Deep Learning Basics
Lecture 9: Recurrent Neural Networks

Princeton University COS 495

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Introduction

Recurrent neural networks

- Dates back to (Rumelhart *et al.*, 1986)
- A family of neural networks for handling sequential data, which involves variable length inputs or outputs

- Especially, for natural language processing (NLP)

Sequential data

- Each data point: A sequence of vectors $x^{(t)}$, for $1 \leq t \leq \tau$
- Batch data: many sequences with different lengths τ
- Label: can be a scalar, a vector, or even a sequence

- Example
  - Sentiment analysis
  - Machine translation

Example: machine translation

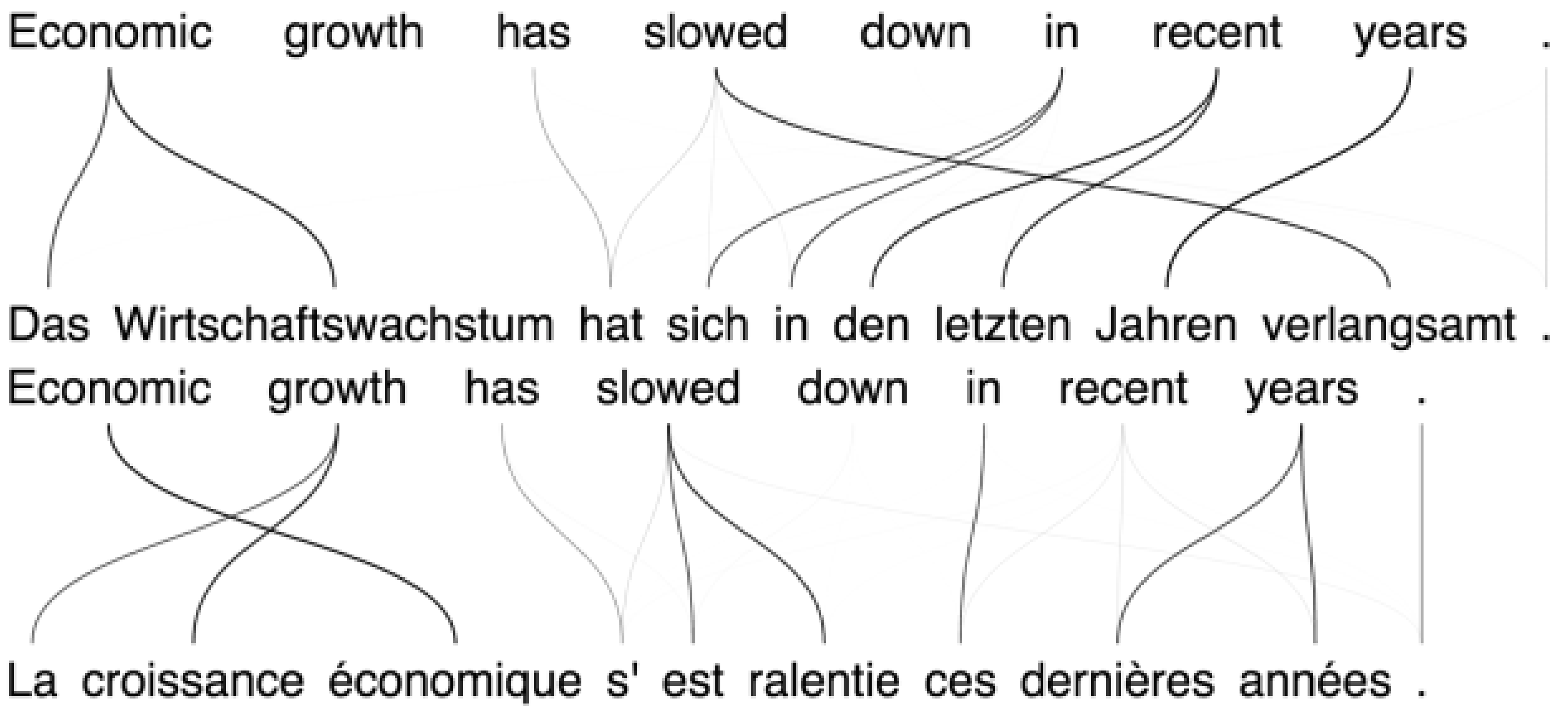


Figure from: devblogs.nvidia.com

More complicated sequential data

- Data point: two dimensional sequences like images
- Label: different type of sequences like text sentences

- Example: image captioning

Image captioning

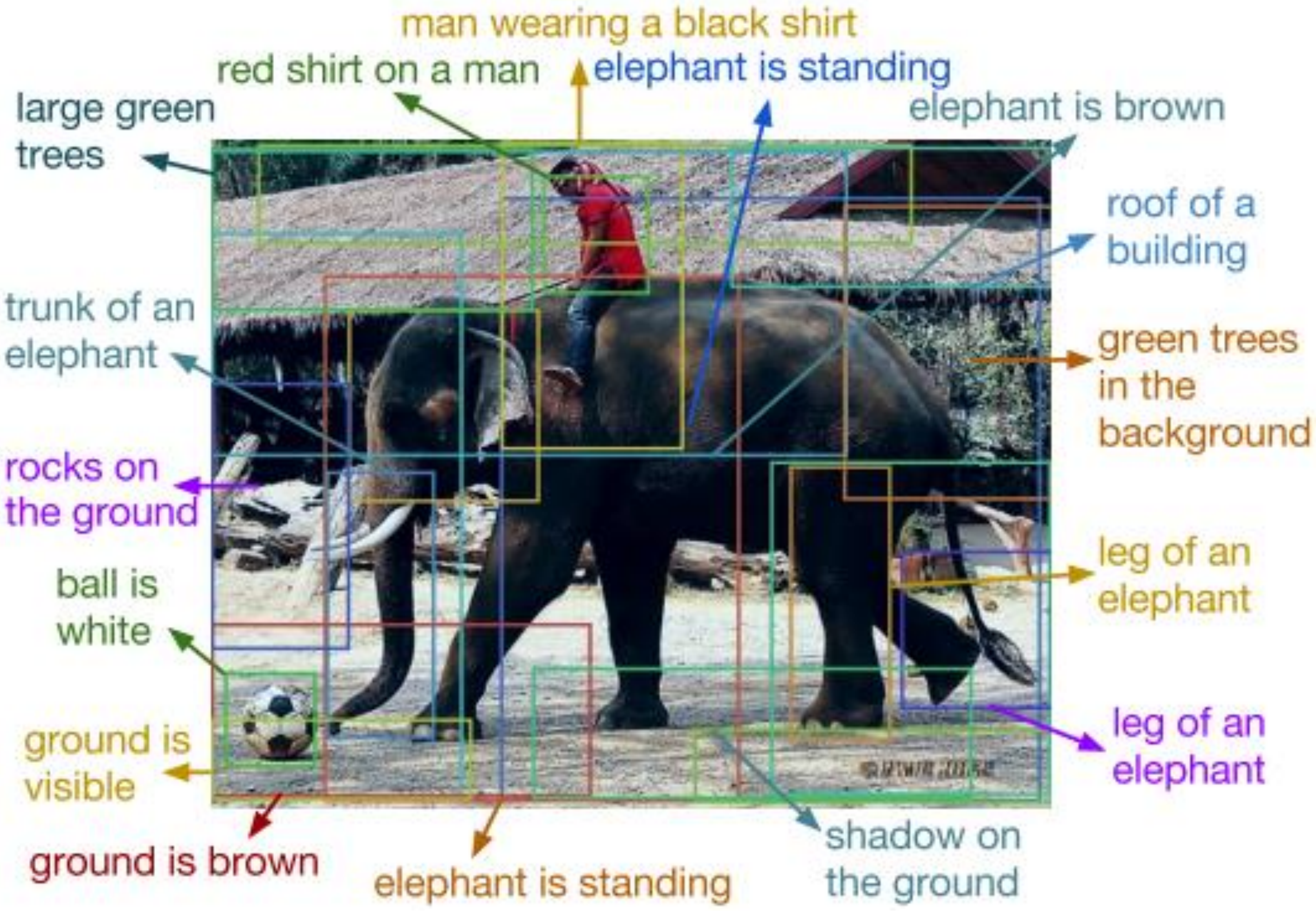
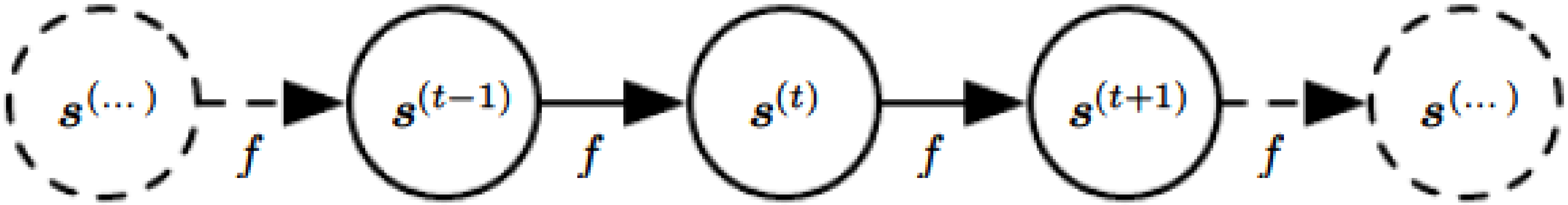


Figure from the paper “DenseCap: Fully Convolutional Localization Networks for Dense Captioning”, by Justin Johnson, Andrej Karpathy, Li Fei-Fei

Computational graphs

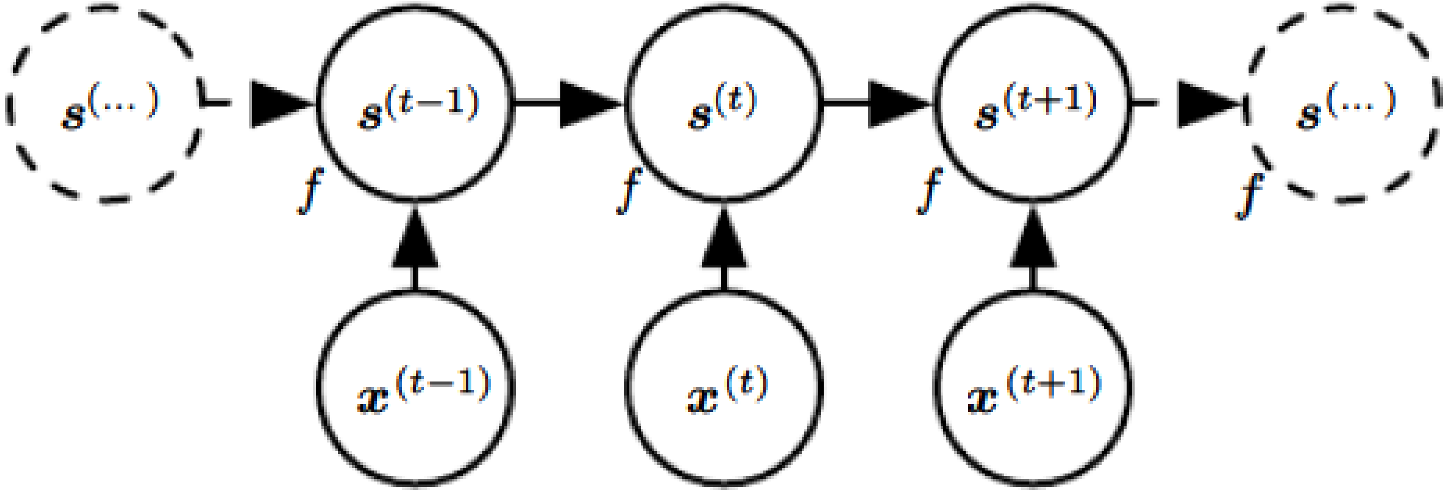
A typical dynamic system



$$s^{(t+1)} = f(s^{(t)}; \theta)$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

A system driven by external data



$$s^{(t+1)} = f(s^{(t)}, x^{(t+1)}; \theta)$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Compact view

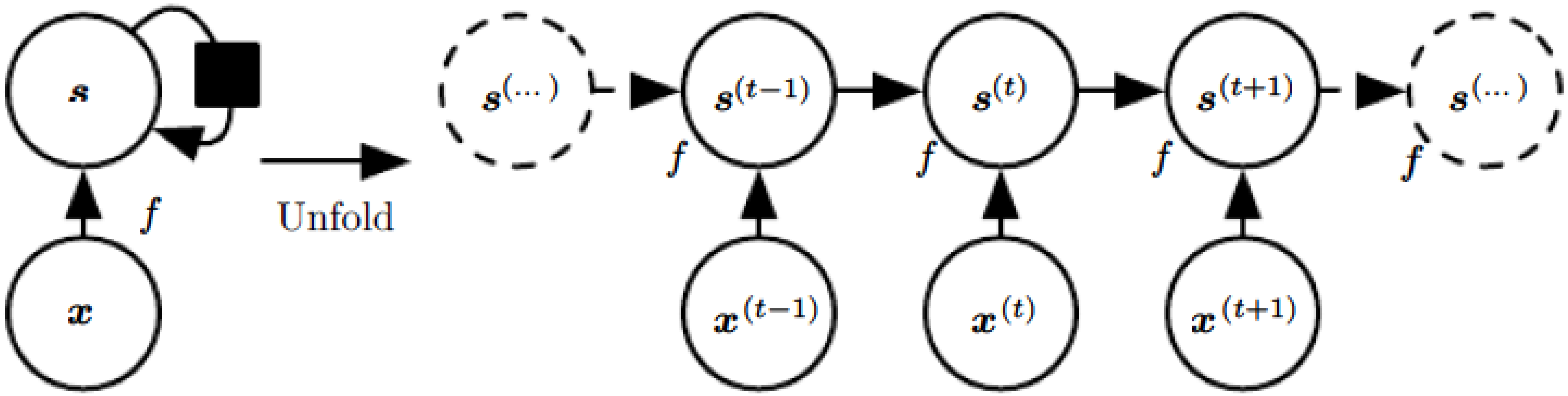


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Figure from *Deep Learning*,
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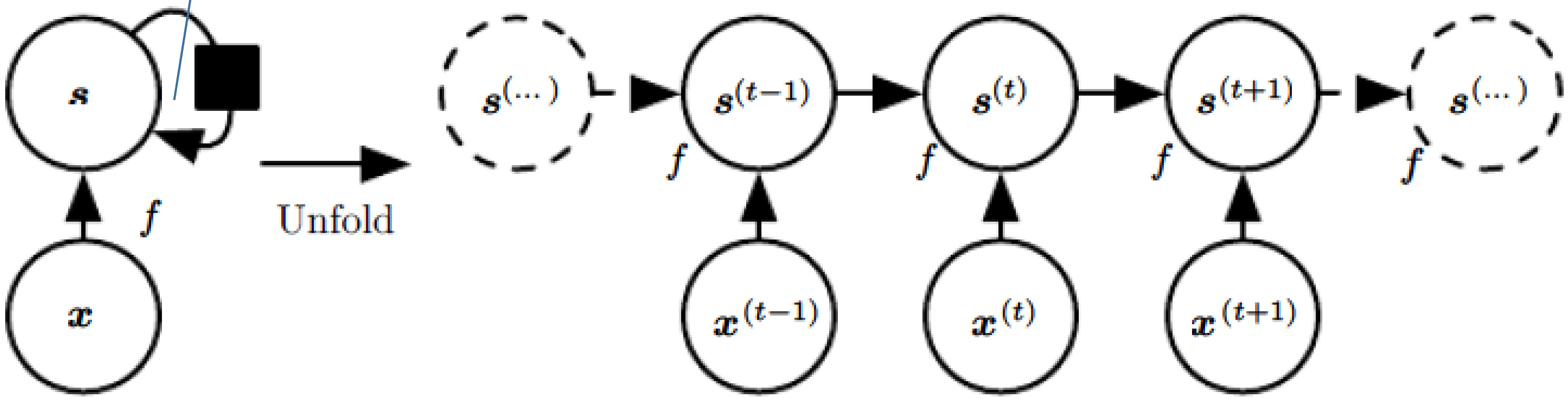
Compact view

square: one step time delay



$$s^{(t+1)} = f(s^{(t)}, x^{(t+1)}; \theta)$$

Key: the same f and θ
for all time steps

Figure from *Deep Learning*,
Goodfellow, Bengio and Courville

Recurrent neural networks (RNN)

Recurrent neural networks

- Use the same computational function and parameters across different time steps of the sequence
- Each time step: takes the input entry and the previous hidden state to compute the output entry
- Loss: typically computed every time step

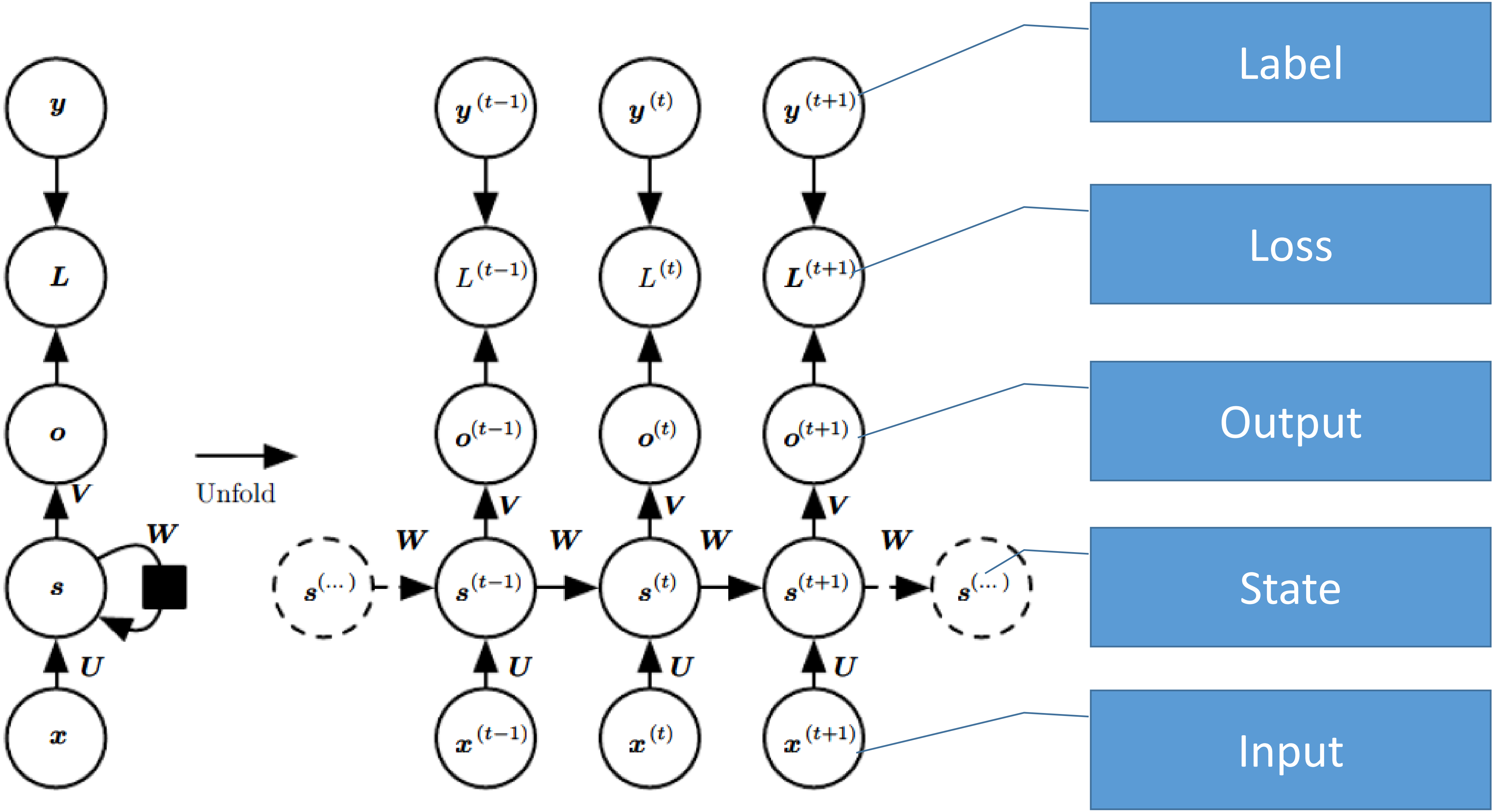
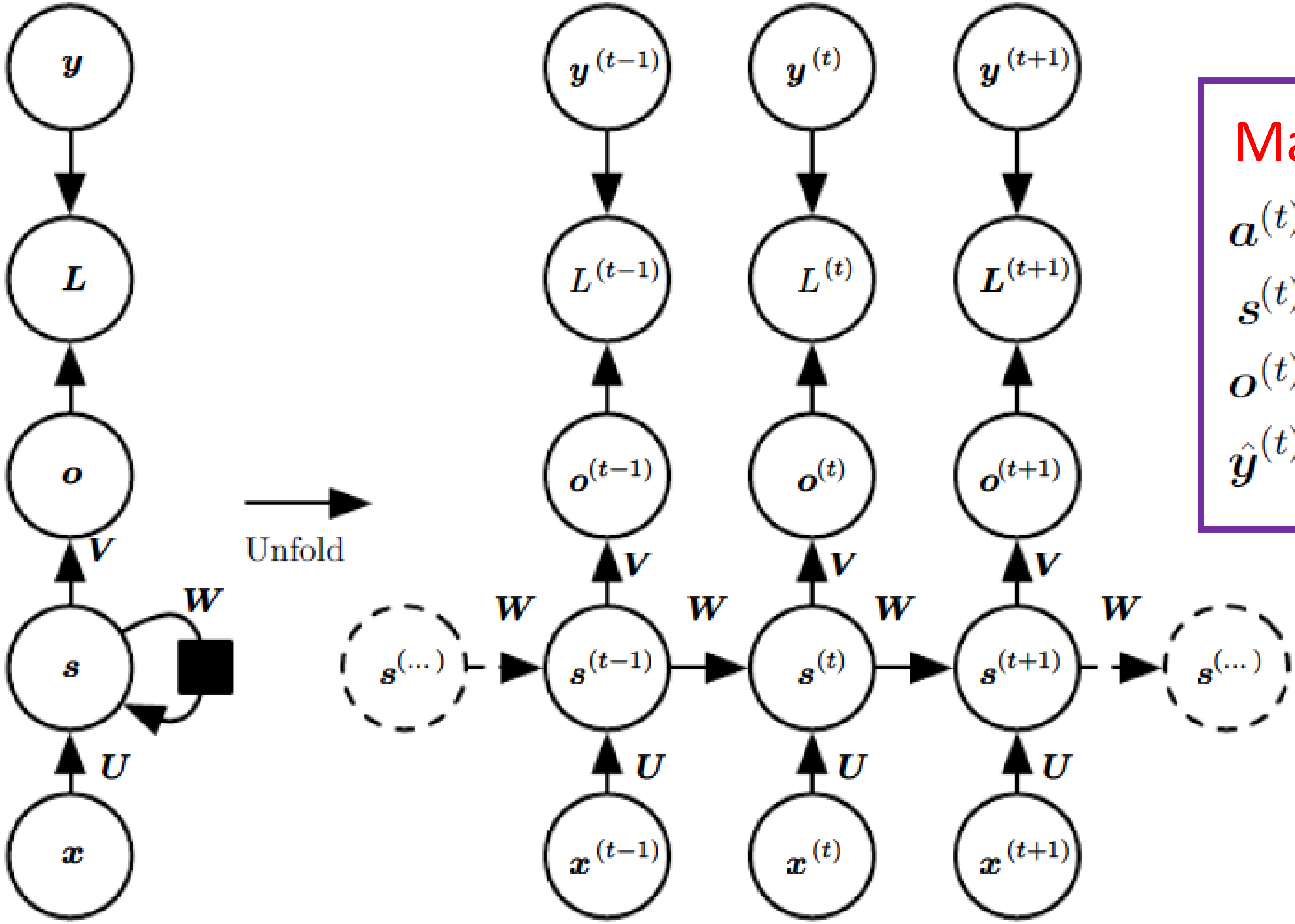


Figure from *Deep Learning*, by Goodfellow, Bengio and Courville



Math formula:

$$
\begin{aligned}
\boldsymbol{a}^{(t)} &= \boldsymbol{b} + \boldsymbol{W}\boldsymbol{s}^{(t-1)} + \boldsymbol{U}\boldsymbol{x}^{(t)} \\
\boldsymbol{s}^{(t)} &= \tanh(\boldsymbol{a}^{(t)}) \\
\boldsymbol{o}^{(t)} &= \boldsymbol{c} + \boldsymbol{V}\boldsymbol{s}^{(t)} \\
\hat{\boldsymbol{y}}^{(t)} &= \mathrm{softmax}(\boldsymbol{o}^{(t)})
\end{aligned}
$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Advantage

- Hidden state: a lossy summary of the past
- Shared functions and parameters: greatly reduce the capacity and good for generalization in learning
- Explicitly use the prior knowledge that the sequential data can be processed by in the same way at different time step (e.g., NLP)

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- Yet still powerful (actually universal): any function computable by a Turing machine can be computed by such a recurrent network of a finite size (see, e.g., Siegelmann and Sontag (1995))

Training RNN

- Principle: unfold the computational graph, and use backpropagation
- Called back-propagation through time (BPTT) algorithm
- Can then apply any general-purpose gradient-based techniques

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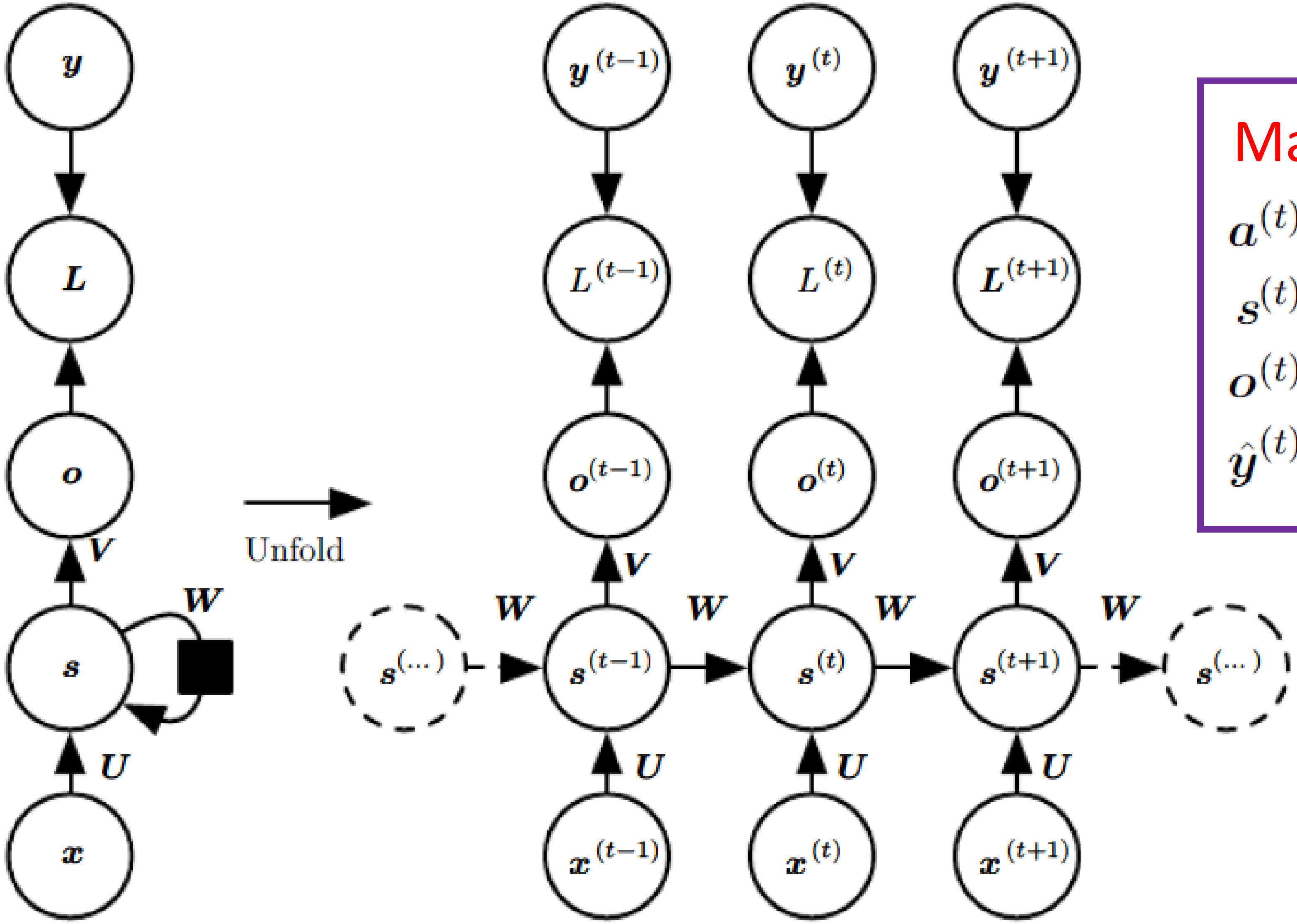
- Conceptually: first compute the gradients of the internal nodes, then compute the gradients of the parameters



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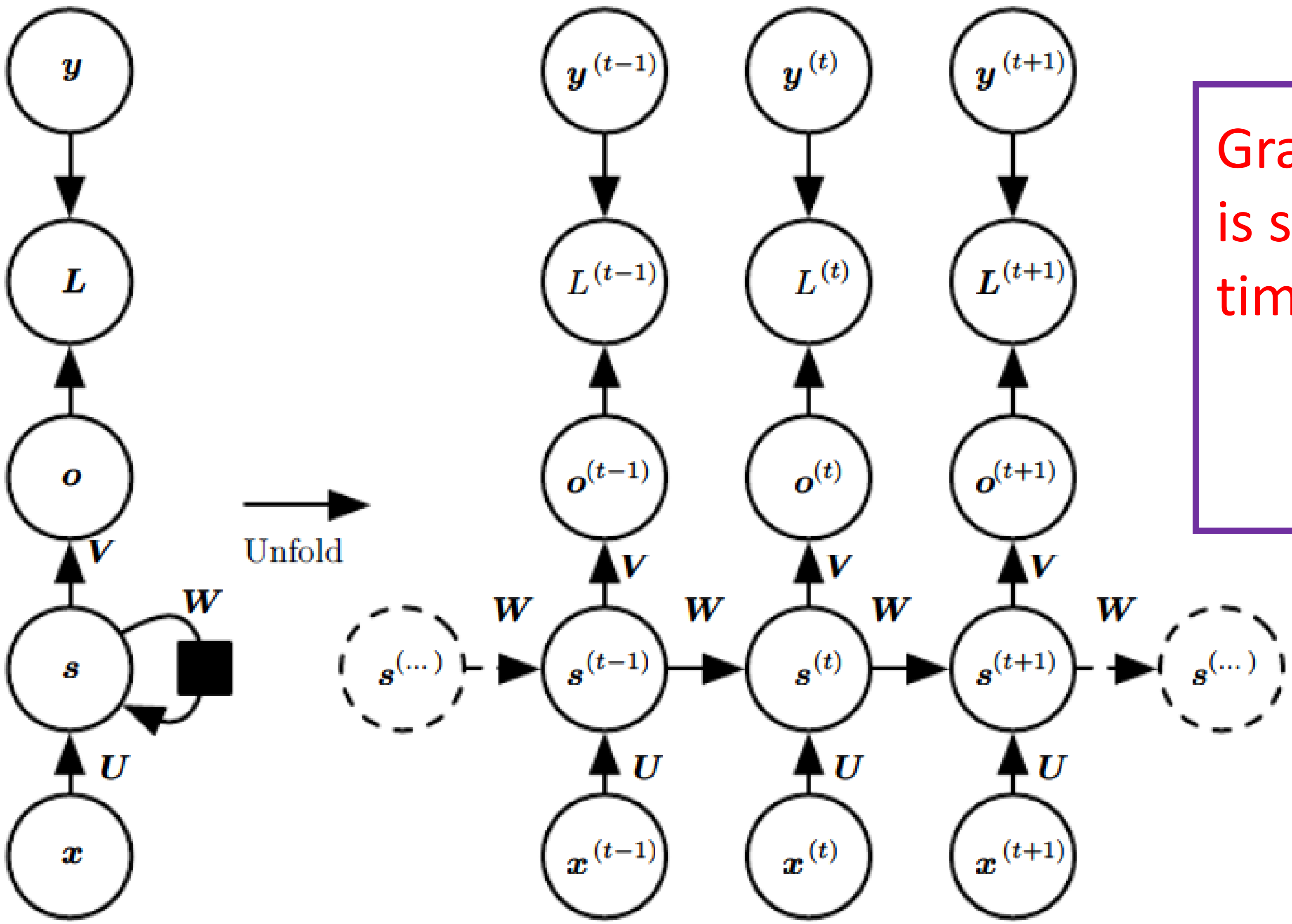
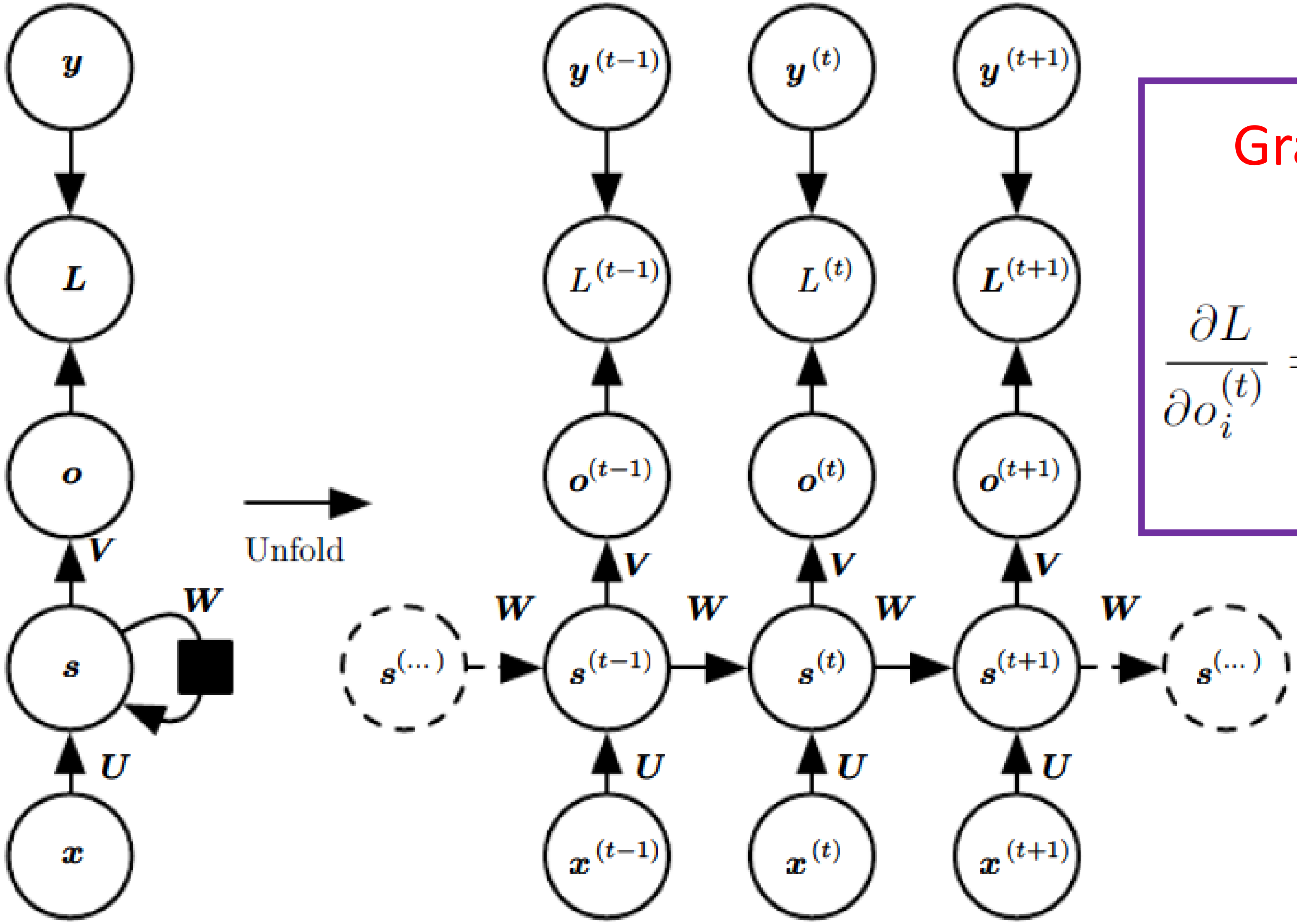


Gradient at $L^{(t)}$: (total loss is sum of those at different time steps)

$$\frac{\partial L}{\partial L^{(t)}} = 1.$$

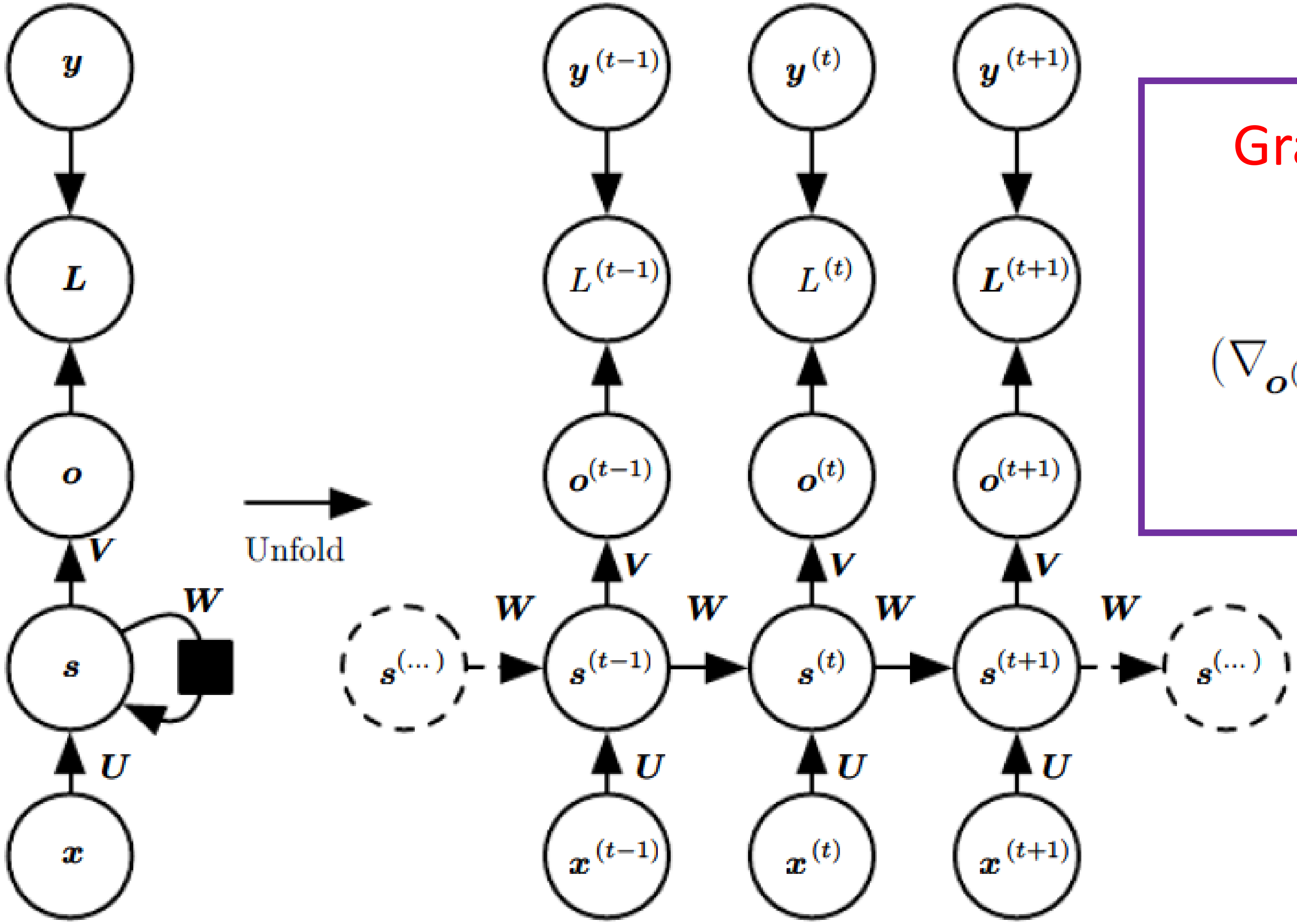
Figure from *Deep Learning*, Goodfellow, Bengio and Courville



Gradient at $o^{(t)}$:

$$\frac{\partial L}{\partial o_i^{(t)}} = \frac{\partial L}{\partial L^{(t)}} \frac{\partial L^{(t)}}{\partial o_i^{(t)}} = \hat{y}_i^{(t)} - \mathbf{1}_{i,y^{(t)}}$$

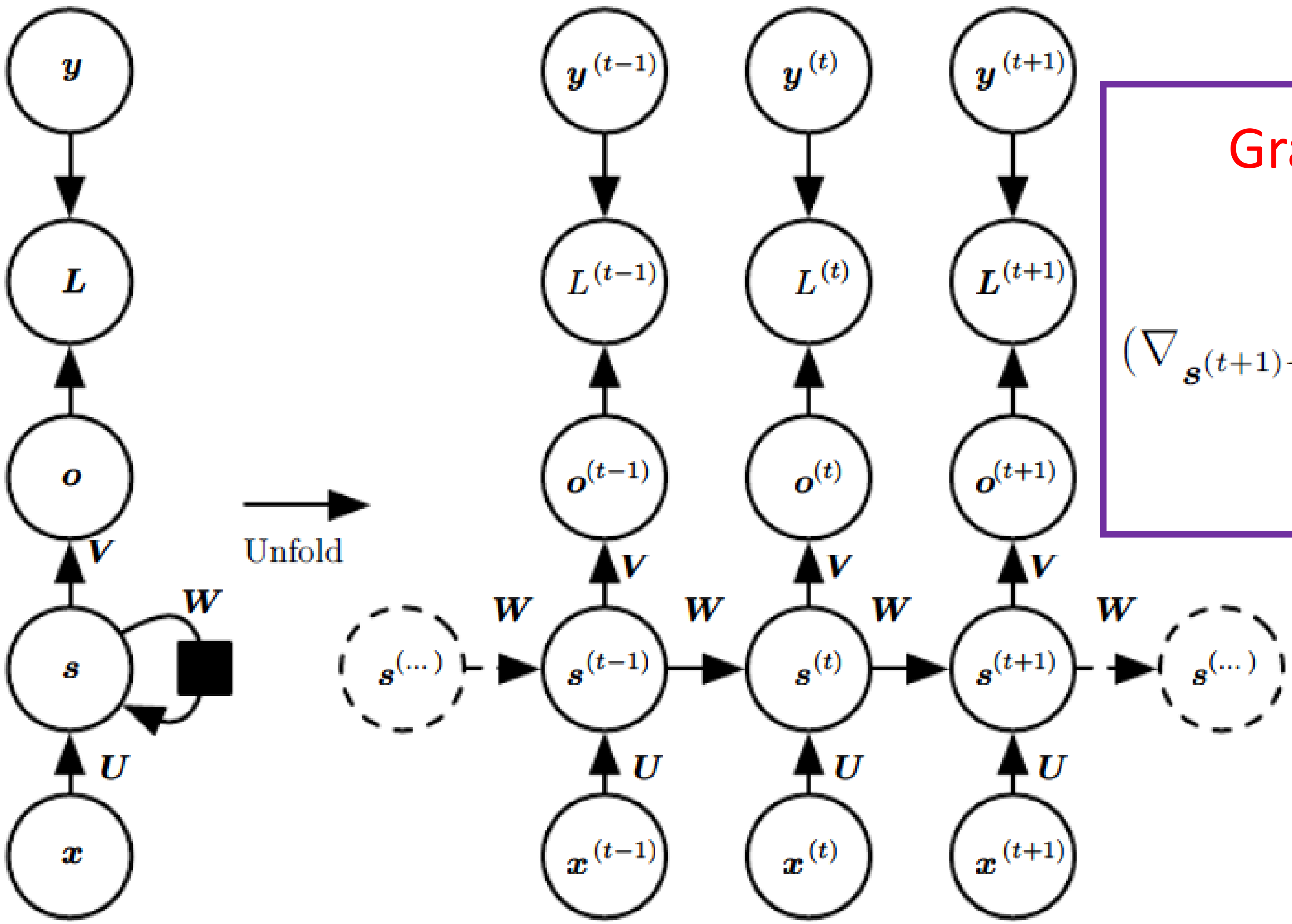
Figure from *Deep Learning*, Goodfellow, Bengio and Courville



Gradient at $s^{(\tau)}$:

$$\left(\nabla_{\boldsymbol{o}^{(\tau)}} L\right) \frac{\partial \boldsymbol{o}^{(\tau)}}{\partial \boldsymbol{s}^{(\tau)}} = \left(\nabla_{\boldsymbol{o}^{(\tau)}} L\right) \boldsymbol{V}$$

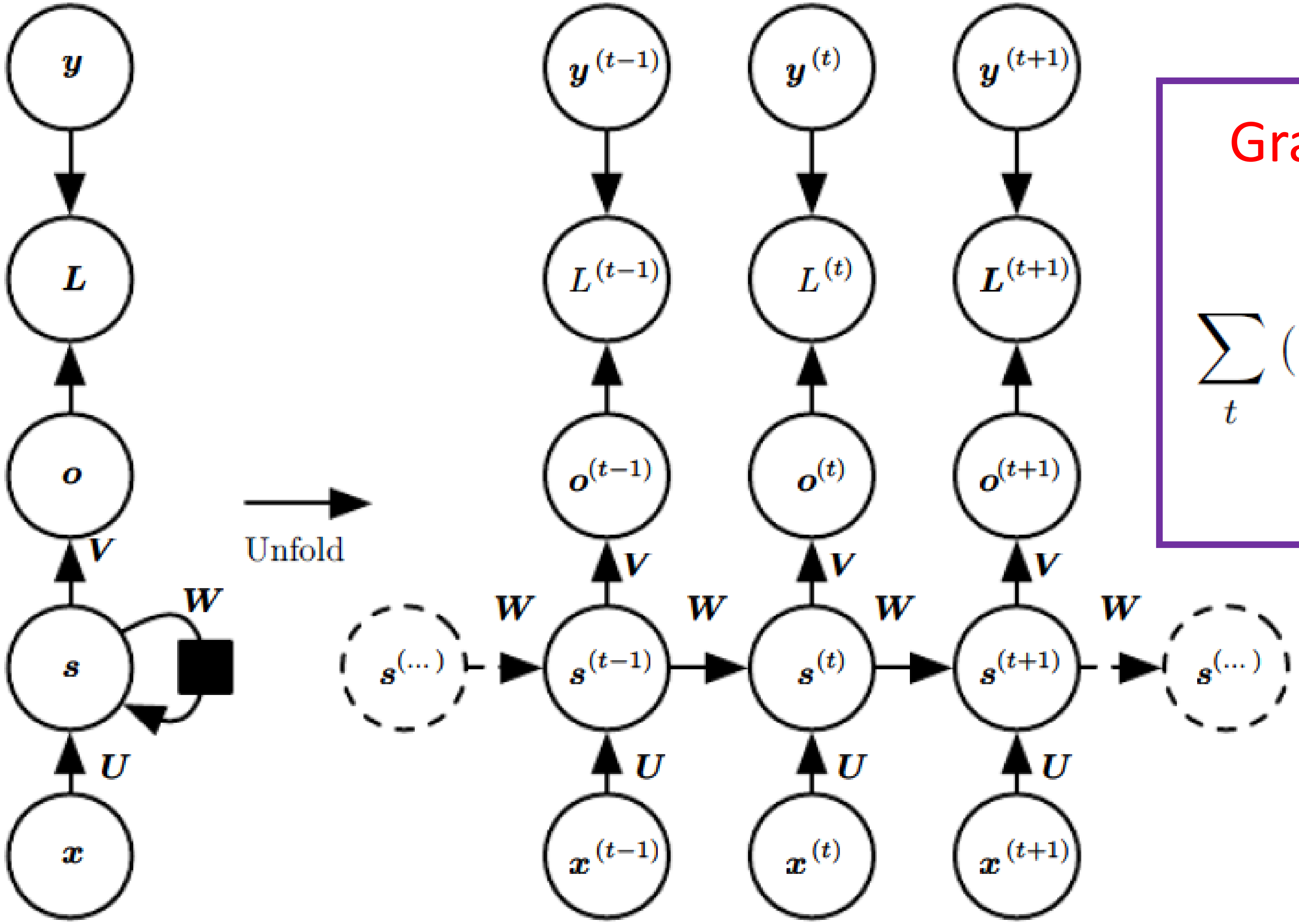
Figure from *Deep Learning*, Goodfellow, Bengio and Courville



Gradient at $s^{(t)}$:

$$(\nabla_{\boldsymbol{s}^{(t+1)}} L) \frac{\partial \boldsymbol{s}^{(t+1)}}{\partial \boldsymbol{s}^{(t)}} + (\nabla_{\boldsymbol{o}^{(t)}} L) \frac{\partial \boldsymbol{o}^{(t)}}{\partial \boldsymbol{s}^{(t)}}$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville



Gradient at parameter V:

$$\sum_t \left(\nabla_{\boldsymbol{o}^{(t)}} L\right) \frac{\partial \boldsymbol{o}^{(t)}}{\partial \boldsymbol{V}} = \sum_t \left(\nabla_{\boldsymbol{o}^{(t)}} L\right) \boldsymbol{s}^{(t)\top}$$

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Variants of RNN

RNN

- Use the same computational function and parameters across different time steps of the sequence
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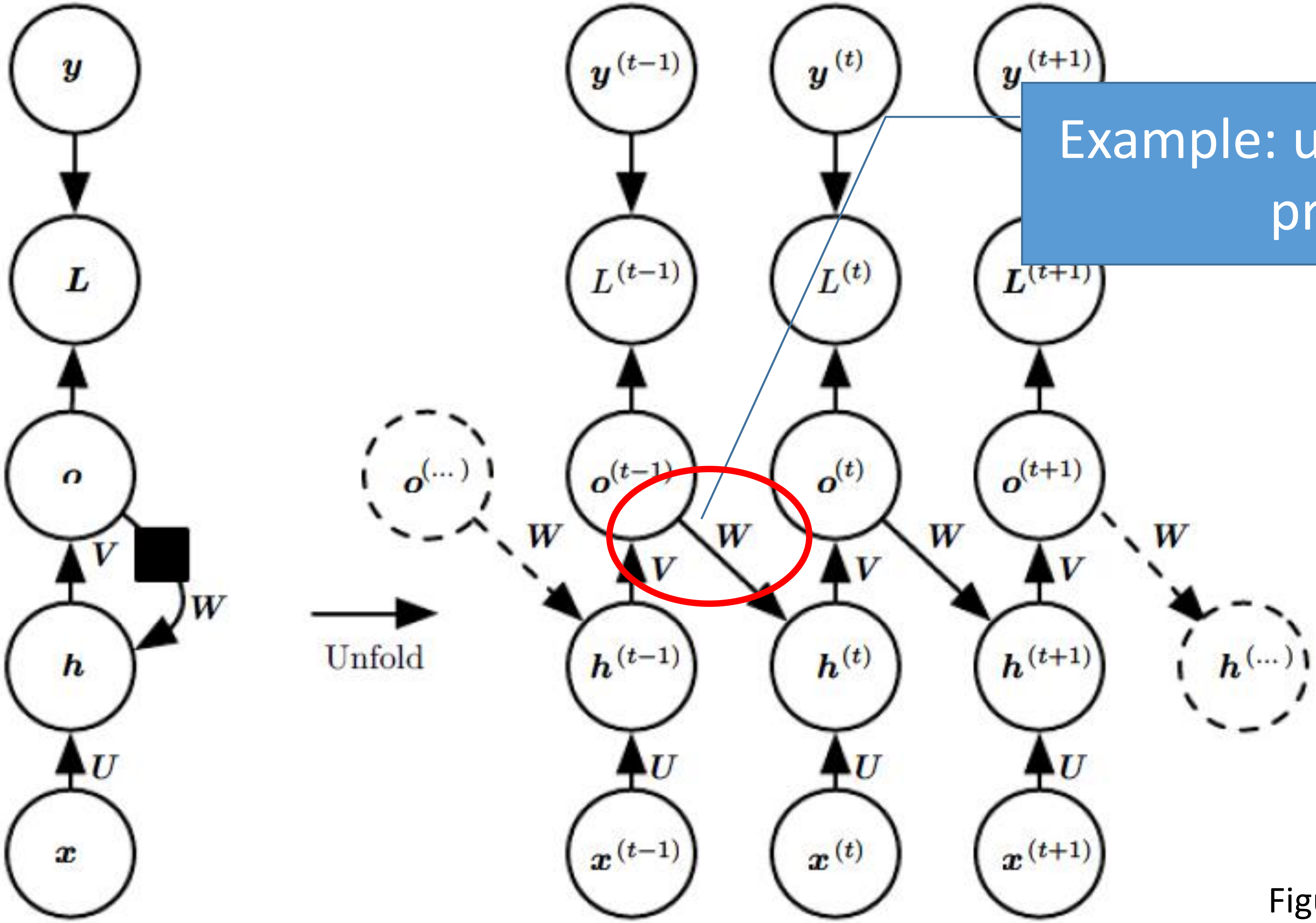
- Loss: typically computed every time step

- Many variants
  - Information about the past can be in many other forms
  - Only output at the end of the sequence



Example: use the output at the previous step

Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Example: only output at the end

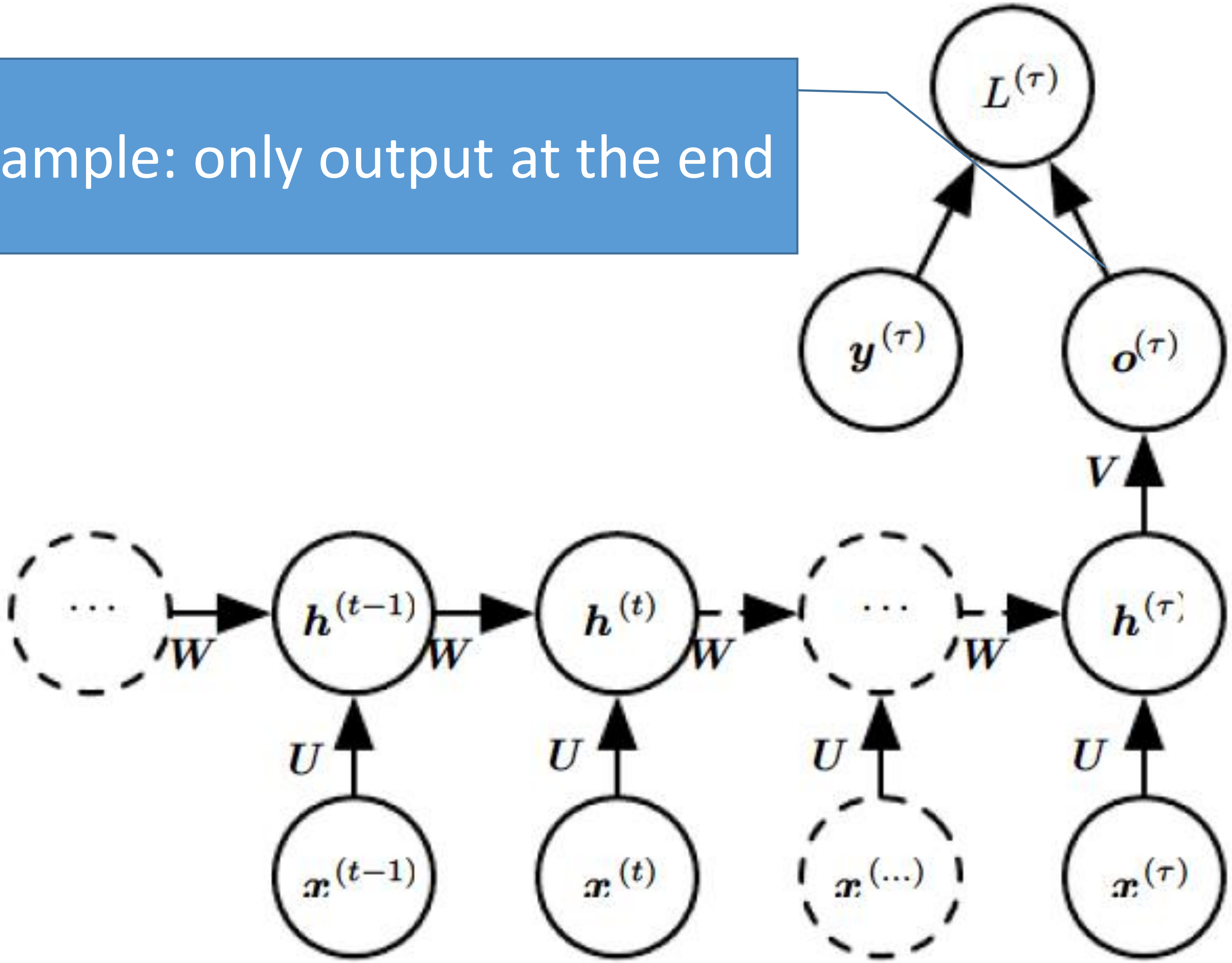


Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Bidirectional RNNs

- Many applications: output at time t may depend on the whole input sequence
- Example in speech recognition: correct interpretation of the current sound may depend on the next few phonemes, potentially even the next few words
- Bidirectional RNNs are introduced to address this

BiRNNs

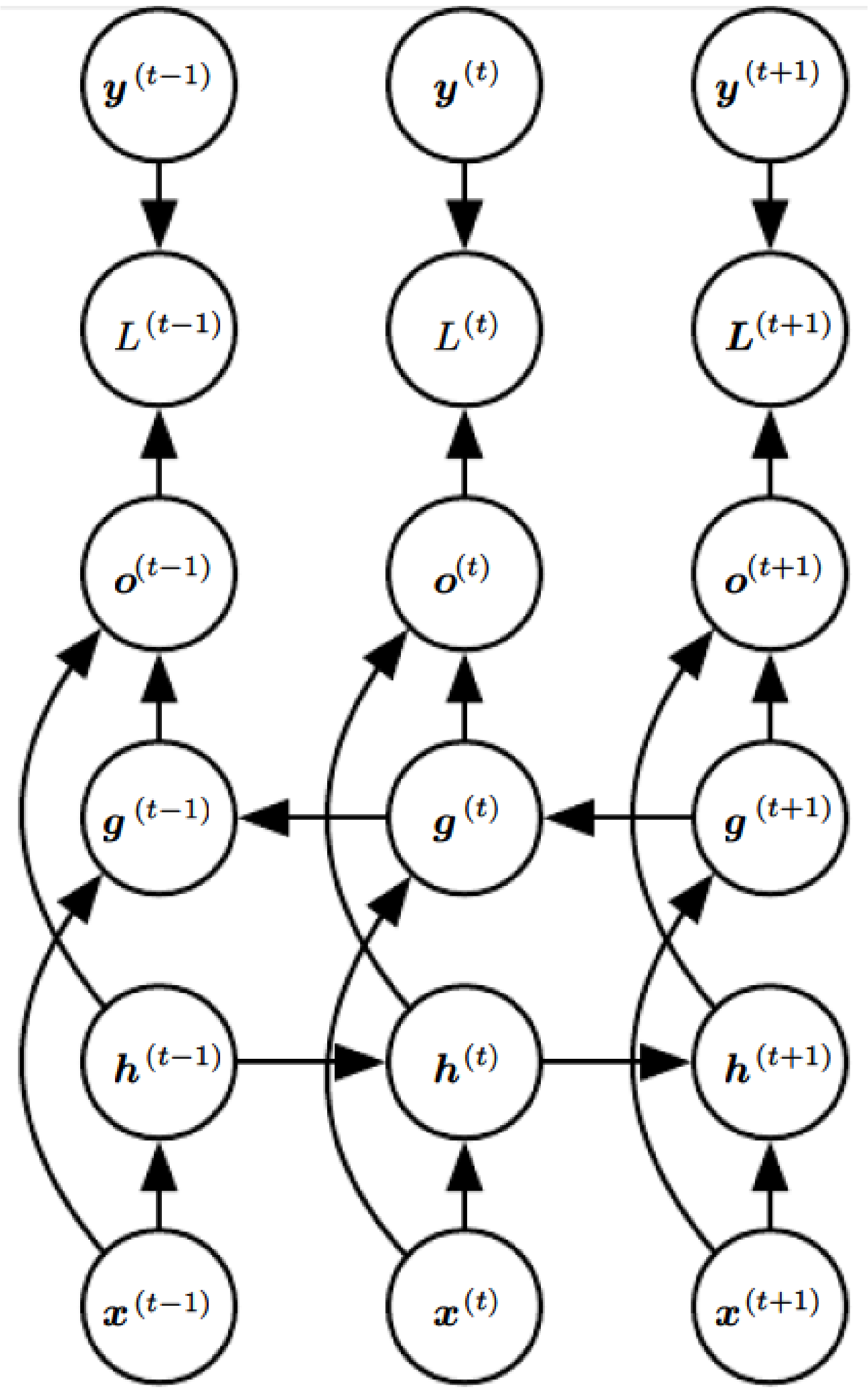
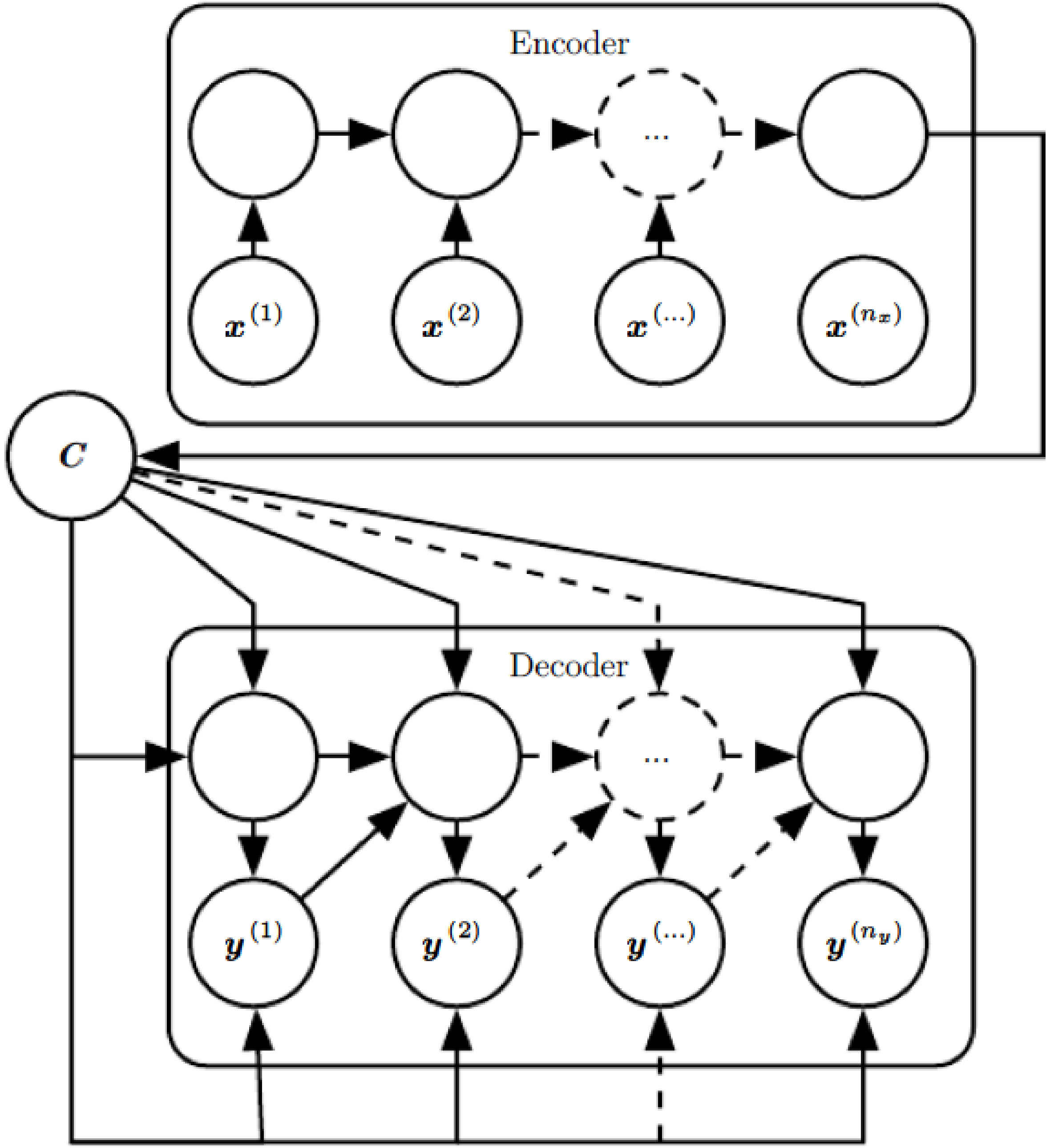


Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Encoder-decoder RNNs

- RNNs: can map sequence to one vector; or to sequence of same length
- What about mapping sequence to sequence of different length?
- Example: speech recognition, machine translation, question answering, etc



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