Typing

Fun has a strong type system with subtyping. The following sections define the typing rules for the language.

Subtyping

The judgement (tp1 $<$ tp2) states that tp1 is a subtype of tp2

1. $$\frac{}{\text{tp} < \text{tp}}$$

2. $$\frac{\text{tp1} < \text{tp2} \qquad \text{tp2} < \text{tp3}}{\text{tp1} < \text{tp3}}$$

3. $$\frac{\text{tp0} < \text{tp0'} \ \ldots\ \text{tpn} < \text{tpn'}}{\langle \text{tp0}, \ldots, \text{tpn} \rangle < \langle \text{tp0'}, \ldots, \text{tpn'} \rangle}$$

4. $$\frac{(0 <= \text{n} < \text{k})}{\langle \text{tp0}, \ldots, \text{tpn}, \ldots, \text{tpk} \rangle < \langle \text{tp0}, \ldots, \text{tpn} \rangle}$$

5. $$\frac{\text{tp1'} < \text{tp1} \qquad \text{tp2} < \text{tp2'}}{\text{tp1} \rightarrow \text{tp2} < \text{tp1'} \rightarrow \text{tp2'}}$$

6. $$\frac{\text{tp} < \text{tp'} \qquad \text{tp'} < \text{tp}}{\text{tp ref} < \text{tp' ref}}$$

Expression Typing

The judgement ($\Gamma \vdash$ exp : tp) states that expression exp has type tp in context Γ. A context Γ is a finite partial map from identifiers to types. The context ”nil” is the empty context. The context Γ [x : tp] is the same as Γ except it maps idenfier x to tp. The following rules define the expression typing judgement.

1. $$\frac{}{\Gamma \vdash \text{id} : \Gamma(\text{id})}$$

2. $$\frac{}{\Gamma \vdash \text{num} : \text{int}}$$

3. $$\frac{\Gamma \vdash \text{exp1} : \text{tp1} \quad \Gamma \vdash \text{exp2} : \text{tp2}}{\Gamma \vdash \text{exp1 ; exp2} : \text{tp2}}$$

4. $$\frac{\text{optype (un)} = \text{tp1} \rightarrow \text{tp2} \quad \Gamma \vdash \text{exp} : \text{tp1}}{\Gamma \vdash \text{un exp} : \text{tp2}}$$

5. $$\frac{\text{optype (bin)} = \text{tp1} \rightarrow \text{tp2} \rightarrow \text{tp3} \quad \Gamma \vdash \text{exp1} : \text{tp1} \quad \Gamma \vdash \text{exp2} : \text{tp2}}{\Gamma \vdash \text{exp1 bin exp2} : \text{tp3}}$$

6. $$\frac{\Gamma \vdash \text{exp0} : \text{tp0} \ \ldots \ \Gamma \vdash \text{expn} : \text{tpn}}{\Gamma \vdash <\text{exp0, ..., expn}> : <\text{tp0, ..., tpn}>}$$

7. $$\frac{\Gamma \vdash \text{exp} : <\text{tp0, ..., tpn}> \ (0 <= \text{i} <= \text{n})}{\Gamma \vdash \text{\#i exp} : \text{tpi}}$$

8. $$\frac{\Gamma \vdash \text{exp1} : \text{tp1} \rightarrow \text{tp2} \quad \Gamma \vdash \text{exp2} : \text{tp1}}{\Gamma \vdash \text{exp1 exp2} : \text{tp2}}$$

9. $$\frac{\Gamma \vdash \text{exp1} : \text{int} \quad \Gamma \vdash \text{exp2} : tp_2 \quad \Gamma \vdash \text{exp3} : tp_3 \quad \Gamma \vdash tp_2 < tp_1 \quad tp_3 < tp_1}{\Gamma \vdash \textbf{if} \text{ exp1 } \textbf{then} \text{ exp2 } \textbf{else} \text{ exp3} : tp_1}$$

10. $$\frac{\Gamma \vdash \text{exp1} : \text{int} \quad \Gamma \vdash \text{exp2} : <>}{\Gamma \vdash \textbf{if} \text{ exp1 } \textbf{then} \text{ exp2} : <>}$$

11. $$\frac{\Gamma \vdash \text{exp1} : \text{int} \quad \Gamma \vdash \text{exp2} : <>}{\Gamma \vdash \textbf{while} \text{ exp1 } \textbf{do} \text{ exp2} : <>}$$

12. $$\frac{\Gamma \vdash \text{exp1} : \text{tp1} \quad \Gamma\,[\text{x} : \text{tp1}] \vdash \text{exp2} : \text{tp2}}{\Gamma \vdash \textbf{let} \text{ x} = \text{exp1 } \textbf{in} \text{ exp2} : \text{tp2}}$$

13.
$$\frac{\Gamma \vdash \text{exp} : \text{tp}}{\Gamma \vdash \textbf{ref}\ (\text{exp} : \text{tp}) : \text{tp}\ \textbf{ref}}$$

14.
$$\frac{\Gamma \vdash \text{exp} : \text{tp}\ \textbf{ref}}{\Gamma \vdash !\text{exp} : \text{tp}}$$

15.
$$\frac{\Gamma \vdash \text{exp1} : \text{tp}\ \textbf{ref} \qquad \Gamma \vdash \text{exp2} : \text{tp}}{\Gamma \vdash \text{exp1} := \text{exp2} : <>}$$

16.
$$\frac{\text{tp} = \text{tp'} \qquad \Gamma \vdash \text{exp} : \text{tp'}}{\Gamma \vdash \text{exp} : \text{tp} : \text{tp}} \quad \text{(type constraint)}$$

(Note that in the rule above, the first colon is part of program syntax and the second colon is part of our description of typing rules)

17.
$$\frac{\Gamma \vdash \text{exp} : \text{tp'} \qquad \text{tp'} < \text{tp}}{\Gamma \vdash \text{exp} : \text{tp}} \quad \text{(subsumption rule)}$$

Function Declaration Typing

1.
$$\frac{\Gamma\ [\text{id2} : \text{tp1}]\ [\text{id1} : \text{tp1} \rightarrow \text{tp2}] \vdash \text{exp} : \text{tp2}}{\Gamma \vdash (\ \textbf{fun}\ \text{id1}\ (\ \text{id2} : \text{tp1}\ ) : \text{tp2} = \text{exp}\ ) : \text{tp1} \rightarrow \text{tp2}}$$

Program Typing

1.
$$\frac{\Gamma \vdash \text{decl1}\ \textit{valid}\ \ldots\ \Gamma \vdash \text{decln}\ \textit{valid}}{\vdash \text{decl1}\ \ldots\ \text{decln}\ \textit{valid}}$$