
};
```
Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees
Ray Intersection

• Ray Intersection
  ➢ Sphere
  ◦ Triangle
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  ◦ Scene

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  ◦ BSP trees
Ray-Sphere Intersection

$P_0$, $v$, $P$, $P'$, $O$, $r$
Ray-Sphere Intersection

Ray: \( P = P_0 + tV \)
Sphere: \( |P - O|^2 - r^2 = 0 \)
Ray-Sphere Intersection I

Ray: \( P = P_0 + tV \)
Sphere: \( |P - O|^2 - r^2 = 0 \)

Substituting for \( P \), we get:
\[ |P_0 + tV - O|^2 - r^2 = 0 \]

Solve quadratic equation:
\[ at^2 + bt + c = 0 \]
where:
\[ a = V^2 \]
\[ b = 2 \cdot V \cdot (P_0 - O) \]
\[ c = |P_0 - O|^2 - r^2 = 0 \]

\( P = P_0 + tV \)
Ray-Sphere Intersection II

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

$L = O - P_0$

$t_{ca} = L \cdot V$
if ($t_{ca} < 0$) return 0

d^2 = L \cdot L - t_{ca}^2$
if ($d^2 > r^2$) return 0

$t_{hc} = \sqrt{r^2 - d^2}$
$t = t_{ca} - t_{hc}$ and $t_{ca} + t_{hc}$

$P = P_0 + tV$
Ray-Sphere Intersection

- Need normal vector at intersection for lighting calculations (next lecture)

\[ N = \frac{(P - O)}{||P - O||} \]
Ray Intersection

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Ray-Triangle Intersection
Ray-Triangle Intersection

- First, intersect ray with plane
- Then, check if intersection point is inside triangle
Ray-Plane Intersection

Ray: \( P = P_0 + tv \)
Plane: \( P \cdot N + d = 0 \)

Substituting for \( P \), we get:
\[ (P_0 + tv) \cdot N + d = 0 \]

Solution:
\[ t = -\frac{(P_0 \cdot N + d)}{(V \cdot N)} \]
\[ P = P_0 + tv \]
Ray-Triangle Intersection I

• Check if point is inside triangle algebraically

For each side of triangle

\[ V_1 = T_1 - P_0 \]
\[ V_2 = T_2 - P_0 \]
\[ N_1 = V_2 \times V_1 \]

Normalize \( N_1 \)

Plane \( p(P_0, N_1) \)

if (SignedDistance(p, P) < 0)
  return FALSE
end

return TRUE
Ray-Triangle Intersection II

• Check if point is inside triangle algebraically

For each side of triangle

\[ V_1 = T_1 - P \]
\[ V_2 = T_2 - P \]
\[ N_1 = V_2 \times V_1 \]
if \( (V \cdot N_1 < 0) \)
    return FALSE
end
return TRUE
Ray-Triangle Intersection II

• Check if point is inside triangle algebraically

For each side of triangle

\[ V_1 = T_1 - P \]
\[ V_2 = T_2 - P \]
\[ N_1 = V_2 \times V_1 \]

if \((V \cdot N_1 < 0)\)

return FALSE

end

return TRUE
Ray-Triangle Intersection III

- Check if point is inside triangle parametrically

“Barycentric coordinates” \( \alpha, \beta, \gamma \):

\[
P = \alpha T_3 + \beta T_2 + \gamma T_1
\]

where \( \alpha + \beta + \gamma = 1 \)

\[
\alpha = \frac{\text{Area}(T_1 T_2 P)}{\text{Area}(T_1 T_2 T_3)}
\]

\[
\beta = \frac{\text{Area}(T_1 P T_3)}{\text{Area}(T_1 T_2 T_3)}
\]

\[
\gamma = \frac{\text{Area}(P T_2 T_3)}{\text{Area}(T_1 T_2 T_3)}
\]

\[
= 1 - \alpha - \beta
\]
Ray-Triangle Intersection III

• Check if point is inside triangle parametrically

Compute "barycentric coordinates" $\alpha$, $\beta$:
$$
\alpha = \frac{\text{Area}(T_1T_2P)}{\text{Area}(T_1T_2T_3)}
\beta = \frac{\text{Area}(T_1PT_3)}{\text{Area}(T_1T_2T_3)}
$$

Area$(T_1T_2T_3) = \frac{1}{2} \| (T_2-T_1) \times (T_3-T_1) \|$

check if backfacing:
$$
((T_2-T_1) \times (T_3-T_1)) \cdot N < 0
$$

Check if point inside triangle.
$$
0 \leq \alpha \leq 1 \text{ and } 0 \leq \beta \leq 1
$$
and $\alpha + \beta \leq 1$
Ray Intersection

- Ray Intersection
  - Sphere
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  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees
Ray-Box Intersection

- Check front-facing sides for intersection with ray and return closest intersection (least t)
Ray-Box Intersection

- Check front-facing sides for intersection with ray and return closest intersection (least $t$)
  - Find intersection with plane
  - Check if point is inside rectangle

![Diagram of ray-box intersection](image-url)
Ray-Box Intersection

- **Check** front-facing sides for intersection with ray and return closest intersection (least t)
  - Find intersection with plane
  - Check if point is inside rectangle
Other Ray-Primitive Intersections

- Cone, cylinder:
  - Similar to sphere
  - Must also check end caps

- Convex polygon
  - Same as triangle (check point-in-polygon algebraically)
  - Or, decompose into triangles, and check all of them

- Mesh
  - Compute intersection for all polygons
  - Return closest intersection (least t)
Ray Intersection

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  ➢ Scene

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Ray-Scene Intersection

- Intuitive method
  - Compute intersection for all nodes of scene graph
  - Return closest intersection (least t)
Ray-Scene Intersection

- Scene graph is a DAG
  - Traverse with recursion

DIAGRAM:
- Camera
- Light
- Surfaces
- Sphere
- Cylinder
- Box
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray) {
    // Check for intersection with shape
    shape_intersection = Intersect node’s shape with ray
    if (shape_intersection is a hit) closest_intersection = shape_intersection
    else closest_intersection = infinitely far miss

    // Check for intersection with children nodes
    for each child node
        // Check for intersection with child contents
        child_intersection = ComputeIntersection(scene, child, ray);
        if (child_intersection is a hit and is closer than closest_intersection)
            closest_intersection = child_intersection;

    // Return closest intersection in tree rooted at this node
    return closest_intersection
}
Ray-Scene Intersection

- Scene graph can have transformations
Ray-Scene Intersection

• Scene graph node can have transformations
  ○ Transform ray (not primitives) by inverse of M
  ○ Intersect in coordinate system of node
  ○ Transform intersection by M
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Transform ray by inverse of node’s transformation

    // Check for intersection with shape

    // Check for intersection with children nodes

    // Transform intersection by node’s transformation

    // Return closest intersection in tree rooted at this node
}
Ray-Scene Intersection II

R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray) {
    // Transform ray by inverse of node’s transformation

    // Check for intersection with shape

    // Check for intersection with children nodes

    // Transform intersection by node’s transformation

    // Return closest intersection in tree rooted at this node
}

Note: directions (including ray direction and surface normal N) must be transformed by inverse transpose of M
Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees
Ray Intersection Acceleration

- What if there are a lot of nodes?

[Diagram of a tree-like structure]

http://www.3dm3.com
Bounding Volumes

• Check for intersection with simple bounding volume first
Bounding Volumes

- Check for intersection with bounding volume first
Bounding Volumes

- Check for intersection with bounding volume first
  - If ray doesn’t intersect bounding volume, then it can’t intersect its contents
Bounding Volumes

• Check for intersection with bounding volume first
  ◦ If already found a primitive intersection closer than intersection with bounding box, then skip checking contents of bounding box
Bounding Volume Hierarchies

- Scene graph has hierarchy of bounding volumes
  - Bounding volume of interior node contains all children
Bounding Volume Hierarchies

- Checking bounding volumes hierarchically (within each node) can greatly accelerate ray intersection
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Transform ray by inverse of node’s transformation
    // Check for intersection with shape

    // Check for intersection with children nodes
    for each child node
        // Check for intersection with child bounding box first
        bbox_intersection = Intersect child’s bounding box with ray
        if (bbox_intersection is a miss or further than closest_intersection) continue

        // Check for intersection with child contents
        child_intersection = ComputeIntersection(scene, child, ray);
        if (child_intersection is a hit and is closer than closest_intersection)
            closest_intersection = child_intersection;

    // Transform intersection by node’s transformation
    // Return closest intersection in tree rooted at this node
}
Sort Bounding Volume Intersections

• Sort child bounding volume intersections and then visit child nodes in front-to-back order

• Why?
Cache Node Intersections

- For each node, store closest child intersection from previous ray and check that node first
Bounding Volumes

• Common primitives are:
  ◦ Axis-aligned bounding box
  ◦ Sphere

• What are the tradeoffs?
  ◦ Sphere has simple/efficient intersection code
  ◦ Bounding box is generally “tighter”
Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
    - Uniform grids
  - Octrees
  - BSP trees
Uniform Grid

- Construct uniform grid over scene
  - Index primitives according to overlaps with grid cells
Uniform Grid

- Trace rays through grid cells
  - Fast
  - Incremental

Only check primitives in intersected grid cells
Uniform Grid

• Potential problem:
  - How choose suitable grid resolution?

Too little benefit if grid is too coarse

Too much cost if grid is too fine
Ray Intersection

• Ray Intersection
  ◦ Sphere
  ◦ Triangle
  ◦ Box
  ◦ Scene

• Ray Intersection Acceleration
  ◦ Bounding volumes
  ◦ Uniform grids
  ➢ Octrees
  ◦ BSP trees
Octree

- Construct adaptive grid over scene
  - Recursively subdivide box-shaped cells into 8 octants
  - Index primitives by overlaps with cells

Generally fewer cells
Octree

- Trace rays through neighbor cells
  - Fewer cells

Trade-off fewer cells for more expensive traversal
• Or, check rays versus octree boxes hierarchically
  ◦ Computing octree boxes while descending tree
  ◦ Sort eight boxes front-to-back at each level
  ◦ Check primitives/children inside box
Ray Intersection

• Ray Intersection
  ○ Sphere
  ○ Triangle
  ○ Box
  ○ Scene

• Ray Intersection Acceleration
  ○ Bounding volumes
  ○ Uniform grids
  ○ Octrees
  ➢ BSP trees
Binary Space Partition (BSP) Tree

- Recursively partition space by planes
  - BSP tree nodes store partition plane and set of polygons lying on that partition plane
  - Every part of every polygon lies on a partition plane
Binary Space Partition (BSP) Tree

- Traverse nodes of BSP tree front-to-back
  - Visit halfspace (child node) containing $P_0$
  - Intersect polygons lying on partition plane
  - Visit halfspace (other child node) not containing $P_0$
R3Intersection
ComputeBSPIntersection(R3Ray *ray, BspNode *node, double min_t, double max_t)
{
    // Compute parametric value of ray-plane intersection
    t = ray parameter for intersection with split plane of node
    if (t < min_t) || (t < max_t)) return no_intersection;

    // Compute side of partition plane that contains ray start point
    int side = (SignedDistance(node->plane, ray.Start()) < 0) ? 0 : 1;
    intersection1 = ComputeBSPIntersection(ray, node->child[side], min_t, t);
    if (intersection1 is a hit) return intersection1;
    intersection2 = ComputePolygonsIntersection(ray, node->polygons);
    if (intersection2 is a hit) return intersection2;
    intersection3 = ComputeBSPIntersection(ray, node->child[1-side], t, max_t);
    return intersection 3;
}
Other Accelerations

• Screen space coherence – check > 1 ray at once
  ◦ Beam tracing
  ◦ Pencil tracing
  ◦ Cone tracing

• Memory coherence
  ◦ Large scenes

• Parallelism
  ◦ Ray casting is “embarrassingly parallelizable”
  ◦ Assignment 3 (raytracer) runs program per-pixel

• etc.
Acceleration

• Intersection acceleration techniques are important
  ◦ Bounding volume hierarchies
  ◦ Spatial partitions

• General concepts
  ◦ Sort objects spatially
  ◦ Make trivial rejections quick
  ◦ Perform checks hierarchically
  ◦ Utilize coherence when possible

Expected time is sub-linear in number of primitives
Summary

- Writing a simple ray casting renderer is easy
  - Generate rays
  - Intersection tests
  - Lighting calculations

```cpp
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene, &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```
Heckbert’s Business Card Ray Tracer

- typedef struct{double x,y,z}vec;vec U,black,amb={.02,.02,.02};struct sphere{ vec cen,color; double rad,kd,ks,kt,kl,ir}*s,*best,sph[]={0.,6..,5,1.,1.,1.,9,.05,2.,85,0.,1.7,-1.8,-.5,1.,5.,2,1.,.7,.3,0.,05,1.2,1.,8.,-.5,1.,8,8,1.,3,7,0.,0.,1.,2,3.,6,15,1.,8,1,7,0,0,0,0,6,1,5,3,3,12,.8,1.,1.,5,0,0,0,0,5,1,5,};yx;double u,b,tmin,sqrt(),tan();double vdot(A,B)vec A ,B;{return A.x*B.x+A.y*B.y+A.z*B.z;}vec vcomb(a,A,B)double a;vec A,B;{B.x+=a* A.x;B.y+=a*A.y;B.z+=a*A.z; return B;}vec vunit(A)vec A;{return vcomb(1./sqrt( vdot(A,A)),A,black);}struct sphere*intersect(P,D)vec P,D;{best=0;tmin=1e30;s=sph+5;while(s-- >sph)b=vdot(D,U=vcomb(-1.,P,s->cen)),u=b*b-vdot(U,U)+s-rad*s->rad,u>0?sqrt(u):1e31,u=b-u>1e-7?b-u:b+u,tmin=u>=1e-7&&u<tmin?best=s,u: tmin;return best;}vec trace(level,P,D)vec P,D;{double d,eta,e;vec N,color; struct sphere*s,*l;if(!level--)return black;if(s=intersect(P,D));else return amb;color=amb;eta=s->ir;d= -vdot(D,N=vunit(vcomb(-1.,P=vcomb(tmin,D,P),s->cen )));if(d<0)N=vcomb(-1.,N,black), eta=1/eta,d= -d;l=sph+5;while(l---> sph)if((e=l-1*kl*vdot(N,U=vunit(vcomb(-1.,P,l->cen))))>0&& intersect(P,U)==l)color=vcomb(e ,l->color,color);U=s->color;color.x*=U,x;color.y*=U.y;color.z *=U.z;e=1-eta* eta*(1-d*d);return vcomb(s->kt,e>0?trace(level,P,vcomb(eta,D,vcomb(eta*d-sqrt (e),N,black)))),black,vcomb(s->ks,trace(level,P,vcomb(2*d,N,D)),vcomb(s->kd, color,vcomb(s->kl,U,black))));}main(){printf("%d %d\n",32,32);while(yx<32*32) U.x=yx%32-32/2,U.z=32/2-yx+/32, U.y=32/2/tan(25/114.5915590261),U=vcomb(255., trace(3,black,vunit(U)),black),printf ("%.0f %.0f %.0f\n",U);}/*minray!*/
Next Time is Illumination!

Without Illumination

With Illumination