Deep Learning Basics
Lecture 9: Recurrent Neural Networks

Princeton University COS 495
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Introduction
Recurrent neural networks

• Dates back to (Rumelhart et al., 1986)
• A family of neural networks for handling sequential data, which involves variable length inputs or outputs
• Especially, for natural language processing (NLP)
Sequential data

• Each data point: A sequence of vectors $x^{(t)}$, for $1 \leq t \leq \tau$
• Batch data: many sequences with different lengths $\tau$
• Label: can be a scalar, a vector, or even a sequence

• Example
  • Sentiment analysis
  • Machine translation
Example: machine translation

Economic growth has slowed down in recent years.

Das Wirtschaftswachstum hat sich in den letzten Jahren verlangsamt.

Economic growth has slowed down in recent years.

La croissance économique s'est ralentie ces dernières années.
More complicated sequential data

• Data point: two dimensional sequences like images
• Label: different type of sequences like text sentences

• Example: image captioning
Image captioning

Figure from the paper “DenseCap: Fully Convolutional Localization Networks for Dense Captioning”, by Justin Johnson, Andrej Karpathy, Li Fei-Fei
Computational graphs
A typical dynamic system

\[ s(t+1) = f(s(t); \theta) \]

Figure from *Deep Learning*, Goodfellow, Bengio and Courville
A system driven by external data

\[ s^{(t+1)} = f(s^{(t)}, x^{(t+1)}; \theta) \]

Figure from *Deep Learning*, Goodfellow, Bengio and Courville
Compact view

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Figure from *Deep Learning*, Goodfellow, Bengio and Courville
Compact view

\[ s(t+1) = f(s(t), x(t+1); \theta) \]

Key: the same \( f \) and \( \theta \) for all time steps

square: one step time delay

Figure from *Deep Learning*, Goodfellow, Bengio and Courville
Recurrent neural networks (RNN)
Recurrent neural networks

• Use the same computational function and parameters across different time steps of the sequence
• Each time step: takes the input entry and the previous hidden state to compute the output entry
• Loss: typically computed every time step
Recurrent neural networks

Figure from *Deep Learning*, by Goodfellow, Bengio and Courville
Math formula:

\[ a^{(t)} = b + W s^{(t-1)} + U x^{(t)} \]
\[ s^{(t)} = \tanh(a^{(t)}) \]
\[ o^{(t)} = c + V s^{(t)} \]
\[ \hat{y}^{(t)} = \text{softmax}(o^{(t)}) \]
Advantage

• Hidden state: a lossy summary of the past
• Shared functions and parameters: greatly reduce the capacity and good for generalization in learning
• Explicitly use the prior knowledge that the sequential data can be processed by in the same way at different time step (e.g., NLP)
Advantage

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• Shared functions and parameters: greatly reduce the capacity and good for generalization in learning
• Explicitly use the prior knowledge that the sequential data can be processed by in the same way at different time step (e.g., NLP)

• Yet still powerful (actually universal): any function computable by a Turing machine can be computed by such a recurrent network of a finite size (see, e.g., Siegelmann and Sontag (1995))
Training RNN

- Principle: unfold the computational graph, and use backpropagation
- Called back-propagation through time (BPTT) algorithm
- Can then apply any general-purpose gradient-based techniques
Training RNN

• Principle: unfold the computational graph, and use backpropagation
• Called back-propagation through time (BPTT) algorithm
• Can then apply any general-purpose gradient-based techniques

• Conceptually: first compute the gradients of the **internal nodes**, then compute the gradients of the **parameters**
Recurrent neural networks

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Gradient at $L^{(t)}$: (total loss is sum of those at different time steps)

$$\frac{\partial L}{\partial L^{(t)}} = 1.$$
Gradient at $o^{(t)}$:

$$\frac{\partial L}{\partial o_i^{(t)}} = \frac{\partial L}{\partial L^{(t)}} \frac{\partial L^{(t)}}{\partial o_i^{(t)}} = \hat{y}_i^{(t)} - 1_{i,y^{(t)}}$$

Figure from Deep Learning, Goodfellow, Bengio and Courville
Gradient at $s^{(τ)}$:

$\left(\nabla_{o^{(τ)}} L\right) \frac{∂O^{(τ)}}{∂s^{(τ)}} = \left(\nabla_{o^{(τ)}} L\right) V$
Gradient at $s^{(t)}$:

$$\left( \nabla_{s^{(t+1)}} L \right) \frac{\partial s^{(t+1)}}{\partial s^{(t)}} + \left( \nabla_{o^{(t)}} L \right) \frac{\partial o^{(t)}}{\partial s^{(t)}}$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville
Gradient at parameter $V$:

$$\sum_t \left( \nabla_{o(t)} L \right) \frac{\partial o(t)}{\partial V} = \sum_t \left( \nabla_{o(t)} L \right) s^{(t)} \top$$

Figure from *Deep Learning*, Goodfellow, Bengio and Courville
Variants of RNN
RNN

• Use **the same** computational function and parameters across different time steps of the sequence
• Each time step: takes the input entry **and the previous hidden state** to compute the output entry
• Loss: typically computed every time step

• Many variants
  • Information about the past can be in many other forms
  • Only output at the end of the sequence
Example: use the output at the previous step
Example: only output at the end
Bidirectional RNNs

• Many applications: output at time $t$ may depend on the whole input sequence

• Example in speech recognition: correct interpretation of the current sound may depend on the next few phonemes, potentially even the next few words

• Bidirectional RNNs are introduced to address this
BiRNNs

Figure from *Deep Learning*, Goodfellow, Bengio and Courville
Encoder-decoder RNNs

• RNNs: can map sequence to one vector; or to sequence of same length

• What about mapping sequence to sequence of different length?

• Example: speech recognition, machine translation, question answering, etc