COS 423	Theory of Algorithms	Spring 2018

Precept 8

These problems will be solved in precept.

1. *Difference constraints.* (CLRS 24.4) Given a vector $b = (b_1, b_2, \ldots, b_m) \in \Re^m$ and m inequalities of the form $x_i - x_j \leq b_k$, is ther a vector $x = (x_1, x_2, \ldots, x_n) \in \Re^n$ that satisfies all of the inequalities?

   For example, the system at left has a solution $x = (0, 2, 5, 4, 1)$ while the system at right has no solution (add inequalties 5, 3, 4, 8, and 2).

$$\begin{array}{rcr} x_1 - x_2 & \leq & 0 \\ x_1 - x_5 & \leq & -1 \\ x_2 - x_5 & \leq & 1 \\ x_3 - x_1 & \leq & 5 \\ x_4 - x_1 & \leq & 4 \\ x_4 - x_3 & \leq & -1 \\ x_5 - x_3 & \leq & -3 \\ x_5 - x_4 & \leq & -3 \end{array} \qquad \begin{array}{rcr} x_1 - x_2 & \leq & 4 \\ x_1 - x_5 & \leq & 5 \\ x_2 - x_4 & \leq & -6 \\ x_3 - x_2 & \leq & 1 \\ x_4 - x_1 & \leq & 3 \\ x_4 - x_3 & \leq & 5 \\ x_4 - x_5 & \leq & 10 \\ x_5 - x_3 & \leq & -4 \end{array}$$

   (a) Given a system of difference constraints with m inequalities and n variables, design an algorithm to find a solution x (or report that no such solution exists). Your algorithm should take $O(mn)$ time and use $O(m + n)$ space.

   (b) Suppose b is integer-valued. Design an algorithm to find an integer-valued solution x (or report that no such solution exists). Your algorithm should take $O(mn)$ time and use $O(m + n)$ space.

   (c) How would you add an equality constraint of the form $x_i - x_j = b_k$?

2. Exercise 7.10 in *Kleinberg–Tardos* (*reduce capacity of edge by 1*).

   Repeat the exercise but do not assume that the set of edges with positive flow is acyclic.