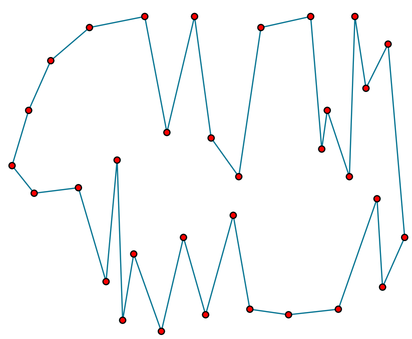
COS 423	Theory of Algorithms	Spring 2018

Precept 7

These problems will be solved in precept.

1. *Bitonic TSP.* (CLRS 15-3) Given n points in the plane $p_1, p_2, \ldots, p_n$, the *Euclidean traveling salesperson* problem is to find a shortest closed tour that connects all n points, where the distance between two points is the Euclidean distance between them. The general problem is NP-hard, so we will restrict our attention to *bitonic* tours—tours that start at the rightmost point, go strictly leftward to the leftmost point, and then go strictly rightward back to the starting point. Design a dynamic-programming algorithm that finds an bitonic tour of minimum length. Your algorithm should take $O(n^2)$ time and use $O(n^2)$ space.

   Assume that no two points have the same x-coordinate and that all operations on real numbers (such as computing the Euclidean distance between two points) take unit time.

   A bitonic TSP tour of 33 points in the plane.

2. *Local sequence alignment.* Often two DNA sequences are significantly different, but contain regions that are very similar and are highly conserved. Design an algorithm that takes two strings $x = x_1 x_2 \ldots x_m$ and $y = y_1 y_2 \ldots y_n$ as input and finds a min-cost alignment between any *substring* of x with any *substring* of y. As usual, let δ denote the gap cost and α_{pq} denote the cost of aligning character p with character q.

   For example, consider the two strings

   $$x = TTAAAACTTTGGTTTTT, \quad y = CCCAAAAATTGGCCC$$

   and suppose the gap cost is 2, the mismatch cost for aligning two different characters is 1, and match cost is -1. Then, the minimum cost of a local alignment of x and y is $5 = 8 - 1 - 2$ (align $AAAACTTTGG$ with $AAAAATTGG$).

   (a) Design an algorithm that takes $O(m^3 n^3)$ time by solving many global sequence alignment problems.

   (b) Design a dynamic programming algorithm that takes $O(mn)$ time.