Topic 9: Static Single Assignment

COS 320

Compiling Techniques

Princeton University
Spring 2018

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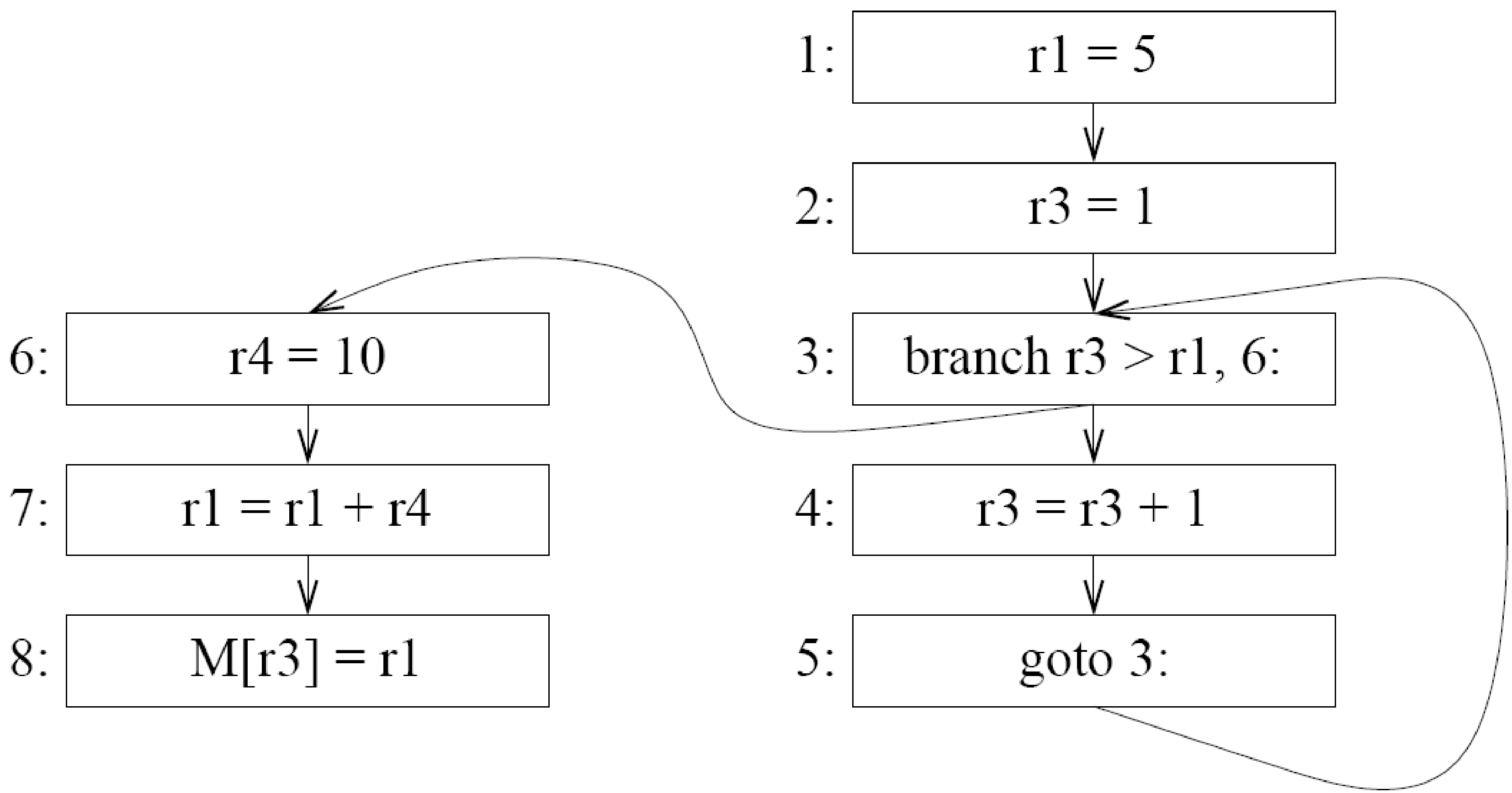
Def-Use Chains, Use-Def Chains

- Many optimizations need to find all use-sites for each definition, and all definition-sites for each use.
  - Constant propagation must refer to the definition-site of the unique reaching definition.
  - Copy propagation, reverse copy propagation, common sub-expression elimination...
- Information connecting all use-sites to corresponding definition-sites can be stored as *def-use chains* and/or *use-def chains*.
- *def-use chains*: for each definition d of r, list of pointers to all uses of r that d reaches.
- *use-def chains*: for each use u of r, list of pointers to all definitions of r that reach u.

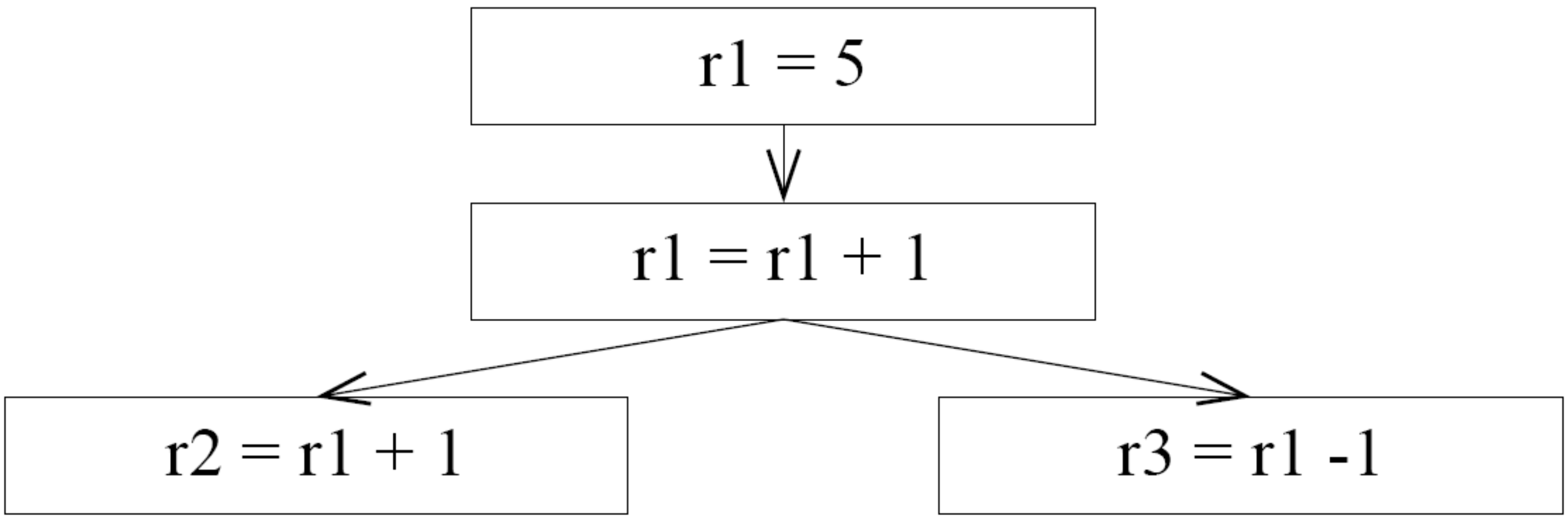
Use-Def Chains, Def-Use Chains



Static Single Assignment

Static Single Assignment (SSA):

- improvement on def-use chains
- each register has only one definition in program
- for each use u of r, only one definition of r reaches u



Why SSA?

Static Single Assignment Advantages:

- Dataflow analysis and code optimization made simpler.
  - Variables have only one definition - no ambiguity.
  - Dominator information is encoded in the assignments.
- Less space required to represent def-use chains. For each variable, space is proportional to uses * defs.
- Eliminates unnecessary relationships:

```
for i = 1 to N do A[i] = 0
for i = 1 to M do B[i] = 1
```

  - No reason why both loops should be forced to use same register to hold index register.
  - SSA renames second `i` to new register which may lead to better register allocation/optimization.

(Dynamic Single Assignment is also proposed in the literature.)

Conversion to SSA Code

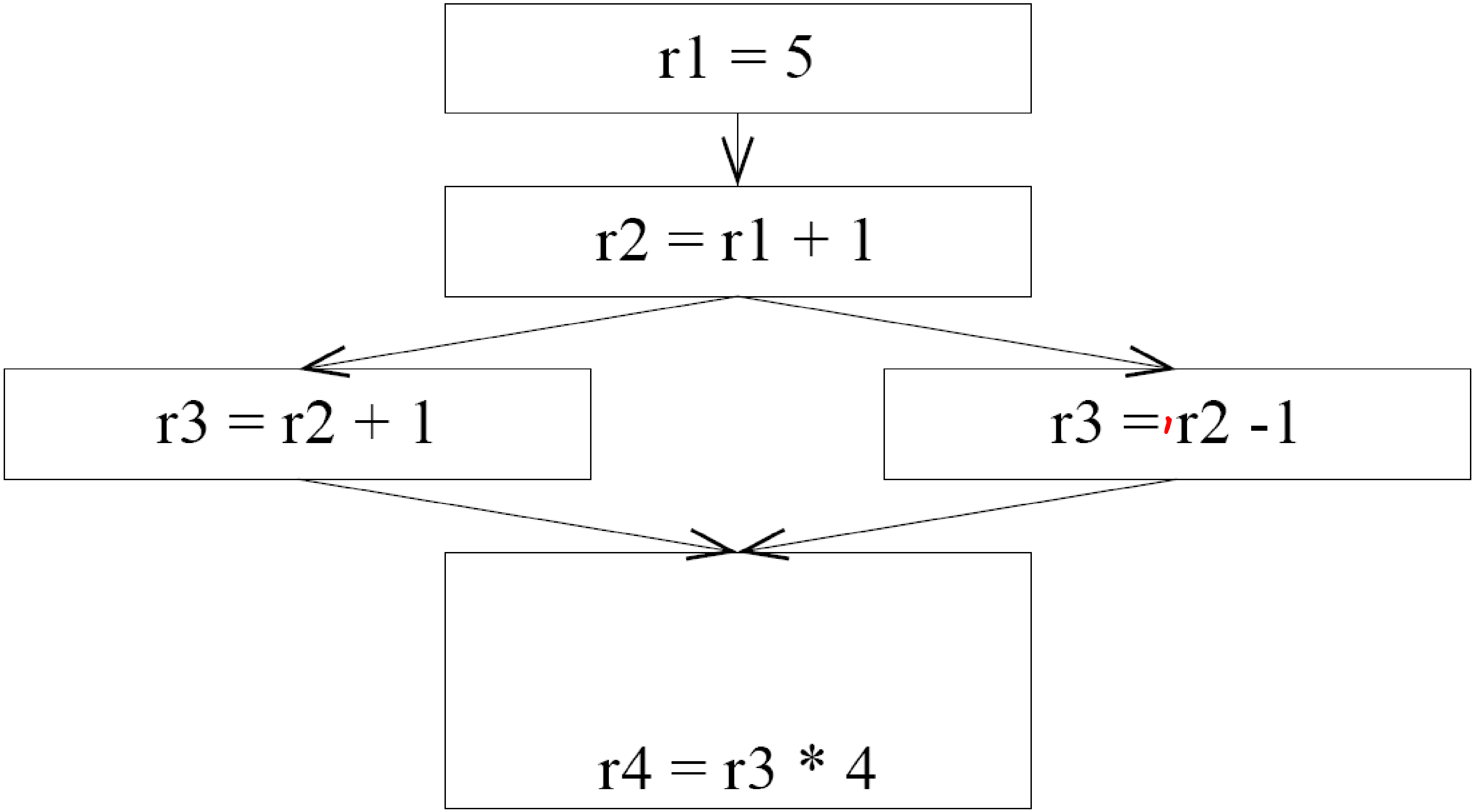
Easy to convert basic blocks into SSA form:

- Each definition modified to define brand-new register, instead of redefining old one.
- Each use of register modified to use most recently defined version.

```
r1 = r3 + r4

r2 = r1 - 1

r1 = r4 + r2

r2 = r5 * 4

r1 = r1 + r2
```

Control flow introduces problems.

Conversion to SSA Form



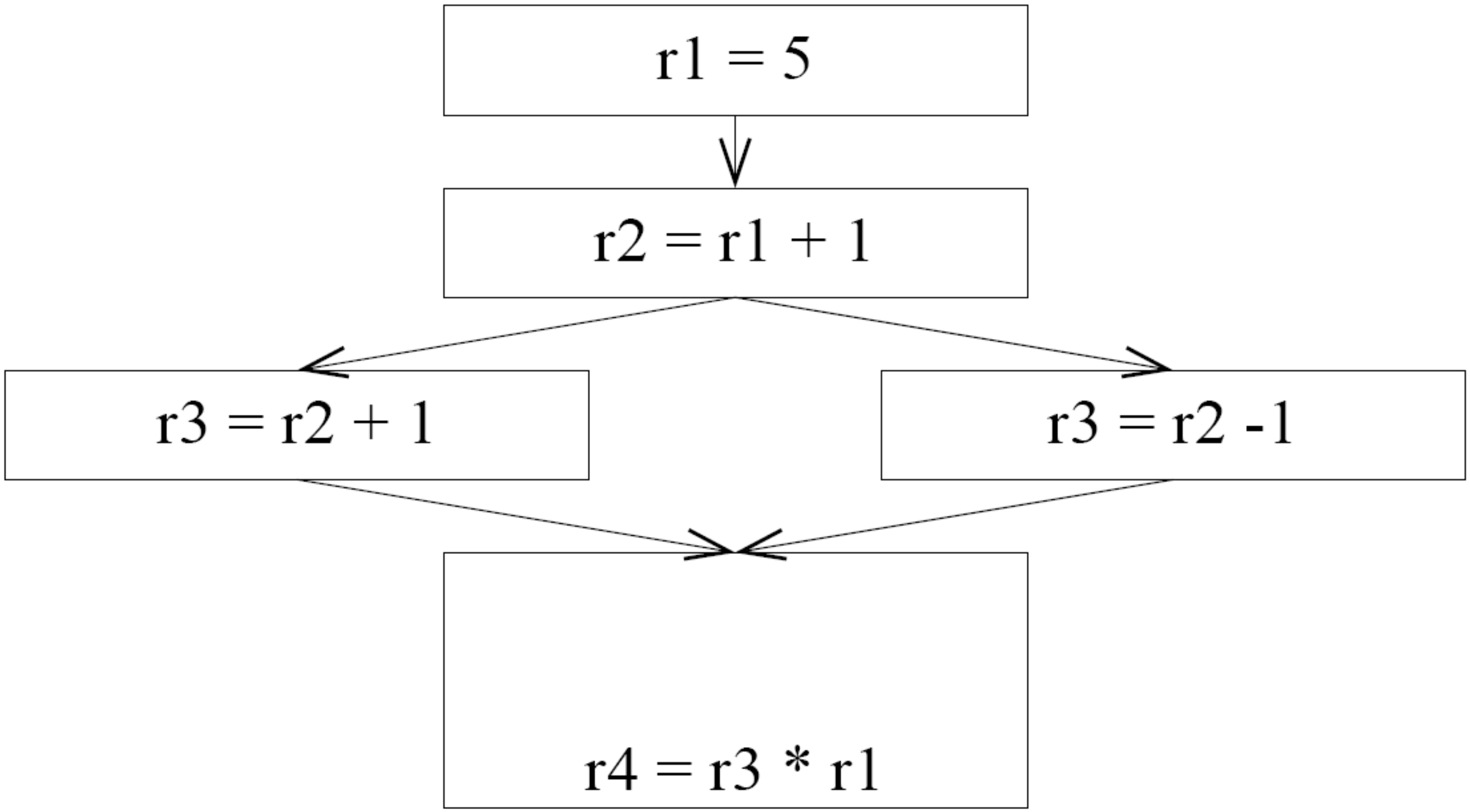
Use ϕ functions.

Conversion to SSA Form

- ϕ-functions enable the use of r3 to be reached by exactly one definition of r3.
- $r3'' = \phi(r3, r3')$:
  - $r3'' = r3$ if control enters from left
  - $r3'' = r3'$ if control enters from right
- Can implement ϕ-functions as set of move operations on each incoming edge.
- In practice, ϕ-functions are just used as notation.

Conversion to SSA Form

Can insert ϕ-functions for each register at each node with more than two predecessors.



We can do better...

Conversion to SSA Form

Path-Convergence Criterion: Insert a ϕ-function for a register r at node z of the flow graph if ALL of the following are true:

1. There is a block x containing a definition of r.
2. There is a block $y \neq x$ containing a definition of r.
3. There is a non-empty path P_{xz} of edges from x to z.
4. There is a non-empty path P_{yz} of edges from y to z.
5. Paths P_{xz} and P_{yz} do not have any node in common other than z.
6. The node z does not appear within both P_{xz} and P_{yz} prior to the end, though it may appear in one or the other.

Assume CFG entry node contains implicit definition of each register:

- r = actual parameter value
- r = undefined

ϕ-functions are counted as definitions.

Conversion to SSA Form

Solve path-convergence iteratively:

WHILE (there are nodes x, y, z satisfying conditions 1-6) &&
(z does not contain a phi-function for r) DO:
insert $r = \phi(r, r, ..., r)$ (one per predecessor) at node z.

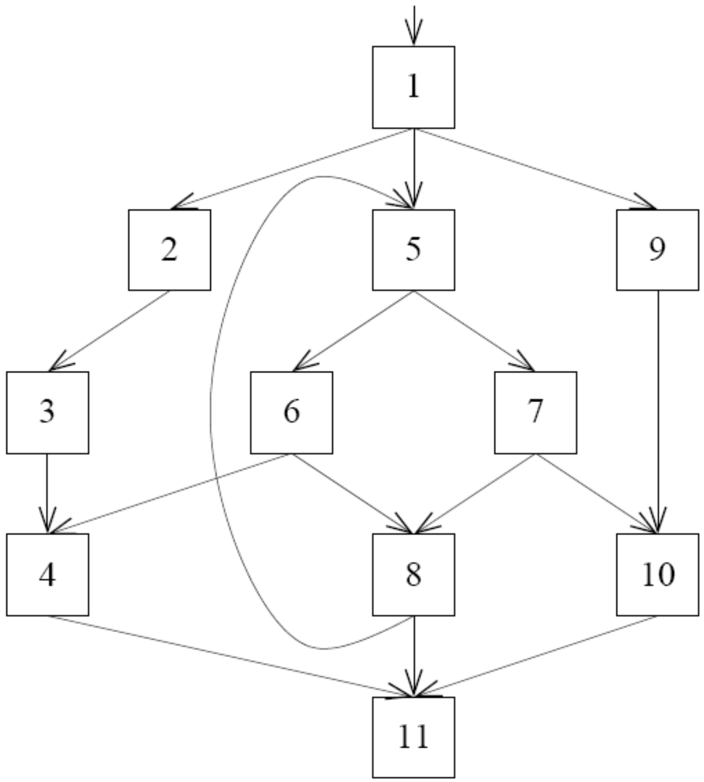
- Costly to compute.
- Since definitions dominate uses, use domination to simplify computation.

Use *Dominance Frontier...*

Dominance Frontier

Definitions:

- x *strictly dominates* w if x dominates w and $x \neq w$.
- *dominance frontier* of node x is set of all nodes w such that x dominates a predecessor of w, but does not strictly dominate w.



Dominance Frontier

- *Dominance Frontier Criterion:* Whenever node x contains definition of some register r, then need to insert ϕ-function for r in all nodes z in dominance frontier of x.
- *Iterated Dominance Frontier:* Need to repeatedly apply since ϕ-function counts as a definition.

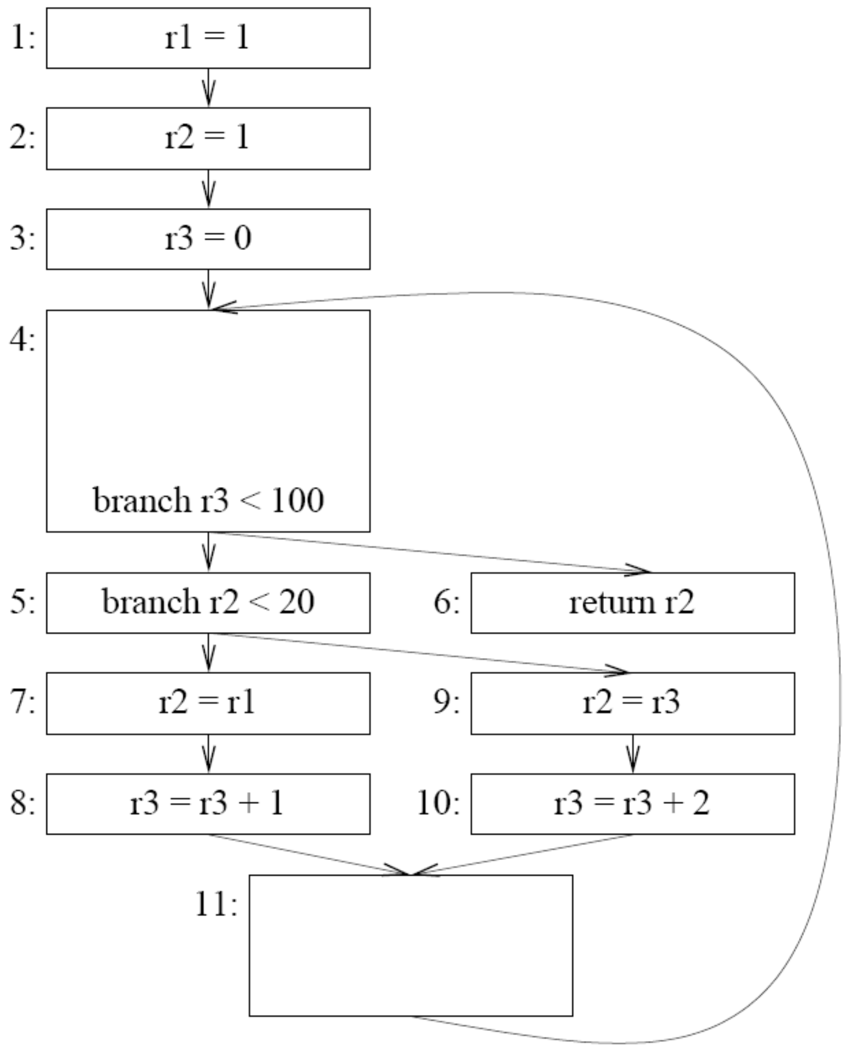
Dominance Frontier Computation

- Use dominator tree
- $DF[n]$: dominance frontier of n
- $DF_{local}[n]$: successors of n in CFG that are not strictly dominated by n
- $DF_{up}[c]$: nodes in dominance frontier of c that are not strictly dominated by c's immediate dominator

$$DF[n] = DF_{local}[n] \cup \left(\cup_{c \in children[n]} DF_{up}[c]\right)$$

- where $children[n]$ are the nodes whose idom is n.
- Work bottom up in dominator tree.

SSA Example



Node	$DOM[n]$	$IDOM[n]$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		

Dominator Analysis

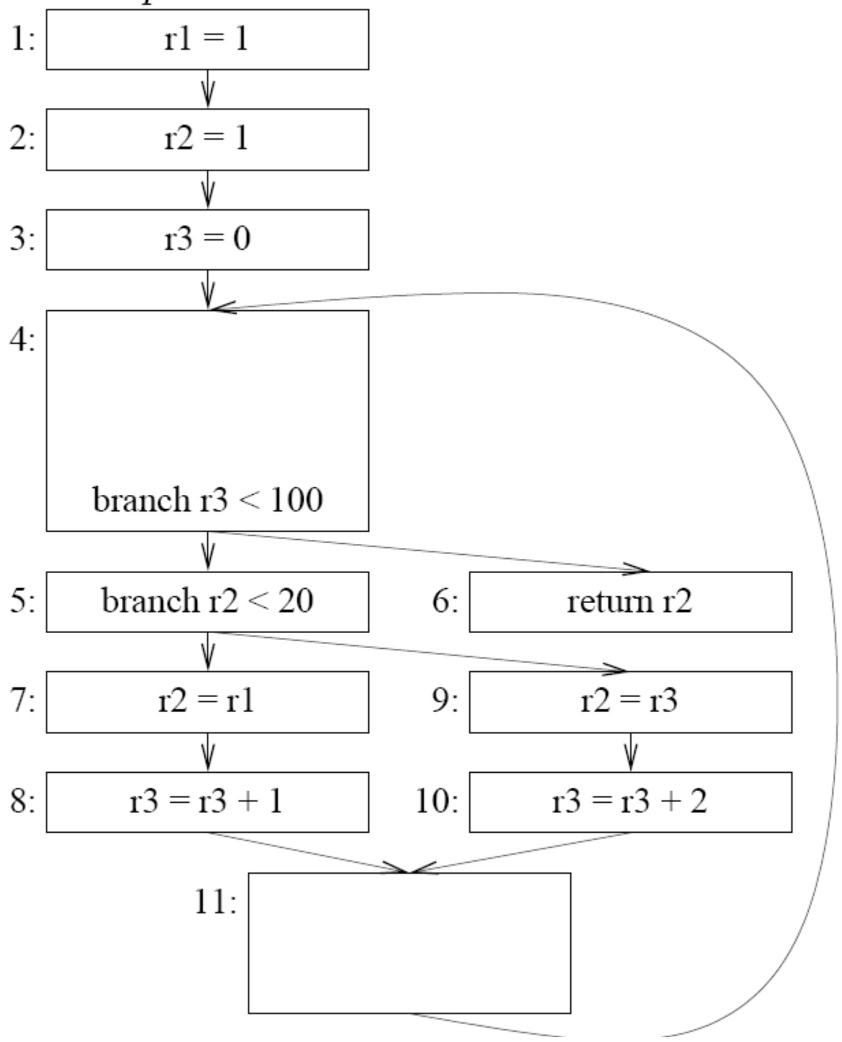
- If d dominates each of the p_i, then d dominates n.
- If d dominates n, then d dominates each of the p_i.
- $Dom[n]$ = set of nodes that dominate node n.
- N = set of all nodes.
- Computation:

  1. $Dom[s_0] = \{s_0\}$.
  2. **for** $n \in N - \{s_0\}$ **do** $Dom[n] = N$
  3. **while** (changes to any $Dom[n]$ occur) **do**
  4.   **for** $n \in N - \{s_0\}$ **do**
  5. $Dom[n] = \{n\} \cup \left(\bigcap_{p \in pred[n]} Dom[p]\right)$.

SSA Example

SSA Example

Insert *phi*-functions:



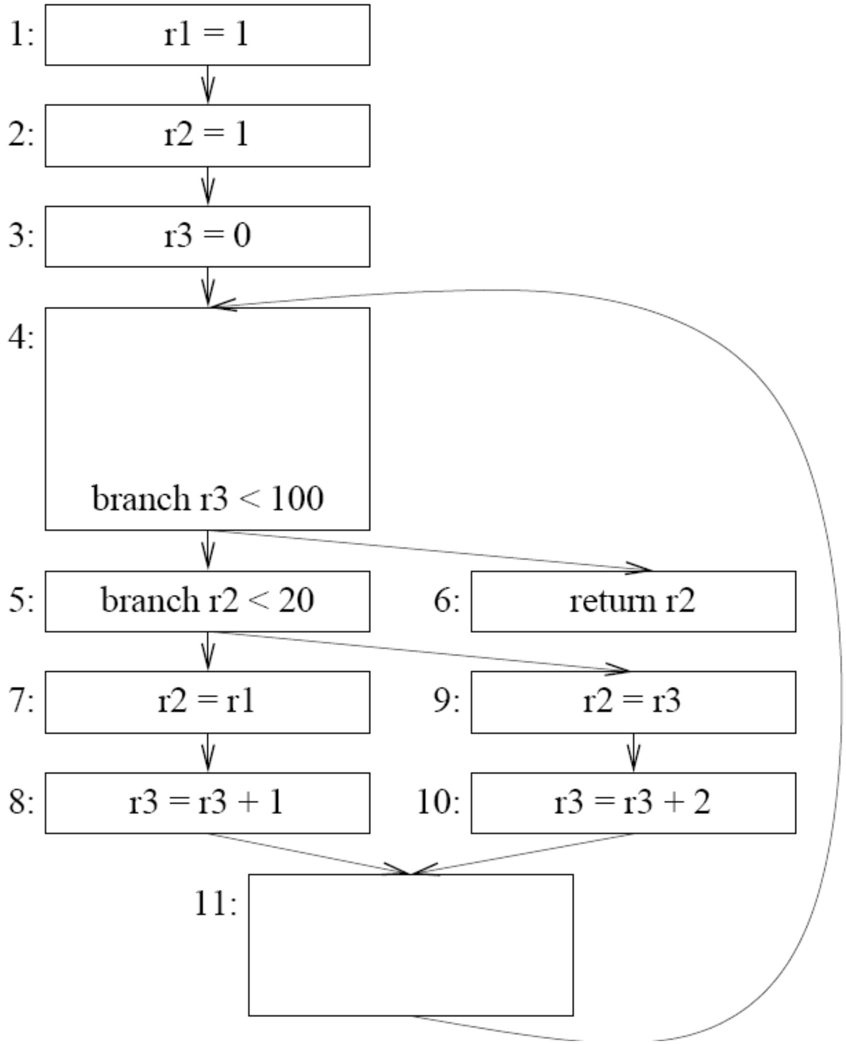
SSA Example

Rename Variables:

1. traverse dominator tree, renaming different definitions of r to r_1, r_2, r_3...
2. rename each regular use of r to most recent definition of r
3. rename ϕ-function arguments with each incoming edge's unique definition

SSA Example

Rename Variables:



Static Single Assignment

Static Single Assignment Advantages:

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- Eliminates unnecessary relationships:

```
for i = 1 to N do A[i] = 0
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```

  - No reason why both loops should be forced to use same register to hold index register.
  - SSA renames second `i` to new register which may lead to better register allocation.
- SSA form make certain optimizations quick and easy $\rightarrow$ dominance property.
  - Variables have only one definition - no ambiguity.
  - Dominator information is encoded in the assignments.

SSA Dominance Property

Dominance property of SSA form: definitions dominate uses

- If x is i^{th} argument of ϕ-function in node n, then definition of x dominates i^{th} predecessor of n.
- If x is used in non-ϕ statement in node n, then definition of x dominates n.

SSA Dead Code Elimination

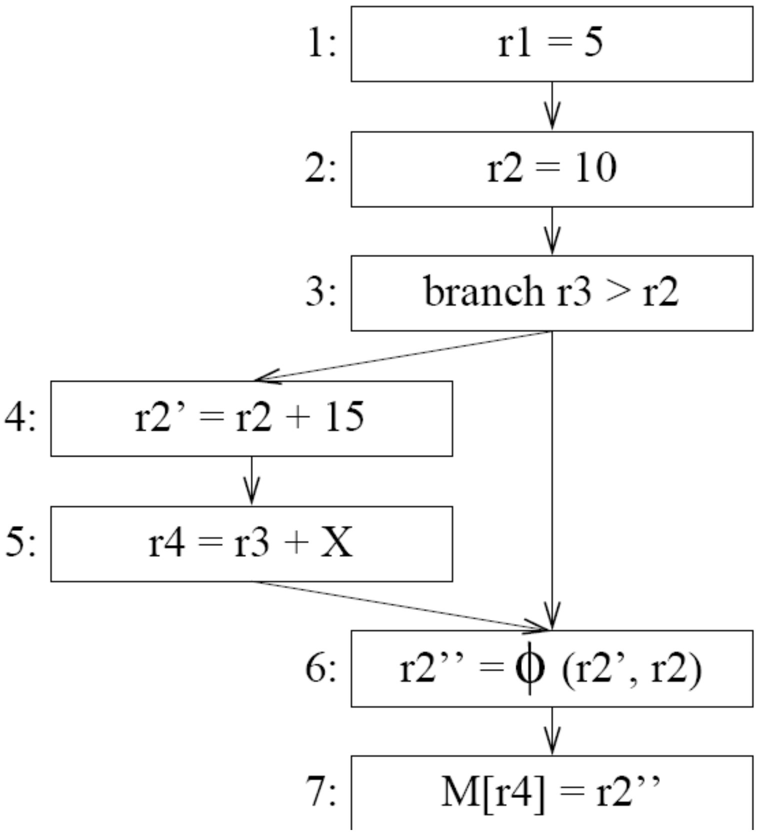
Given d: `t = x op y`

- `t` is live at end of node d if there exists path from end of d to use of `t` that does not go through definition of `t`.
- if program not in SSA form, need to perform liveness analysis to determine if `t` live at end of d.
- if program is in SSA form:
  - cannot be another definition of `t`
  - if there exists use of `t`, then path from end of d to use exists, since definitions dominate uses.
    - every use has a unique definition
    - `t` is live at end of node d if `t` is used at least once

SSA Dead Code Elimination

Algorithm:

WHILE (for each temporary `t` with no uses &&
statement defining `t` has no other side-effects) DO
delete statement definition `t`



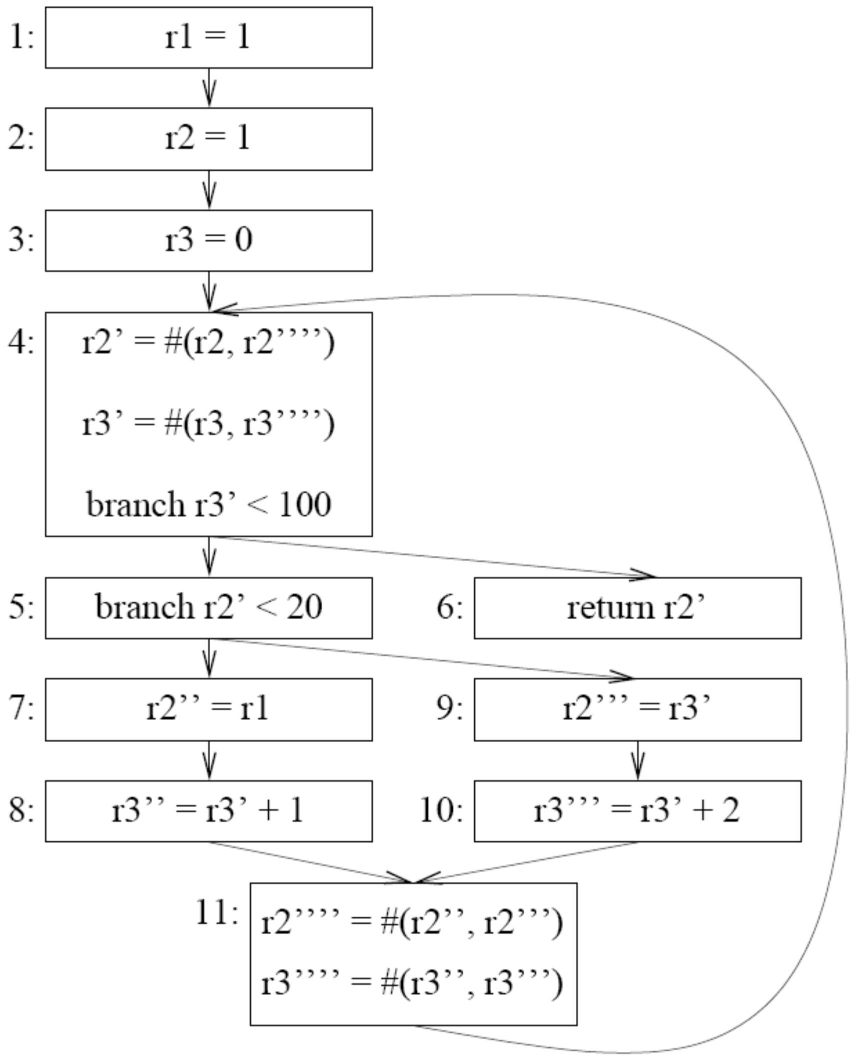
SSA Simple Constant Propagation

Given d: `t = c`, c is constant Given u: `x = t op b`

- if program not in SSA form:
  - need to perform reaching definition analysis
  - use of `t` in u may be replaced by `c` if d reaches u and no other definition of `t` reaches u
- if program is in SSA form:
  - d reaches u, since definitions dominate uses, and no other definition of `t` exists on path from d to u
  - d is only definition of `t` that reaches u, since it is the only definition of `t`.
    - any use of `t` can be replaced by `c`
    - any ϕ-function of form `v` = $\phi(c_1, c_2, ..., c_n)$, where $c_i = c$, can be replaced by `v = c`

SSA Simple Constant Propagation

SSA Conditional Constant Propagation



- `r2` always has value of 1
- nodes 9, 10 never executed
- "simple" constant propagation algorithms assumes (through reaching definitions analysis) nodes 9, 10 may be executed.
- cannot optimize use of `r2` in node 5 since definitions 7 and 9 both reach 5.

SSA Conditional Constant Propagation

Much smarter than "simple" constant propagation:

- Does not assume a node can execute until evidence exists that it can be.
- Does not assume register is non-constant unless evidence exists that it is.

Track run-time value of each register `r` using *lattice* of values:

- $V[r] = \perp$ (bottom): compiler has seen no evidence that any assignment to `r` is ever executed.
- $V[r] = 4$: compiler has seen evidence that an assignment `r = 4` is executed, but has seen no evidence that `r` is ever assigned to another value.
- $V[r] = \top$ (top): compiler has seen evidence that `r` will have, at various times, two different values, or some value that is not predictable at compile-time.

Also:

- all registers start at bottom of lattice
- new information can only move registers up in lattice

SSA Conditional Constant Propagation

Track executability of each node in N:

- $E[N] = \text{false}$: compiler has seen no evidence that node N can ever be executed.
- $E[N] = \text{true}$: compiler has seen evidence that node N can be executed.

Initially:

- $V[r] = \perp$, for all registers r
- $E[s_0] = \text{true}$, s_0 is CFG start node
- $E[N] = \text{false}$, for all CFG nodes $N \neq s_0$

SSA Conditional Constant Propagation

Algorithm: apply following conditions until no more changes occur to E or V values:

1. Given: register `r` with no definition (formal parameter, uninitialized).
   Action: $V[r] = \top$
2. Given: executable node B with only one successor C
   Action: $E[C] = \text{true}$
3. Given: executable assignment `r = x op y`, $V[x] = c_1$ and $V[y] = c_2$
   Action: $V[r] = c_1 \text{op} c_2$
4. Given: executable assignment `r = x op y`, $V[x] = \top$ or $V[y] = \top$
   Action: $V[r] = \top$
5. Given: executable assignment `r =` $\phi(x_1, x_2, ..., x_n)$, $V[x_i] = c_1$, $V[x_j] = c_2$, and predecessors i and j are executable
   Action: $V[r] = \top$

SSA Conditional Constant Propagation

6. Given: executable assignment `r = M[..]` or `r = f(..)`
   Action: $V[r] = \top$

7. Given: executable assignment `r =` $\phi(x_1, x_2, ..., x_n)$, $V[x_i] = \top$, and predecessor i is executable
   Action: $V[r] = \top$

8. Given: executable assignment `r =` $\phi(x_1, x_2, ..., x_n)$, $V[x_i] = c_i$, and predecessor i is executable; and for all $j \neq i$ predecessor j is not executable, or $V[x_j] = \bot$, or $V[x_j] = c_i$
   Action: $V[r] = c_i$

9. Given: executable branch `branch x bop y, L1 (else L2)`, $V[x] = \top$ or $V[y] = \top$
   Action: $E[L1] = \text{true}$, $E[L2] = \text{true}$

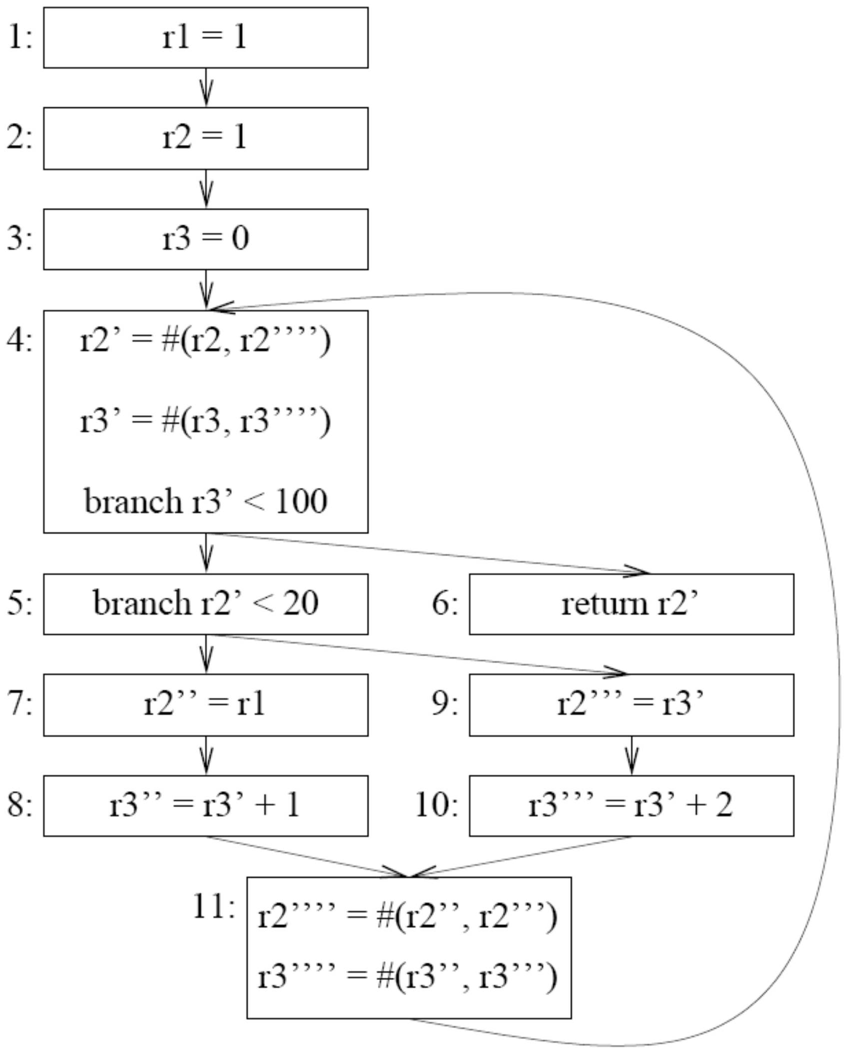
10. Given: executable branch `branch x bop y, L1 (else L2)`, $V[x] = c_1$ and $V[y] = c_2$
    Action: $E[L1] = \text{true}$ *OR* $E[L2] = \text{true}$ depending on c_1 `bop` c_2.

SSA Conditional Constant Propagation

Given V, E values, program can be optimized as follows:

- if $E[B] = $ false, delete node B form CFG.
- if $V[r] = c$, replace each use of `r` by c, delete assignment to r.

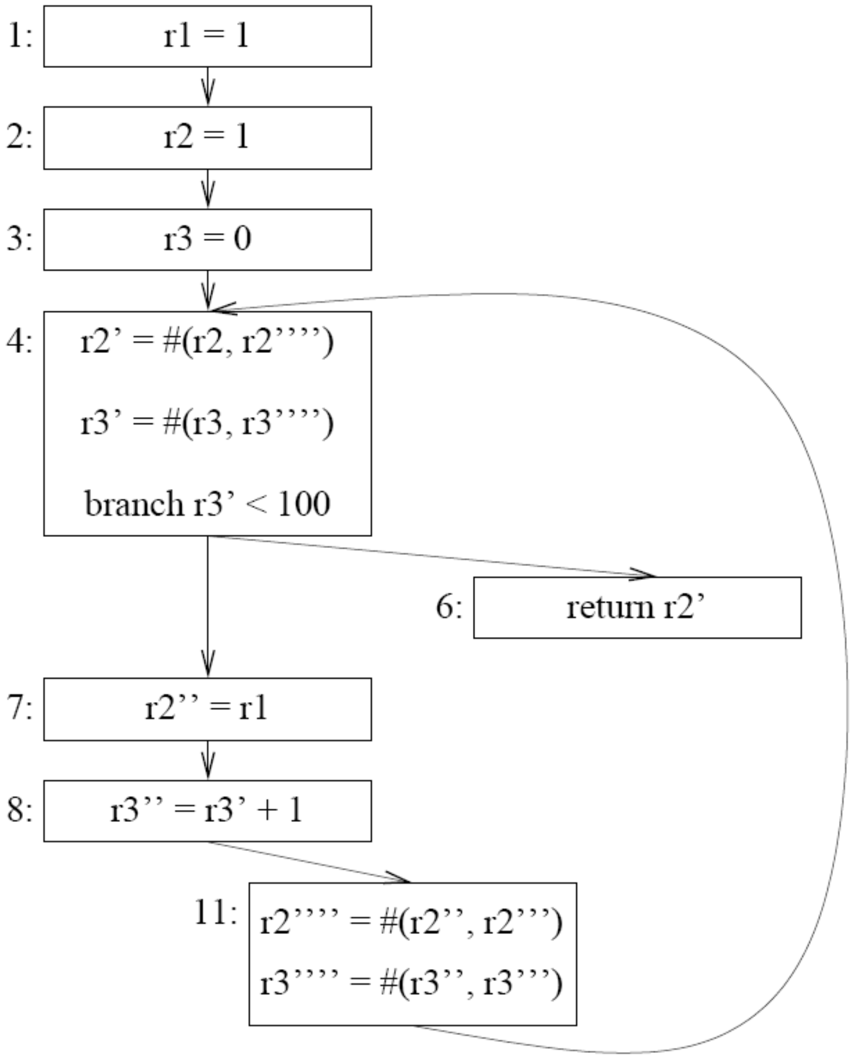
SSA Conditional Constant Propagation Example



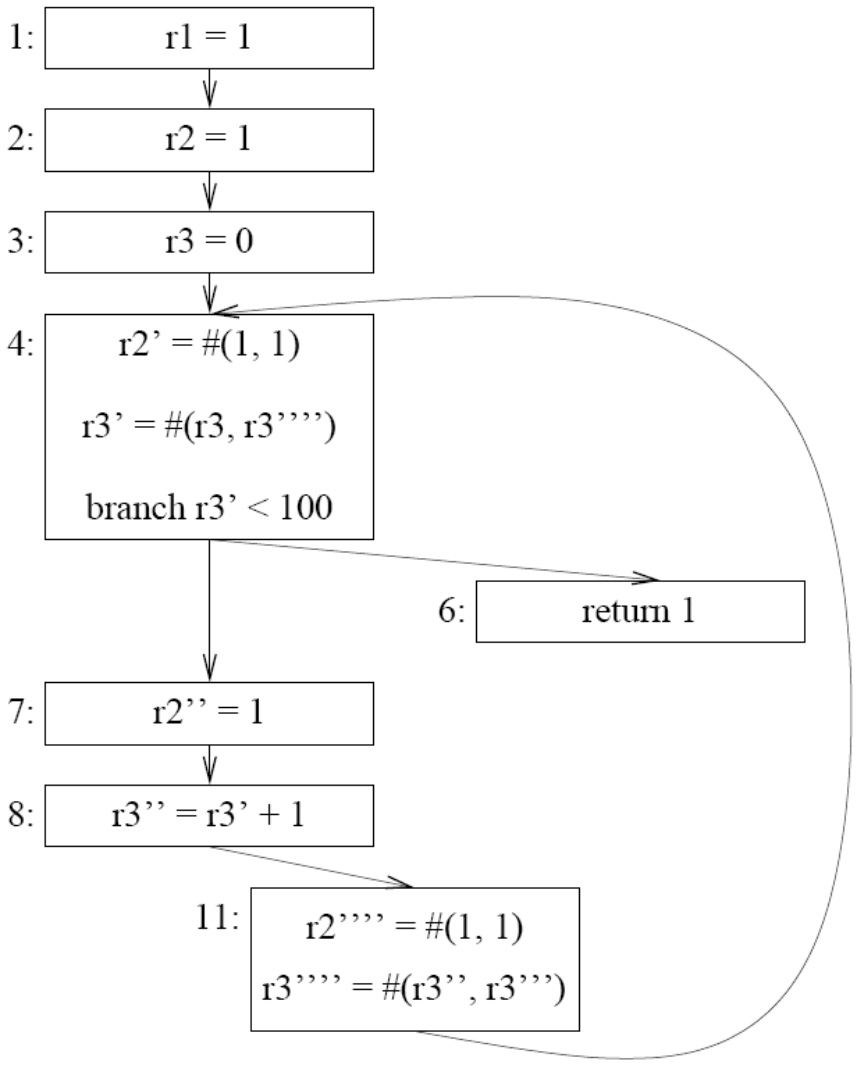
N	$E[N]$	r	$V[r]$
1	t	r1	$\perp$
2	f	r2	$\perp$
3	f	r2’	$\perp$
4	f	r2’’	$\perp$
5	f	r2’’’	$\perp$
6	f	r2’’’’	$\perp$
7	f	r3	$\perp$
8	f	r3’	$\perp$
9	f	r3’’	$\perp$
10	f	r3’’’	$\perp$
11	f	r3’’’’	$\perp$

SSA Conditional Constant Propagation

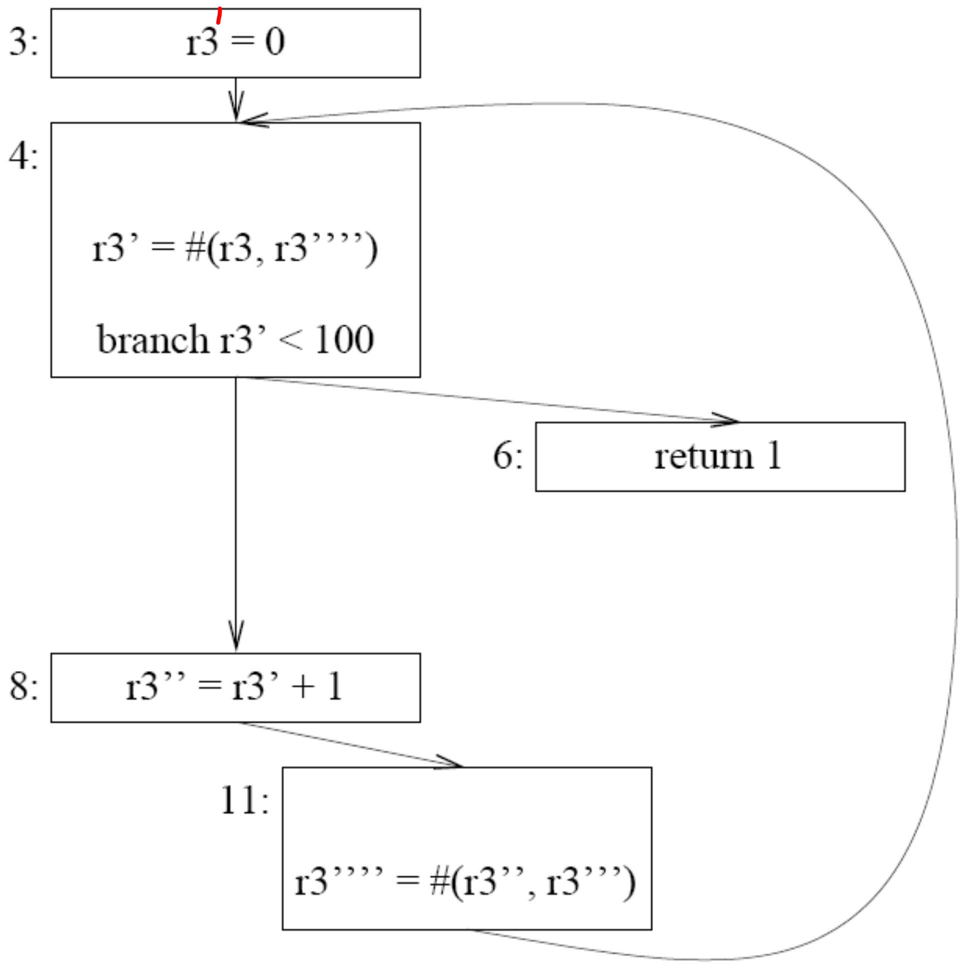
Example



SSA Conditional Constant Propagation Example



SSA Conditional Constant Propagation Example



SSA Conditional Constant Propagation

Example