Topic 11: Loops

COS 320

Compiling Techniques

Princeton University
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Loop Preheaders

Recall:

- A *loop* is a set of CFG nodes S such that:
  1. there exists a *header* node h in S that dominates all nodes in S.
     - there exists a path of directed edges from h to any node in S.
     - h is the only node in S with predecessors not in S.
  2. from any node in S, there exists a path of directed edges to h.
- A loop is a single entry, multiple exit region.

Loop Preheaders:

- Some loop optimizations (loop invariant code removal) need to insert statements immediately before loop header.
- Create a loop *preheader* - a basic block before the loop header block.

Loop Preheader Example

Loop Invariant Computation

- Given statements in loop s: `t = ` a_1 `op` a_2:
  - s is loop-invariant if a_1, a_2 have same value each loop iteration.
  - may sometimes be possible to hoist s outside loop.
- Cannot always tell whether a will have same value each iteration $\rightarrow$ conservative approximation.
- d: `t = ` a_1 `op` a_2 is loop-invariant within loop L if for each a_i:
  1. a_i is constant, or
  2. all definitions of a_i that reach d are outside L, or
  3. only one definition of a_i reaches d, and is loop-invariant.

Loop Invariant Computation

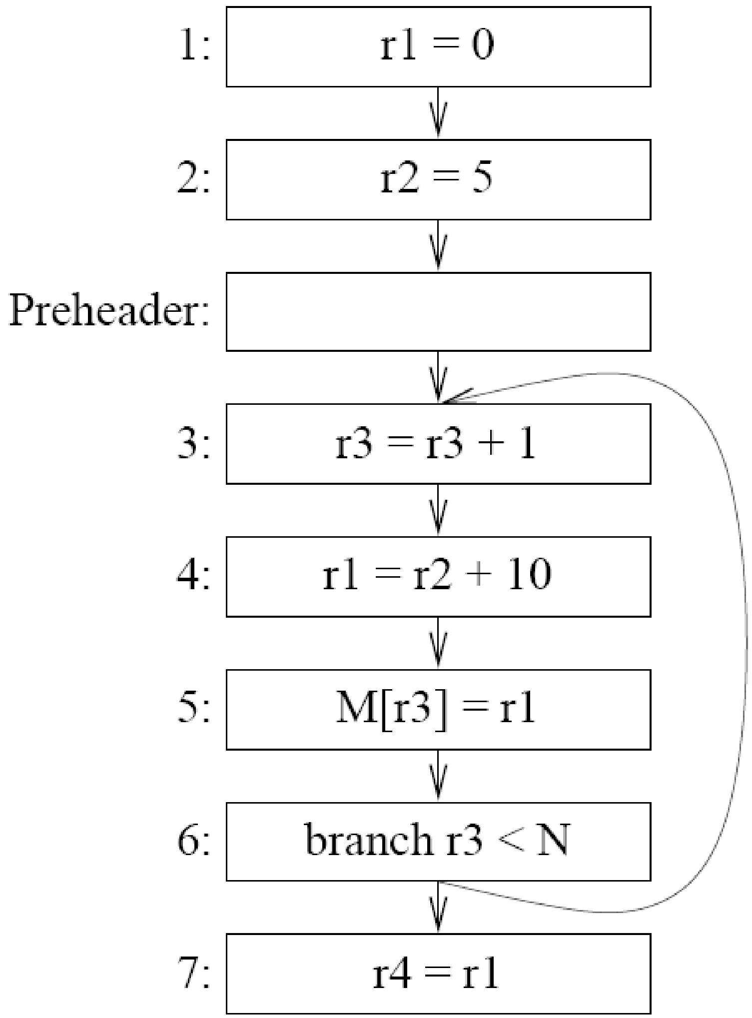
Iterative algorithm for determining loop-invariant computations:

```
mark "invariant" all definitions whose operands
- are constant, or
- whose reaching definitions are outside loop.

WHILE (changes have occurred)
  mark "invariant" all definitions whose operands
  - are constant,
  - whose reaching definitions are outside loop, or
  - which have a single reaching definition in loop
    marked invariant.
```

Loop Invariant Code Motion (LICM)

After detecting loop-invariant computations, perform code motion.

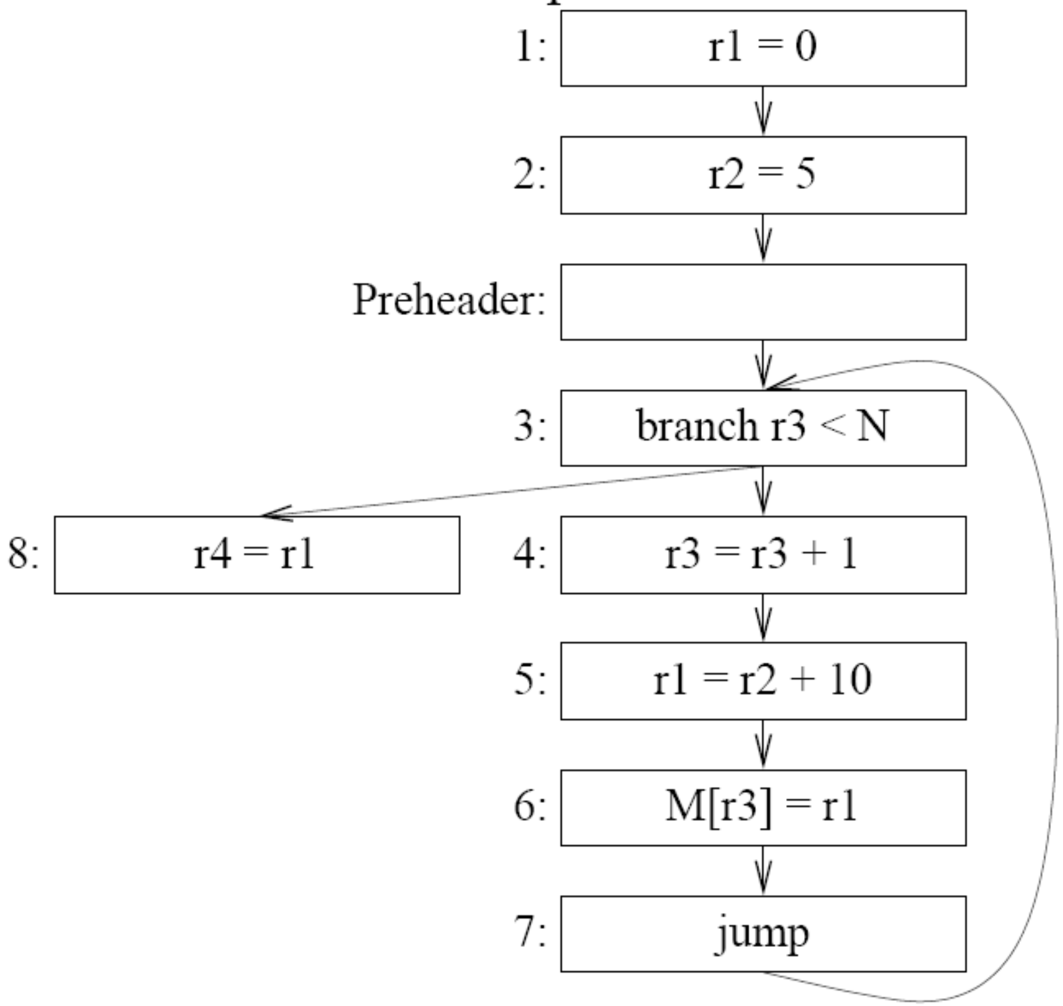


Subject to some constraints.

LICM: Constraint 1

d: `t = a op b`

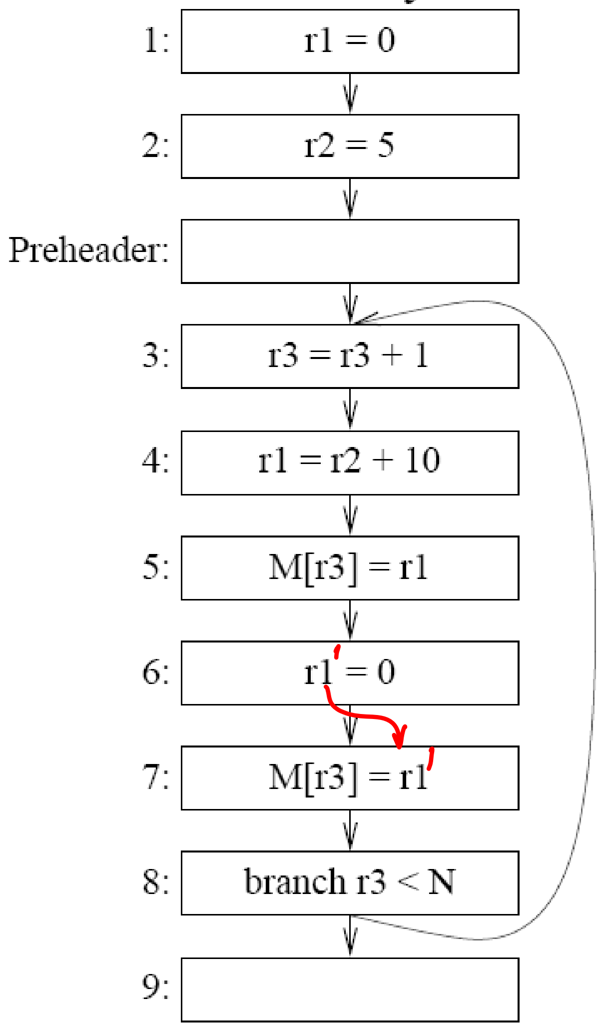
d must dominate all loop exit nodes where `t` is live out.



LICM: Constraint 2

d: `t = a op b`

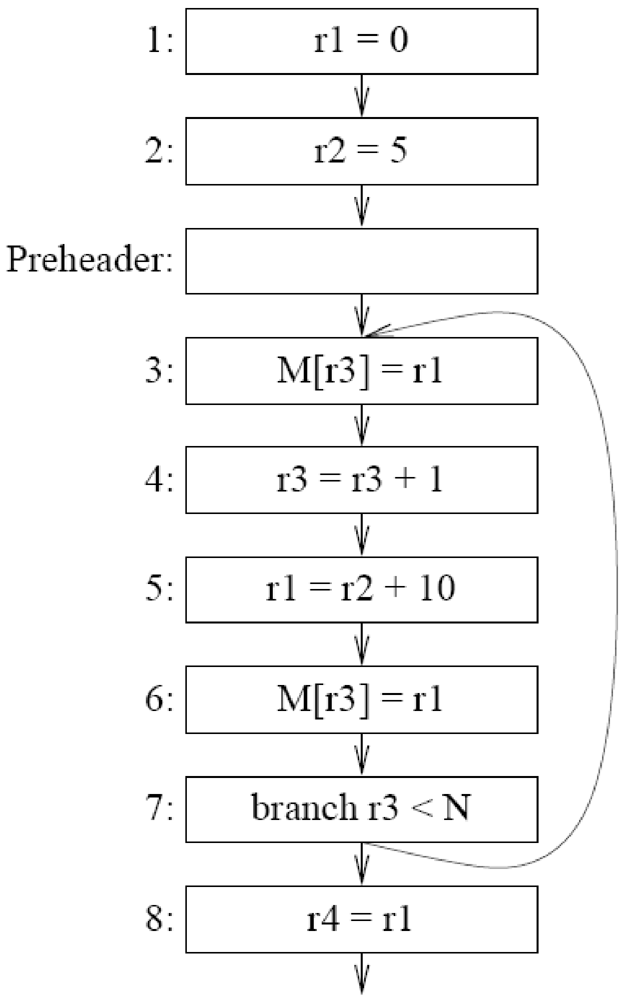
there must be only one definition of `t` inside loop.



LICM: Constraint 3

d: `t = a op b`

t must not be live-out of loop preheader node (live-in to loop)



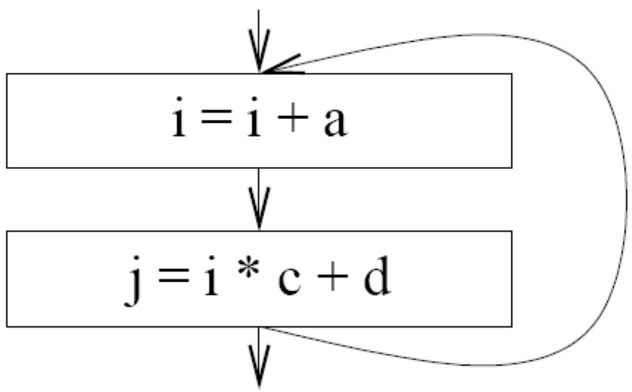
LICM

Algorithm for code motion:

- Examine invariant statements of L in same order in which they were marked.
- If invariant statement s satisfies three criteria for code motion, remove s from L, and insert into preheader node of L.

Induction Variables

Variable `i` in loop L is called induction variable of L if each time `i` changes value in L, it is incremented/decremented by loop-invariant value.



Assume `a, c` loop-invariant.

- `i` is an induction variable
- `j` is an induction variable
  - `j = i * c` is equivalent to `j = j + a * c`
  - compute `e = a * c` outside loop: `j = j + e` $\Rightarrow$ strength reduction
  - may not need to use `i` in loop $\Rightarrow$ induction variable elimination

Induction Variable Detection

Scan loop L for two classes of induction variables:

- *basic* induction variables - variables (`i`) whose only definitions within L are of the form `i = i + c` or `i = i - c`, `c` is loop invariant.
- *derived* induction variables - variables (`j`) defined only once within L, whose value is linear function of some basic induction variable L.

Associate triple `(i, a, b)` with each basic induction variable `j`

- `i` is basic induction variable; `a` and `b` are loop invariant.
- value of `j` at point of definition is `a + b * i`
- `j` belongs to the family of `i`

Induction Variable Detection: Algorithm

Algorithm for induction variable detection:

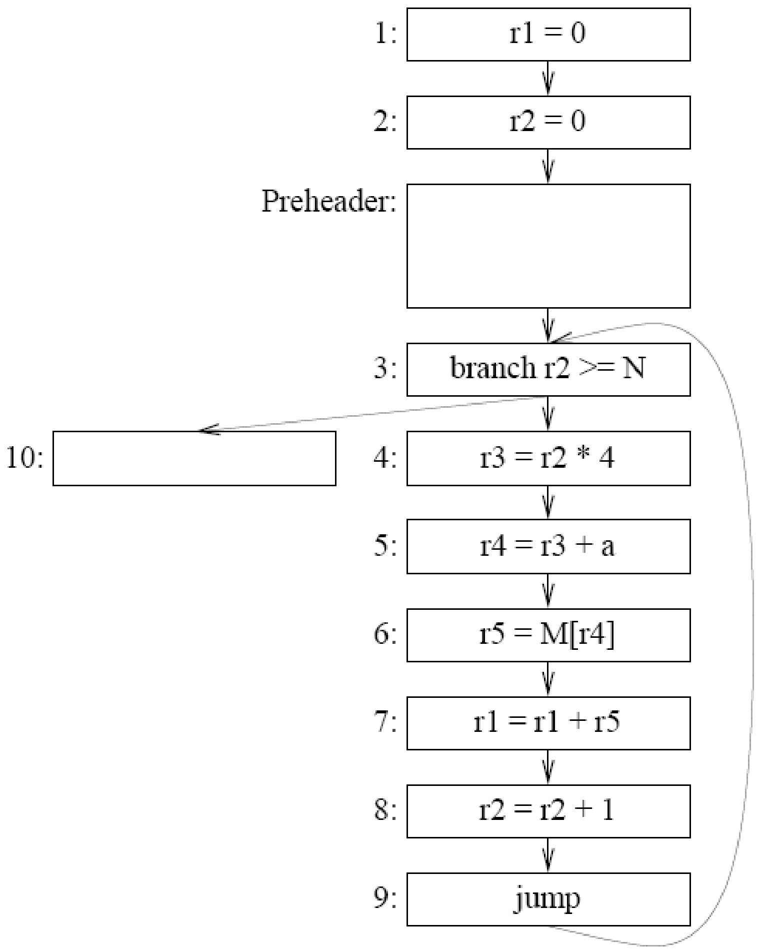
- Scan statements of L for basic induction variables `i`
  - for each `i`, associate triple `(i, 0, 1)`
  - `i` belongs to its own family.

$1 \cdot i + 0 = i$

- Scan statements of L for derived induction variables `k`:
  1. there must be single assignment to `k` within L of the form `k = j * c` or `k = j + d`, `j` is an induction variable; `c`, `d` loop-invariant, and
  2. if `j` is a derived induction variable belonging to the family of i, then:
     - the only definition of `j` that reaches `k` must be one in L, and
     - no definition of `i` must occur on any path between definition of `j` and definition of `k`
- Assume `j` associated with triple `(i, a, b)`: `j = a + b * i` at point of definition.
- Can determine triple for `k` based on triple for `j` and instruction defining `k`:
  - `k = j * c` $\rightarrow$ `(i, a*c, b*c)`
  - `k = j + d` $\rightarrow$ `(i, a + d, b)`

Induction Variable Detection: Example

```
s = 0;
for(i = 0; i < N; i++)
  s += a[i];
```



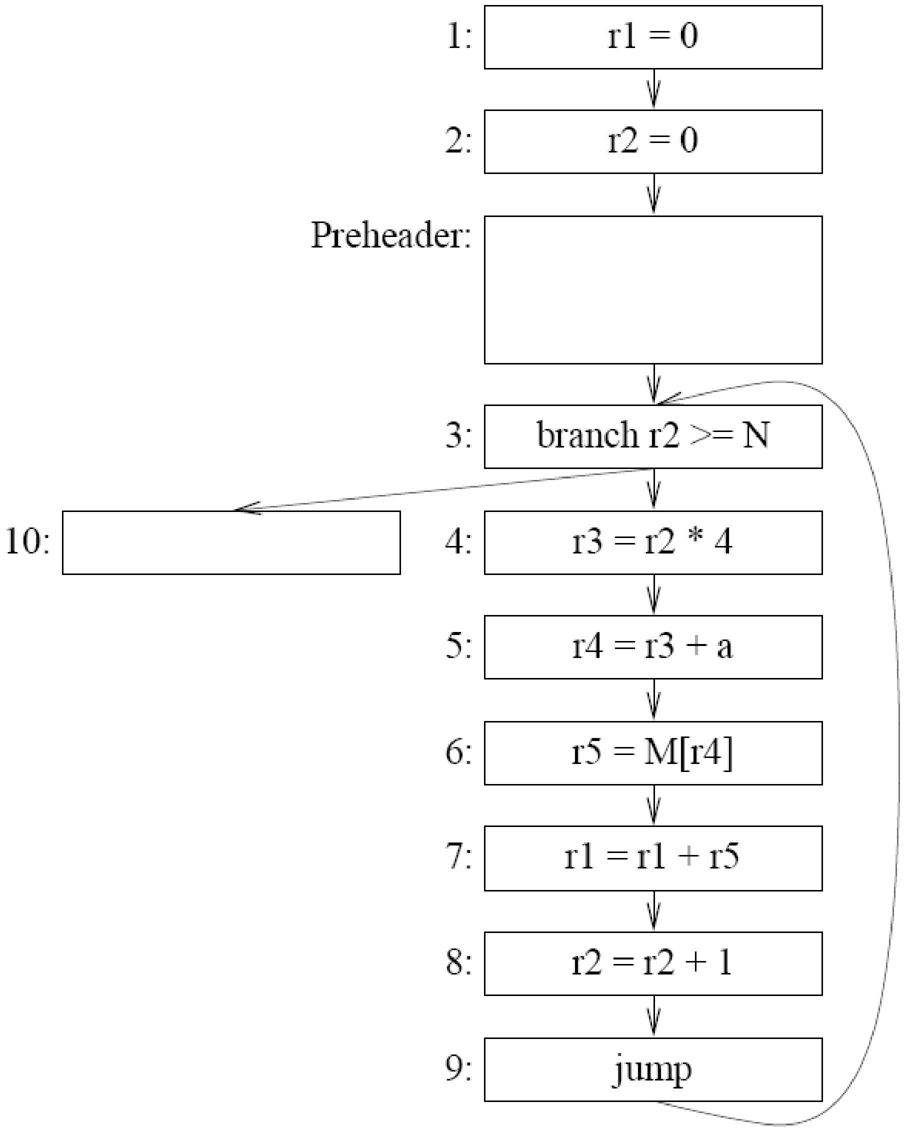
Strength Reduction

1. For each derived induction variable `j` with triple `(i, a, b)`, create new `j'`.
    - all derived induction variables with same triple `(i, a, b)` may share `j'`
2. After each definition of `i` in L, `i = i + c`, insert statement:
   `j' = j' + b * c`
    - `b * c` is loop-invariant and may be computed in preheader or during compile time.
3. Replace unique assignment to `j` with `j = j'`.
4. Initialize `j'` at end of preheader node:

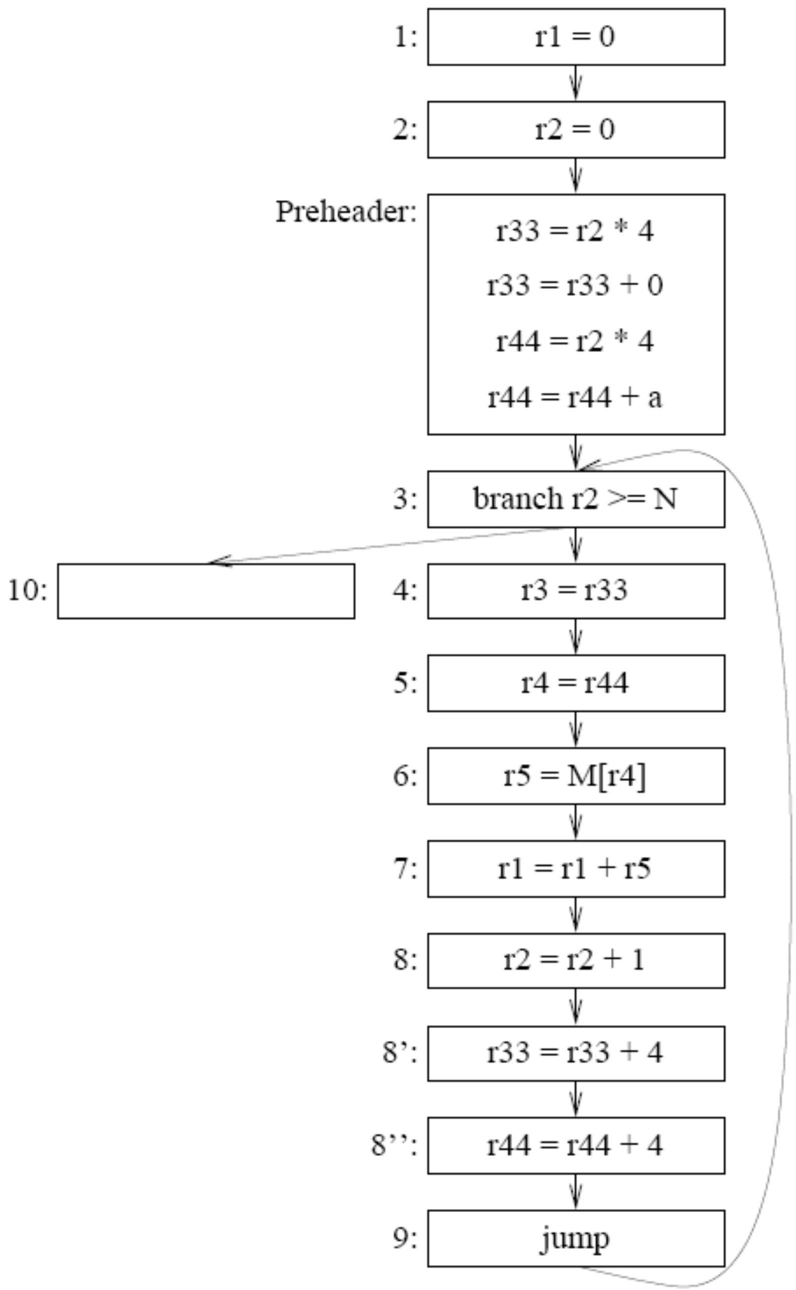
```
j' = b * i
j' = j' + a
```

- Strength reduction still requires multiplication, but multiplication now performed outside loop.
- `j'` also has triple `(i, a, b)`

Strength Reduction Example



Strength Reduction Example

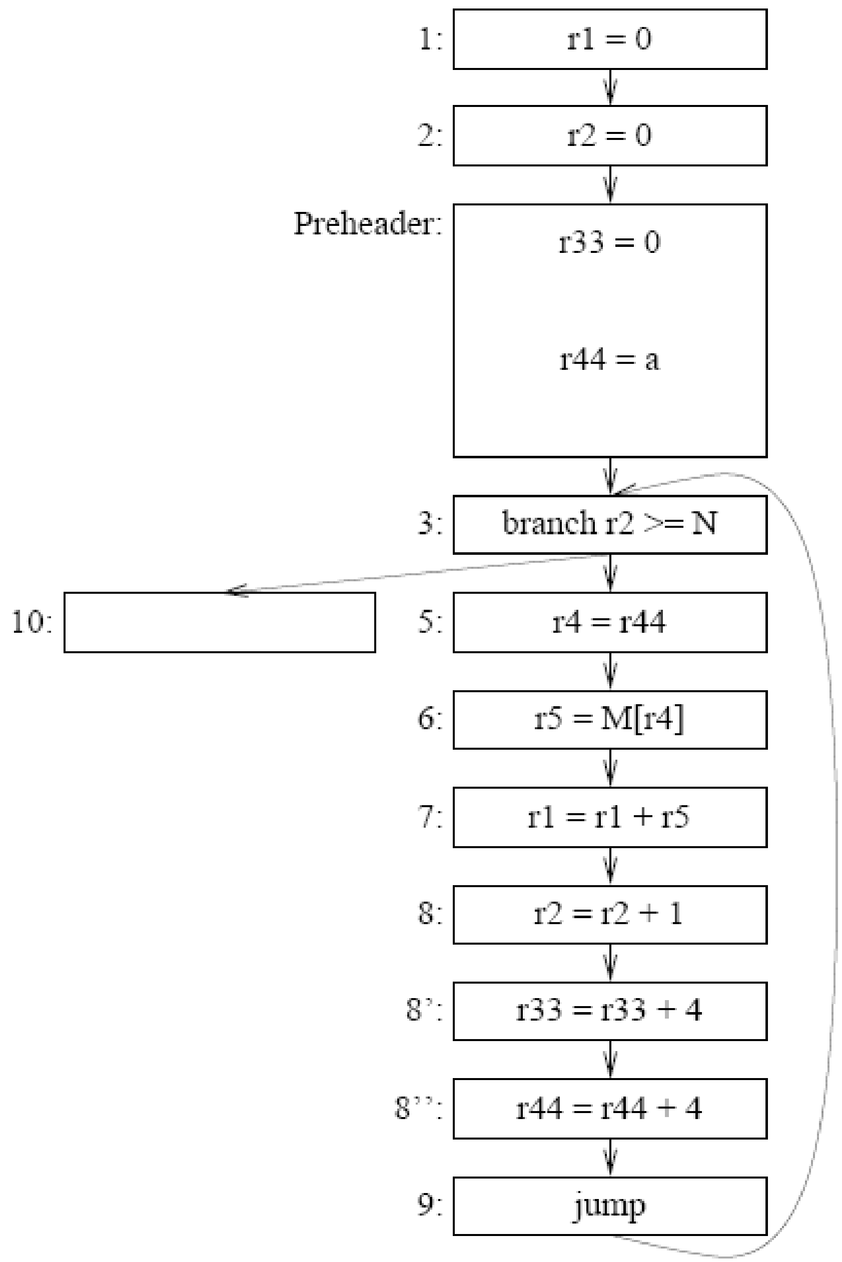


Induction Variable Elimination

After strength reduction has been performed:

- some induction variables are only used in comparisons with loop-invariant values.
- some induction variables are *useless*
  - dead on all loop exits, used only in definition of itself.
  - dead code elimination will not remove useless induction variables.

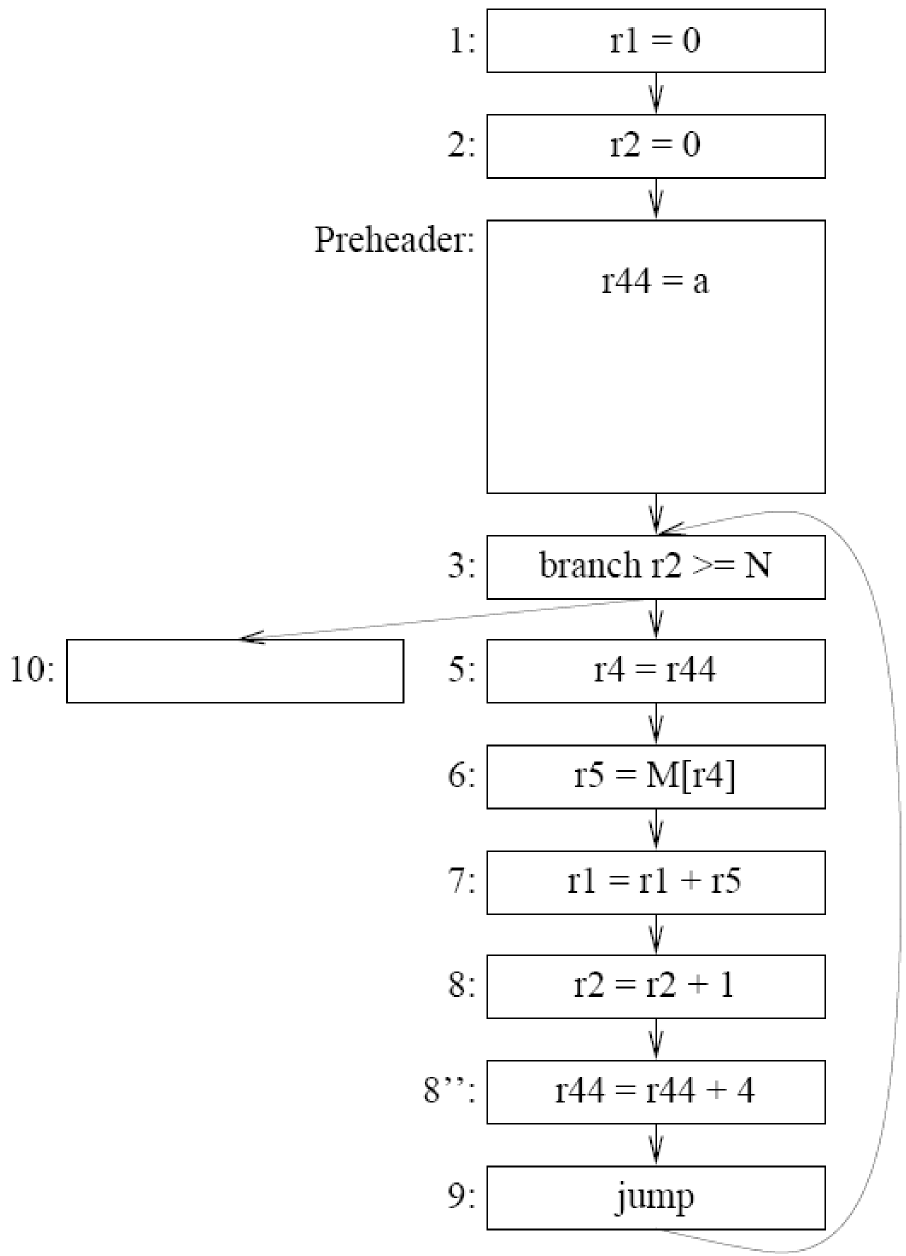
Induction Variable Elimination Example



Induction Variable Elimination

- Variable `k` is *almost useless* if it is only used in comparisons with loop-invariant values, and there exists another induction variable `t` in the same family as `k` that is not useless.
- Replace `k` in comparison with `t`
  $\rightarrow$ `k` is useless

Induction Variable Elimination: Example



Induction Variable Elimination: Example