Topic 10: Dataflow Analysis

COS 320

Compiling Techniques

Princeton University
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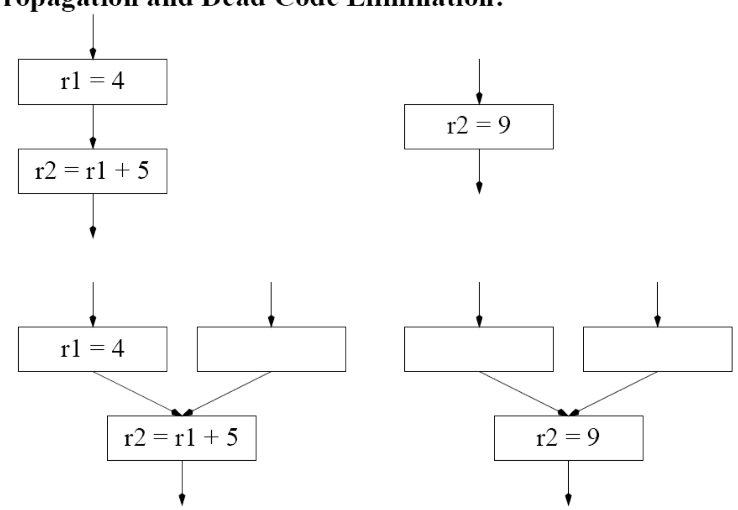
Analysis and Transformation

- Analysis:
  - Control Flow Analysis
  - Dataflow Analysis
- Transformation:
  - Register Allocation
  - Optimization
    - Machine dependent/independent
    - Local/Global/Interprocedural
    - Acyclic/Cyclic
  - Scheduling

Dataflow Analysis Motivation

Constant Propagation and Dead Code Elimination:



Needs dominator, liveness, and reaching definition information.

Dataflow Analysis Motivation

Register Allocation:

- Infinite number of registers (virtual registers) must be mapped to a limited number of real registers.
- Pseudo-assembly must be examined by *live variable analysis* to determine which virtual registers contain values which may be used later.
- Virtual registers which are not simultaneously *live* may be mapped onto the same real register.

```
r2 = r1 + 1

r3 = M[r2]

r4 = r3 + 4

LOAD   r5 = M[r2 + r4]
```

Dataflow Analysis

Three types we will cover:

- Live Variable
  - Live range for register allocation
  - Scheduling
  - Dead code elimination
- Reaching Definitions
  - Constant propagation
  - Constant folding
  - Copy propagation
- Available expressions
  - Common subexpression elimination

Iterative Dataflow Analysis Framework

- These dataflow analyses are all very similar $\rightarrow$ define a framework.
- Specify:
  - Two *set definitions* - $A[n]$ and $B[n]$
  - A *transfer function* - $f\,(A, B, IN/OUT)$
  - A *confluence operator* - $\vee$.
  - A *direction* - FORWARD or REVERSE.
- For forward analyses:

$$IN[n] = \vee_{p \in PRED[n]} OUT[p]$$
$$OUT[n] = f\,(A, B, IN)$$

- For reverse analyses:

$$OUT[n] = \vee_{s \in SUCC[n]} IN[s]$$

$$IN[n] = f\,(A, B, OUT)$$

Definitions

Control Flow Definitions:

- CFG node has *out-edges* leading to *successor nodes*.
- CFG node has *in-edges* coming from *predecessor nodes*.
- For each CFG node n, $PRED[n]$ = set of all predecessors of n.
- For each CFG node n, $SUCC[n]$ = set of all successors of n.

Iterative Dataflow Analysis Framework

- Iterative dataflow analysis equations are applied in an iterative fashion until IN and OUT sets do not change.
- Typically done in (FORWARD or REVERSE) topological sort order of CFG for efficiency.
- IN and OUT sets initialized to $\emptyset$.

```
For each node n {
  IN[n] = OUT[n] = {};
}
Repeat {
  For each node n in forward/reverse topological order {
    IN'[n] = IN[n];
    OUT'[n] = OUT[n];
    IN[n], OUT[n] = (Equations);
  }
} until IN'[n] = IN[n] and OUT'[n] = OUT[n] for all n.
```

Definitions for Liveness Analysis

Liveness Definitions:

- A source (RHS) register t is a *use* of t.
- A destination (LHS) register t is a *definition* of t.
- A register t is *live* on edge e if there exists a path from e to a use of t that does not go through a definition of t.
- Register t is *live-in* at CFG node n if t is live on any in-edge of n.
- Register t is *live-out* at CFG node n if t is live on any out-edge of n.

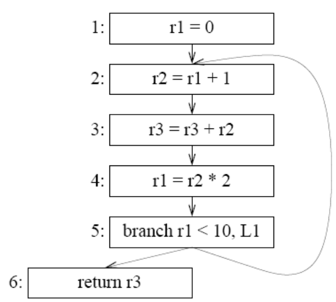
Definitions for Liveness Analysis

Live Variable Analysis Equation:

- Set definition ($A[n]$): $USE[n]$ - the set of registers that n uses.
- Set definition ($B[n]$): $DEF[n]$ - the set of registers that n defines.
- Transfer function ($f(A, B, OUT)$): $USE[n] \cup (OUT[n] - DEF[n])$
- Confluence operator ($\vee$): $\cup$
- Direction: REVERSE

$$OUT[n] = \cup_{s \in SUCC[n]} IN[s]$$
$$IN[n] = USE[n] \cup (OUT[n] - DEF[n])$$

Live Variable Analysis Example



Node	USE	DEF	OUT	IN	OUT	IN	OUT	IN
1								
2								
3								
4								
5								
6								

Live Variable Application 1: Register Allocation

Register Allocation:

1. Perform live variable analysis.
2. Build *interference graph*.
3. Color interference graph with real registers.

Interference Graph

- Node t corresponds to virtual register t.
- Edge $\langle t_i, t_j \rangle$ exists if registers t_i, t_j have overlapping live ranges.
- For some node n, if $DEF[n] = \{a\}$ and $OUT[n] = \{b_1, b_2, ...b_k\}$, then add interference edges: $\langle a, b_1 \rangle$, $\langle a, b_2 \rangle$, $\langle a, b_k \rangle$

Interference Graph For Example:

Node	DEF	OUT	IN
1	r1	r1,r3	r3
2	r2	r2,r3	r1,r3
3	r3	r2,r3	r2,r3
4	r1	r1,r3	r2,r3
5	-	r1, r3	r1,r3
6	-		r3

Virtual registers r1 and r2 may be mapped to same real registers.

Live Variable Application 2: Dead Code Elimination

- Given statement s with a definition and no side-effects:

  `r1 = r2 + r3, r1 = M[r2], or r1 = r2`

  If r1 is *not* live at the end of s, then the s is *dead*
- Dead statements can be deleted.
- Given statement s without a definition or side-effects:

  `r1 = call FUN_NAME, M[r1] = r2`

  Even if r1 is not live at the end of s, it is not dead.

Example:

```
r1 = r2 + 1
r2 = r2 + 2
r1 = r2 + 3
M[r1] = r2
```

Reaching Definition Analysis

Determines whether definition of register t directly affects use of t at some point in program.

Reaching Definition Definitions:

- *unambiguous* - instruction explicitly defines register t.
- *ambiguous* - instruction may or may not define register t.
  - Global variables in a function call.
  - No ambiguous definitions in tiger since all globals are stored in memory.
- Definition of d (of t) *reaches* statement u if a path of CFG edges exists from d to u that does not pass through an unambiguous definition of t.
- One unambiguous and many ambiguous definitions of t may reach u on a single path.

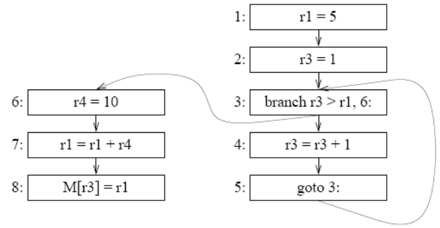
Reaching Definition Analysis

Reaching Definition Analysis Equation:

- Set definition ($A[n]$): $GEN[n]$ - the set of *definition id's* that n creates.
- Set definition ($B[n]$): $KILL[n]$ - the set of *definition id's* that n kills.
  - $defs(t)$ - set of all *definition id's* of register t.
- Transfer function ($f(A, B, IN)$): $GEN[n] \cup (IN[n] - KILL[n])$
- Confluence operator ($\vee$): $\cup$
- Direction: FORWARD

$$IN[n] = \cup_{p \in PRED[n]} OUT[p]$$
$$OUT[n] = GEN[n] \cup (IN[n] - KILL[n])$$

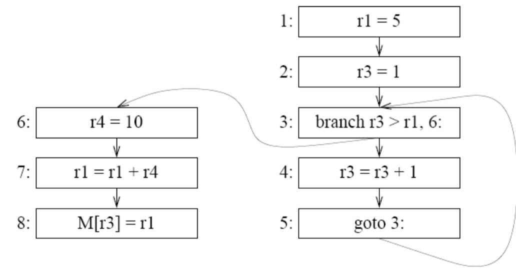
Reaching Definition Analysis Example



Node	GEN	$KILL$	IN	OUT	IN	OUT	IN	OUT
1								
2								
3								
4								
5								
6								
7								
8								

Reaching Definition Application 1: Constant Propagation

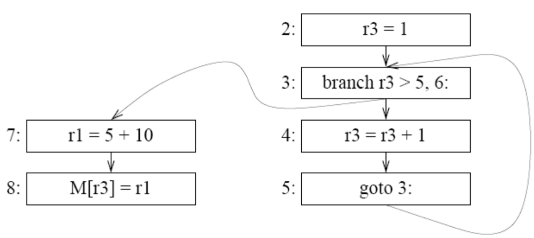
- Given Statement d: `a = c` where `a` is constant
- Given Statement u: `t = a op b`
- If statement d reach u and no other definition of `a` reaches u, then replace u b `c op b`.



Statements 1 and 6 are dead.

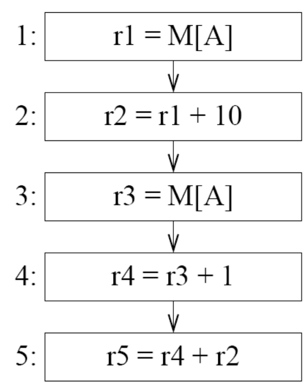
Constant Folding

- Given Statement d: `t = a op b`
- If `a` and `b` are constant, compute `c` as `a op b`, replace d by `t = c`



Common Subexpression Elimination

If `x op y` is computed multiple times, *common subexpression elimination* (CSE) attempts to eliminate some of the duplicate computations.



Need to track expression propagation → available expression analysis

Definitions

- Expression `x op y` is *available* at CFG node n if, on every path from CFG entry node to n, `x op y` is computed at least once, and neither `x` nor `y` are defined since last occurrence of `x op y` on path.
- Can compute set of expressions available at each statement using system of dataflow equations.
- Statement `r1 = M[r2]`:
  - *generates* expression `M[r2]`.
  - *kills* all expressions containing `r1`.
- Statement `r1 = r2 + r3`:
  - *generates* expression `r2 + r3`.
  - *kills* all expressions containing `r1`.

Iterative Dataflow Analysis Framework

- Specify:
  - Two *set definitions* - $A[n]$ and $B[n]$
  - A *transfer function* - $f\,(A, B, IN/OUT)$
  - A *confluence operator* - $\vee$.
  - A *direction* - FORWARD or REVERSE.
- For forward analyses:

$$IN[n] = \vee_{p \in PRED[n]} OUT[p]$$
$$OUT[n] = f\,(A, B)$$

- For reverse analyses:

$$OUT[n] = \vee_{s \in SUCC[n]} IN[s]$$
$$IN[n] = f\,(A, B)$$

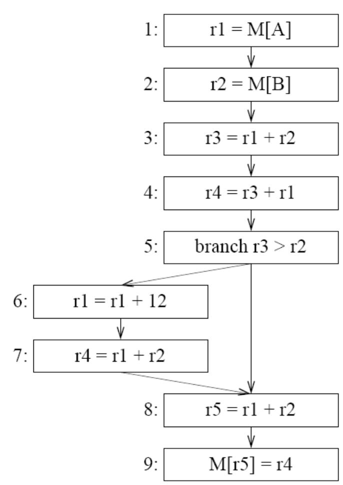
Available Expression Analysis

Available Expression Analysis:

- $exp(t)$ - set of all expressions containing t.
- Set definition ($A[n]$): $GEN[n]$ - the set of all expressions generated by n.
- Set definition ($B[n]$): $KILL[n]$ - the set of all expressions that n kills - $exp(n)$.
- Transfer function ($f(A, B, IN/OUT)$): $GEN[n] \cup (IN[n] - KILL[n])$
- Confluence operator ($\vee$): $\cap$
  - Use of $\cup$, required initialization of IN and OUT sets to $\emptyset$.
  - Use of $\cap$, requires initialization of IN and OUT sets to U (except for IN of entry node).
- Direction: FORWARD

$$IN[n] = \cap_{p \in PRED[n]} OUT[p]$$
$$OUT[n] = GEN[n] \cup (IN[n] - KILL[n])$$

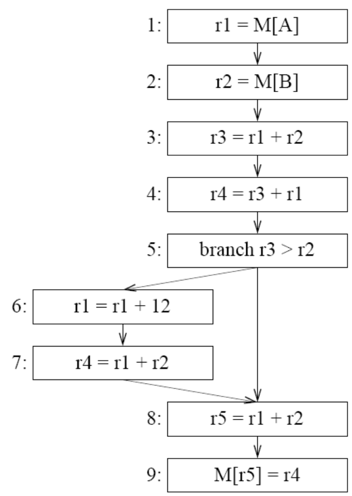
Example



Node	GEN	$KILL$	IN	OUT
1	M[A]	r1+r2, r1+12, r3+r1	-	U
2	M[B]	r1+r2	U	U
3	r1+r2	r3+r1	U	U
4	r3+r1		U	U
5			U	U
6		r1+r2, r3+r1, r1+12	U	U
7	r1+r2		U	U
8	r1+r2	M[r5]	U	U
9		M[A], M[B], M[r5]	U	U

Node	GEN	$KILL$	IN	OUT
1	1	378, 6, 4	-	U
2	2	378	U	U
3	378	4	U	U
4	4		U	U
5			U	U
6		378, 4, 6	U	U
7	378		U	U
8	378	9	U	U
9		1, 2, 9	U	U

Example



Node	GEN	$KILL$	IN	OUT
1	1	378, 4, 6	-	U
2	2	378	U	U
3	378	4	U	U
4	4		U	U
5			U	U
6		378, 4, 6	U	U
7	378		U	U
8	378	9	U	U
9		1, 2, 9	U	U

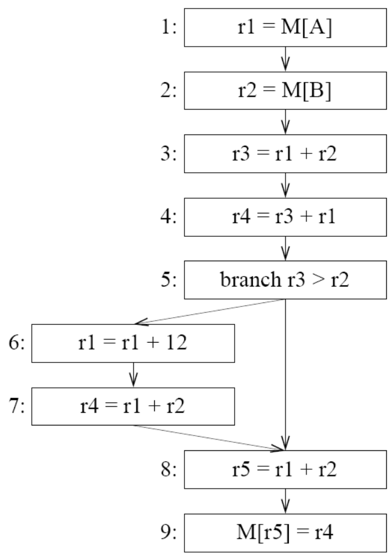
Node	IN	OUT	IN	OUT
1				
2				
3				
4				
5				
6				
7				
8				
9				

Common Subexpression Elimination (CSE)

Given statement s: `t = x op y`:
If expression `x op y` is available at beginning of node s then:

1. starting from node s, traverse CFG edges backwards to find last occurrence of `x op y` on each path from entry node to s.
2. create new temporary `w`.
3. for each statement s': `v = x op y` found in (1), replace s' by:
   `w = x op y`
   `v = w`
4. replace statement s by: `t = w`

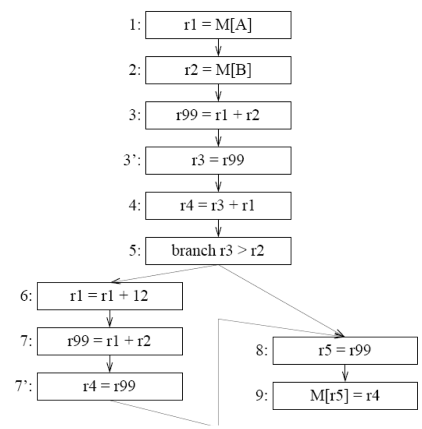
CSE Example



`r1 + r2` in node 8 is a common subexpression.

Copy Propagation

- Given statement d: `a = z` (`a` and `z` are both register temps) $\rightarrow$ d is a copy statement.
- Given statement u: `t = a op b`.
- If d reaches u, no other definition of `a` reaches u, and no definition of `z` exists on any path from d to u, then replace u by: `t = z op b`.



Sets

- Sets have been used in all the dataflow and control flow analyses presented.
- There are at least 3 representations which can be used:
  - Bit-Arrays:
    - Each *potential* member is stored in a bit of some array.
    - Insertion, Member is $O(1)$.
    - Assuming set size of N and word size of W - Union (OR) and Intersection (AND) is $O(N/W)$.
  - Sorted Lists/Trees:
    - Each member is stored in a list element.
    - Insertion, Member, Union, Intersection is $O(size)$. (Insertion, Member is $O(\log_2 size)$ in trees.)
    - Better for sparse sets than bit-arrays.
  - Hybrids: - Trees with bit-arrays
    - Use Tree to hold elements containing bit-arrays.
    - Union, Intersection is $O(size/W)$. Insertion, Member is $O(\log_2 size/W)$.

Basic Block Level Analysis

- To improve performance of dataflow, process at basic block level.
  - Represent the entire basic block by a single *super-instruction* which has any number of destinations and sources.
  - Run dataflow at basic block level.
  - Expand result to the instruction level.
- Example:

```
p:  r1 = r2 + r3     ->    r1, r2 = r2, r3
n:  r2 = r1
```

Basic Block Level Analysis

- Example:

```
p:  r1 = r2 + r3     ->    r1, r2 = r2, r3
n:  r2 = r1
```

- For reaching definitions:

$$OUT[n] = GEN[n] \cup (IN[n] - KILL[n])$$

But $IN[n] = OUT[p]$:

$$OUT[n] = GEN[n] \cup ((GEN[p] \cup (IN[p] - KILL[p])) - KILL[n])$$

Which (clearly) yields:

$$OUT[n] = GEN[n] \cup (GEN[p] - KILL[n]) \cup (IN[p] - (KILL[p] \cup KILL[n]))$$

So:

$$GEN[pn] = GEN[n] \cup (GEN[p] - KILL[n])$$
$$KILL[pn] = KILL[p] \cup KILL[n]$$

- Can we do this at the loop or general region level?

Reducible Flow Graphs Revisited

Definition

- A flow graph is reducible iff each edge exists in exactly one class:
  1. Forward edges (forms an acyclic graph where every node is reachable from start node)
  2. Back edges (head dominates tail)

Algorithm:

1. Remove all backedges
2. Check for cycles:
   - Cycles: Irreducible.
   - No Cycles: Reducible.

Think:

- All loop entry arcs point to header.

Reducible Flow Graphs – Structured Programs

Motivation:

- Structured programs are always reducible programs.
- Reducible programs are not always structured programs.
- Exploit the structured or reducible property in dataflow analysis.

Structures:

- Lists of instructions
- Conditionals/Hammocks
- While Loops (no breaks)

Method:

- Represent structures by a single *super-instruction* which has any number of destinations and sources.
- Run dataflow at structure level.
- Expand result to the instruction level.

Structured Program Analysis

- Lists of instructions - Basic Blocks!

$$GEN[pn] = GEN[n] \cup (GEN[p] - KILL[n])$$
$$KILL[pn] = KILL[p] \cup KILL[n]$$

- Conditionals/Hammocks

$$GEN[lr] = GEN[l] \cup GEN[r]$$
$$KILL[lr] = KILL[l] \cap KILL[r]$$

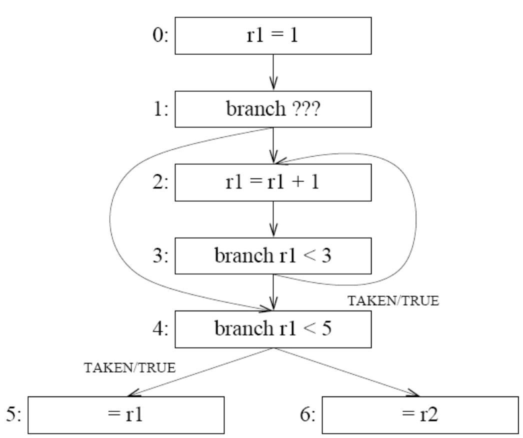
- While Loops

$$GEN[loop] = GEN[l]$$
$$KILL[loop] = KILL[l]$$

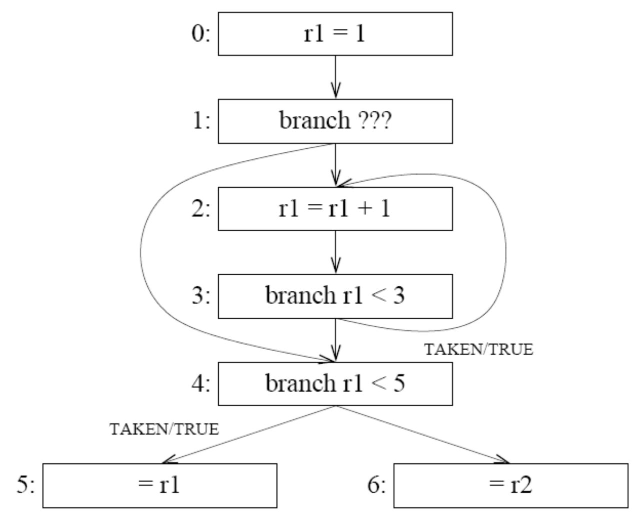
Try this on an irreducible flow graph...

Conservative Approximations Example

Register Allocation:



New Dataflow Analysis



Limitation of Dataflow Analysis