Problem 1: (12%)
Build a Deterministic Finite Automaton (DFA) that recognizes the following regular expression:

\[(aa)^* \mid (aaa)^*\]

Problem 2: (11%)
Derive a context free grammar for the regular expression in Problem 1.

Problem 3: (11%)
Derive a regular expression describing all possible sequences of entries (E) and exits (X) for a room that can hold no more than three people. The room begins and ends empty and can be empty many times in between. Use the alphabet: \{E, X\}. 
Problem 4: (11%)

Prove that the following grammar is ambiguous:

\[
\begin{align*}
S &\rightarrow a S \\
S &\rightarrow c \\
S &\rightarrow a S S \\
S &\rightarrow a S b S
\end{align*}
\]

Problem 5: (11%)

Is the following grammar in LR(0)? Prove your answer in an organized manner.

\[
\begin{align*}
S' &\rightarrow S $ \\
S &\rightarrow T \\
T &\rightarrow b \\
S &\rightarrow S a T \\
T &\rightarrow T c b
\end{align*}
\]

Problem 6: (11%)

Is the Problem 5 grammar in LR(1)? Prove your answer in an organized manner.

Problem 7: (11%)

Is the Problem 5 grammar in SLR(1)? Prove your answer in an organized manner.

Problem 8: (11%)

Is the Problem 5 grammar in LALR(1)? Prove your answer in an organized manner.

Problem 9: (11%)

Is the Problem 5 grammar in LL(1)? Prove your answer in an organized manner.