1. Consider the following decision and optimization versions of the longest path problem:

- **LONGEST-PATH**: Given an undirected $G$ with integer edge weights $w(e) \geq 1$ and an integer $L$, does there exist a simple path (no repeated nodes) whose length is $\geq L$?
- **FIND-LONGEST-PATH**: Given an undirected graph $G$ with integer edge weights $w(e) \geq 1$, find a longest simple path.

Prove that FIND-LONGEST-PATH $\equiv_p$ LONGEST-PATH.

2. Consider the following two related problems:

- **SUBSET-SUM**: Given $n$ natural numbers $w_1, \ldots, w_n$ and an integer $W$, is there a subset that adds up to exactly $W$? A subset may contain each number at most once.
- **COIN-CHANGING**: Given $m$ coin denominations $1 = c_1 < \ldots < c_m$ and an amount $S$, can you make change for the amount $S$ using at most $T$ coins? You may use as many coins of each coin denomination as desired.

(a) Prove that SUBSET-SUM $\leq_p$ COIN-CHANGING.

*Hint: as in the reduction from 3-Sat to SUBSET-SUM, use the individual digits of the COIN-CHANGING instance to impose any desired constraints (e.g., that you will take at most one coin of each denomination). Express the digits in base $b$ for a value of $b$ that is sufficiently large that there are no carries.*

(b) Prove that COIN-CHANGING is NP-complete.

3. Design a linear-time algorithm for FIND-LONGEST-PATH (defined above) when $G$ is a tree.