NAME:

## Computer Science 426 Exam 2 4/30/09, 1:30PM-2:50PM

This test is 5 questions. Do all of your work on these pages, giving the answers in the spaces provided (use the back for scratch space). This is a closed-book exam -- you may use one-page of notes with writing on one side during the exam. Please write out and sign the Honor Code pledge before turning in the test.
"I pledge my honor that I have not violated the Honor Code during this examination."

| Question | Score |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| Total |  |

## Q1:Transformations (20 Points)

Describe the transformation that takes the 2 D shape shown on the left to the 2 D shape shown on the right (e.g., $\mathrm{P} \rightarrow \mathrm{P}$ ', for every point P ). For each transformation, tell whether it is linear, affine, or projective, and provide a matrix M that applies the transformation (i.e., $\mathrm{P}^{\prime}=\mathrm{MP}$ ). Assume all coordinates are integers. You may express your matrices in terms of other matrices without multiplying them together, and you may represent matrices executing translations by $\mathrm{T}(\mathrm{tx}, \mathrm{ty})$, scales by $S(s x$, sy), and counterclockwise rotations by $R(\Theta)$.



## Q2: Lighting (15 Points)

a) Given the Phong lighting equation and the scene drawn below, calculate the radiance resulting from the ray of light leaving the point light source (shown on the right), reflecting off the surface at point P , and arriving at the camera (shown on the left). You can assume that the point light source emits white light with intensity $=1$ and that parameters governing light attenuation with distance are $c a=0, l a=1$, and $q a=2$. Also, you can assume that the surface reflects $50 \%$ of the light diffusely and $20 \%$ specularly for all wavelengths with a shininess coefficient of 2. Ignoring $\mathrm{I}_{\mathrm{E}}$ and $\mathrm{K}_{\mathrm{A}} \mathrm{I}_{\mathrm{AL}}$, what is the radiance along the ray shown as the light arrives at the camera (for any wavelength of light)? Your answer should be a mathematical value or an expression with no variables -- you may have trigonometric expressions as part of your answer -- e.g., $\sin \left(45^{\circ}\right)$. Show your work in the space on the right.

$$
I=I_{E}+K_{A} I_{A L}+K_{D}(N \bullet L) I_{L}+K_{S}(V \bullet R)^{n} I_{L}
$$


b) What is the justification for $\mathrm{K}_{\mathrm{A}} \mathrm{I}_{\mathrm{AL}}$ in the Phong lighting equation above? (one sentence)
c) What is the justification for $\mathrm{N} \cdot \mathrm{L}$ in the Phong lighting equation above (one sentence, and provide a picture to support your answer)?

## Q3: Polygon Rendering Pipeline (20 Points)

a) What is Gouraud shading? (one phrase or sentence)
b) Please list three distinct, significant problems with Gouraud shading. For each one, please provide a quick sketch showing an instance of the problem, a phrase or sentence describing the visual artifacts that can result, and a phrase or sentence describing the cause of the problem.
i)
ii)
iii)
c) Besides hardware acceleration, why is rendering a scene using the triangle rendering pipeline with Gouraud shading and z-buffering usually faster than ray casting? (one or two sentences)
d) Give a description of a scene for which ray casting will be faster than the triangle rendering pipeline with Gouraud shading and z-buffering. (one or two sentences)

## Q4: Radiosity (20 Points)

The radiosity equation is provided below in matrix form ( $\mathrm{Ax}=\mathrm{b}$ ):

$$
\left[\begin{array}{ccccc}
1-\rho_{1} F_{1,1} & \cdot & \cdot & \cdot & -\rho_{1} F_{1, n} \\
-\rho_{2} F_{2,1} & 1-\rho_{2} F_{2,2} & \cdot & \cdot & -\rho_{2} F_{2, n} \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
-\rho_{n-1} F_{n-1,1} & \cdot & \cdot & \cdot & -\rho_{n-1} F_{n-1, n} \\
-\rho_{n} F_{n, 1} & \cdot & \cdot & \cdot & 1-\rho_{n} F_{n, n}
\end{array}\right]\left[\begin{array}{c}
B_{1} \\
B_{2} \\
\cdot \\
\cdot \\
\cdot \\
B_{n}
\end{array}\right]=\left[\begin{array}{c}
E_{1} \\
E_{2} \\
\cdot \\
\cdot \\
\cdot \\
E_{n}
\end{array}\right]
$$

a) What is the meaning of each variable (a short phrase):
$\rho_{\mathrm{i}}=$
$\mathrm{F}_{\mathrm{ij}}=$
$\mathrm{B}_{\mathrm{i}}=$
$\mathrm{E}_{\mathrm{i}}=$
b) Give a reason why $\mathrm{F}_{\mathrm{ij}}$ could be zero (a short phrase):
c) Give a reason why $\mathrm{E}_{\mathrm{i}}$ could be non-zero (a short phrase):
d) Do you expect that it takes longer to compute all the elements of this matrix equation or to solve the system of equations after the elements have been computed? Please explain your answer.

## Q5: Passive Dynamics (25 Points)

This question concerns computation of the path of a particle with passive dynamics. Assume that a particle has very little mass (ignore momentum) and starts with zero velocity and acceleration at the position marked "Start" $(5,1)$ in the force field below (where the vector originating at each point on a grid represents the direction and magnitude of the force at that point). For each of the following methods, please draw the paths of the simulated particle trajectory as computed with the given method and step size on the provided grids (draw a clear dot at every computed position and connect those dots with a curve). Then, describe in a sentence or two how the method works in the space provided under the grids.
a) Euler's method (with step size 1 meter):

b) Midpoint method (with step size 1 meter):

(with step size 2 meters):

(with step size 2 meters):

c) Adaptive step size method (with initial step size 2 meters):

d) Imagine that an obstacle represented by a mesh with N triangles (where $\mathrm{N}>10^{6}$ ) were present in the middle of the space above. Please describe an efficient way to detect collisions between the particle's path and the obstacle during the passive dynamic simulation. Your method must have expected running time less than $\mathrm{O}(\mathrm{N})$ per update to the particle's position.

