

Precept 10

These problems will be solved in precept.

1. Suppose that you are given a maximum flow f^* in a flow network G . Design an algorithm to determine whether f^* is the unique maximum flow (i.e., a flow whose value is strictly larger than that of every other flow). Your algorithm should take $O(m^2)$ time. As usual, let $m = |E|$ and $n = |V|$ and assume $n = O(m)$.

Note: it's also possible to do in $O(m)$ time by finding a cycle in a mixed graph.

2. Consider the following three related problems:

- HAMILTON-CYCLE: Given an undirected graph G , does G contain a cycle that visits every node exactly once?
- HAMILTON-PATH: Given an undirected graph G , does G contain a path that visits every node exactly once?
- LONGEST-PATH: Given an undirected G with integer edge weights $w(e) \geq 0$ and an integer L , is there a simple path (no repeated nodes) whose length is $\geq L$?

Prove that HAMILTON-CYCLE \leq_P HAMILTON-PATH \leq_P LONGEST-PATH.