NAME:

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## Computer Science 426 Midterm 2 4/29/04, 1:30PM-2:50PM

This test is 5 questions of equal weight. Do all of your work on these pages (use the back for scratch space), giving the answer in the space provided. This is a closed-book exam -- you may use one-page of notes with writing on both sides during the exam. Please write out and sign the Honor Code pledge before turning in the test.
"I pledge my honor that I have not violated the Honor Code during this examination."

| Question Score |
| :--- |
| 1  <br> 2  <br> 3  <br> 4  <br> 5  <br> Total  |

## Q1: 3D Object Representations

We learned about several different 3D object representations with different properties. For each of the following scenarios, please describe which 3D object representation you think would be best for the object written in capital letters and give a one sentence explanation why.
(a) Rendering images of a CLOUD for a movie.
(b) Visualization of the body exterior of the next model MERCEDES for design evaluation.
(c) Animation of a deforming BLOBBY OBJECT (e.g., a clay ball) with very smooth surfaces.
(d) Rank the following object representations from smallest (" 1 ") to largest (" 6 ") according to how much storage they require to represent a 1 m sphere within 1 mm accuracy. For each representation, give a brief explanation for your ranking.

## Rank

$\nabla$
Cubic tensor product spline surface

Subdivision surface

Voxels

Quadric implicit surface

Octree

Triangle mesh

## Q1: 3D Object Representations (cont)

(e) Describe a simple algorithm to render a CSG model containing only "union" operators with OpenGL (one or two sentences).
(f) Why is it harder to render a CSG model containing "intersection" and "difference" operators in addition to "union" operators? (one or two sentences)
(g) What algorithm is simplest to implement for rendering CSG models with all three types of operators? (one phrase)

## Q2: Parametric Curves

(a) Why do computer graphics applications use cubic polynomials for their piecewise-polynomial parametric curves rather than higher-order polynomials?
(b) On the axes below, draw the blending functions $\left(B_{0}(u), B_{1}(u), B_{2}(u), B_{3}(u)\right.$, and $\left.B_{4}(u)\right)$ used to define the piecewise-linear curve shown on the right through the control vertices $V_{0}, V_{1}, V_{2}, V_{3}$, and $\mathrm{V}_{4}$, where: $\mathrm{Q}(\mathrm{u})=\mathrm{B}_{0}(\mathrm{u})^{*} \mathrm{~V}_{0}+\mathrm{B}_{1}(\mathrm{u}) * \mathrm{~V}_{1}+\mathrm{B}_{2}(\mathrm{u})^{*} \mathrm{~V}_{2}+\mathrm{B}_{3}(\mathrm{u}) * \mathrm{~V}_{3}+\mathrm{B}_{4}(\mathrm{u}) * \mathrm{~V}_{4}$.


(c) Circle all the properties that these blending functions imply:

Local control
Curve lies within convex hull of control vertices
Interpolation of control vertices
$\mathrm{G}^{0}$ continuity at the joints
$\mathrm{G}^{1}$ continuity at the joints
$\mathrm{G}^{2}$ continuity at the joints
$\mathrm{C}^{0}$ continuity at the joints
$\mathrm{C}^{1}$ continuity at the joints
$\mathrm{C}^{2}$ continuity at the joints

## Q2: Parametric Curves (cont)

(d) Draw the uniform piecewise-cubic B-Spline defined by the following user-specified control points (be careful to satisfy all properties).

(e) Draw the piecewise-cubic Catmull-Rom spline defined by the following user-specified control points (be careful to satisfy all properties). Hint: include new "phantom" control vertices.

(f) The equation defining a bi-cubic tensor-product B-Spline surface patch is given below, where $B_{i}(u)$ and $B_{j}(v)$ are the $B-S p l i n e ~ b l e n d i n g ~ f u n c t i o n s ~ p r o v i d e d ~ o n ~ t h e ~ n e x t ~ p a g e, ~$ and $V_{i, j}$ are control vertices. Show that the curve defined along any constant value of " $u$ " is a cubic B-Spline.

$$
Q(u, v)=\sum_{i=0}^{3} \sum_{j=0}^{3} B_{i}(u) B_{j}(v) V_{i, j}
$$



## Q3: B-Spline Curves

The uniform piecewise-cubic B-Spline blending functions for each of ( $\mathrm{n}-3$ ) curve segments are given below for $0 \leq \mathrm{u} \leq 1$ and $3 \leq \mathrm{i} \leq \mathrm{n}$, where n is the number of control vertices:

$$
\begin{aligned}
& \mathrm{B}_{\mathrm{i}-3}(\mathrm{u})=1 / 6-1 / 2 \mathrm{u}+1 / 2 \mathrm{u}^{2}-1 / 6 \mathrm{u}^{3} \\
& \mathrm{~B}_{\mathrm{i}-2}(\mathrm{u})=4 / 6-\mathrm{u}^{2}+1 / 2 \mathrm{u}^{3} \\
& \mathrm{~B}_{\mathrm{i}-1}(\mathrm{u})=1 / 6+1 / 2 \mathrm{u}+1 / 2 \mathrm{u}^{2}-1 / 2 \mathrm{u}^{3} \\
& \mathrm{~B}_{\mathrm{i}-0}(\mathrm{u})=1 / 6 \mathrm{u}^{3}
\end{aligned}
$$

(a) Write the matrix equation for a point $\mathrm{Q}(\mathrm{u})$ on a uniform piecewise-cubic $\mathrm{B}-$ Spline curve based on these blending functions.
(b) Explain how you know that uniform piecewise-cubic B-Spline curves have local control (one sentence).
(c) Use the blending functions provided above to prove that all uniform piecewise-cubic B-Spline curve segments must lie within the convex hull of their control vertices.

## Q3: B-Spline Curves (cont)

(d) Use the blending functions provided above to prove that all uniform piecewise-cubic B-Splines are $\mathrm{C}^{0}$ continuous at their joints.
(d) Use the blending functions provided above to prove that all uniform piecewise-cubic B-Splines are $\mathrm{C}^{1}$ continuous at their joints.

## Q4: Rendering Equation

(a) Describe the rendering equation in words (one or two sentences - no equations and no pictures).
(b) Two forms of the rendering equation are provided here.

Please describe why one form has terms V and G and the other does not.

$$
\begin{aligned}
& L\left(x^{\prime} \rightarrow x^{\prime \prime}\right)=L_{e}\left(x^{\prime} \rightarrow x^{\prime \prime}\right)+\int_{S} f_{r}\left(x \rightarrow x^{\prime} \rightarrow x^{\prime \prime}\right) L\left(x \rightarrow x^{\prime}\right) V\left(x, x^{\prime}\right) G\left(x, x^{\prime}\right) d A \\
& L_{o}\left(x^{\prime}, \vec{\omega}^{\prime}\right)=L_{e}\left(x^{\prime}, \vec{\omega}^{\prime}\right)+\int_{\Omega} f_{r}\left(x^{\prime}, \vec{\omega}, \vec{\omega}^{\prime}\right) L_{i}\left(x^{\prime}, \vec{\omega}\right)(\vec{\omega} \bullet \vec{n}) d \vec{\omega}
\end{aligned}
$$

(c) When solving the rendering equation, which of the following terms is most expensive to compute for typical real-world scenes: $\mathrm{G}\left(\mathrm{x}, \mathrm{x}^{\prime}\right), \mathrm{V}\left(\mathrm{x}, \mathrm{x}^{\prime}\right)$, or $\mathrm{f}_{\mathrm{r}}\left(\mathrm{x}->\mathrm{x}^{\prime}->\mathrm{x}^{\prime}\right)$ )? Please explain with one or two sentences.

## Q4: Rendering Equation (cont)

(d) Explain why rendering images with soft shadows requires integration. What quantity is being integrated? What is the integration domain?
(e) For each of the following three rendering algorithms studied in this class, explain how it provides an approximate solution to the rendering equation. In each case, make sure to state the assumptions made in the approximation.

## Ray Tracing:

## Radiosity:

Polygon Rendering Pipeline (OpenGL):

## Q5: Animation

(a) What are space-time constraints (one sentence)?
(b) Please list the input that would be required of a user to specify a single hop for Luxo Jr. using space-time constraints.
(c) Under what conditions are space-time constraints more useful than keyframe animation? (one or two sentences)
(d) Why aren't space-time constraints commonly used at Pixar? (one or two sentences)
(e) Give three examples of natural phenomena that are well modeled by passive dynamics.
(f) What was your favorite in-class video demonstrating active dynamics?

