Theoretical Machine Learning - COS 511

Homework Assignment 5

Due Date: two weeks from announcement, in class

(1) Consulting other students from this course is allowed. In this case - clearly state whom you consulted with for each problem separately.

(2) Searching the internet or literature for solutions, other than the course lecture notes, is NOT allowed.

Ex. 1:

Prove that for $m \geq d$,

$$\sum_{i=0}^{d} \binom{m}{i} \leq \left(\frac{em}{d}\right)^d$$

Ex. 2:

In this exercise we consider the attribute set $\mathcal{X} = \mathbb{R}^d$ for some $d \geq 1$, and the label set $\mathcal{Y} = \{-1, 1\}$. In this classification problem, we consider the set of all hyperplanes as candidate hypotheses.

More accurately, we define a hyperplane $H \in \mathbb{R}^d$ as $H = \{x | a^\top x = b\}$ for some $a \in \mathbb{R}^d$ and $b \in \mathbb{R}$ (For example, in $\mathbb{R}^2$ a hyperplane is simply a line), and its corresponding hypothesis $h : \mathbb{R}^d \to \{-1, 1\}$ as a function $h(x) = sign\{a^\top x - b\}$. We assume that $sign\{0\} = 1$. We denote by $\mathcal{H}$ the set of all hypotheses of this kind. Show that:

(1) For $d = 2$, the VC dimension of $\mathcal{H}$ is 3.

(2) For $d = 3$, the VC dimension of $\mathcal{H}$ is 4.

(3) For any value of d, it exists that $VC(\mathcal{H}) \geq d + 1$. I.e., show that there exists a set of $d + 1$ points that can be perfectly classified for any labelling.

(4) For any value of d, it exists that $VC(\mathcal{H}) \leq d + 1$. I.e., show that there does not exist a set of $d + 2$ points that can be perfectly classified for any labeling.

Ex. 3:

For $m \geq d$, let $\left\{ {m \atop d} \right\} = \sum_{i=0}^{d} \binom{m}{i}$. Prove that

$$\left\{ {m \atop d} \right\} = \left\{ {m-1 \atop d} \right\} + \left\{ {m-1 \atop d-1} \right\}$$

Ex. 4-7:

The following questions are taken from the book draft on online convex optimization (reading material number 1).

(1) problem 3 in chapter 6.

(2) problem 4 in chapter 6.

(3) problem 5 in chapter 6.

(4) problem 6 in chapter 6.