Def-Use Chains, Use-Def Chains

- Many optimizations need to find all use-sites for each definition, and all definition-sites for each use.
  - Constant propagation must refer to the definition-site of the unique reaching definition.
  - Copy propagation, reverse copy propagation, common sub-expression elimination...
- Information connecting all use-sites to corresponding definition-sites can be stored as def-use chains and/or use-def chains.
- def-use chains: for each definition $d$ of $r$, list of pointers to all uses of $r$ that $d$ reaches.
- use-def chains: for each use $u$ of $r$, list of pointers to all definitions of $r$ that reach $u$.

Use-Def Chains, Def-Use Chains

```
6: r4 = 10
7: r1 = r1 + r4
8: M[r3] = r1
```

```
1: r1 = 5
2: r3 = 1
3: branch r3 > r1, 6:
4: r3 = r3 + 1
goto 3:
```
Static Single Assignment (SSA):

- improvement on def-use chains
- each register has only one definition in program
- for each use of \( r \), only one definition of \( r \) reaches \( u \)

\[
\begin{align*}
  r1 &= 5 \\
  \downarrow \quad & \\
  r1 &= r1 + 1 \\
  r2 &= r1 + 1 \\
  r3 &= r1 - 1
\end{align*}
\]

Why SSA?

Static Single Assignment Advantages:

- Dataflow analysis and code optimization made simpler.
  - Variables have only one definition - no ambiguity.
  - Dominator information is encoded in the assignments.
- Less space required to represent def-use chains. For each variable, space is proportional to uses * defs.
- Eliminates unnecessary relationships:
  
  \[
  \begin{align*}
  &\text{for } i = 1 \text{ to } N \text{ do } A[i] = 0 \\
  &\text{for } i = 1 \text{ to } M \text{ do } B[i] = 1
  \end{align*}
  \]

  - No reason why both loops should be forced to use same register to hold index register.
  - SSA renames second \( i \) to new register which may lead to better register allocation/optimization.

(Dynamic Single Assignment is also proposed in the literature.)

Conversion to SSA Code

Easy to convert basic blocks into SSA form:

- Each definition modified to define brand-new register, instead of redefining old one.
- Each use of register modified to use most recently defined version.

\[
\begin{align*}
  r1 &= r3 + r4 \\
  r2 &= r1 - 1 \\
  r1 &= r4 + r2 \\
  r2 &= r5 * 4 \\
  r1 &= r1 + r2
\end{align*}
\]

Control flow introduces problems.
Conversion to SSA Form

- \( \phi \)-functions enable the use of \( r_3 \) to be reached by exactly one definition of \( r_3 \).
- \( r_3'' = \phi(r_3, r_3') \):
  - \( r_3'' = r_3 \) if control enters from left
  - \( r_3'' = r_3' \) if control enters from right
- Can implement \( \phi \)-functions as set of move operations on each incoming edge.
- In practice, \( \phi \)-functions are just used as notation.

Conversion to SSA Form

Can insert \( \phi \)-functions for each register at each node with more than two predecessors.

We can do better...
Conversion to SSA Form

**Path-Convergence Criterion:** Insert a $\phi$-function for a register $r$ at node $z$ of the flow graph if ALL of the following are true:

1. There is a block $x$ containing a definition of $r$.
2. There is a block $y \neq x$ containing a definition of $r$.
3. There is a non-empty path $P_x$, of edges from $x$ to $z$.
4. There is a non-empty path $P_y$, of edges from $y$ to $z$.
5. Paths $P_x$ and $P_y$ do not have any node in common other than $z$.
6. The node $z$ does not appear within both $P_x$ and $P_y$ prior to the end, though it may appear in one or the other.

Assume CFG entry node contains implicit definition of each register:

- $r = \text{actual parameter value}$
- $r = \text{undefined}$

$\phi$-functions are counted as definitions.

**Conversion to SSA Form**

Solve path-convergence iteratively:

```
WHILE (there are nodes $x$, $y$, $z$ satisfying conditions 1-6) 
AND
 (z does not contain a phi-function for $r$) DO:
    insert \( r = \phi(r, r, ..., r) \) (one per predecessor) at node $z$.
```

- Costly to compute.
- Since definitions dominate uses, use domination to simplify computation.

**Use Dominance Frontier...**

**Dominance Frontier**

**Definitions:**

- $x$ **strictly dominates** $w$ if $x$ dominates $w$ and $x \neq w$.
- **dominance frontier** of node $x$ is set of all nodes $w$ such that $x$ dominates a predecessor of $w$, but does not strictly dominate $w$.

![Diagram of a control flow graph with nodes labeled 1 to 11 and edges connecting them, illustrating the concept of dominance frontier.]
Dominance Frontier

- **Dominance Frontier Criterion**: Whenever node $x$ contains definition of some register $r$, then need to insert $\phi$-function for $r$ in all nodes $z$ in dominance frontier of $x$.
- **Iterated Dominance Frontier**: Need to repeatedly apply since $\phi$-function counts as a definition.

**Dominance Frontier Computation**

- Use dominator tree
- $DF[n]$: dominance frontier of $n$
- $DF_{local}[n]$: successors of $n$ in CFG that are not strictly dominated by $n$
- $DF_{up}[c]$: nodes in dominance frontier of $c$ that are not strictly dominated by $c$’s immediate dominator
  \[
  DF[n] = DF_{local}[n] \cup (\cup_{c \in \text{children}[n]} DF_{up}[c])
  \]
- where $\text{children}[n]$ are the nodes whose idom is $n$.
- Work bottom up in dominator tree.

**SSA Example**

```
<table>
<thead>
<tr>
<th>Node</th>
<th>DOM[n]</th>
<th>IDOM[n]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>r1</td>
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<tr>
<td>2</td>
<td></td>
<td>r2</td>
</tr>
<tr>
<td>3</td>
<td>r3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>branch r3 &lt; 100</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>branch r2 &lt; 20</td>
<td>r2</td>
</tr>
<tr>
<td>6</td>
<td>return r2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>r2 = r1</td>
<td>r3</td>
</tr>
<tr>
<td>8</td>
<td>r3 = r3 + 1</td>
<td>r3 = r3 + 2</td>
</tr>
<tr>
<td>9</td>
<td>r2 - r3</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>r3 = r3 + 2</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>r3 = r3 + 2</td>
<td></td>
</tr>
</tbody>
</table>
```
Dominator Analysis

- If $d$ dominates each of the $p_i$, then $d$ dominates $n$.
- If $d$ dominates $n$, then $d$ dominates each of the $p_i$.
- $\text{Dom}[n] =$ set of nodes that dominate node $n$.
- $\mathcal{N} =$ set of all nodes.
- Computation:
  1. $\text{Dom}[s_0] = \{s_0\}$.
  2. for $n \in \mathcal{N} - \{s_0\}$ do $\text{Dom}[n] = \mathcal{N}$
  3. while (changes to any $\text{Dom}[n]$ occur) do
  4. for $n \in \mathcal{N} - \{s_0\}$ do
  5. $\text{Dom}[n] = \{n\} \cup (\cap_{p \in \text{pred}[n]} \text{Dom}[p])$.

SSA Example

Insert phi-functions:

1. r1 = 1
2. r2 = 1
3. r3 = 0
4. if r3 < 100
5. if r2 < 20
6. return r2
7. r2 = r1
8. r3 = r3 + 1
9. r2 = r3
10. r3 = r3 + 2
11. 

SSA Example
**Static Single Assignment**

**Static Single Assignment Advantages:**

- Less space required to represent def-use chains. For each variable, space is proportional to uses * defs.
- Eliminates unnecessary relationships:
  
  ```
  for i = 1 to N do A[i] = 0
  for i = 1 to M do B[i] = 1
  ```
  
  – No reason why both loops should be forced to use same register to hold index register.
  – SSA renames second i to new register which may lead to better register allocation.
- SSA form make certain optimizations quick and easy — dominance property.
  – Variables have only one definition - no ambiguity.
  – Dominator information is encoded in the assignments.
SSA Dominance Property

Dominance property of SSA form: definitions dominate uses
- If \( x \) is \( i^{th} \) argument of \( \phi \)-function in node \( n \), then definition of \( x \) dominates \( i^{th} \) predecessor of \( n \).
- If \( x \) is used in non-\( \phi \) statement in node \( n \), then definition of \( x \) dominates \( n \).

SSA Dead Code Elimination

Given \( d: \tau = x \, \text{op} \, y \)
- \( \tau \) is live at end of node \( d \) if there exists path from end of \( d \) to use of \( \tau \) that does not go through definition of \( \tau \).
- if program not in SSA form, need to perform liveness analysis to determine if \( \tau \) live at end of \( d \).
- if program is in SSA form:
  - cannot be another definition of \( \tau \)
  - if there exists use of \( \tau \), then path from end of \( d \) to use exists, since definitions dominate uses.
  - every use has a unique definition
  - \( \tau \) is live at end of node \( d \) if \( \tau \) is used at least once

SSA Dead Code Elimination

Algorithm:
WHILE (for each temporary \( \tau \) with no uses && statement defining \( \tau \) has no other side-effects) DO
  delete statement definition \( \tau \)
  1: \( r1 = 5 \)
  2: \( r2 = 10 \)
  3: branch \( r3 > r2 \)
  4: \( r2' = r2 + 15 \)
  5: \( r4 = r3 + X \)
  6: \( r2'' = \emptyset (r2', r2) \)
  7: \( M[r4] = r2'' \)
SSA Simple Constant Propagation

Given \( d: t = c \), \( c \) is constant

Given \( u: x = t \ \text{op} \ b \)

- if program not in SSA form:
  - need to perform reaching definition analysis
  - use of \( t \) in \( u \) may be replaced by \( c \) if \( d \) reaches \( u \) and no other definition of \( t \) reaches \( u \)

- if program is in SSA form:
  - \( d \) reaches \( u \), since definitions dominate uses, and no other definition of \( t \) exists on path from \( d \) to \( u \)
  - \( d \) is only definition of \( t \) that reaches \( u \), since it is the only definition of \( t \).
    * any use of \( t \) can be replaced by \( c \)
    * any \( \phi \)-function of form \( \nu = \phi(c_1, c_2, ..., c_n) \), where \( c_i = c \), can be replaced by \( \nu = c \)

SSA Conditional Constant Propagation

- \( t \equiv 2 \) always has value of 1
- nodes 9, 10 never executed
- "simple" constant propagation algorithms assumes (through reaching definitions analysis) nodes 9, 10 may be executed.
- cannot optimize use of \( t \equiv 2 \) in node 5 since definitions 7 and 9 both reach 5.
Much smarter than “simple” constant propagation:
- Does not assume a node can execute until evidence exists that it can be.
- Does not assume register is non-constant unless evidence exists that it is.

Track run-time value of each register \( r \) using lattice of values:
- \( V[r] = \bot \) (bottom): compiler has seen no evidence that any assignment to \( r \) is ever executed.
- \( V[r] = 4 \): compiler has seen evidence that an assignment \( r = 4 \) is executed, but has seen no evidence that \( r \) is ever assigned to another value.
- \( V[r] = \top \) (top): compiler has seen evidence that \( r \) will have, at various times, two different values, or some value that is not predictable at compile-time.

Also:
- all registers start at bottom of lattice
- new information can only move registers up in lattice

Track executability of each node in \( N \):
- \( E[N] = \text{false} \): compiler has seen no evidence that node \( N \) can ever be executed.
- \( E[N] = \text{true} \): compiler has seen evidence that node \( N \) can be executed.

Initially:
- \( V[r] = \bot \), for all registers \( r \)
- \( E[s_0] = \text{true} \), \( s_0 \) is CFG start node
- \( E[N] = \text{false} \), for all CFG nodes \( N \neq s_0 \)

Algorithm: apply following conditions until no more changes occur to \( E \) or \( V \) values:
1. Given: register \( r \) with no definition (formal parameter, uninitialized).
   Action: \( V[r] = \top \)
2. Given: executable node \( B \) with only one successor \( C \)
   Action: \( E[C] = \text{true} \)
3. Given: executable assignment \( r = x \text{ op } y \), \( V[x] = c_1 \) and \( V[y] = c_2 \)
   Action: \( V[r] = c_1 \text{op} c_2 \)
4. Given: executable assignment \( r = x \text{ op } y \), \( V[x] = \top \) or \( V[y] = \top \)
   Action: \( V[r] = \top \)
5. Given: executable assignment \( r = \phi(x_1, x_2, \ldots, x_n) \), \( V[x_i] = c_1 \), \( V[x_j] = c_2 \), and predecessors \( i \) and \( j \) are executable
   Action: \( V[r] = \top \)
6. Given: executable assignment $x = M[.]$ or $x = f(\ldots)$
   Action: $V[x] = T$

7. Given: executable assignment $x = \phi(x_1, x_2, \ldots, x_n)$, $V[x_i] = T$, and predecessor $i$ is executable
   Action: $V[x] = T$

8. Given: executable assignment $x = \phi(x_1, x_2, \ldots, x_n)$, $V[x_i] = c_i$, and predecessor $i$ is executable; and for all $j \neq i$ predecessor $j$ is not executable, or $V[x_j] = \perp$, or $V[x_j] = c_j$
   Action: $V[x] = c_i$

9. Given: executable branch $\text{branch } x \text{ bop } y$, L1 (else L2), $V[x] = T$ or $V[y] = T$
   Action: $E[L1] = \text{true, } E[L2] = \text{true}$

10. Given: executable branch $\text{branch } x \text{ bop } y$, L1 (else L2), $V[x] = c_1$ and $V[y] = c_2$
    Action: $E[L1] = \text{true OR } E[L2] = \text{true}$ depending on $c_1 \text{ bop } c_2$.

### SSA Conditional Constant Propagation

Given $V$, $E$ values, program can be optimized as follows:

- if $E[B] = \text{false}$, delete node $B$ form CFG.
- if $V[r] = c$, replace each use of $x$ by $c$, delete assignment to $r$.

### Example

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<td>r3''</td>
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**Note:** The image contains a table and a diagram illustrating the example, but the text does not include the entire table or the full diagram. The table and diagram are related to the example provided in the text, showing the process of SSA Conditional Constant Propagation.
SSA Conditional Constant Propagation

Example

1: \( r_1 = 1 \)
2: \( r_2 = 1 \)
3: \( r_3 = 0 \)
4: \( r_2' = \text{#}(r_2, r_2'') \)
   \( r_3' = \text{#}(r_3, r_3''') \)
   branch \( r_3' < 100 \)
6: \( \text{return } r_2' \)
7: \( r_2''' = r_1 \)
8: \( r_3''' = r_3' + 1 \)
11: \( r_2'''' = \text{#}(r_2'', r_2''') \)
    \( r_3'''' = \text{#}(r_3'', r_3''') \)

SSA Conditional Constant Propagation

Example

1: \( r_1 = 1 \)
2: \( r_2 = 1 \)
3: \( r_3 = 0 \)
4: \( r_2'' = \text{#}(1, 1) \)
   \( r_3'' = \text{#}(r_3, r_3''') \)
   branch \( r_3' < 100 \)
6: \( \text{return } 1 \)
7: \( r_2''' = 1 \)
8: \( r_3''' = r_3' + 1 \)
11: \( r_2'''' = \text{#}(1, 1) \)
    \( r_3'''' = \text{#}(r_3', r_3'''') \)

SSA Conditional Constant Propagation

Example

3: \( r_3 = 0 \)
4: \( r_3' = \text{#}(r_3, r_3''') \)
   branch \( r_3' < 100 \)
6: \( \text{return } 1 \)
8: \( r_3''' = r_3' + 1 \)
11: \( r_3'''' = \text{#}(r_3'', r_3'''') \)
SSA Conditional Constant Propagation

Example

3: \( r3 = 0 \)

4: \( \text{branch } r3 < 100 \)

6: \( \text{return } 1 \)

8: \( r3 = r3 + 1 \)