These problems will be solved in precept.

1. Let \( G = (V, E) \) be a connected graph with positive edge weights. Let \( x_1 \leq x_2 \leq \ldots \leq x_{n-1} \) be the weights of the edges in any MST \( T^* \) of \( G \) and let \( y_1 \leq y_2 \leq \ldots \leq y_{n-1} \) be the weights of the edges in any spanning tree \( T \) of \( G \). Prove that \( x_i \leq y_i \) for every \( i \). In other words, the \( i^{th} \) smallest edge in any MST is no larger than the \( i^{th} \) smallest edge in any spanning tree.

Using this property, quickly prove that 9 of the following 10 statements are corollaries (and identify the one that is false).

(a) \( T^* \) is a spanning tree that minimizes the minimum edge weight.
(b) \( T^* \) is a spanning tree that minimizes the maximum edge weight.
(c) \( T^* \) is a spanning tree that minimizes the median edge weight.
(d) \( T^* \) is a spanning tree that minimizes the arithmetic mean of the edge weights.
(e) \( T^* \) is a spanning tree that minimizes the geometric mean of the edge weights.
(f) \( T^* \) is a spanning tree that minimizes the harmonic mean of the edge weights.
(g) \( T^* \) is a spanning tree that minimizes the variance of the edge weights.
(h) \( T^* \) is a spanning tree that minimizes the sum of the squares of the edge weights.
(i) \( T^* \) is a spanning tree that minimizes the product of the edge weights.
(j) \( T^* \) is a spanning tree that minimizes the sum of the logarithms of the edge weights.

Prove these two corollaries as well.

(k) Any two MSTs have the same multiset of edge weights.
(l) If the edge weights are distinct, then there is a unique MST.

2. Given a directed acyclic graph \( G \) with edge costs \( c_e > 0 \), design a greedy algorithm to find a minimum-cost arborescence. The running time should be \( O(m + n) \).