COS 423	Theory of Algorithms	Spring 2018

Problem Set 4

This assignment is due Wednesday, March 14 at 11pm via electronic submission. Collaboration is permitted, according to the rules specified in the syllabus.

Read CHAPTER 5.1–5.4 in *Algorithm Design*.

1. Suppose that $T(n)$ is a function that satisfies the following recurrence:

$$T(n) = \begin{cases} 0 & \text{if } n = 1 \\ T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + n \log_2 n & \text{if } n > 1 \end{cases}$$

   Prove from first principles that $T(n)$ is $\Theta\left(n \log^2 n\right)$ when n is a power of 2.

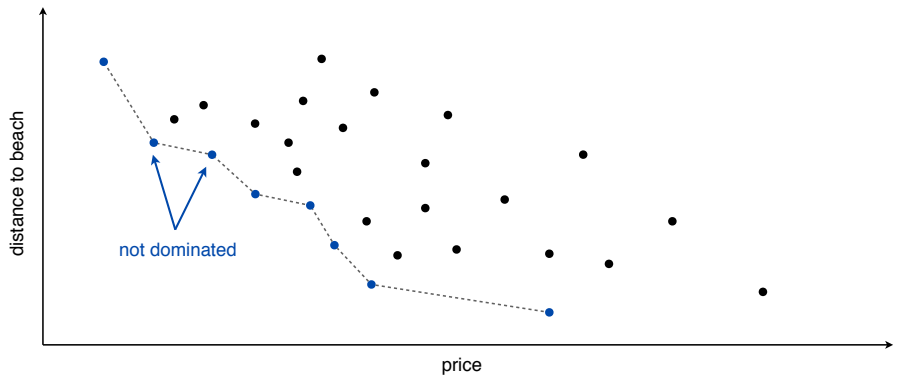
2. Given a list of n integers $A[1..n]$, your goal is to rearrange the integers to be in ascending order via a sequence of *reversal operations*: pick two indices $i < j$ and reverse the sublist $A[i..j]$. The *cost* of a reversal operation is $j - i + 1$—the length of the sublist.

   (a) Given a list $A[1..n]$ containing only 0s and 1s, design a divide-and-conquer algorithm that sorts A via a sequence of reversal operations of $O(n \log n)$ cost.

   (b) Given a list $A[1..n]$ of n integers, design a divide-and-conquer algorithm that sorts A via a sequence of reversal operations of $O(n \log^2 n)$ cost.
   Hint: use your algorithm from (a) as a subroutine and analyze it using Problem 4.1.

   Context: sorting-by-reversals plays a key role in inferring the evolutionary relationship between two genomes.

3. Consider a database of n hotels in which each hotel has two fields of interest to a customer: the price of a standard room and the distance to the beach. We say that hotel i *dominates* hotel j if hotel i is both cheaper and closer to the beach that hotel j. Design a divide-and-conquer algorithm to identify all hotels that are not dominated by any other hotel. In database terminology, this is known as a *skyline query*. The running time of your algorithm should be $O(n \log n)$ in the worst case.



ok