Polygonal Meshes

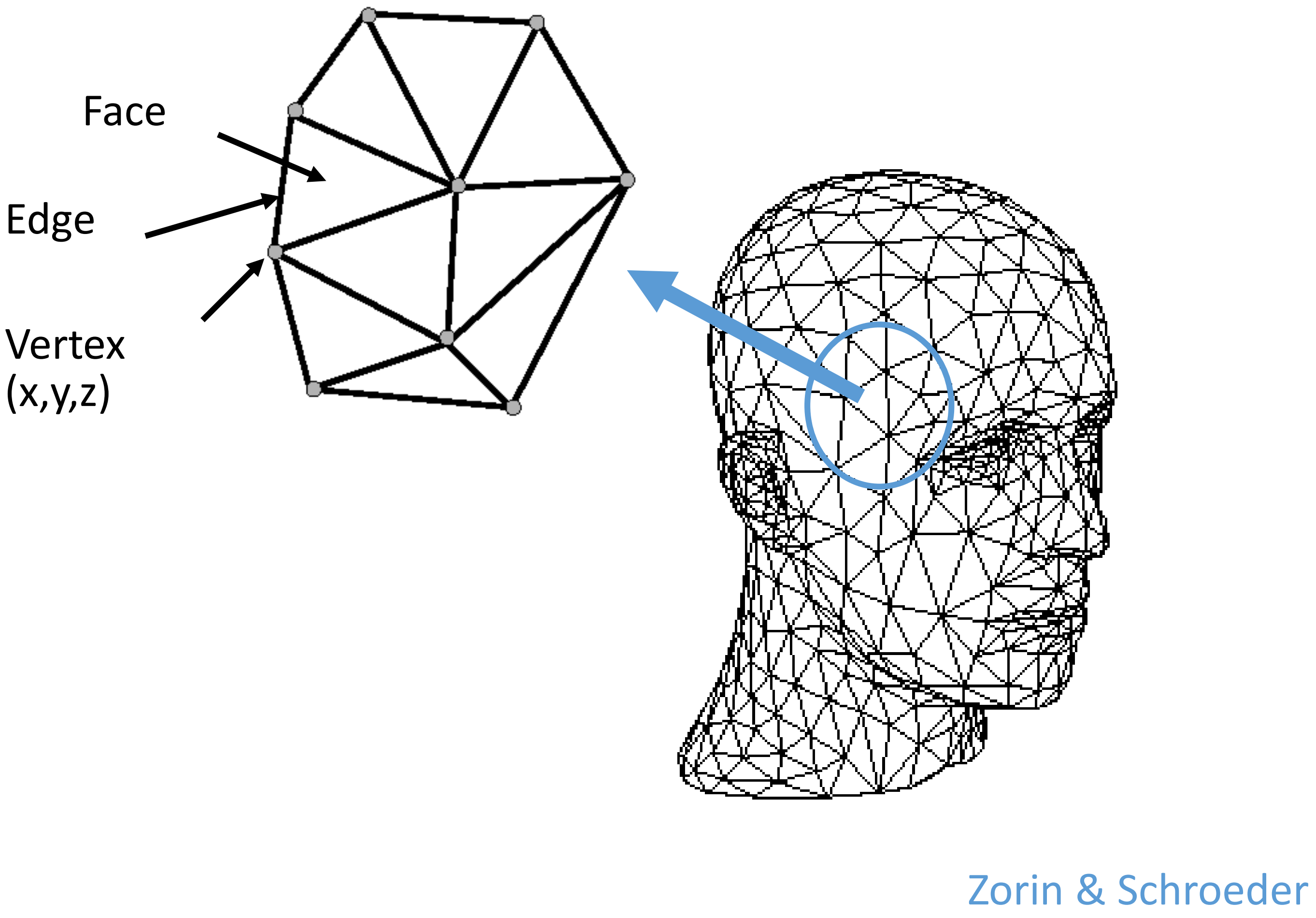
COS 426, Spring 2018
Princeton University
Amit Bermano

3D Object Representations

- Points
  - Range image
  - Point cloud
- Surfaces
  - ➢ Polygonal mesh
  - Parametric
  - Subdivision
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

3D Polygonal Mesh

- Set of polygons representing a 2D surface embedded in 3D



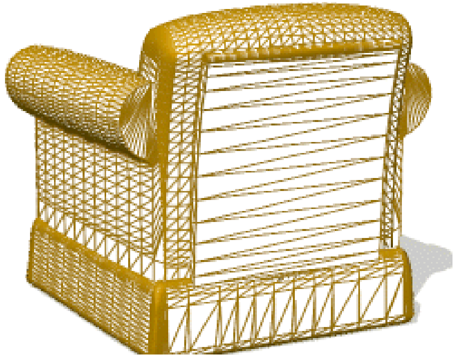
3D Polygonal Mesh

- The power of polygonal meshes

DEADPOOL
20TH CENTURY FOX (2016)

3D Polygonal Meshes

- Why are they of interest?
  - Simple, common representation
  - Rendering with hardware support
  - Output of many acquisition tools
  - Input to many simulation/analysis tools



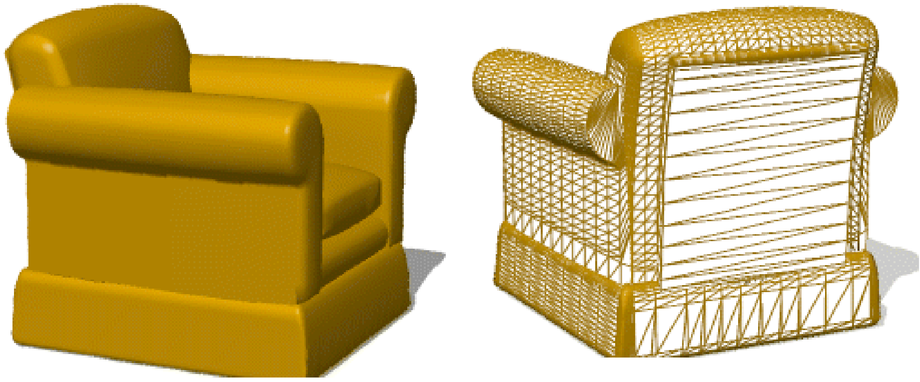
Viewpoint

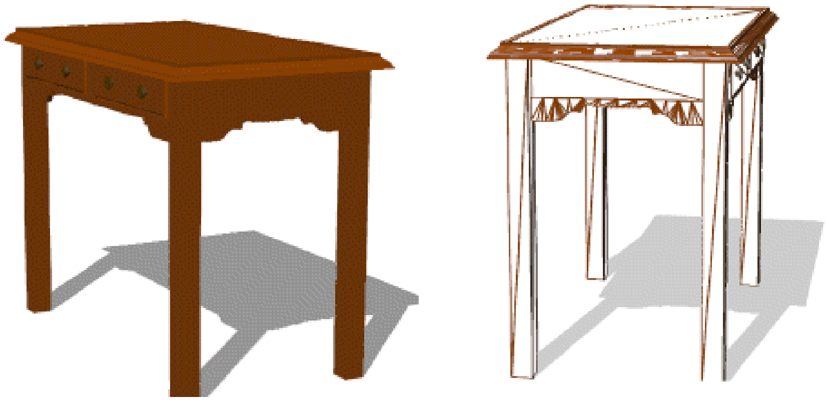
3D Polygonal Meshes

Meshlab Demo

3D Polygonal Meshes

- Properties
  - ? Efficient display
  - ? Easy acquisition
  - ? Accurate
  - ? Concise
  - ? Intuitive editing
  - ? Efficient editing
  - ? Efficient intersections
  - ? Guaranteed validity
  - ? Guaranteed smoothness
  - ? etc.





Viewpoint

Outline

- Acquisition
- Representation
- Processing

Polygonal Mesh Acquisition

- Interactive modeling
- Scanners
- Procedural generation
- Conversion
- Simulations

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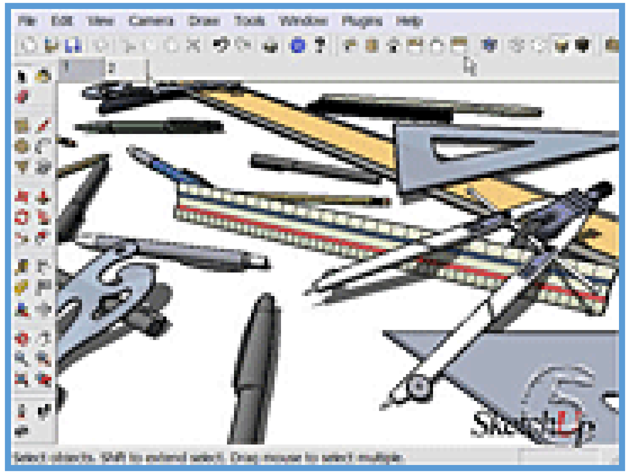
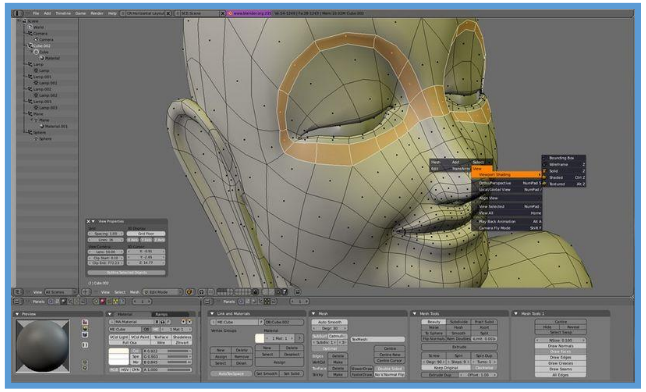


Sketchup



Blender

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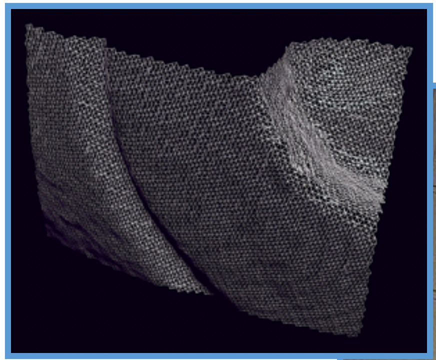

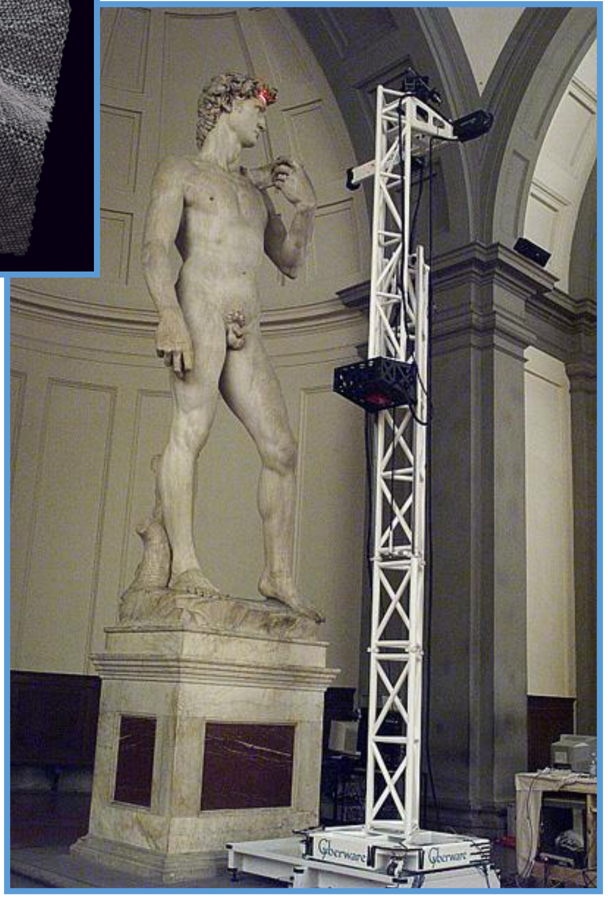

Digital Michelangelo Project
Stanford

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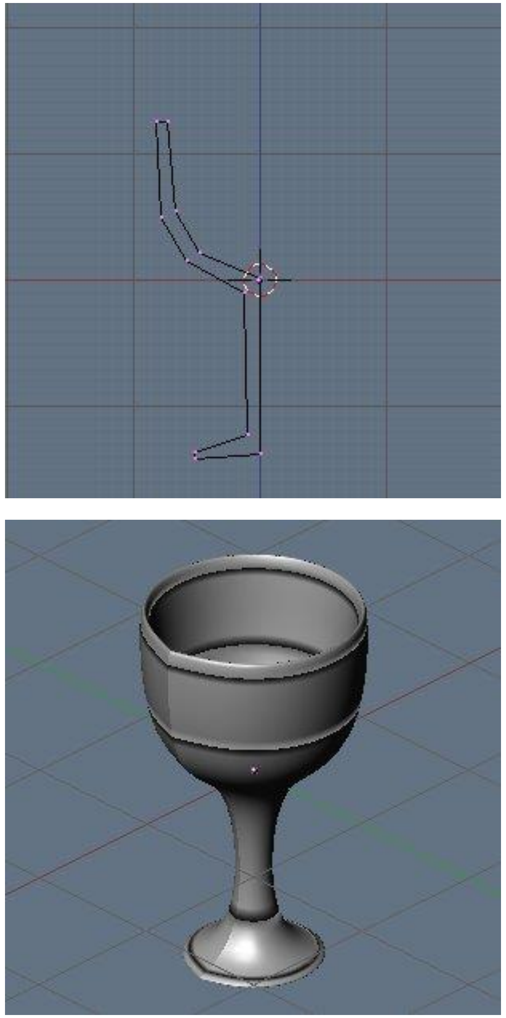

sphynx.co.uk

Polygonal Mesh Acquisition

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Nicky Robinson, COS 426, 2014

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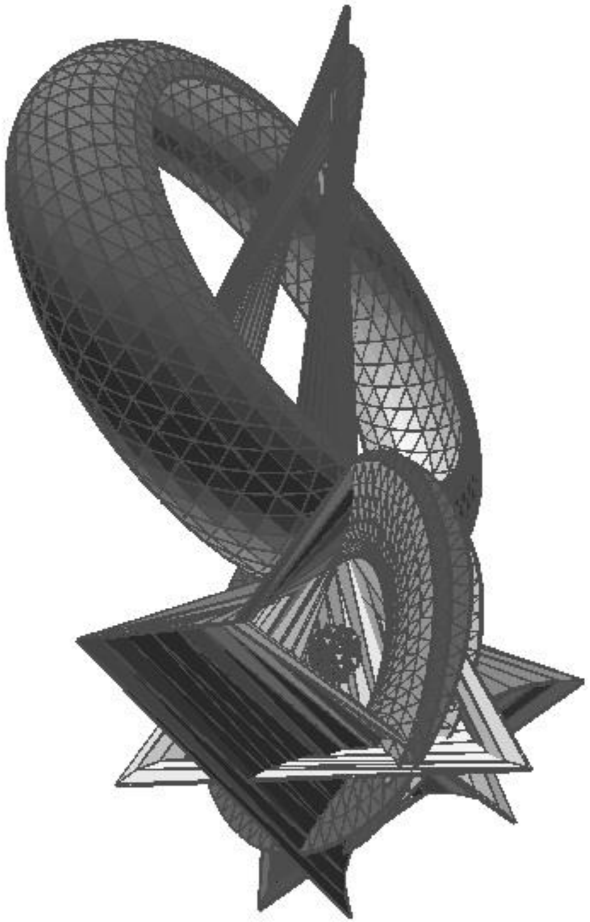

Alejandro Van Zandt-Escobar, COS 426, 2014

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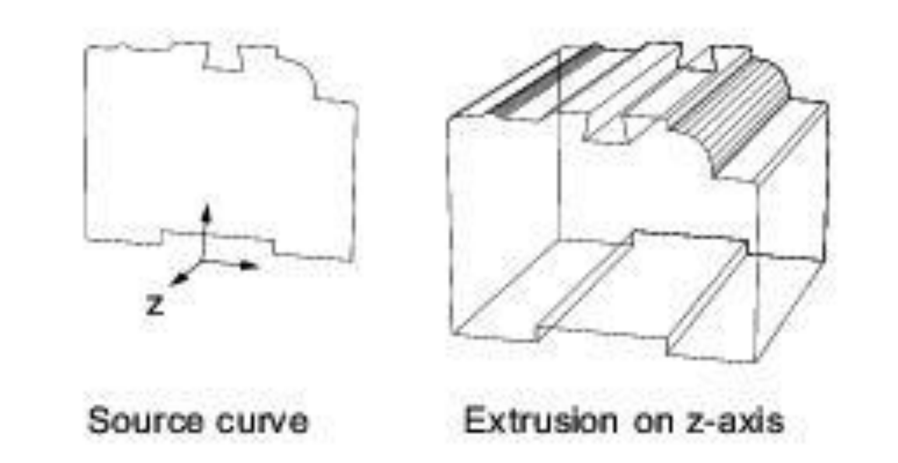

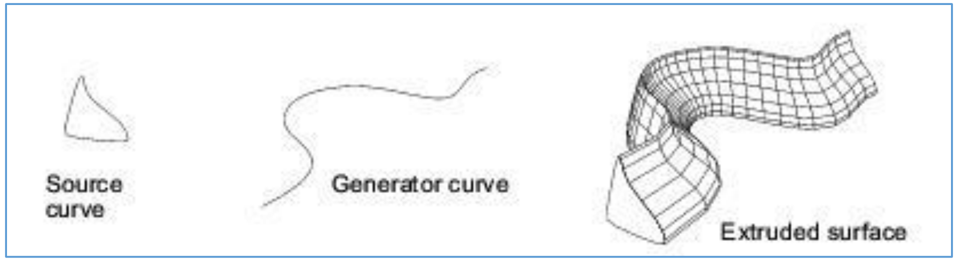

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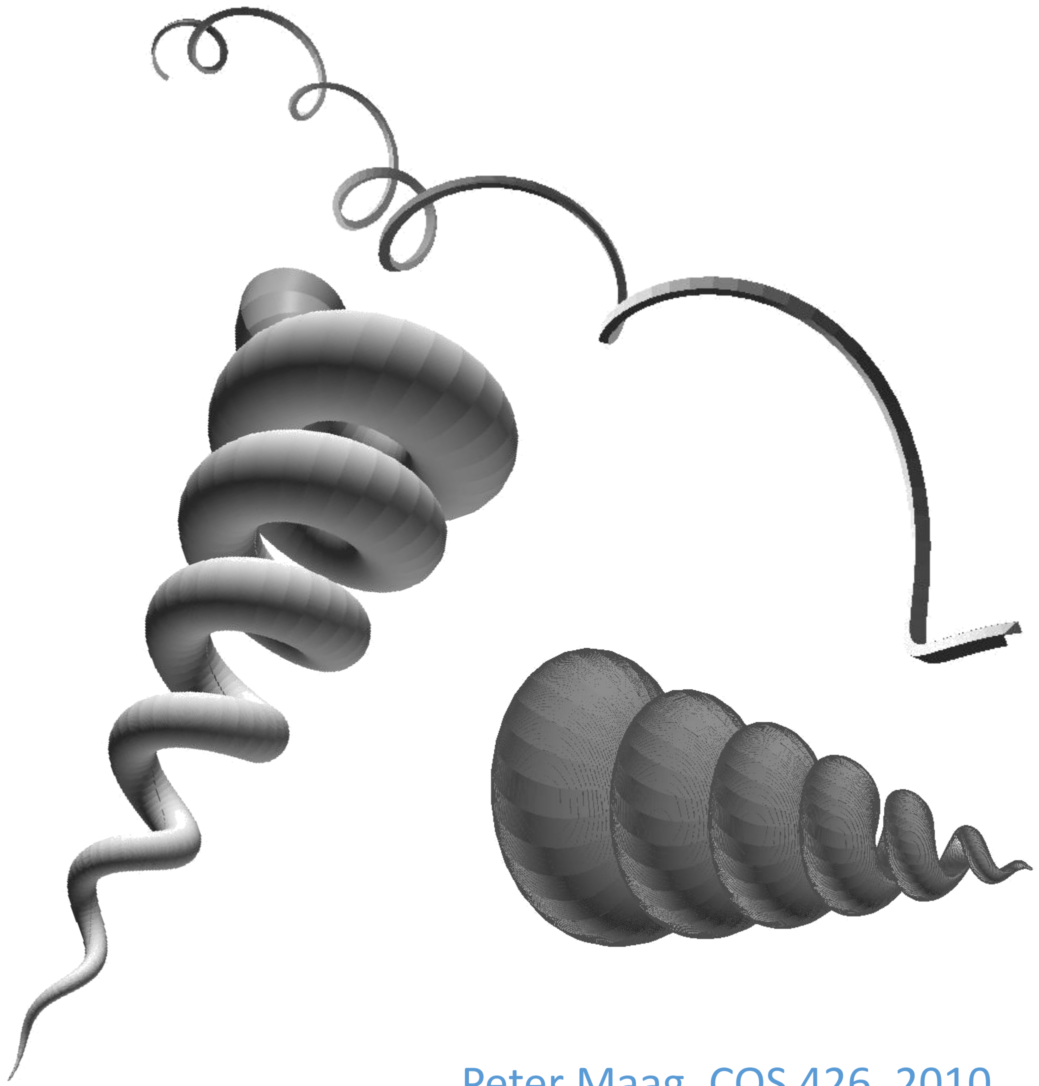

Peter Maag, COS 426, 2010

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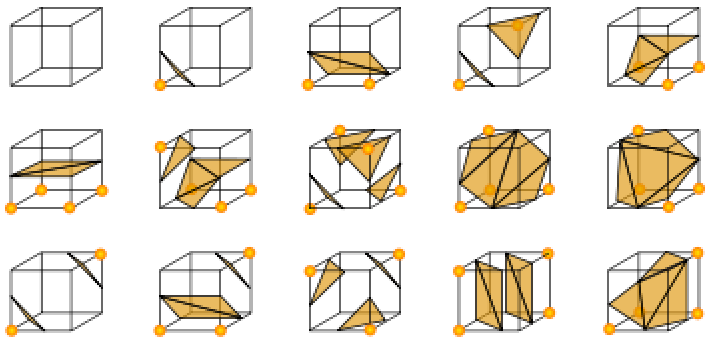
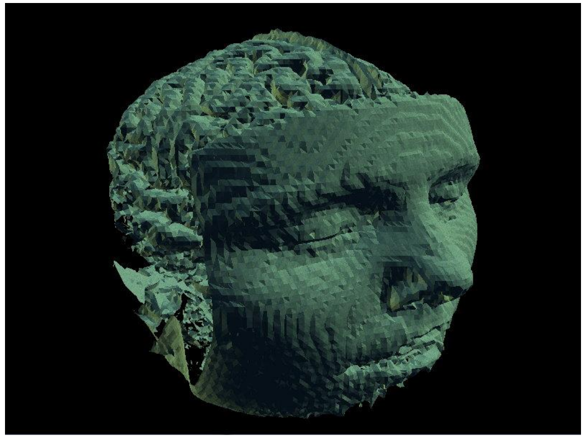
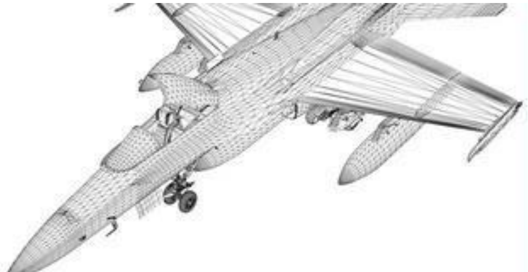



Marching cubes

Jose Maria De Espona

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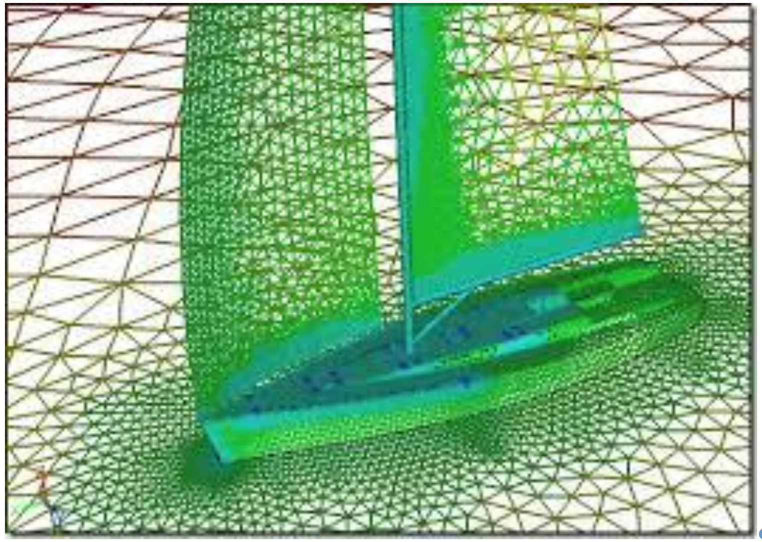
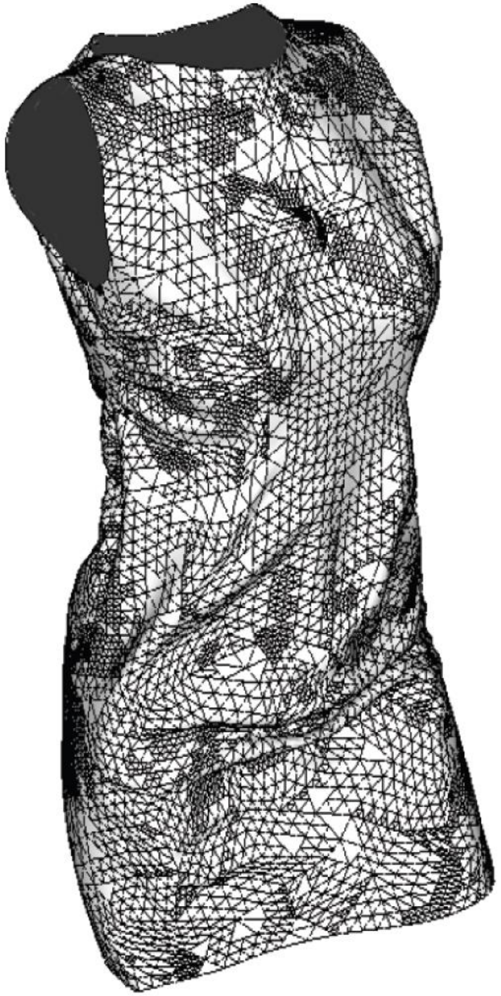


symscape



Lee et. al 2010

Outline

- Acquisition
- Representation
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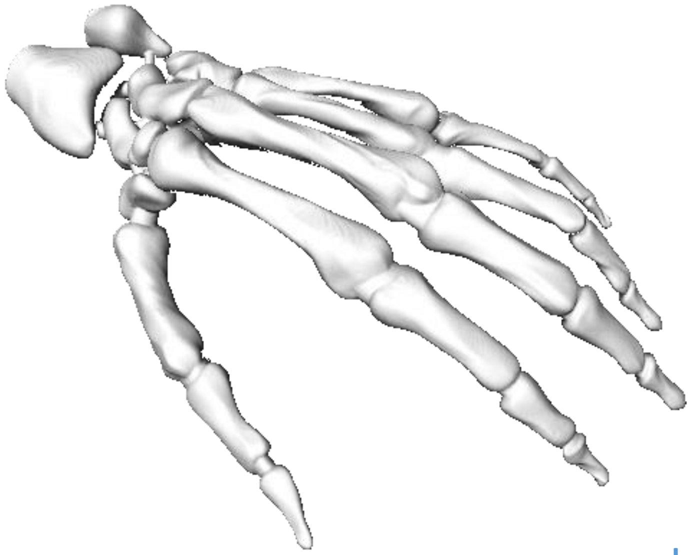
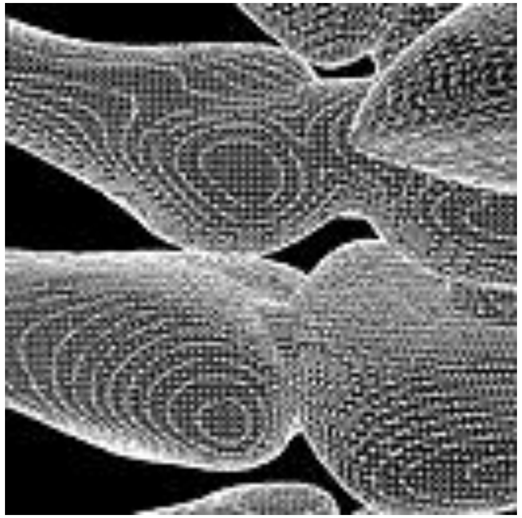
Polygon Mesh Representation

- Important properties of mesh representation?

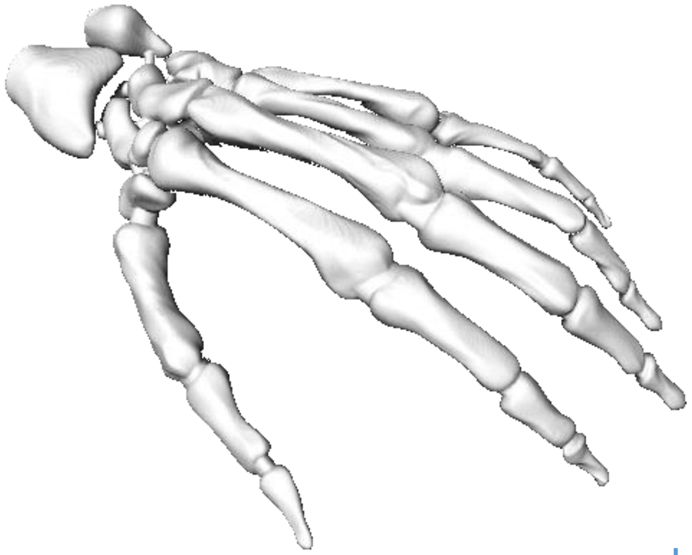


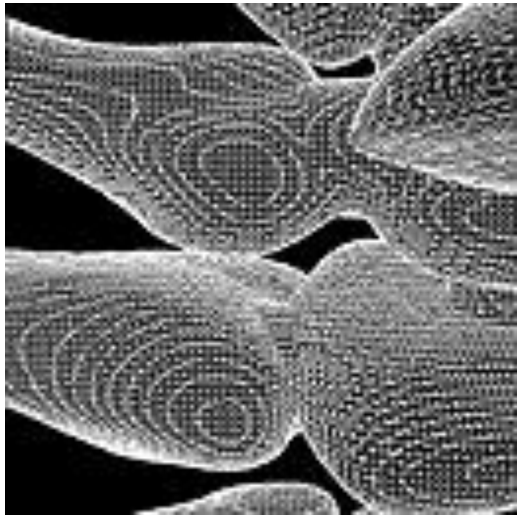


Large Geometric Model Repository
Georgia Tech

Polygon Mesh Representation

- Important properties of mesh representation?
  - Efficient traversal of topology
  - Efficient use of memory
  - Efficient updates





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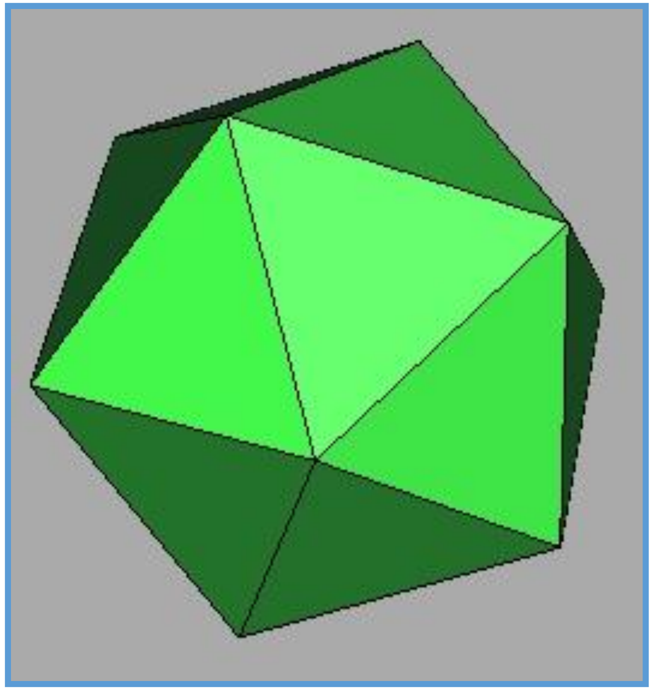
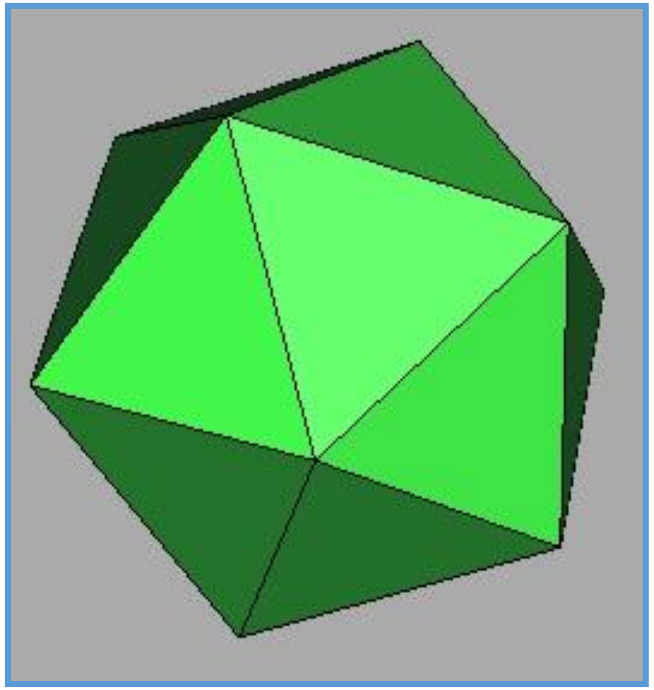
Polygon Mesh Representation

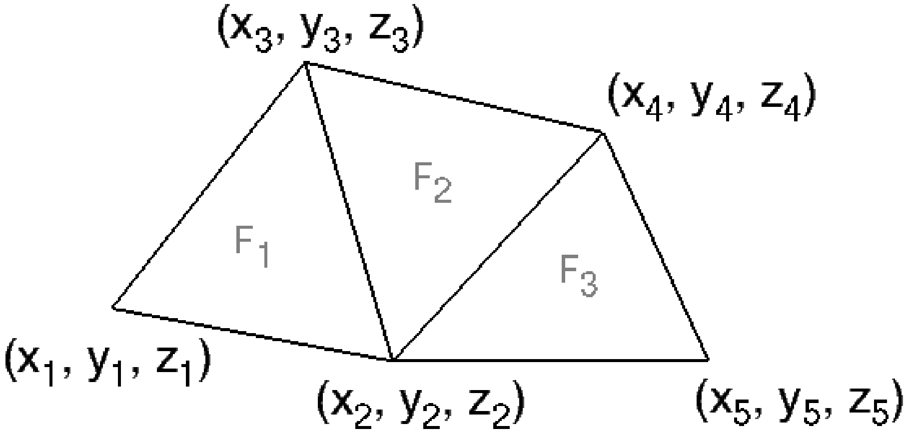
- Possible data structures



Independent Faces

- Each face lists vertex coordinates
  - Redundant vertices
  - No adjacency information

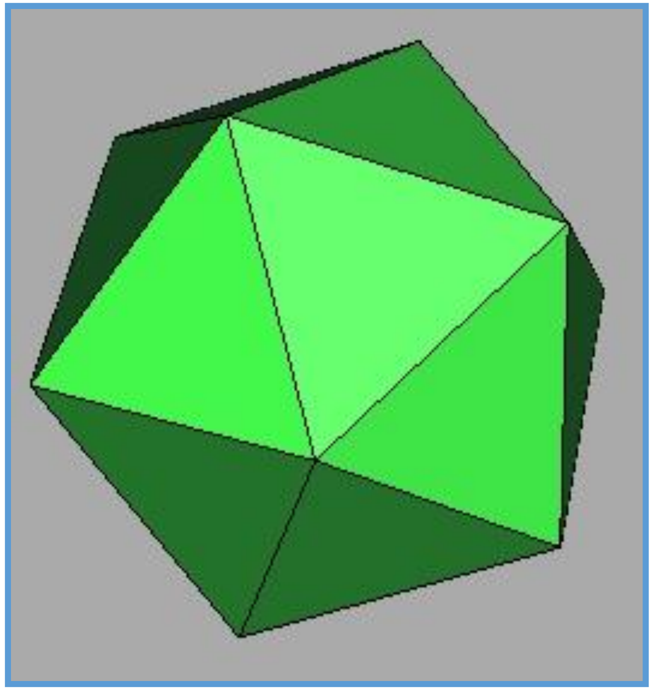


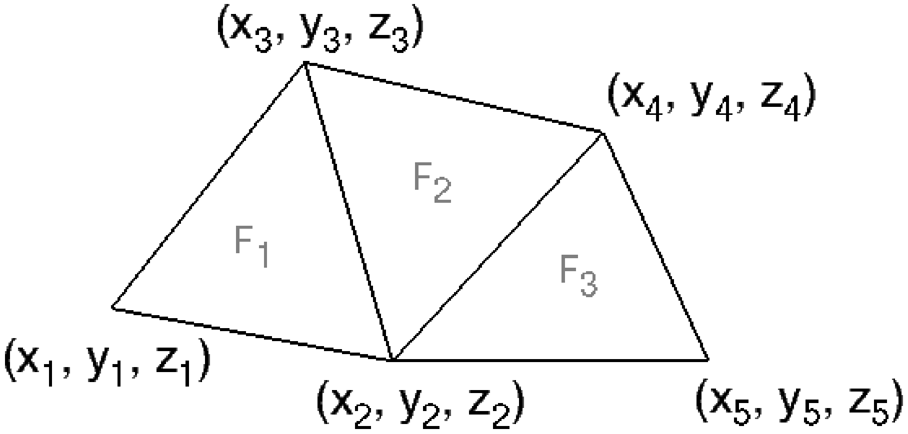


FACE TABLE	
F_1	(x_1, y_1, z_1) (x_2, y_2, z_2) (x_3, y_3, z_3)
F_2	(x_2, y_2, z_2) (x_4, y_4, z_4) (x_3, y_3, z_3)
F_3	(x_2, y_2, z_2) (x_5, y_5, z_5) (x_4, y_4, z_4)

Vertex and Face Tables (Indexed Vertices)

- Each face lists vertex references
  - Shared vertices
  - Still no adjacency information



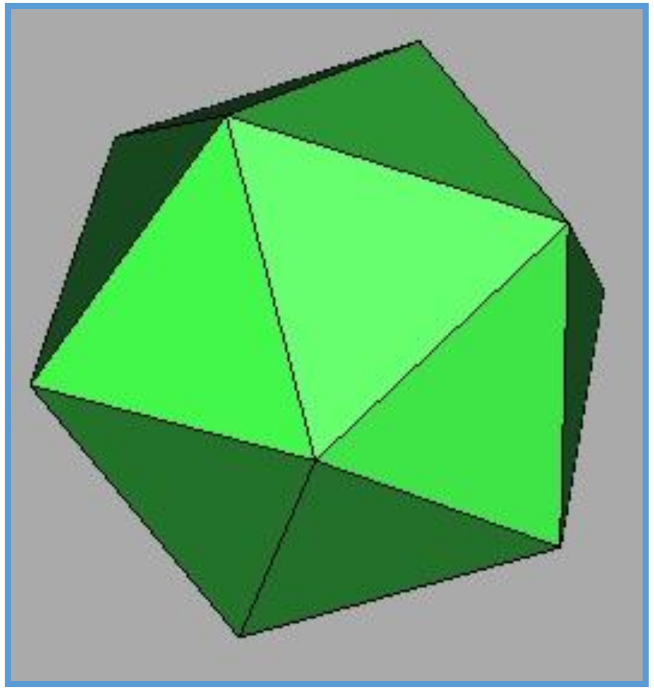


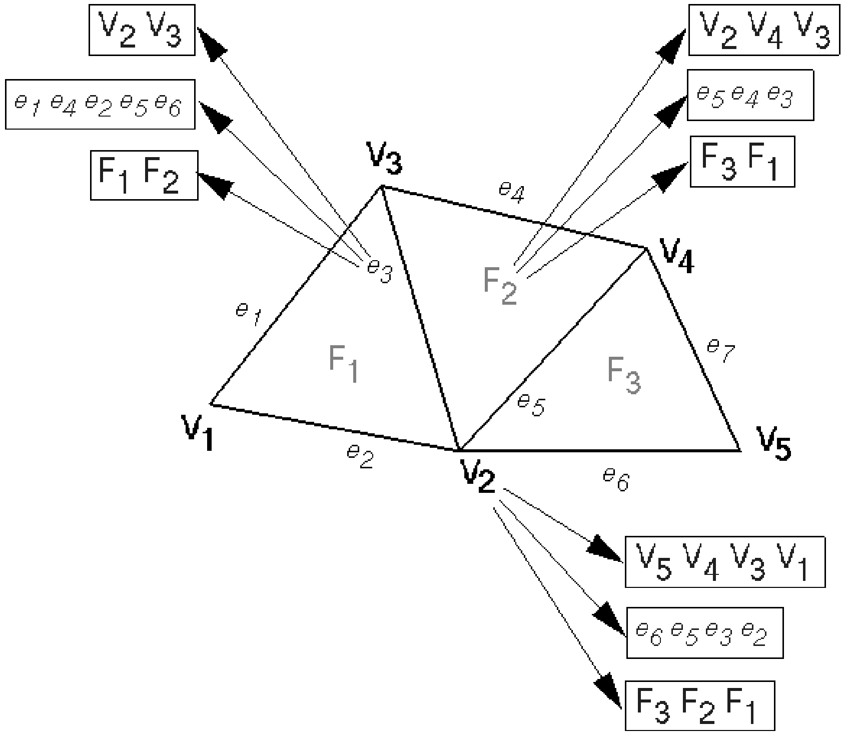
VERTEX TABLE			
V_1	X_1	Y_1	Z_1
V_2	X_2	Y_2	Z_2
V_3	X_3	Y_3	Z_3
V_4	X_4	Y_4	Z_4
V_5	X_5	Y_5	Z_5

FACE TABLE			
F_1	V_1	V_2	V_3
F_2	V_2	V_4	V_3
F_3	V_2	V_5	V_4

Adjacency Lists

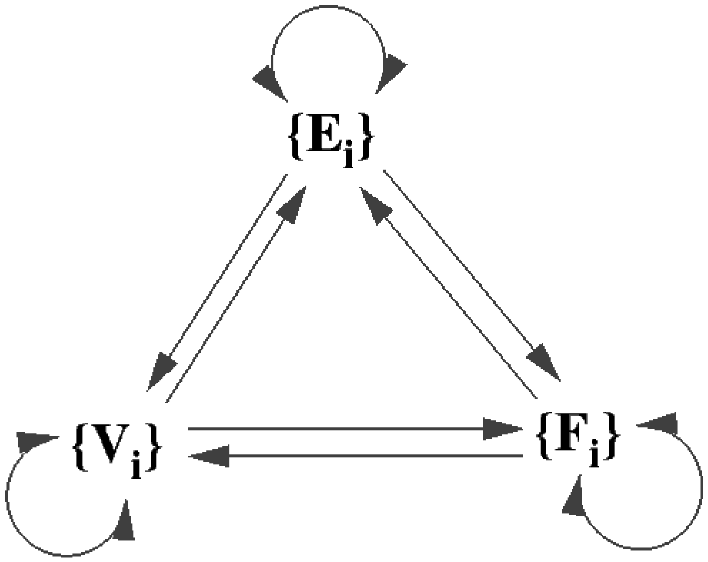
- Store all vertex, edge, and face adjacencies
  - Efficient adjacency traversal
  - Extra storage

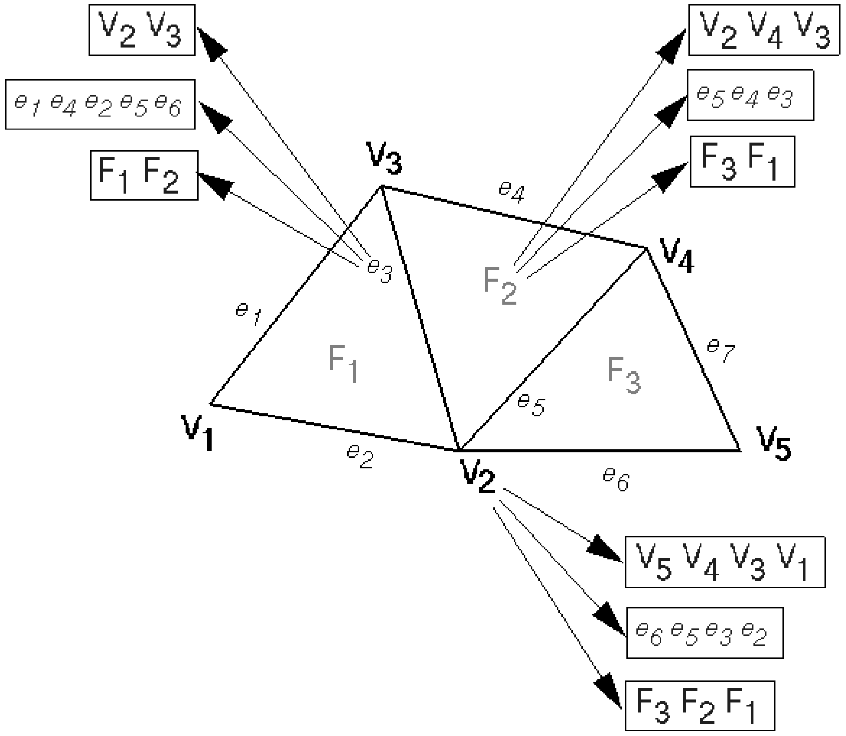




Partial Adjacency Lists

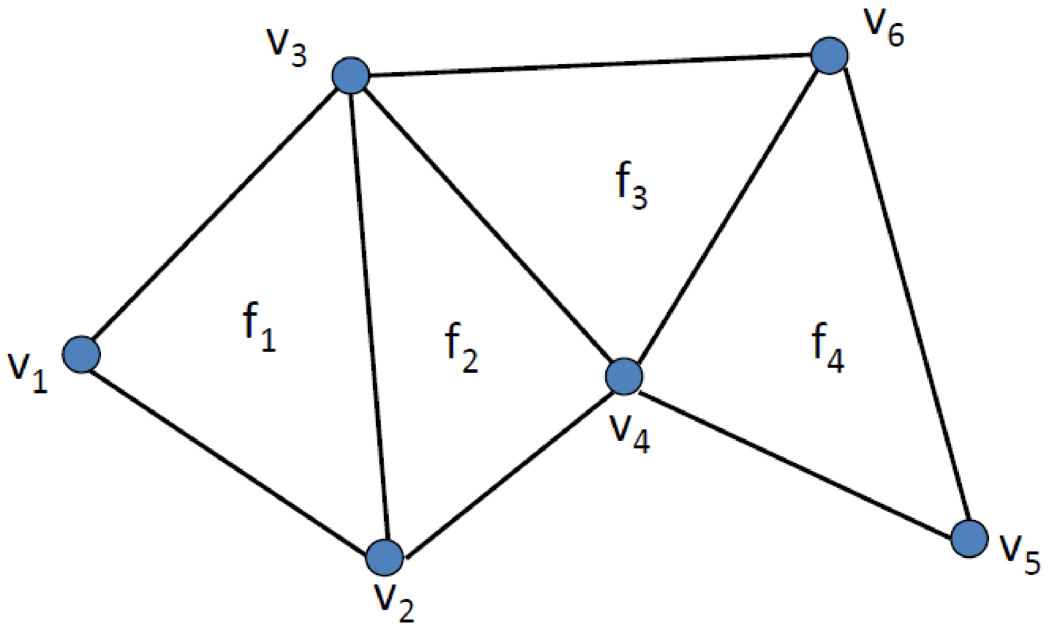
- Can we store only some adjacency relationships and derive others?





Adjacency Matrix

- If there is an edge between v_i & v_j then $A_{ij} = 1$
- Cons:
  - No connection between a vertex and its adjacent faces
  - A lot of storage (should use sparse matrices)

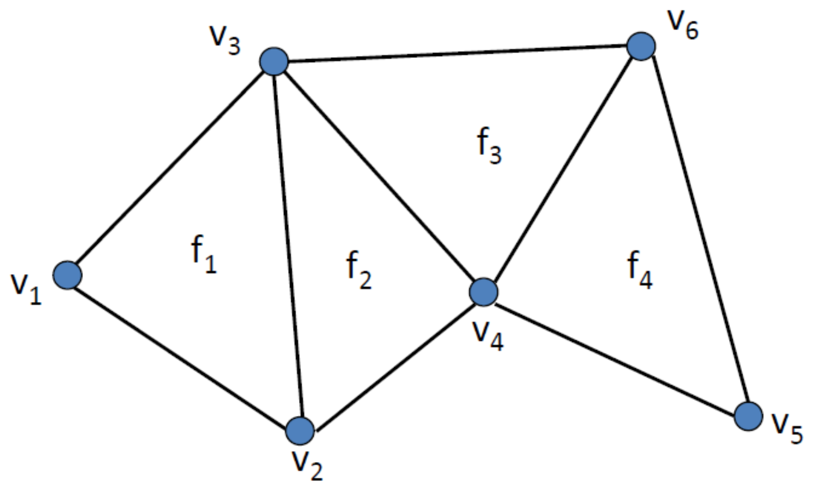


A =

	v_1	v_2	v_3	v_4	v_5	v_6
v_1		1	1			
v_2	1		1	1		
v_3	1	1		1		1
v_4		1	1		1	1
v_5				1		1
v_6			1	1	1	

Adjacency Matrix

- If there is an edge between v_i & v_j then $A_{ij} = 1$
- Pros:
  - No restrictions on mesh topology
  - Mathematical properties:
    - $(A^n)_{ij}$ = # paths of length n from v_i to v_j
    - Eigenvectors are meaningful

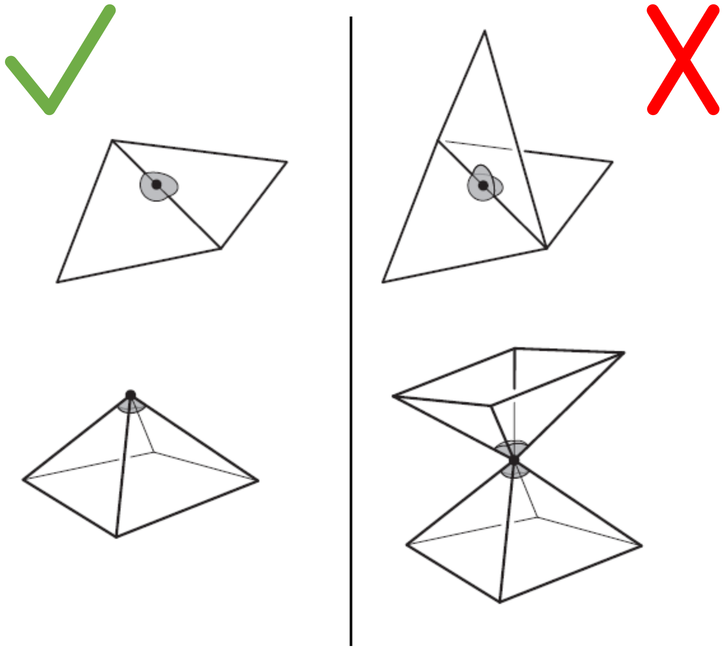


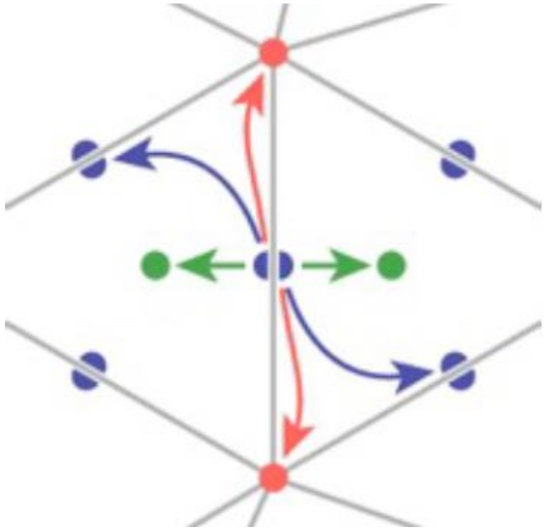
A =

	v_1	v_2	v_3	v_4	v_5	v_6
v_1		1	1			
v_2	1		1	1		
v_3	1	1		1		1
v_4		1	1		1	1
v_5				1		1
v_6			1	1	1	

Half Edge

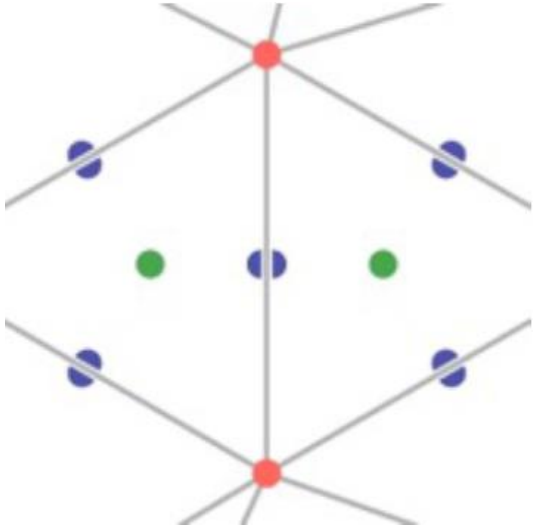
- Adjacency encoded in edges
  - All adjacencies in O(1) time
  - Little extra storage (fixed records)
  - Arbitrary polygons
  - Assumes 2-Manifold surfaces





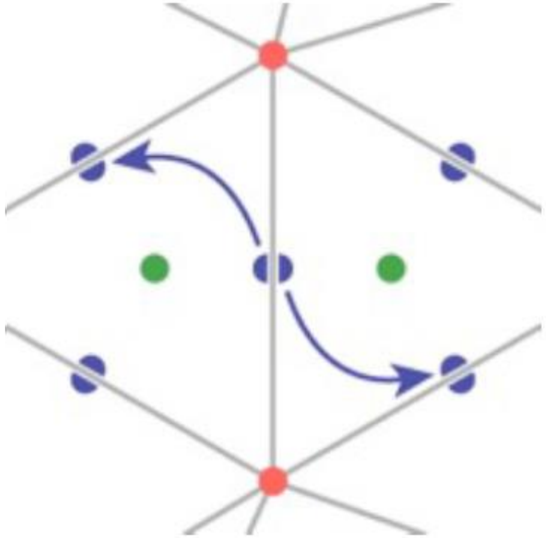
Half Edge

- Each half-edge stores:
  - Its twin half-edge



Half Edge

- Each half-edge stores:
  - Its twin half-edge
  - The next half-edge



Half Edge

- Each half-edge stores:
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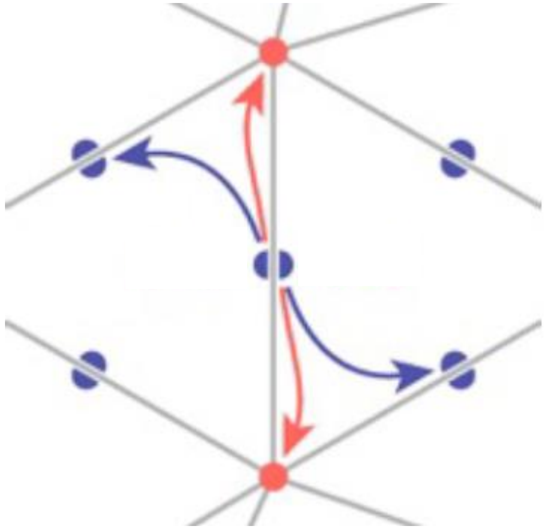
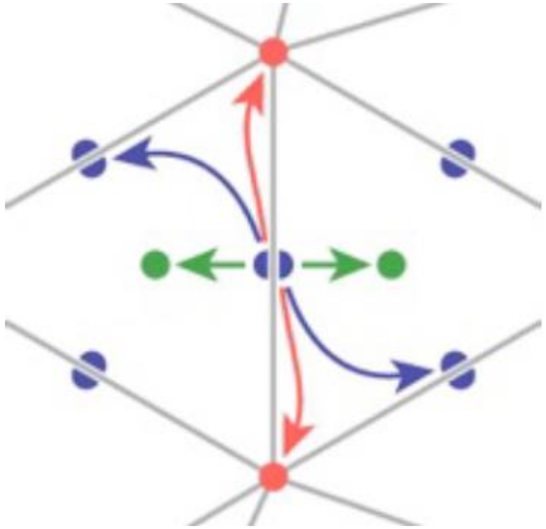
  - The next half-edge
  - The next vertex



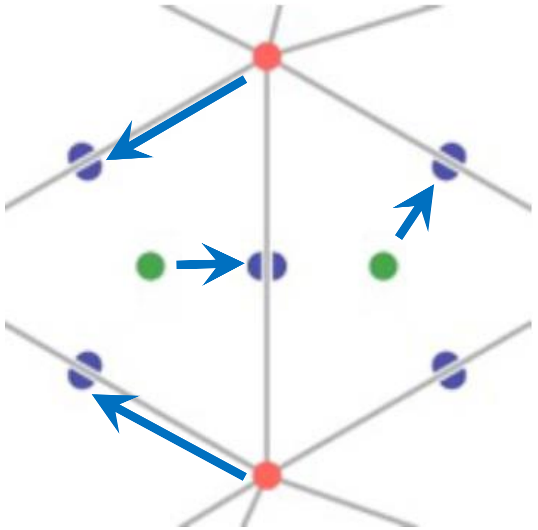
Half Edge

- Each half-edge stores:
  - Its twin half-edge
  - The next half-edge
  - The next vertex
  - The incident face



Half Edge

- Each half-edge stores:
  - Its twin half-edge
  - The next half-edge
  - The next vertex
  - The incident face
- Each face stores:
  - 1 adjacent half-edge
- Each vertex stores:
  - 1 outgoing half-edge



Half Edge

- Queries. How do you find:
  - All faces incident to an edge?
  - All vertices of a face?
  - All faces incident to a face?
  - All vertices incident to a vertex?

Outline

- Acquisition
- Representation
- Processing

Polygonal Mesh Processing

- Analysis
  - Normals
  - Curvature
- Warps
  - Rotate
  - Deform
- Filters
  - Smooth
  - Sharpen
  - Truncate
  - Bevel

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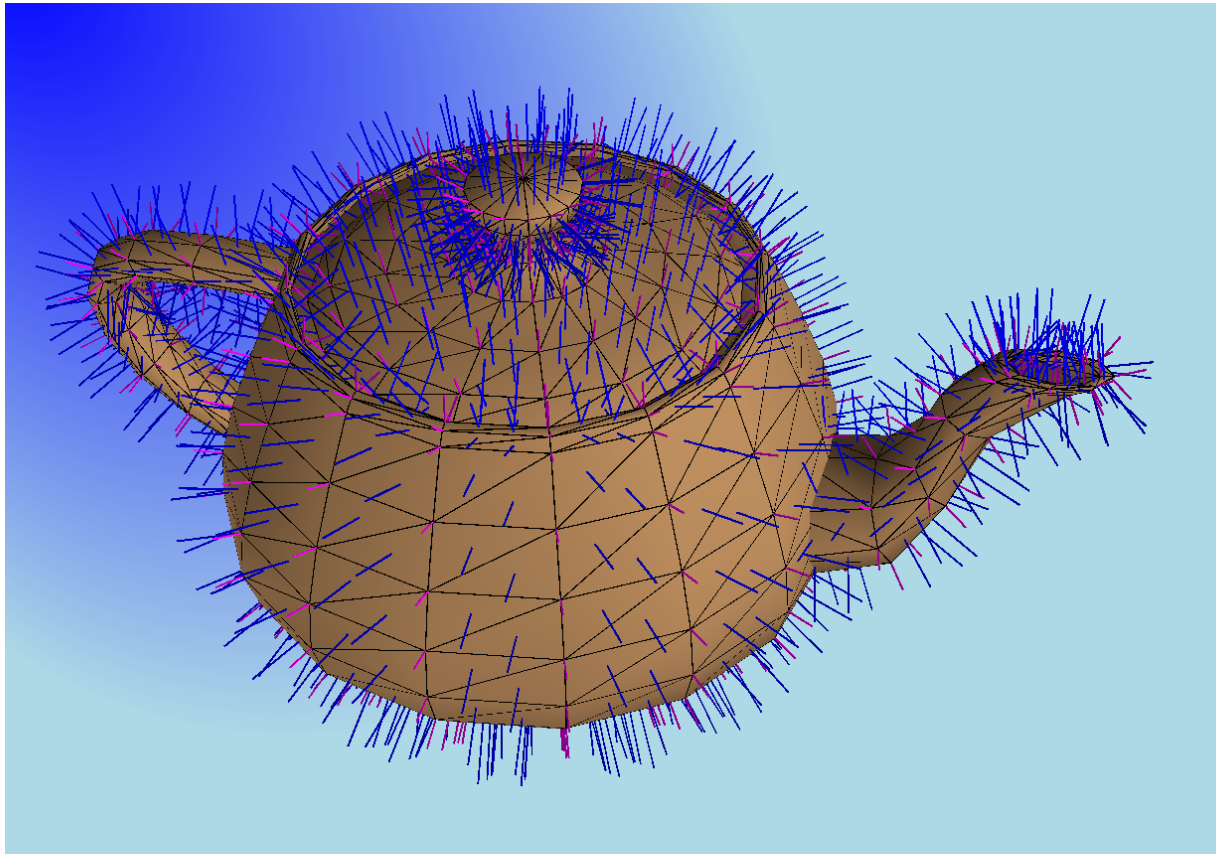

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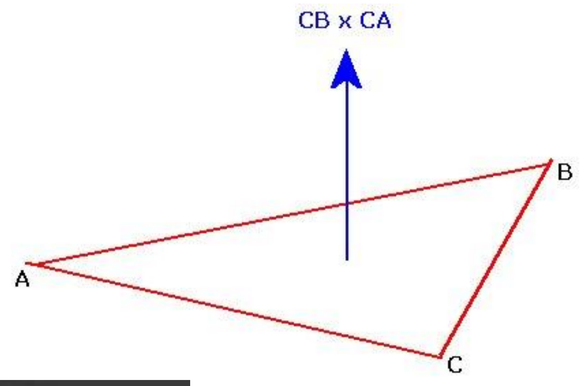
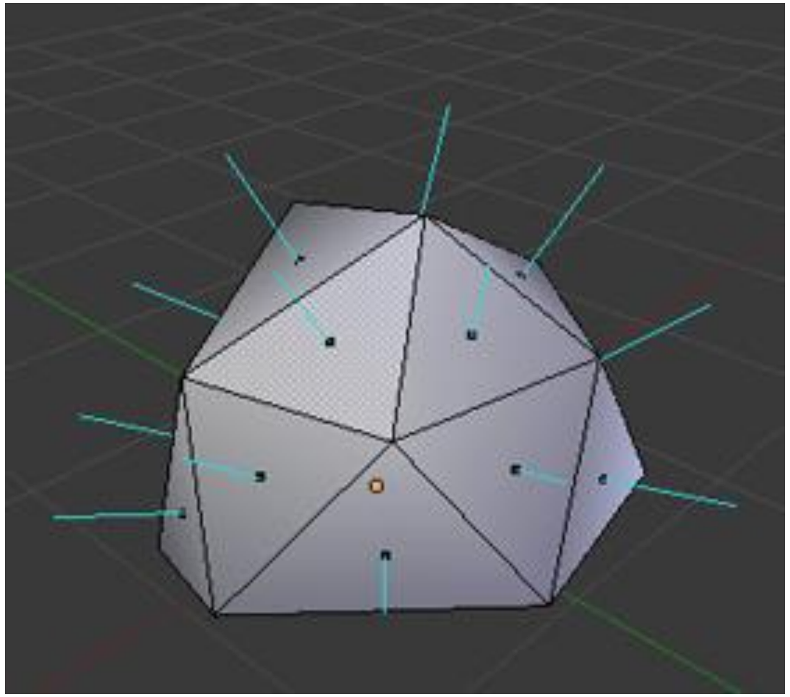
Face normals:





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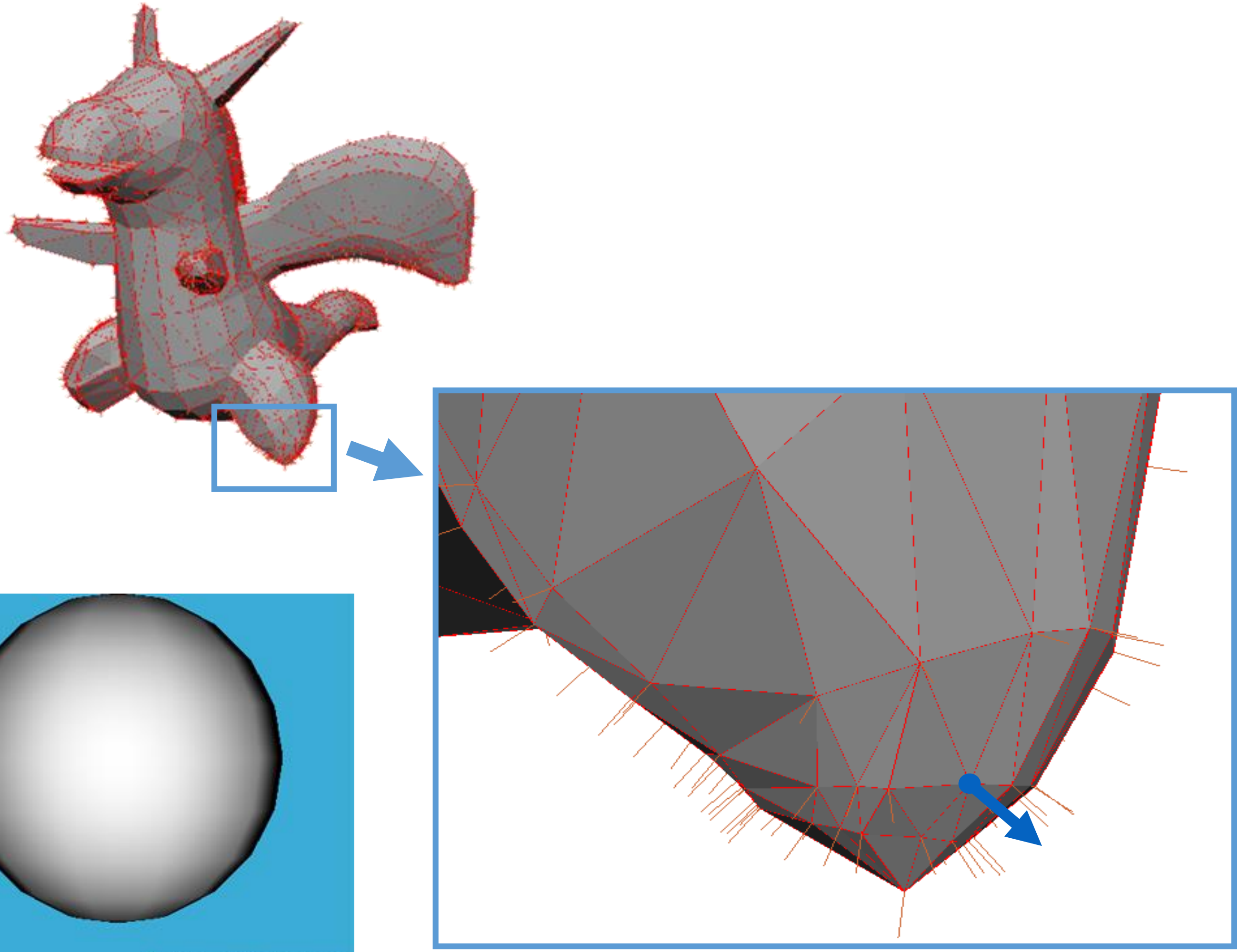
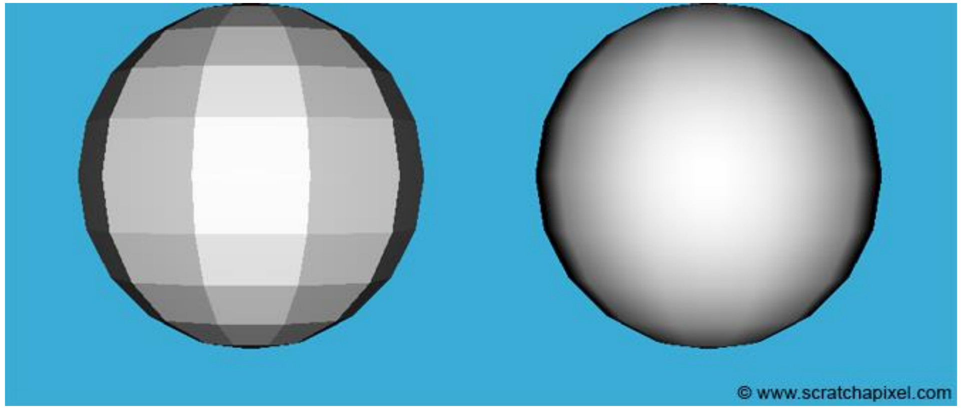
Vertex normals:





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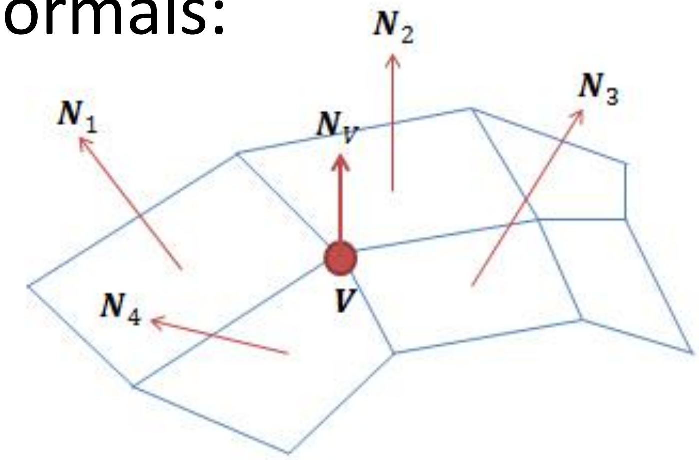
Vertex normals:



$$N_V = \frac{\sum_{k=1}^{n} N_k}{\left|\sum_{k=1}^{n} N_k\right|}$$

for each face
-calculate face normal
-add normal to each connected vertex normal

for each face normal
-normalize

for each vertex normal
-normalize

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"The bunny with normal vertices shown.
Reminded me of an album cover so I made it into one."

Lucas Mayer, COS 426, 2014

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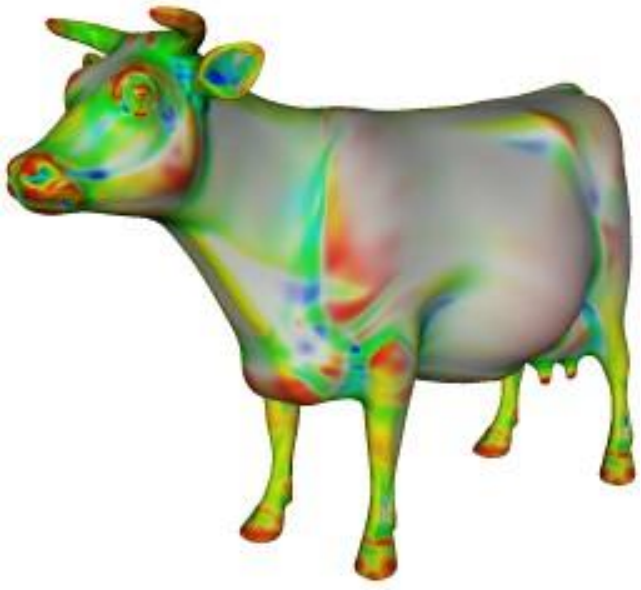

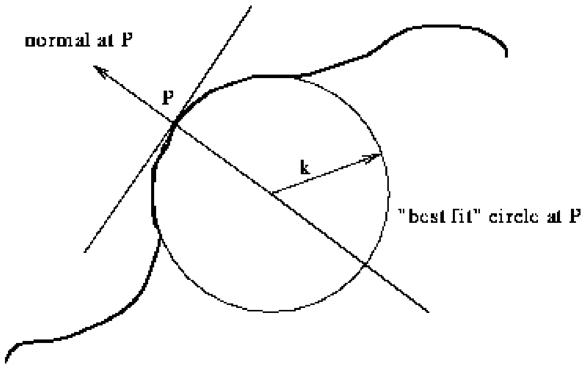

Figure 32: curvature of curve at P is $1/k$

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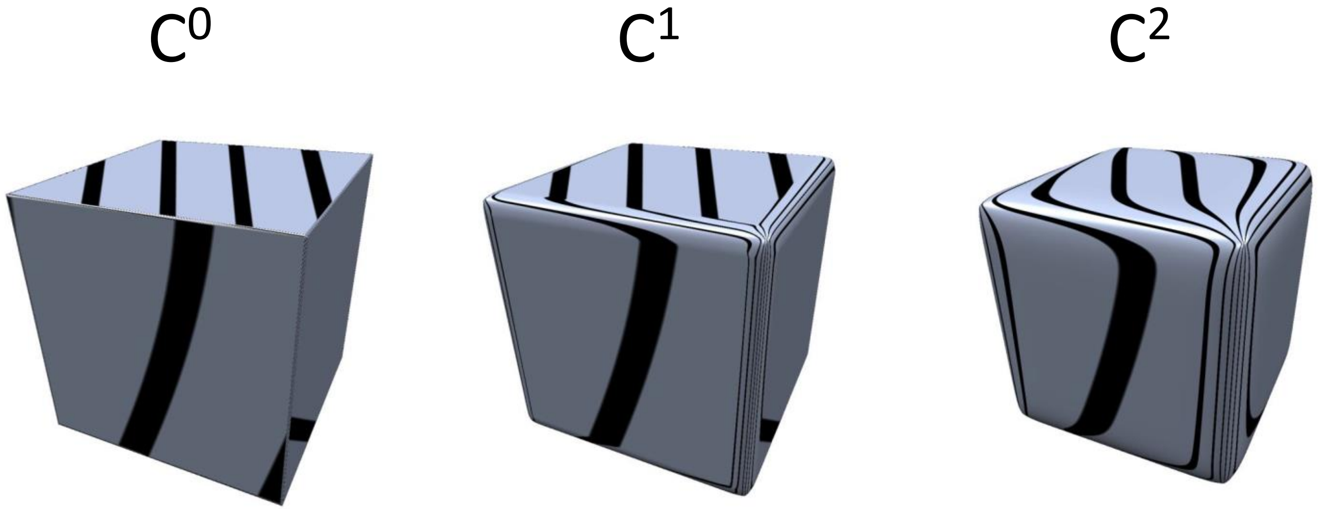

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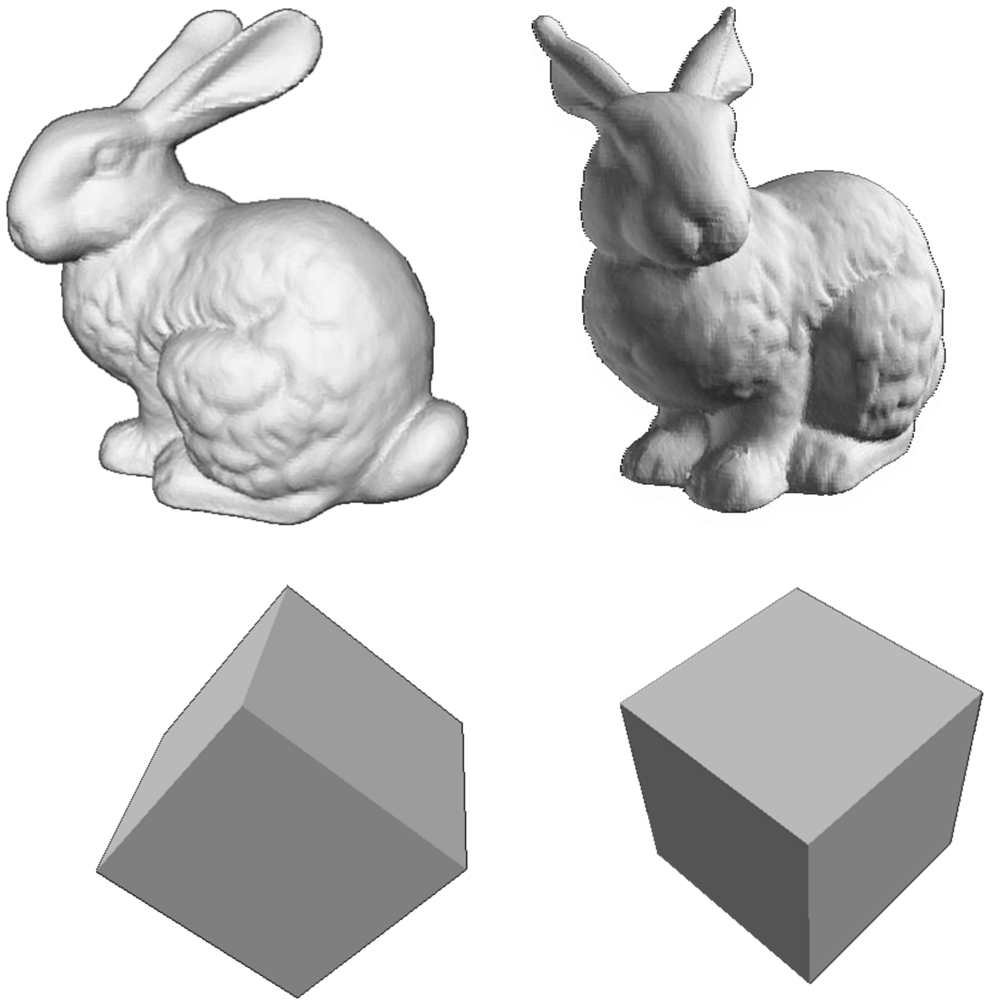

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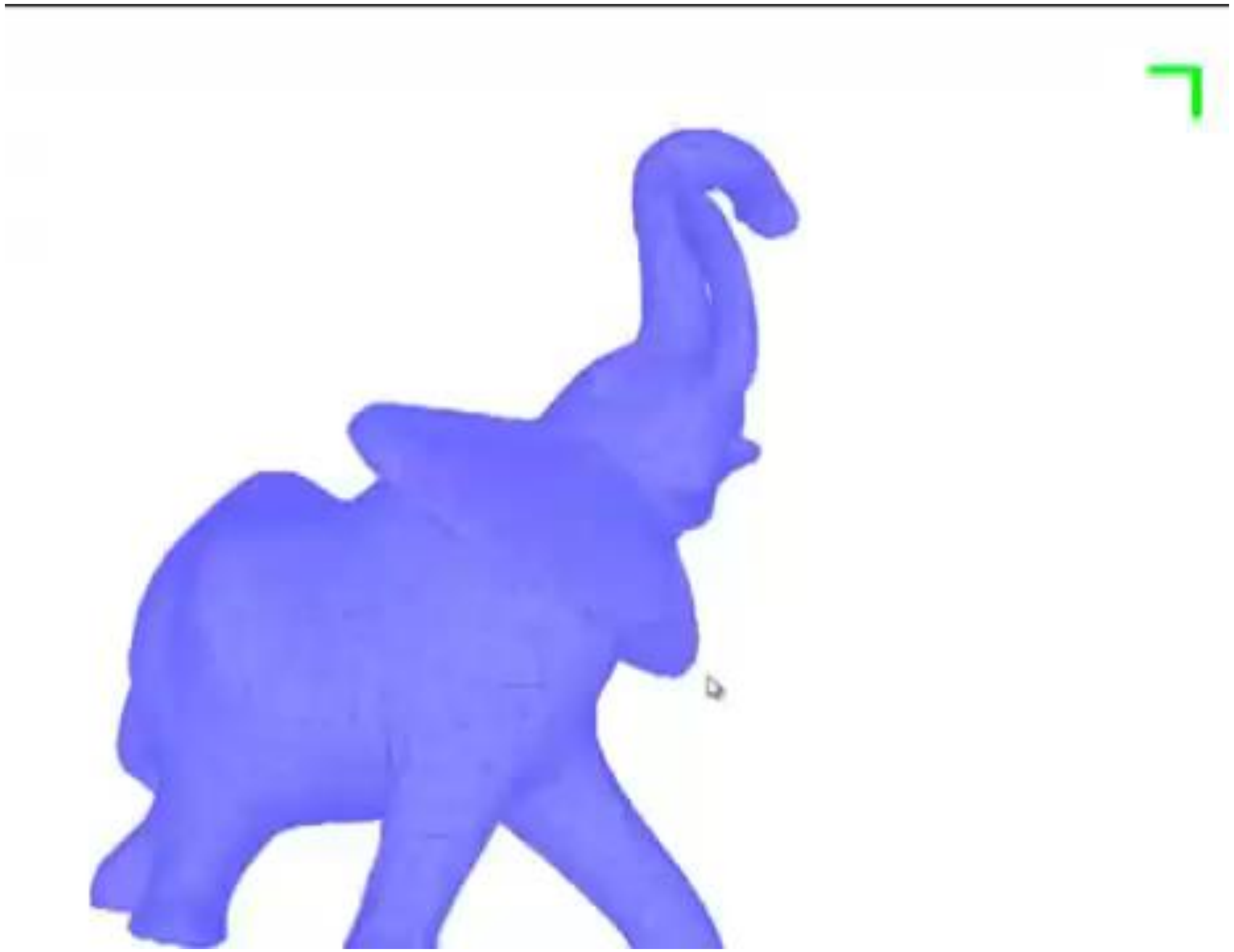

Sorkine

Polygonal Mesh Processing

- Analysis
  - Normals
  - Curvature
- Warps
  - Rotate
  - ➢Deform
- Filters
  - Smooth
  - Sharpen
  - Truncate
  - Bevel

Brendan Chou, COS 426, 2014

Polygonal Mesh Processing

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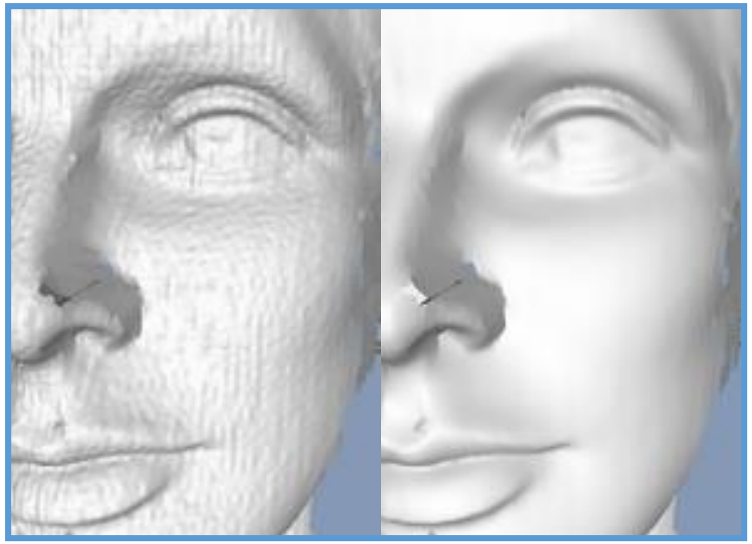


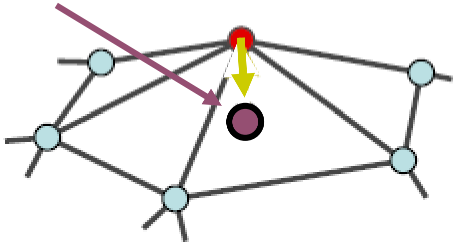
Thouis "Ray" Jones

How?

The Laplacian Operator

- Mesh formulation: Average of Neighboring Vertices

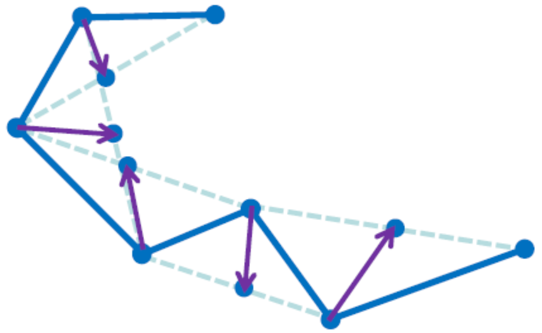
$$p_i = \frac{\Sigma_{j \in 1ring_i} p_j}{\#1ring_i}$$



Olga Sorkine

- Curve (Γ) formulation:

$$p_i = \frac{p_{i+1} + p_{i-1}}{2}$$



$$0 = \frac{\underbrace{(p_{i+1} - p_i)}_{\Gamma'(p_{i+1})} - \underbrace{(p_i - p_{i-1})}_{\Gamma'(p_i)}}{2} \Rightarrow 0 = \Gamma''(p_i)$$

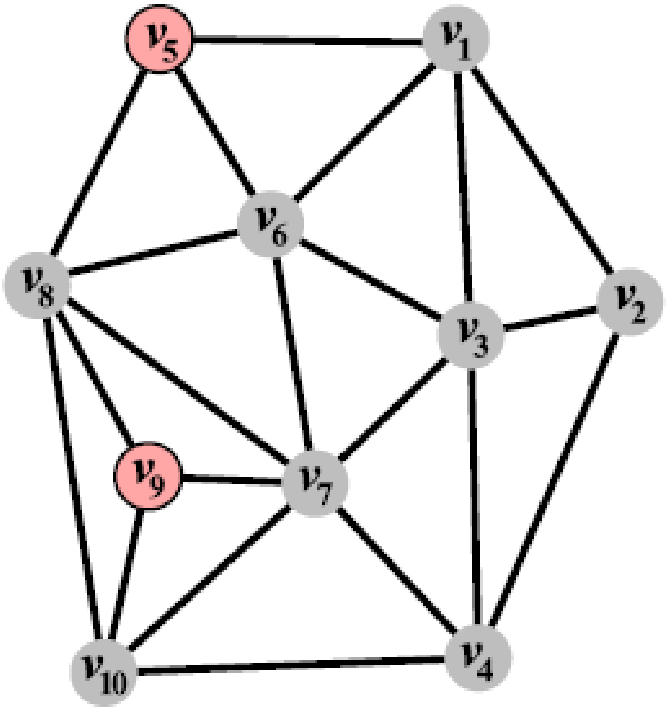
The Laplacian Operator

- Lo and behold: The Laplacian operator Δ

$$L(p_i) = \Delta(p_i) = \frac{\sum_{j \in 1_{ring_i}} p_j - p_i}{\# 1_{ring_i}}$$

- In matricial form:

$$L_{ij} = \begin{cases} -w_{ij} & i \neq j \\ \sum_{j \in 1_{ring_i}} w_{ij} & i = j \\ 0 & else \end{cases}$$



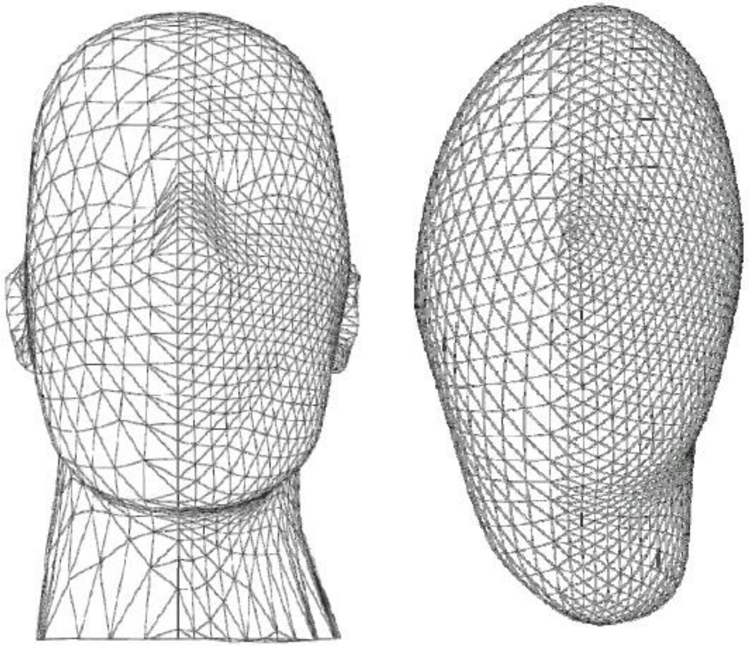
$$\begin{bmatrix} 4 & -1 & -1 & & -1 & -1 & & & & \\ -1 & 3 & -1 & -1 & & & & & & \\ -1 & -1 & 5 & -1 & & -1 & -1 & & & \\ & -1 & -1 & 4 & & & -1 & & & -1 \\ -1 & & & & 3 & -1 & & -1 & & \\ -1 & & -1 & & -1 & 5 & -1 & -1 & & \\ & & -1 & -1 & & -1 & 6 & -1 & -1 & -1 \\ & & & & -1 & -1 & -1 & 5 & -1 & -1 \\ & & & & & & -1 & -1 & 3 & -1 \\ & & & -1 & & & -1 & -1 & -1 & 4 \end{bmatrix}$$

The Laplacian Operator

- Lo and behold: The Laplacian operator Δ

$$L(p_i) = \Delta(p_i) = \frac{\sum_{j \in 1_{ring_i}} p_j - p_i}{\# 1_{ring_i}}$$

- However, Meshes are irregular



The Laplacian Operator

- Lo and behold: The Laplacian operator Δ

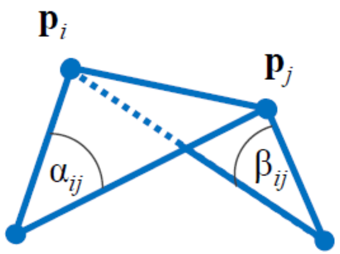
$$L(p_i) = \Delta(p_i) = \frac{\Sigma_{j\in 1_{ring_i}} p_j - p_i}{\#1_{ring_i}}$$

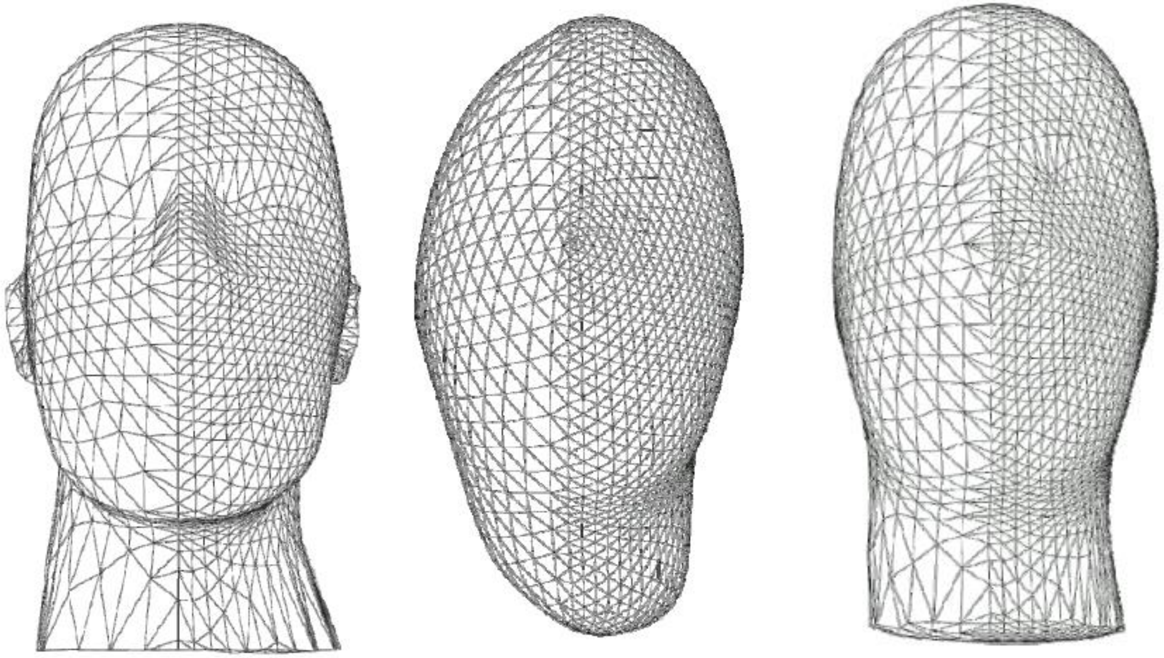
- However, Meshes are irregular

  - Cotangent weights:

$$L(p_i) = \frac{\Sigma_{j\in 1_{ring_i}} w_{ij} \cdot p_j}{\Sigma_{j\in 1_{ring_i}} w_{ij}} - p_i$$

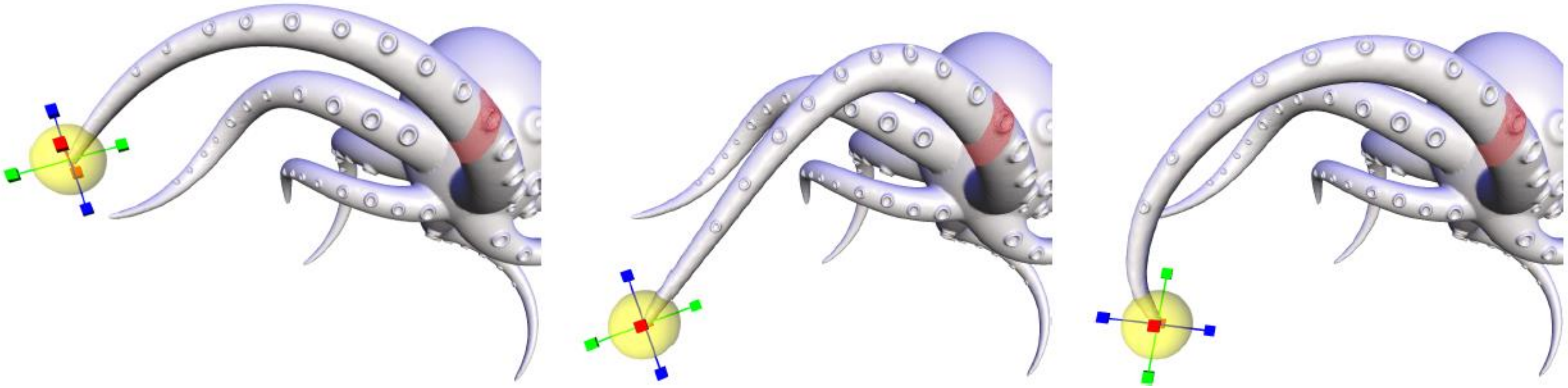
$$w_{ij} = \frac{\cot(\alpha_{ij}) + \cot(\beta_{ij})}{2}$$





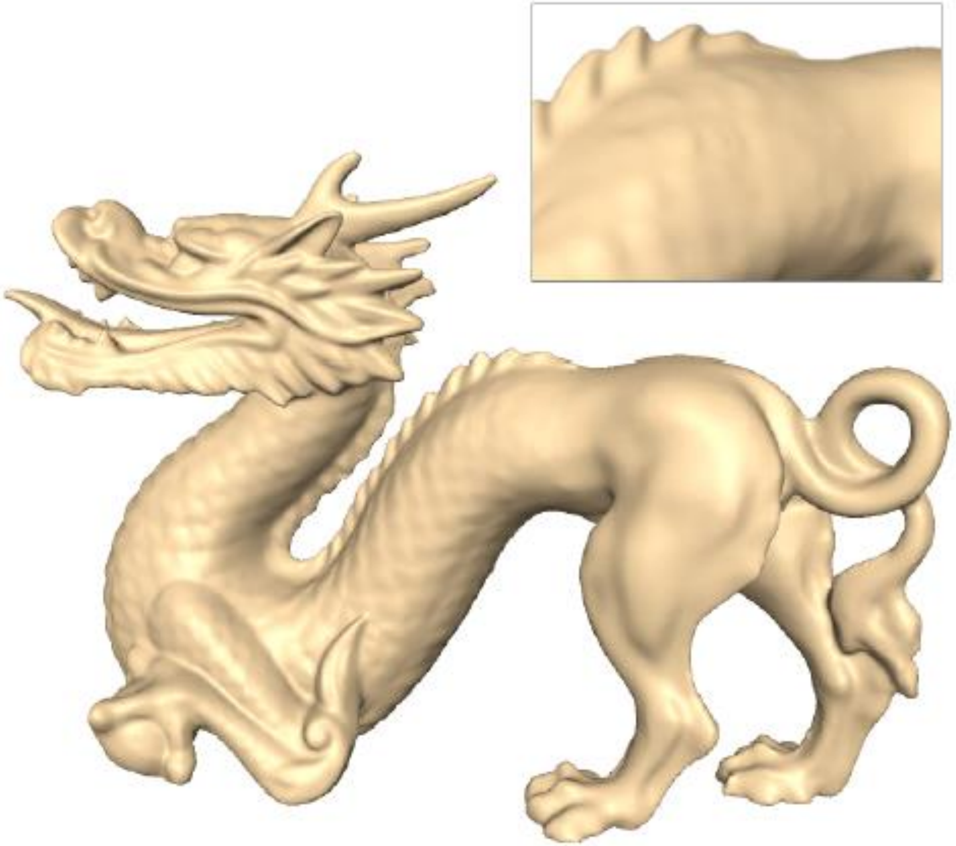
The Laplacian Operator

- Applicable to:
  - Deformation, by adding constraints



The Laplacian Operator

- Applicable to:
  - Deformation, by adding constraints
  - Blending, by concatenating rows



The Laplacian Operator

- Applicable to:
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  - Blending, by concatenating rows
  - Hole filling, by 0’s on the RHS

The Laplacian Operator

- Applicable to:
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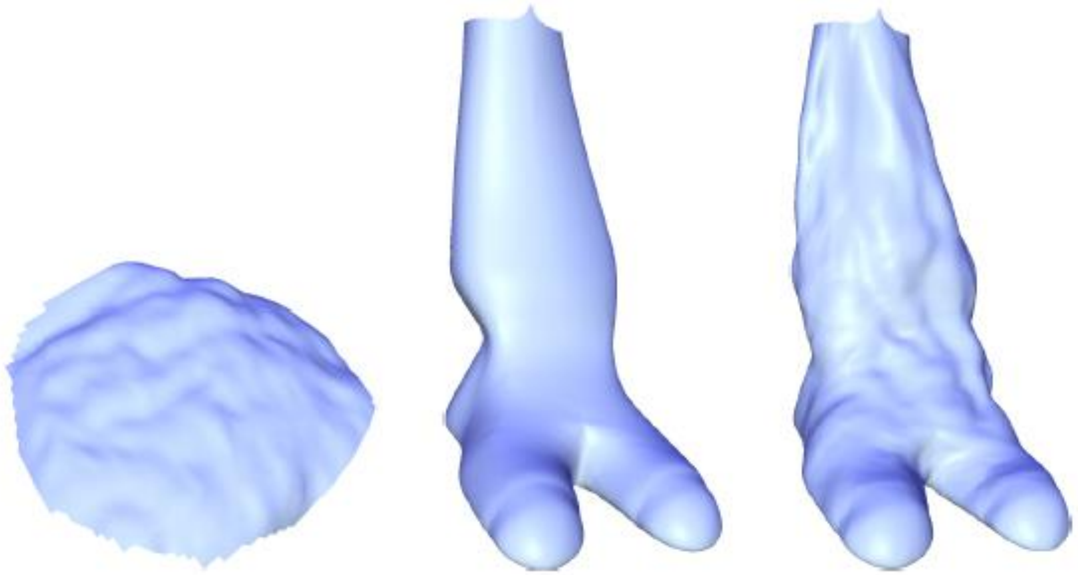
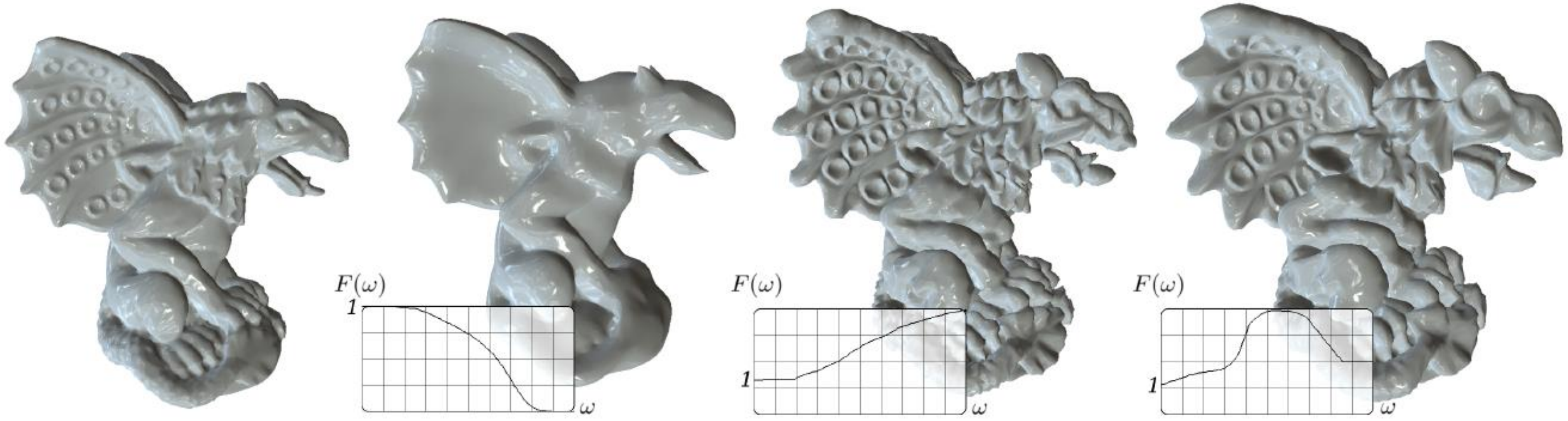
  - Hole filling, by 0’s on the RHS
  - Coating (or detail transfer), by copying RHS values (after filtering)



The Laplacian Operator

- Applicable to:
  - Deformation, by adding constraints
  - Blending, by concatenating rows
  - Hole filling, by 0's on the RHS
  - Coating (or detail transfer), by copying RHS values (after filtering)
  - Spectral mesh processing, through eigen analysis



Polygonal Mesh Processing

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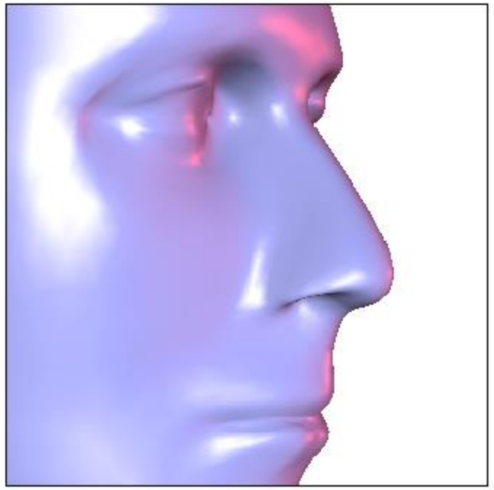
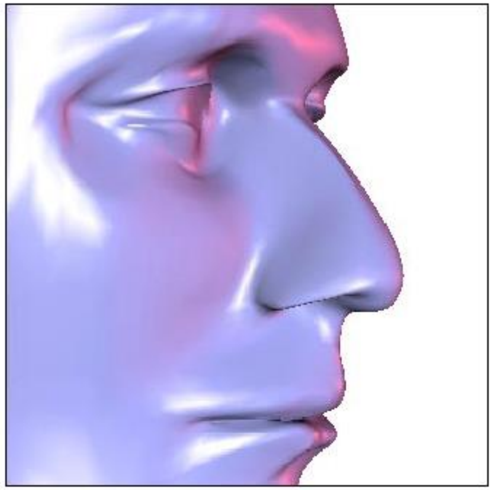
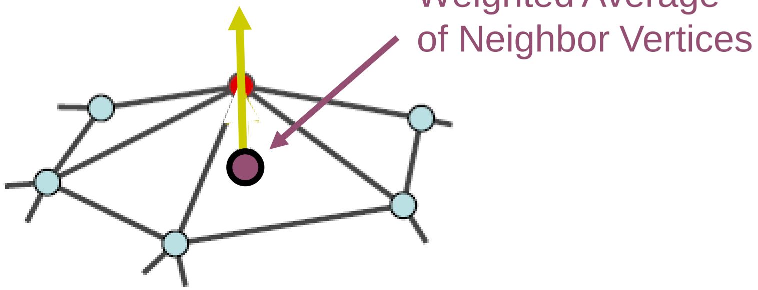




Desbrun



Olga Sorkine

Polygonal Mesh Processing

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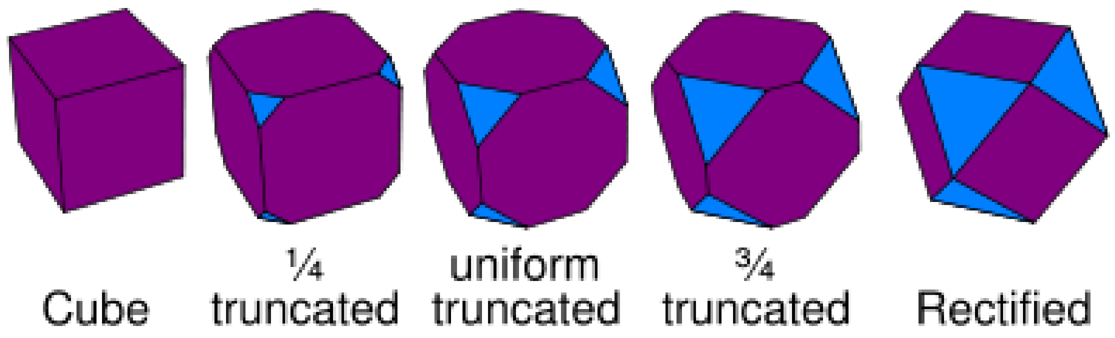

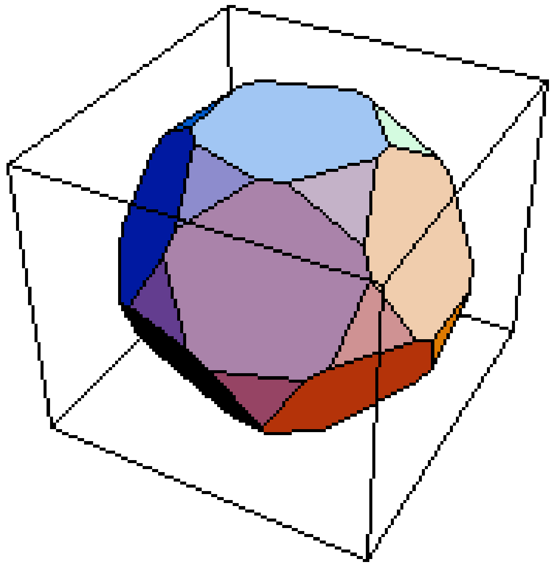

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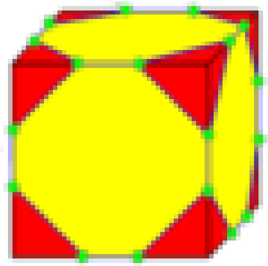

Conway

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Riley Thomasson, COS 426, 2014

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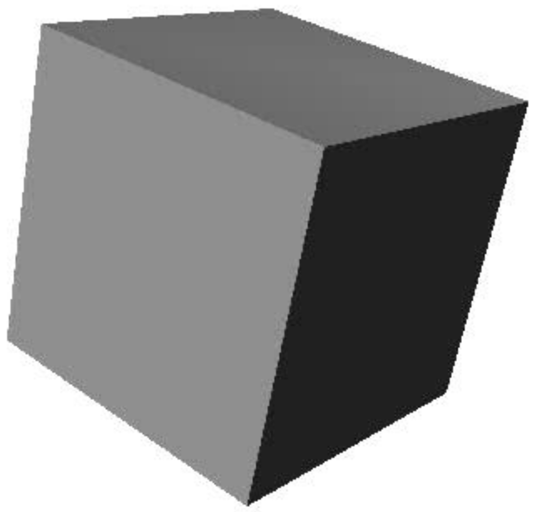

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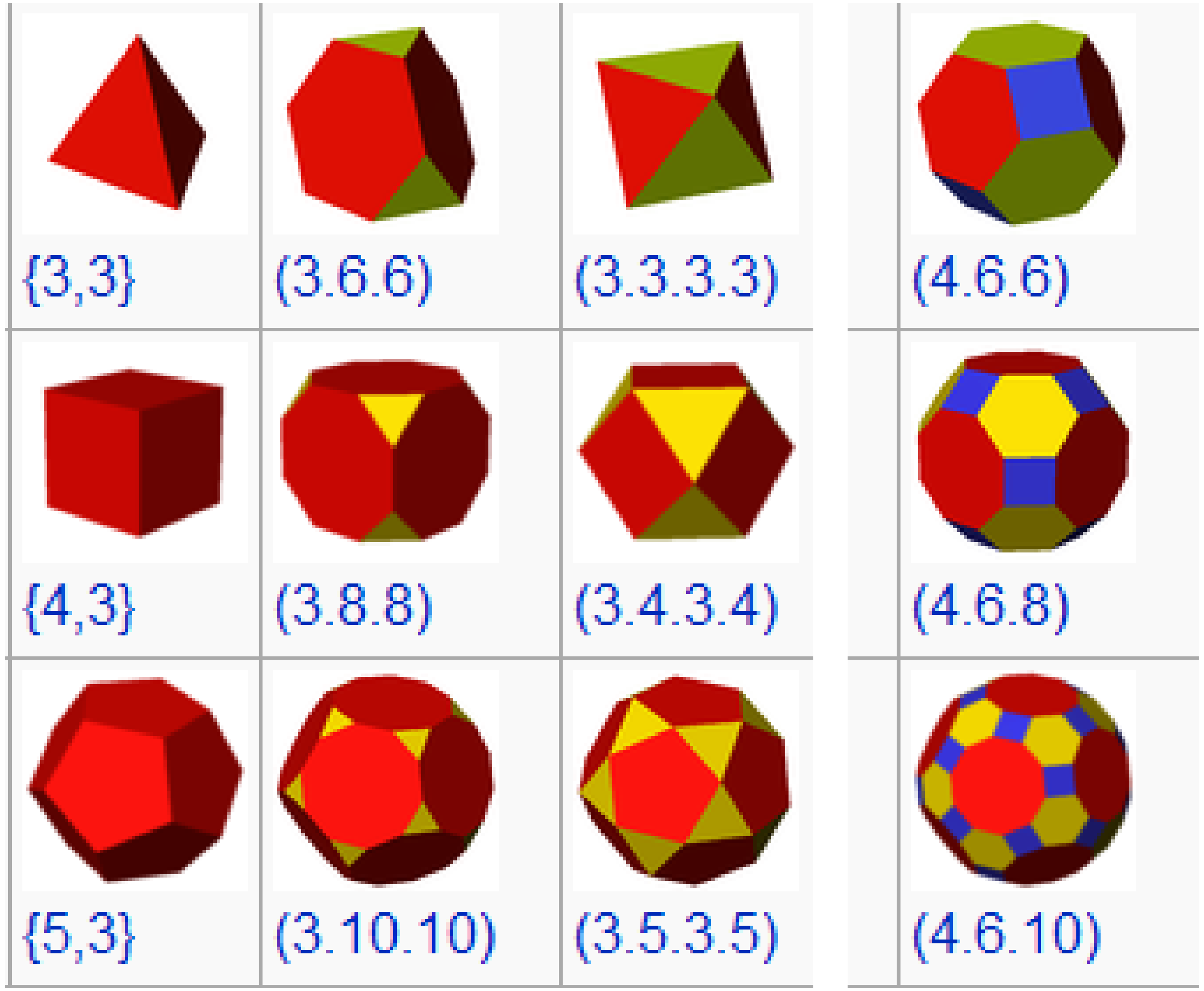

Wikipedia

Polygonal Mesh Processing

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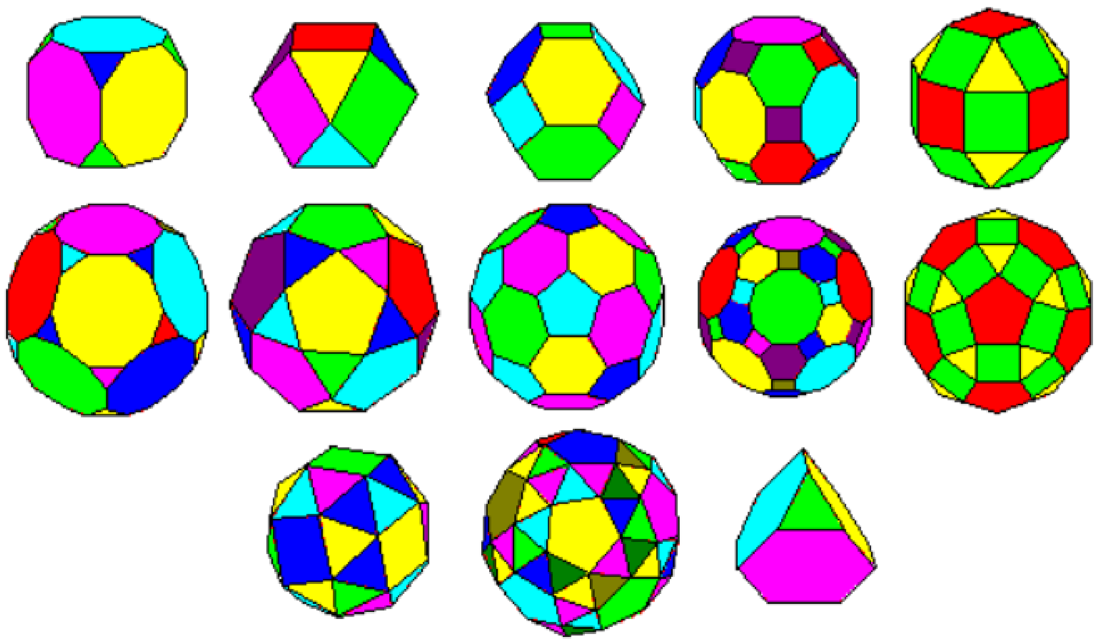

Archimedean Polyhedra

http://www.uwgb.edu/dutchs/symmetry/archpol.htm

Polygonal Mesh Processing

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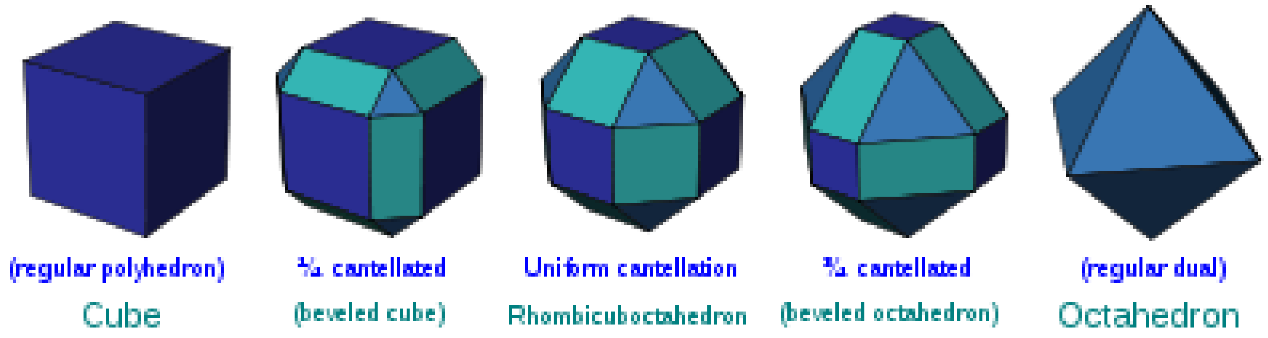
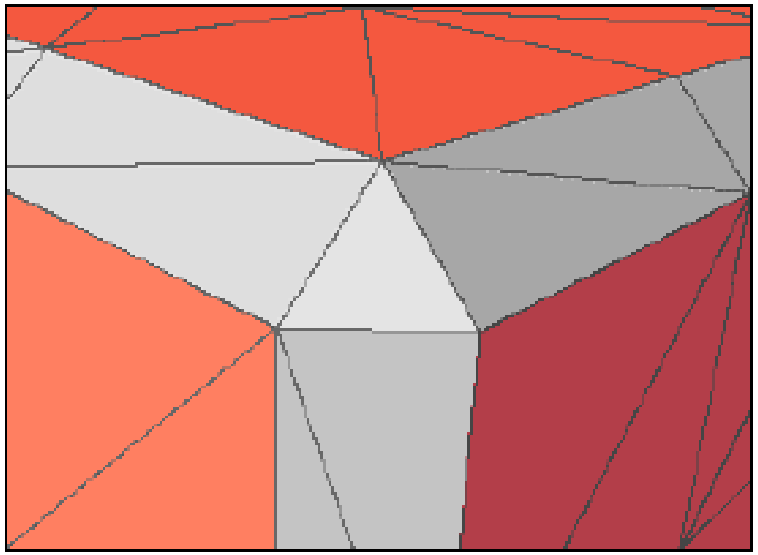
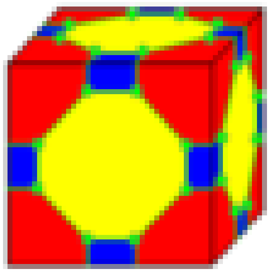
Wikipedia





Jarek Rossignac



Conway

Polygonal Mesh Processing

- Remeshing
  - Subdivide
  - Resample
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  - Fill holes
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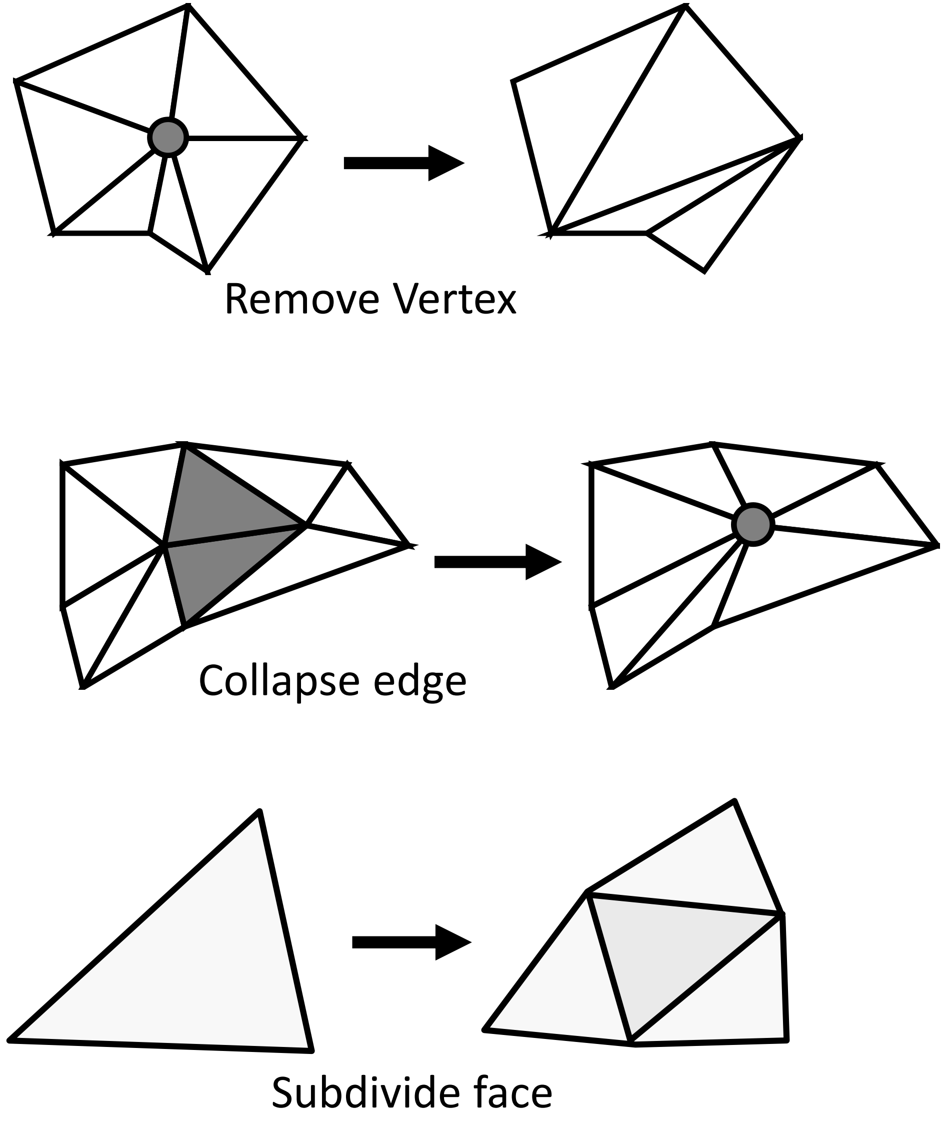

Polygonal Mesh Processing

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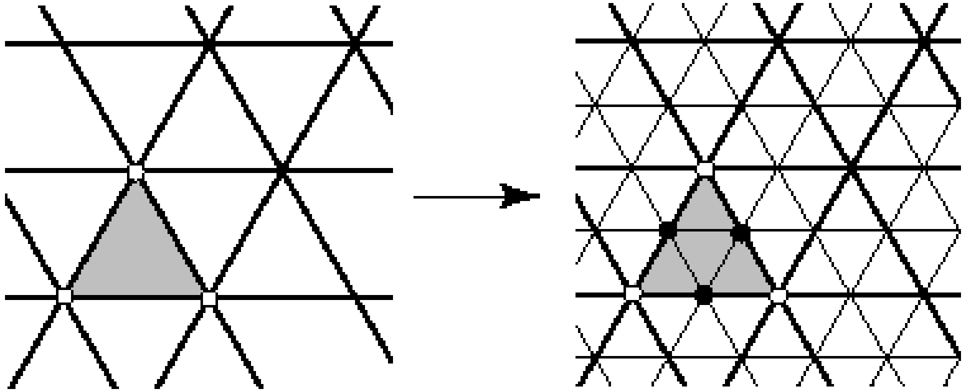

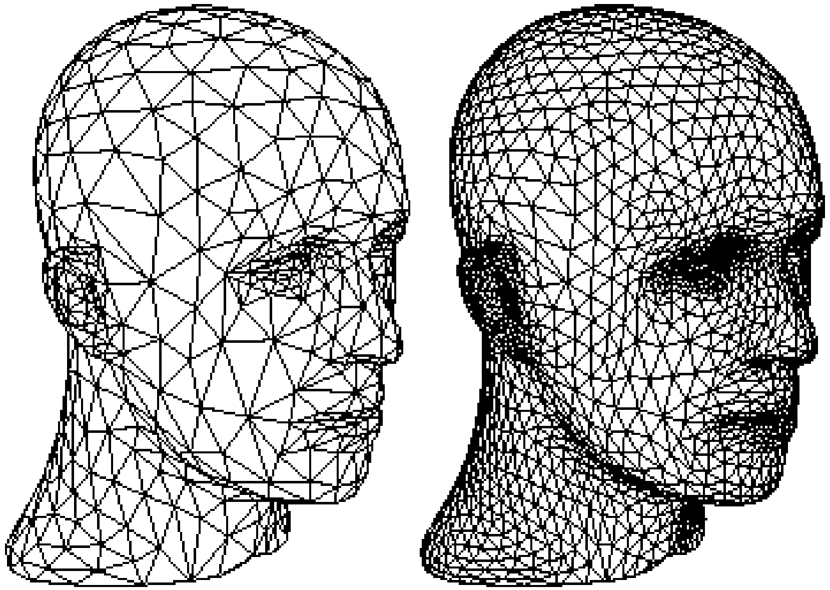

Zorin & Schroeder

Polygonal Mesh Processing

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Matt Matl, COS 426, 2014

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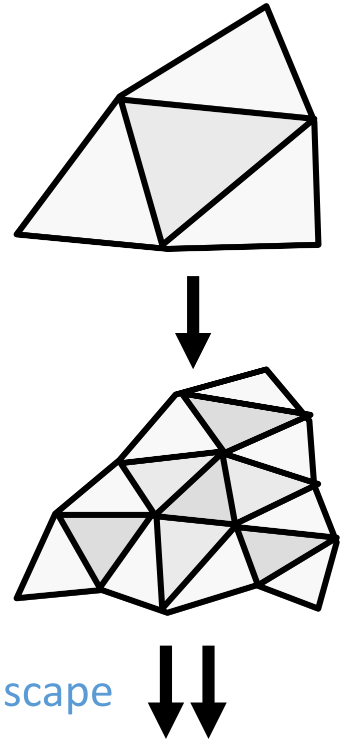
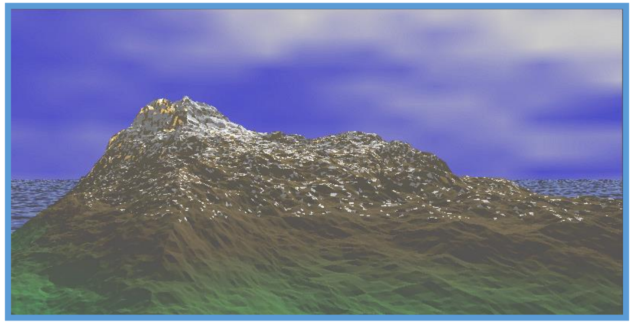


Fractal Landscape



Dirk Balfanz, Igor Guskov,
Sanjeev Kumar, & Rudro Samanta,

Polygonal Mesh Processing

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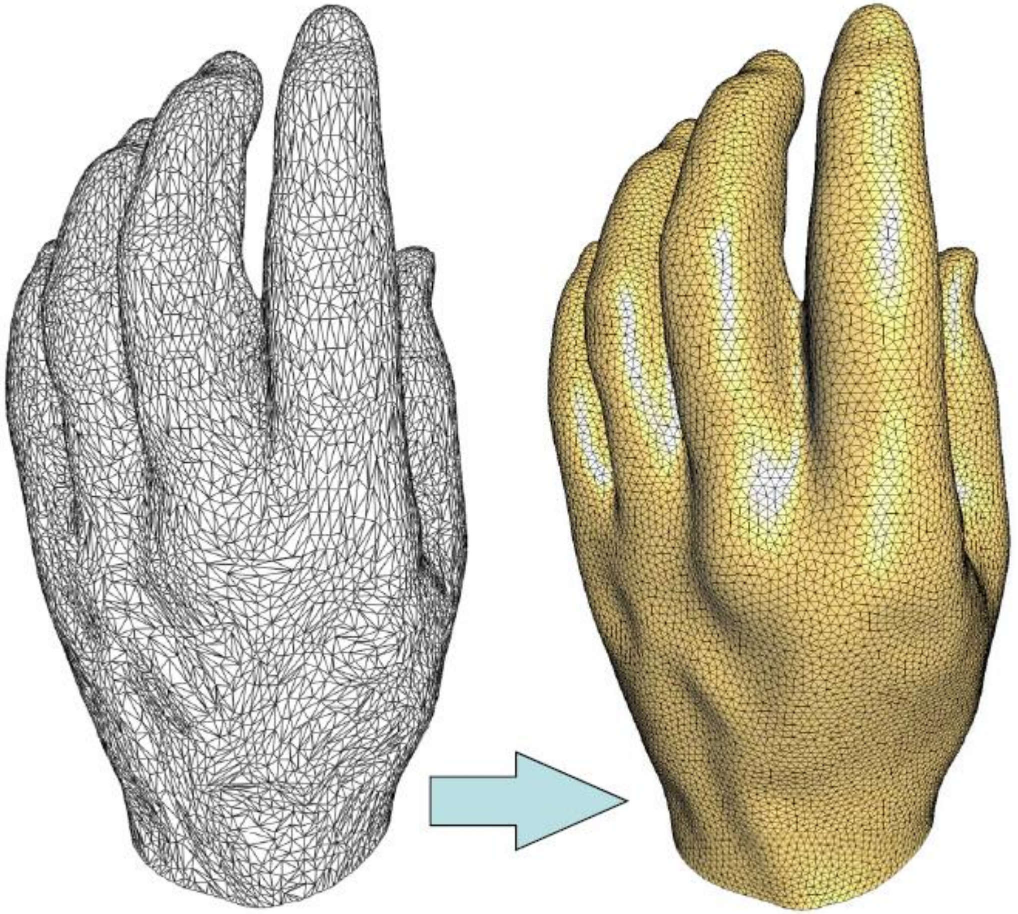

Stanford

- more uniform distribution
- triangles with nicer aspect

Polygonal Mesh Processing

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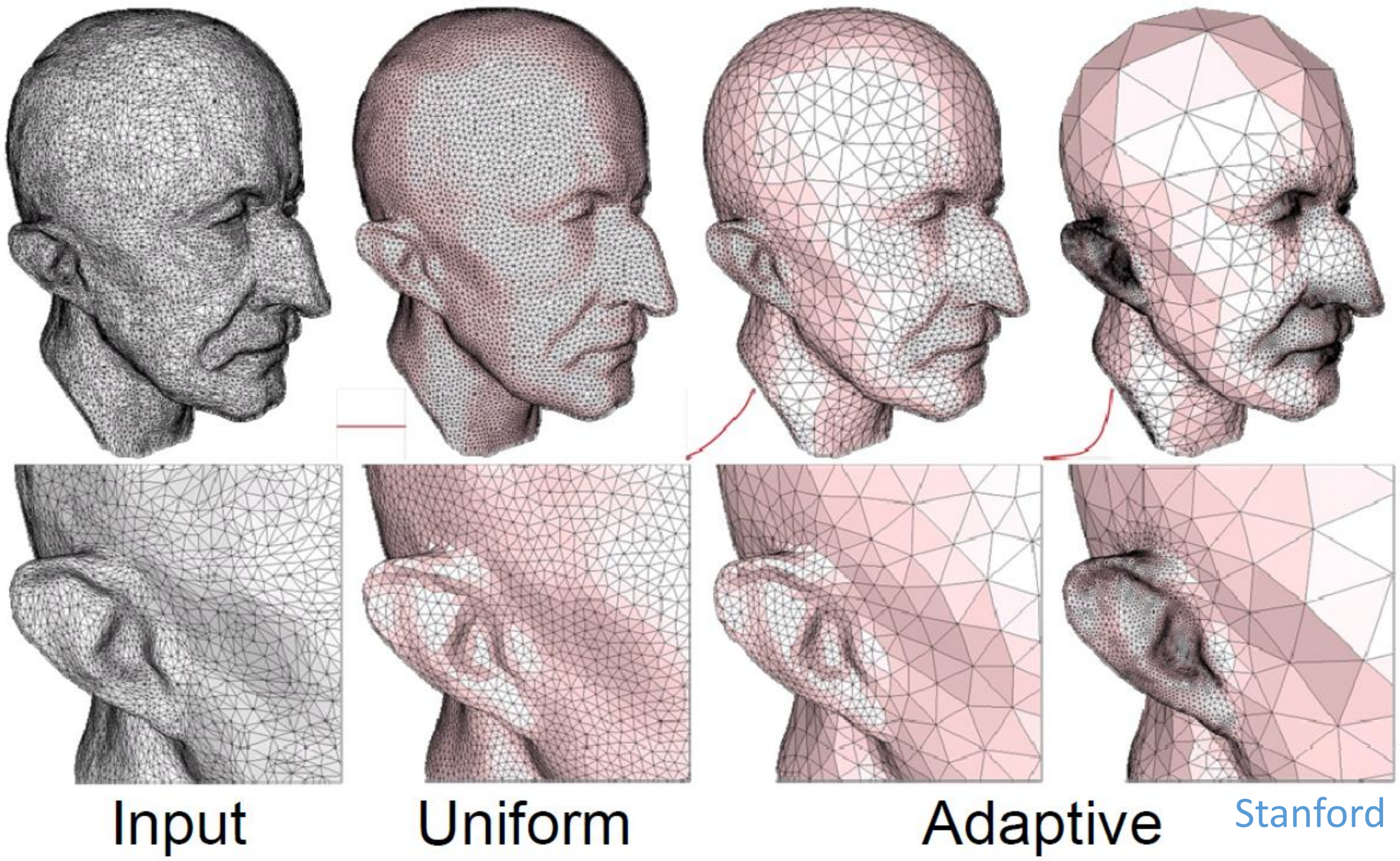

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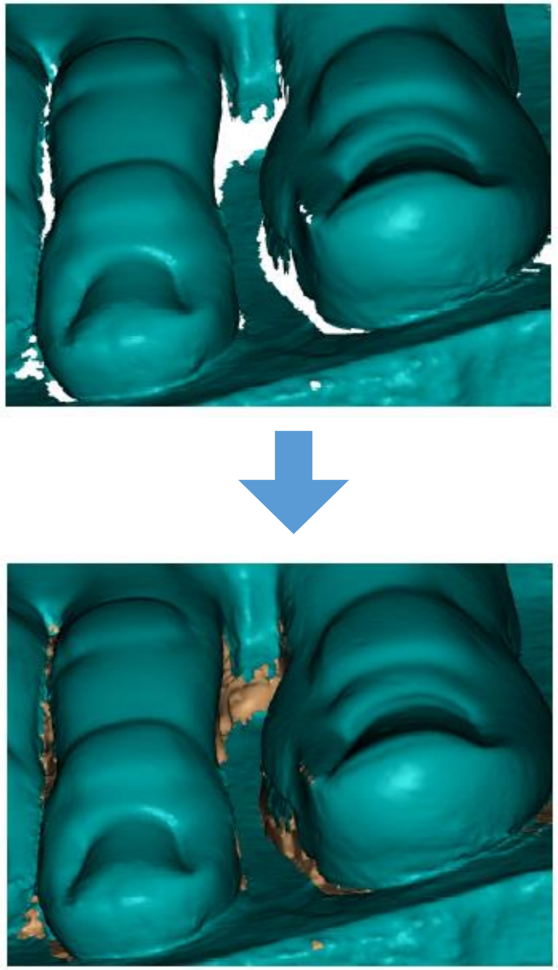

Podolak

Polygonal Mesh Processing

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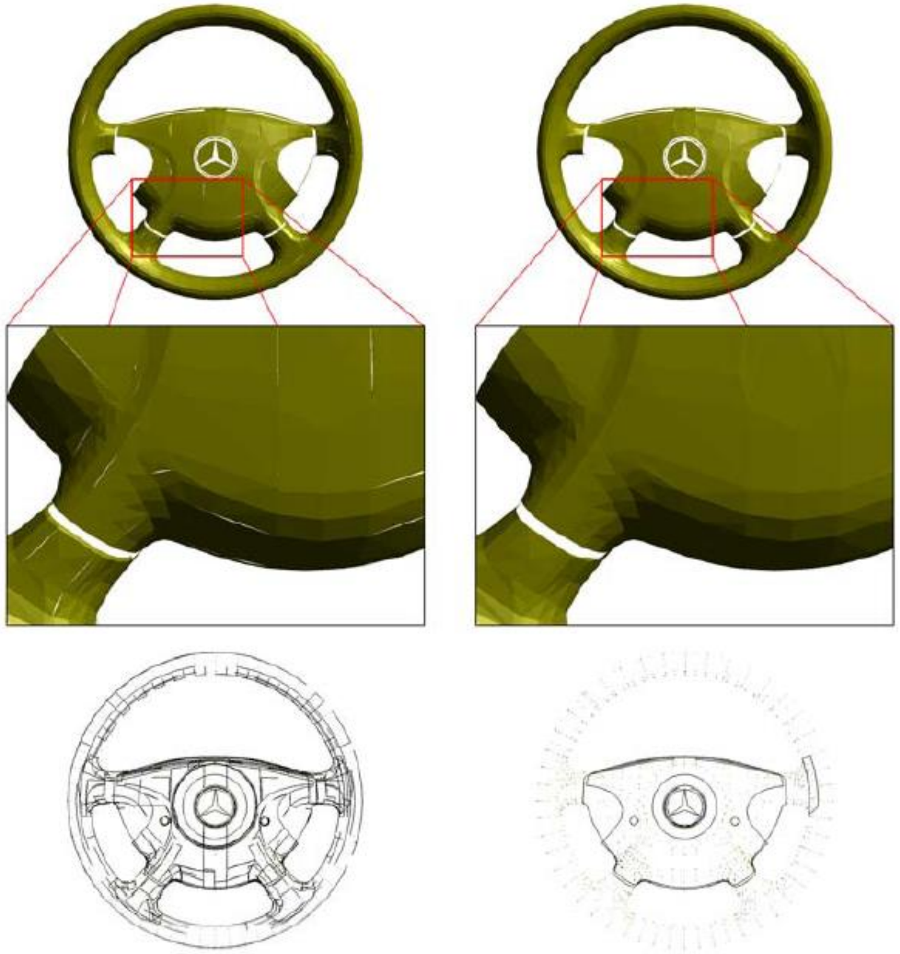

Borodin

Polygonal Mesh Processing

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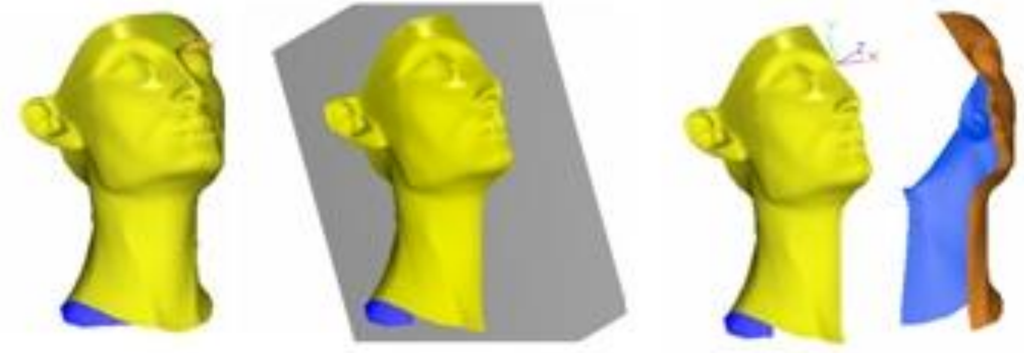
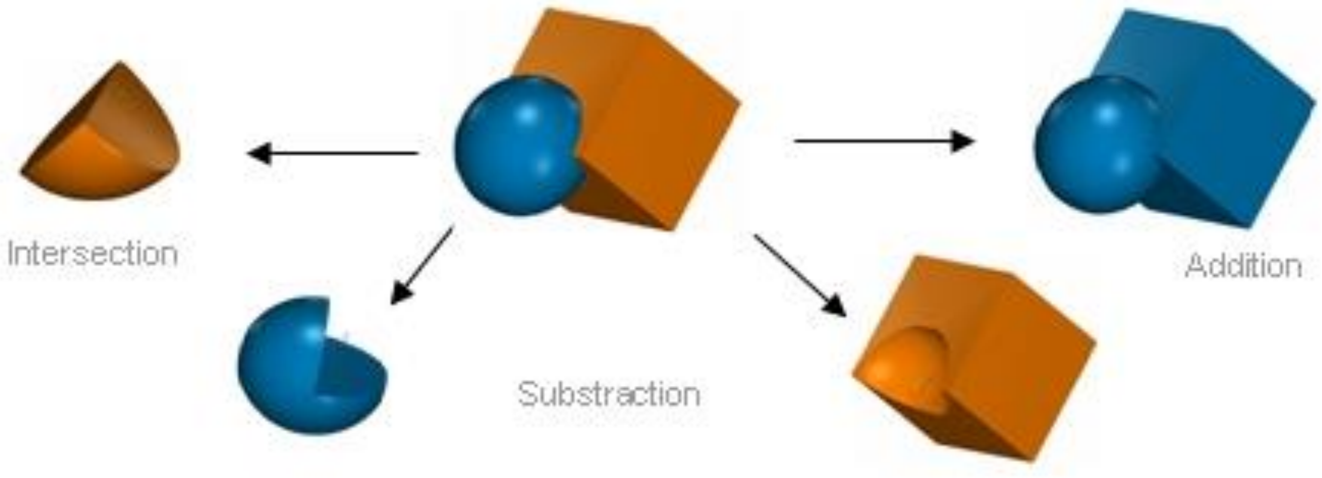
  - ➢Subtract
  - ➢Etc.



Mesh separation processed by a boolean operation.



Several Boolean operations with 3DReshaper®

Summary

- Polygonal meshes
  - Most common surface representation
  - Fast rendering
- Processing operations
  - Must consider irregular vertex sampling
  - Must handle/avoid topological degeneracies
- Representation
  - Which adjacency relationships to store depend on which operations must be efficient

3D Polygonal Meshes

- Properties
  - ? Efficient display
  - ? Easy acquisition
  - ? Accurate
  - ? Concise
  - ? Intuitive editing
  - ? Efficient editing
  - ? Efficient intersections
  - ? Guaranteed validity
  - ? Guaranteed smoothness
  - ? etc.

Viewpoint

3D Polygonal Meshes

- Properties
  - ☺Efficient display
  - ☺Easy acquisition
  - ☹Accurate
  - ☹Concise
  - ☹Intuitive editing
  - ☹Efficient editing
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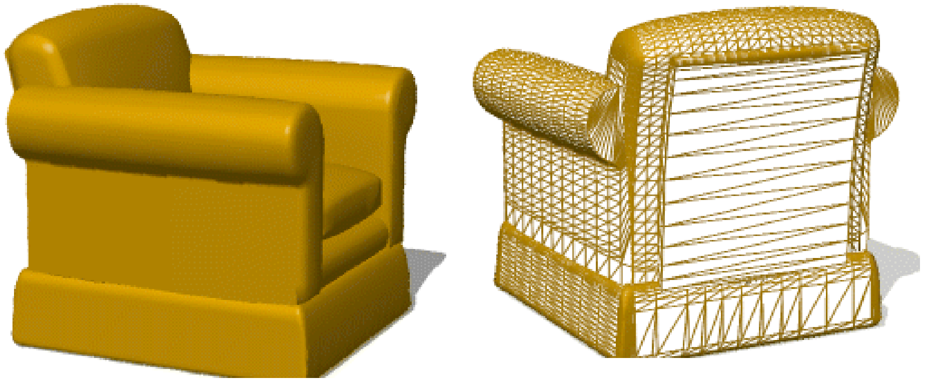

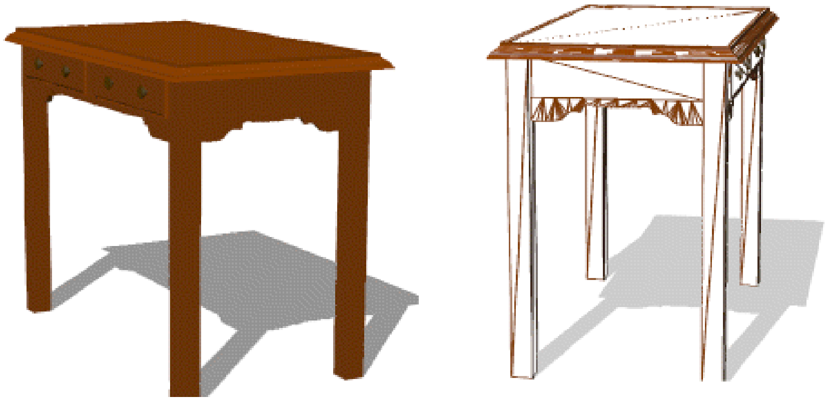

Viewpoint