COS 423	Theory of Algorithms	Spring 2018

Problem Set 1

This problem set is due Wednesday, February 14 ♥ at 11pm via electronic submission. Collaboration is permitted, according to the rules specified in the syllabus.

Read Chapter 1 and review Chapter 2 in *Algorithm Design*. Read the course syllabus, including the collaboration policy and instructions for writing and submitting solutions.

1. Problem 1.5 in *Algorithm Design* (stable matching with ties in preference lists).

2. Let M_1 and M_2 be two different stable matchings for a given instance of the stable matching problem. Suppose that hospital h is matched with student s in M_1 but not in M_2. Prove that h prefers M_1 to M_2 if and only if s prefers M_2 to M_1.

3. For each of the following pairs of functions f and $g : \mathbb{N} \to \mathbb{R}$, determine whether f is $O(g)$, f is $\Theta(g)$, and/or f is $\Omega(g)$. Justify each answer.

   (a) $f(n) = 2^n, \qquad g(n) = 3^n$

   (b) $f(n) = n^{\log_2 e}, \quad g(n) = e^{\log_2(n+1)}$

   (c) $f(n) = n^{3+\sin n}, \quad g(n) = n^2$

   (d) $f(n) = \begin{cases} 0 & \text{if } n = 0 \\ n^n & \text{if } n > 0 \text{ and } n \text{ is even} \\ f(n-1)+1 & \text{if } n \text{ is odd} \end{cases} \qquad g(n) = \begin{cases} 0 & \text{if } n = 0 \\ n^n & \text{if } n \text{ is odd} \\ g(n-1)+1 & \text{if } n > 0 \text{ is } n \text{ is even} \end{cases}$

   Observe that f and g are strictly increasing functions.

4. Given an array of $n > 0$ distinct integers, design an $O(\log n)$ time algorithm to find a local minimum. A *local minimum* in an array is an entry that is smaller than all of its adjacent entries. For example, in the array $[23, 45, 32, 12, 5, 3, 6, 56, 77, 33, 55]$, there are three local minima—23, 3, and 33.