COS 423	Theory of Algorithms	Spring 2018

Problem Set 6

This assignment is due Wednesday, April 18 at 11pm via electronic submission. Collaboration is permitted, according to the rules specified in the syllabus.

Read Chapter 6.6–6.10, 7.1–7.2 in *Algorithm Design.*

1. You have won an airline ticket to fly within the continental United States, beginning in Cape Alava, Washington (westernmost airport), then flying only from west to east until you reach Sail Rock, Maine (easternmost airport) and then flying only from east to west until you return to Cape Alava. No city (other than Cape Alava) may be visited more than once. Given a set of n cities served by the airline, and a set of $m \geq n$ direct flights from one city to another, design an algorithm to find an itinerary that visits as many cities as possible (or report that no such itinerary is possible). Your algorithm should take $O(mn)$ time and use $O(n^2)$ space.

   Note: only a minor deduction for $O(n^3)$ time.

2. You are the entrepreneurial captain of a cargo ship and must plan a recurring voyage that visits a subset of n ports. You earn p_v dollars for visiting port v. Meanwhile, it costs c_{vw} dollars and takes t_{vw} days to travel from port v to port w. Design an algorithm to find a cyclic route which maximizes the mean daily profit (or report that no profitable cyclic route exists). Assuming that the profits, costs, and travel times are all integers between 1 and n, your algorithm should take $O(n^3 \log n)$ time and use $O(n^2)$ space.

   Hint: given a constant $\mu \geq 0$, design a subroutine that determines whether there exists a cyclic route whose mean daily profit is greater than (or equal to) μ.

3. Let $G = (V, E, s, t, c)$ be a flow network in which the capacity of every edge is an integer. Suppose that you are given a maximum flow f^* but the amount of flow on some edges is fractional. Design an algorithm to find a maximum flow in which the flow on every edge is an integer. Your algorithm should take $O(mn)$ time and use $O(m + n)$ space.

   Note: only a minor deduction for $O(m^2)$ time.