Topic 6: Types

COS 320

Compiling Techniques

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Types

What is a type?

Type Checking:

- Helps find language-level errors:
  - Memory Safety – can't dereference something not a pointer
  - Control-Flow Safety – can't jump to something not code
  - Type Safety – redundant specification checked
- Helps find application-level errors:
  - Ensures isolation properties
- Helps generate code:
  - Is that "+" a floating point add or an integer add?

Defining a Type System

- RE → Lexing
- CFG → Parsers
- Inductive Definitions → Type Systems

An inductive definition really has two parts:

1. Specification of the form of *judgments* – A judgment is an assertion/claim, may or may not be true. A *valid judgment* is a true/provable judgment.
2. A collection of *inference rules* – what allow you to conclude whether a judgment is true or false

Inference Rules

An implementation-language-independent way: type system with **inference rules**.

$$\frac{a : \mathit{bool} \quad b : \mathit{bool}}{a \;\&\&\; b : \mathit{bool}}$$

Read: if *a* has type *bool* and *b* has type bool, then *a && b* has type bool.

Inference Rules

An inference rule has a set of **premises** $J_1, \ldots, J_n$ and one **conclusion** J, separated by a horizontal line:

$$\frac{J_1 \quad \ldots \quad J_n}{J}$$

Read:

- If I can establish the truth of the premises $J_1,\ldots,J_n$, I can conclude: J is true.
- To check J, check $J_1,\ldots,J_n$.

An inference rule with no premises is called an **Axiom** – J always true

Judgments

The premises and conclusions are called **judgments**.

The most common judgments in type systems have the form:

$$e : T$$

Read: expression *e* has type *T*.

Means: Based on no outside evidence, *e* is an expression with type *T*

Axioms and Rules

Examples: BT, BF, B&&, B||

Type Checking and Type Inference

Two activities:

- Type checking: Given an expression *e* and a type *T*, decide if *e : T*
- Type inference: Given an expression *e*, find a type *T* such that *e : T*

Both activities necessary. Both originate from typing rules.

Type Checking Implementation

Example: type checking for *&&*:

```
check(a && b, bool):
        check(a, bool)
        check(b, bool)
```

No patterns matching types other than *bool*.

Type Inference Implementation

Example: type checking for *&&*:

```
infer(a && b):
        check(a, bool)
        check(b, bool)
        return bool
```

Inference involves checking.

Recall Symbol Table, Scope Topic

- Generally, a variable can be any type available in the language.
- In C and Java, type determined by the declaration of the variable.
- In inference rules, variables are collected to a context.
- Context is a symbol table of (variable, type) pairs.
- In inference rules, the context is denoted by the Greek letter Γ, Gamma.
- The judgment form for typing is generalized to:

$$\Gamma \vdash e : T$$

- Read: expression *e* has type *T* in context Γ

Context

Consider:

$$x : \texttt{int}, y : \texttt{int} \vdash x \texttt{+} y \texttt{>} y : \texttt{bool}$$

This means:

x + y > y is bool in context where *x* and *y* are ints

Context notation:

$$x_1 : T_1, \ldots, x_n : T_n$$

Adding variable to existing context:

$$\Gamma, x : T$$

Context

Most judgments share the same Γ, because the context doesn't change.

$$\frac{\Gamma \vdash a : \mathit{bool} \quad \Gamma \vdash b : \mathit{bool}}{\Gamma \vdash a \;\&\&\; b : \mathit{bool}}$$

For declarations:

$$\frac{}{\Gamma \vdash x : T} \quad \text{if } x : T \text{ in } \Gamma$$

The condition "if x : T in Γ" is not a judgment – but a sentence in the metalanguage (English). (Condition is a symbol table lookup of *x* in Γ.)

Functions

Function Application:

$$\frac{\Gamma \vdash a_1 : T_1 \quad \cdots \quad \Gamma \vdash a_n : T_n}{\Gamma \vdash f(a_1, \ldots, a_n) : T} \text{ if } f : (T_1, \ldots, T_n) \rightarrow T \text{ in } \Gamma$$

Notation:

$$(T_1, \ldots, T_n) \rightarrow T$$

Proofs

Proof Tree: a trace of the steps that the type checker performs, built up rule by rule.

$$\dfrac{\dfrac{\dfrac{}{x:\texttt{int}, y:\texttt{int} \vdash x:\texttt{int}}\,x \quad \dfrac{}{x:\texttt{int}, y:\texttt{int} \vdash y:\texttt{int}}\,y}{x:\texttt{int}, y:\texttt{int} \vdash x{+}y:\texttt{int}}\,+ \quad \dfrac{}{x:\texttt{int}, y:\texttt{int} \vdash y:\texttt{int}}\,y}{x:\texttt{int}, y:\texttt{int} \vdash x{+}y{>}y:\texttt{bool}}\,>$$

Each judgment is a conclusion from the ones above with some of the rules, indicated beside the line. This tree uses the variable rule and the rules for + and >:

$$\dfrac{}{\Gamma \vdash x : T}\,x \qquad \dfrac{\Gamma \vdash a : int \quad \Gamma \vdash b : int}{\Gamma \vdash a + b : int}\,+ \qquad \dfrac{\Gamma \vdash a : int \quad \Gamma \vdash b : int}{\Gamma \vdash a > b : bool}\,>$$

Overloading

The binary arithmetic operations (+ - * /) and comparisons (== != < > <= >=) are overloaded in many languages.

If the possible types are int, double, and string, the typing rules become:

$$\frac{\Gamma \vdash a : t \quad \Gamma \vdash b : t}{\Gamma \vdash a + b : t} \quad \text{if } t \text{ is } \texttt{int} \text{ or } \texttt{double} \text{ or } \texttt{string}$$

$$\frac{\Gamma \vdash a : t \quad \Gamma \vdash b : t}{\Gamma \vdash a == b : \mathit{bool}} \quad \text{if } t \text{ is } \texttt{int} \text{ or } \texttt{double} \text{ or } \texttt{string}$$

Overloading Implementation

First infer the type of the first operand, then check the second operand with respect to this type:

```
infer (a + b) :
        t := infer(a)
        // check that t ∈ {int, double, string}
        check (b, t)
        return t
```

Type Conversion

Example: an integer can be converted into a double

Generally, integers and doubles have different binary representations operated upon by different instructions.

Compiler generates a conversion instruction (or instructions) for type conversions.

Type Conversion

$$\frac{\Gamma \vdash a : t \quad \Gamma \vdash b : u}{\Gamma \vdash a + b : max(t, u)} \quad \text{if } t, u \in \{\texttt{int}, \texttt{double}, \texttt{string}\}$$

$$\texttt{int} < \texttt{double} < \texttt{string}$$

$$\text{max}(\texttt{int}, \texttt{string}) = \texttt{string}$$

2 + “hello” produces “2hello”

Evaluate: 1 + 2 + "hello" + 1 + 2

Statement Validity

When type-checking a statement, simply check whether the statement is **valid**.

A new judgment form:

$$\Gamma \vdash s \ \mathit{valid}$$

Read: Statement *s* is valid in environment Γ.

Example: **`while`**

$$\frac{\Gamma \vdash e : \mathit{bool} \quad \Gamma \vdash s \ \mathit{valid}}{\Gamma \vdash \texttt{while} \ (e) \ s \ \mathit{valid}}$$

Expression Statements

Some expressions simply need a type inference.

For example: assignments and function calls.

$$\frac{\Gamma \vdash e : t}{\Gamma \vdash e;\ \mathit{valid}}$$

$$\frac{x_1 : T_1, \ldots, x_m : T_m \vdash s_1 \ldots s_n\ \mathit{valid}}{T\ f(T_1\, x_1, \ldots, T_m\, x_m)\{s_1\ \ldots, s_n\}\ \mathit{valid}}$$

Parameters of the function define the context.

The body statements $s_1 \ldots s_n$ are checked in this context.

Context may change within the body from declarations.

Check all variables in the parameter list are distinct.

Return Statements

Return statement should be of expected type.

Control flow makes this interesting:

```
if (fail()) return 1 ; else return 0 ;
```

Declarations and Block Structure

Each declaration has a scope, in a certain block.

Blocks in C and Java correspond (roughly) to parts of code between curly brackets: { }

Two principles regulate the use of variables:

1. A variable declared in a block has its scope till the end of that block.
2. A variable can be declared again in an inner block, but not otherwise.

Declarations and Block Structure

```
{
  int x ;
  {
    x = 3 ;       // x : int
    double x ;    // x : double
    x = 3.14 ;
    int z ;
  }
  x = x + 1 ;    // x : int, receives the value 3 + 1
  z = 8 ;        // ILLEGAL! z is no more in scope
  double x ;     // ILLEGAL! x may not be declared again
  int z ;        // legal, since z is no more in scope
}
```

Stack of Contexts

Context to deal with blocks: Instead of a simple lookup table, Γ must be a stack of lookup tables.

Notation:

$$\Gamma_1 . \Gamma_2$$

where Γ_1 is an old (i.e. outer) context and Γ_2 an inner context.

The innermost context is the top of the stack.

Recall Symbol Table Discussion…

Declarations

A declaration introduces a new variable in the current scope, checked to be fresh with respect to the context.

Rules for sequences of statements, not just individual statements:

$$\Gamma \vdash s_1 \ldots s_n \textit{ valid}$$

A declaration extends the context used for checking the statements that follow:

$$\frac{\Gamma, x : T \vdash s_2 \ldots s_n \textit{ valid}}{\Gamma \vdash T\, x; s_2 \ldots s_n \textit{ valid}} \quad x \text{ not in the top-most context in } \Gamma$$

Example: If Statement Derivation/Proof

Example: sizeof

Example: Function