Topic 2: Lexing and Flexing

COS 320

а

а

а

Compiling Techniques



Princeton University Spring 2016

Lennart Beringer



- Lexical Analysis: Break into tokens (think words, punctuation)
- Syntax Analysis: Parse phrase structure (think document, paragraphs, sentences)
- Semantic Analysis: Calculate meaning

- Goal: break stream of ASCII characters (source/input) into sequence of tokens
- **Token**: sequence of characters treated as a unit (cf. word)

- Goal: break stream of ASCII characters (source/input) into sequence of tokens
- **<u>Token</u>**: sequence of characters treated as a unit (cf. word)
- Each token has a token type (cf. classification verb noun punctuation symbol):

- Goal: break stream of ASCII characters (source/input) into sequence of tokens
- **<u>Token</u>**: sequence of characters treated as a unit (cf. word)
- Each token has a token type (cf. classification verb noun punctuation symbol):

IDENTIFIER foo, x, quicksort, ... NUM 1, 50, -100

REAL	6.7	, 3.9E-33, -4.9		IF	if
SEMI		;		THEN	then
LPAREN	(RPAREN)	EQ =	PLUS +

- Goal: break stream of ASCII characters (source/input) into sequence of tokens
- **Token**: sequence of characters treated as a unit (cf. word)
- Each token has a token type (cf. classification verb noun punctuation symbol):

IDENTIFIERfoo, x, quicksort, ...NUM1, 50, -100REAL6.7, 3.9E-33, -4.9IFifSEMI;THENthenLPAREN(RPAREN)EQ =PLUS +

 Many tokens have associated semantic information: NUM(1), NUM(50), IDENTIFIER(foo), IDENTIFIER(x), but typically not SEMI(;), LPAREN(()

- Goal: break stream of ASCII characters (source/input) into sequence of tokens
- **Token**: sequence of characters treated as a unit (cf. word)
- Each token has a token type (cf. classification verb noun punctuation symbol):

 IDENTIFIER
 foo, x, quicksort, ...
 NUM
 1, 50, -100

 REAL
 6.7, 3.9E-33, -4.9
 IF
 if

 SEMI
 ;
 THEN
 then

 LPAREN
 (
 RPAREN
)
 EQ =
 PLUS +

- Many tokens have associated semantic information: NUM(1), NUM(50), IDENTIFIER(foo), IDENTIFIER(x), but typically not SEMI(;), LPAREN(()
- Definition of tokens (mostly) part of **language definition**
- White space and comments often discarded. Pros/Cons?

Lexical Analysis Example

x = (y + 4.0);

x = (y + 4.0);

IDENTIFIER(x) EQ LPAREN IDENTIFIER(y) PLUS NUM(4.0) RPAREN SEMI

Implementing a Lexical Analyzer (Lexer)

Option 1: write it from scratch:



Implementing a Lexical Analyzer (Lexer)

Option 1: write it from scratch:



Implementing a Lexical Analyzer (Lexer)

Option 1: write it from scratch



Q: how do we describe the tokens for L1, L2, ...?

A: using another language, of course!

Yeah, but how do we describe the tokens of that language???

• An alphabet is a (finite) collection of symbols. Examples: ASCII, {0, 1}, {A, ..Z, a, .. Z}, {0, ..9}

- An alphabet is a (finite) collection of symbols. Examples: ASCII, {0, 1}, {A, ..Z, a, .. Z}, {0, ..9}
- A string/word (over alphabet A) is a <u>finite</u> sequence of symbols from A.

- An alphabet is a (finite) collection of symbols. Examples: ASCII, {0, 1}, {A, ..Z, a, .. Z}, {0, ..9}
- A string/word (over alphabet **A**) is a <u>finite</u> sequence of symbols from **A**.
- A language (over A) is a (finite or infinite) set of strings over A.

- An alphabet is a (finite) collection of symbols. Examples: ASCII, {0, 1}, {A, ..Z, a, .. Z}, {0, ..9}
- A string/word (over alphabet **A**) is a <u>finite</u> sequence of symbols from **A**.
- A language (over A) is a (finite or infinite) set of strings over A.
 Examples:
 - the ML language: set of all strings representing correct ML programs (infinite)
 - the language of ML keywords: set of all strings that are ML keywords (finite)
 - the language of ML tokens: set of all strings that map to ML tokens (infinite)

- An alphabet is a (finite) collection of symbols. Examples: ASCII, {0, 1}, {A, ..Z, a, .. Z}, {0, ..9}
- A string/word (over alphabet A) is a <u>finite</u> sequence of symbols from A.
- A language (over A) is a (finite or infinite) set of strings over A.
 Examples:
 - the ML language: set of all strings representing correct ML programs (infinite)
 - the language of ML keywords: set of all strings that are ML keywords (finite)
 - the language of ML tokens: set of all strings that map to ML tokens (infinite)

<u>Q: How to describe languages?</u> A(for lexing): regular expressions!

REs are **finite descriptions**/representations of (certain) finite or infinite languages, including

- the language of a (programming) language's tokens (eg the language of ML tokens)
- the language describing the language of a (programming) language's tokens,
- the language describing ...

Base cases

Inductive cases: given RE's M and N,

Base cases

- the RE ε (epsilon): the (finite) language containing only the empty string.
- for each symbol a from A, the RE a denotes the (finite) language containing only the string a.

Inductive cases: given RE's M and N,

Base cases

- the RE ε (epsilon): the (finite) language containing only the empty string.
- for each symbol **a** from **A**, the RE **a** denotes the (finite) language containing only the string **a**.

Inductive cases: given RE's M and N,

the RE M | N (alternation, union) describes the language containing the strings in M or N.
 Example: a | b denotes the two-element language {a, b}

Base cases

- the RE ϵ (epsilon): the (finite) language containing only the empty string.
- for each symbol **a** from **A**, the RE **a** denotes the (finite) language containing only the string **a**.

Inductive cases: given RE's M and N,

- the RE M | N (alternation, union) describes the language containing the strings in M or N.
 Example: a | b denotes the two-element language {a, b}
- The RE MN (concatenation) denotes the strings that can be written as the concatenation mn where m in from M and n is from N. Example: (a|b)(a|c) denotes the language {aa, ac, ba, bc}

Base cases

- the RE ϵ (epsilon): the (finite) language containing only the empty string.
- for each symbol **a** from **A**, the RE **a** denotes the (finite) language containing only the string **a**.

Inductive cases: given RE's M and N,

- the RE M | N (alternation, union) describes the language containing the strings in M or N.
 Example: a | b denotes the two-element language {a, b}
- The RE MN (concatenation) denotes the strings that can be written as the concatenation mn where m in from M and n is from N. Example: (a|b)(a|c) denotes the language {aa, ac, ba, bc}
- The RE M* (Kleene closure/star) denotes the (<u>infinitely</u> many) strings obtained by concatenating <u>finitely</u> many elements from M.
 Example: (a|b)* denotes the language {ε, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, ...}

For alphabet $\Sigma = \{a,b\}$:

Strings with an even number of a's: $RE^a =$

Strings that with an odd number of b's:

```
RE<sup>b</sup> =
```

For alphabet $\Sigma = \{a,b\}$:Solutions not unique!Strings with an even number of a's: $RE^a = b^* (a b^* a b^*)^*$ Strings that with an odd number of b's: $RE^b =$

For alphabet $\Sigma = \{a,b\}$:Solutions not unique!Strings with an even number of a's: $RE^a = b^* (a b^* a b^*)^*$ Strings that with an odd number of b's: $RE^b = a^* b a^* (b a^* b a^*)^*$

For alphabet $\Sigma = \{a,b\}$:

Strings with an even number of a's: RE^a = b* (a b* a b*)*

Strings that with an odd number of b's: $RE^{b} = a^{*}b a^{*}(b a^{*}b a^{*})^{*}$

Strings with an even number of a's $RE^{a,b} =$

Strings that can be split into a string with an even number of a's, **followed** by a string with an odd number of b's:

For alphabet $\Sigma = \{a,b\}$:

Strings with an even number of a's: RE^a = b* (a b* a b*)*

Strings that with an odd number of b's: $RE^{b} = a^{*}b a^{*}(b a^{*}b a^{*})^{*}$

Strings with an even number of a's $RE^{a,b} = RE^{a} | RE^{b}$ **OR** an odd number of b's:

Strings that can be split into a string with an even number of a's, **followed** by a string with an odd number of b's:

For alphabet $\Sigma = \{a,b\}$:

Strings with an even number of a's: R	E ^a = b*	(a b* a b*)*
---------------------------------------	---------------------	----------------

Strings that with an odd number of b's: $RE^{b} = a^{*}b a^{*}(b a^{*}b a^{*})^{*}$

Strings with an even number of a's $RE^{a,b} = RE^{a} | RE^{b}$ **OR** an odd number of b's:

 $RE_{a,b} = RE^{a} RE^{b}$

Strings that can be split into a string with an even number of a's, **followed** by a string with an odd number of b's:

For alphabet $\Sigma = \{a,b\}$:

Strings with an even number of a's:	RE ^a = b* (a b* a b*)*
Strings with an odd number of b's:	RE ^b = a* b a* (b a* b a*)*
Strings with an even number of a's OR an odd number of b's:	RE ^{a,b} = RE ^a RE ^b
Strings that can be split into a string with an even number of a's, followed by a str with an odd number of b's:	ring $RE_{a,b} = RE^a RE^b$

Optional Homework:

Strings with an even number of a's and an odd number of b's....

Implementing RE's: finite automata



Finite automata (aka finite state machines, FSM's): a computational model of machines with finite memory

Components of an automaton over alphabet A:

- a finite set **S** of nodes ("<u>states</u>")
- a set of directed edges ("<u>transitions</u>")
 s _a t, each linking two states and labeled with a symbol from A
- a designated <u>start state</u> s0 from S, indicated by "arrow from nowhere"
- a nonempty set of final ("<u>accepting</u>") states (indicated by double circle)

Finite Automata recognize languages

Definition: the language recognized by an FA is the set of (finite) strings it **accepts**.

<u>Q</u>: how does the finite automaton D accept a string A?



- <u>A</u>: follow the transitions:
- 1. start in the start state **s0** and inspect the first symbol, a1
- 2. when in state **s** and inspecting symbol **a**, traverse one edge labeled **a** to get to the next state. Look at the next symbol.
- 3. After reading in all **n** symbols: if the current state **s** is a **final** one, **accept**. Otherwise, **reject**.
- 4. whenever there's no edge whose label matches the next symbol: **reject**.

Deterministic finite automaton (DFA)

• all edges leaving a node are uniquely labeled.

Nondeterministic finite automaton (NFA)

- two (or more) edges leaving a node may be identically uniquely labeled. Any choice that leads to acceptance is fine.
- edges may also be labeled with ε. So can "jump" to the next state without consuming an input symbol.

Deterministic finite automaton (DFA)

• all edges leaving a node are uniquely labeled.

Nondeterministic finite automaton (NFA)

- two (or more) edges leaving a node may be identically uniquely labeled. Any choice that leads to acceptance is fine.
- edges may also be labeled with ε. So can "jump" to the next state without consuming an input symbol.

Strategy for obtaining a DFA that recognizes exactly the language described by an RE:

- 1. convert RE into an NFA that recognizes the language
- 2. transform the NFA into an equivalent DFA

Remember Tuesday's quiz?

Over alphabet {a, b}:

Strings with an even number of a's:

Strings with an odd number of b's:

Over alphabet {a, b}:

Strings with an even number of a's:



Strings with an odd number of b's:

Over alphabet {a, b}:

Strings with an even number of a's:

Strings with an odd number of b's:



Can we systematically generate NFA's from RE's?





F



F



RE to NFA Conversion: examples for | and concat

Strings with an even number of a's **OR** an odd number of b's: **RE**^a | **RE**^b



Strings that can be split into a string with an even number of a's, **followed** by a string with an odd number of b's:

RE^a RE^b

RE to NFA Conversion: examples for | and concat

Strings with an even number of a's **OR** an odd number of b's: **RE**^a | **RE**^b

d

а

d

a

Strings that can be split into a string with an even number of a's **followed** by a string with an odd number of b's:

RF^a RF^b

42



Idea:

- combine identically labeled NFA transitions
- DFA states represent **sets** of "equivalent" NFA states

Auxiliary definitions:

set of NFA states reachable from NFA state s by an a step

$$closure(S) = S \cup (\bigcup_{s \in S} edge(s, \epsilon))$$

set of NFA states reachable from any s ϵ S by an ϵ step

Main calculation:

DFA-edge(D,a) = closure($\bigcup_{s \in D} edge(s,a)$) set of NFA states reachable from D by making one a step and (then) any number of ε steps













Step 1:		Step 2: edge sets					
closure sets				a	b	С	
1	{1}	1		2,3	-	-	
2	{2}	2		-	4	-	
3	{3,5}	3		-	-	-	
4	{4,6}	4		-	-	-	
5	{5}	5		-	2	4,6	
6	{6}	6		-	-	-	



edge(s, a) $\{t \mid s \xrightarrow{a} t\}$

DFA-edge(D,a) closure($\bigcup_{s \in D} edge(s,a)$)

NF	NFA to DFA Example					a	b	С	
					1	L	2,3	-	-
	a	* 2	2 4	\mathbf{c}	2	2	-	4	-
\rightarrow	$(1)_a$			3	(1)	3	-	-	-
		3	$\overline{5}$		۷	1	-	-	-
					5	5	-	2	4,6
			ton 2. F		6	5	-	-	-
			пер э. г	JFA-Sels					
1	{1}	D		a	b			С	
2	{2}	{1}		Cl(2) + Cl(3) = {2,3,5}	{}			{}	
3	{3,5}	{2,3,	5}	{}	CI((2)+C	l(4)	Cl(4) +	Cl(6)
4	{4,6}				=	{2,4,6	5}	= {4,6]	}
5	{5}	{2,4,	5}	{}	CI((4) =	{4,6}	{}	
6	{6}	{4,6}		{}	{}			{}	
	closure(S)			edge(<mark>s</mark> , a)	DFA-edge(D,a)		, <mark>a</mark>)		
$\frac{S \cup (\bigcup_{s \in S} edge(s, \epsilon))}{s \in S}$			{ t	$ s \rightarrow t$		closure(U edge(s,a) s ∈ D			je(<mark>s,a</mark>))



Step 4: Transition matrix

	D	a	b	С
Α	{1}	Cl(2) + Cl(3) = {2,3,5}	{}	{}
В	{2,3,5}	{}	Cl(2)+Cl(4) = {2,4,6}	Cl(4) +Cl(6) = {4,6}
C	{2,4,6}	{}	Cl(4) = {4,6}	{}
D	{4,6}	{}	{}	{}





Step 5: Initial state: closure of initial NFA state

	D	a	b	С
Α	{ 1 }	Cl(2) + Cl(3) = {2,3,5}	{}	{}
В	{2,3,5}	{}	Cl(2)+Cl(4) = {2,4,6}	Cl(4) + Cl(6) = {4,6}
C	{2,4,6}	{}	Cl(4) = {4,6}	{}
D	{4,6}	{}	{}	{}





Step 6: Final state(s): DFA states "containing" a final NFA state

	D	a	b	С
Α	{1}	Cl(2) + Cl(3) = {2,3,5}	{}	{}
В	{2,3,5}	{}	Cl(2)+Cl(4) = {2,4,6}	Cl(4) + Cl(6) = {4,6}
C	{2,4, 6 }	{}	Cl(4) = {4,6}	{}
D	{4 ,6 }	{}	{}	{}

Algorithm in pseudo-code: Appel, page 27

Lexer should identify the **longest matching** token:

ifz8 should be lexed as IDENTIFIER, not as two tokens IF, IDENTIFIER

Hence, the implementation

- saves the most recently encountered accepting state of the DFA (and the corresponding stream position) and
- updates this info when passing through another accepting state
- Uses the order of rules as tie-breaker in case several tokens (of equal length) match

Read Chapters 1 and 2.

Equivalent states:

- Eliminate redundant states, smaller FA.
- Do Exercise 2.6 (hand in optional).
- $FA \rightarrow RE$:
 - Useful to confirm correct RE \rightarrow FA. (see exercise 2.7)
 - GNFAs! (generalized NFA's: transitions may be labeled with RE's)
 - See: Introduction to the Theory of Computation by Michael Sipser



- Motivated use of lexer generators for partitioning input stream into tokens
- Three formalisms for describing and implementing lexers:
 - Regular expressions
 - NFA's
 - DFA's
 - Conversions RE -> NFA -> DFA
- Next lecture: practicalities of lexing (ML-LEX)





- Lexical Analysis: Break into tokens (think words, punctuation)
- Syntax Analysis: Parse phrase structure (think document, paragraphs, sentences)
- Semantic Analysis: Calculate meaning

Practicalities of lexing: ML Lex, Lex, Flex, ...

The first phase of a compiler is called the Lexical Analyzer or Lexer.

Implementation Options:

- 1. Write Lexer from scratch.
- 2. Use Lexical Analyzer Generator.



- ml-lex is a lexical analyzer generator for ML.
- lex and flex are lexical analyzer generators for C.

ML Lex: lexer generator for ML (similar tools for C: lex, flex)

- Input to ml-lex is a set of *rules* specifying a lexical analyzer.
- Output from **ml-lex** is a lexical analyzer in ML.
- A *rule* consists of a pattern and an *action*:
 - Pattern is a regular expression.
 - Action is a fragment of ordinary ML code. (Typically returns a token type to calling function.)
- Examples:

```
if => (print("Found token IF"));
[0-9]+ => (print("Found token NUM"));
```

• General Idea: When prefix of input matches a pattern, the action is executed.

Specification of a lexer has three parts:

User Declarations %% ML-LEX Definitions %% Rules

User declarations:

- definitions of values to be used in the action fragments of rules
- Two values **must** be defined in the section:
 - type lexresult: type of the value returned by the rule actions
 - fun eof(): function to be called by the generated lexer when end of input stream is reached (eg call parser, print "done")

Specification of a lexer has three parts:

User Declarations %% ML-LEX Definitions %% Rules

ML-LEX Definitions

- definitions of regular expressions abbreviations: DIGITS=[0..9]+; LETTER = [a-zA-Z];
- definitions of start states to permit multiple lexers to run together: %s STATE1 STATE2 STATE3;

Example: entering "comment" mode, e.g. for supporting nested comments

Specification of a lexer has three parts:



Rule Patterns

symbol	matches
a	individual character "a" (not for reserved chars ?,*,+,[,{)
\setminus {	reserved character {
[abc]	a b c
[a-zA-Z]	lowercase and capital letters
	any character except new line
$\setminus n$	newline
$\setminus t$	tab
"abc?"	abc? taken literally (reserved chars as well)
{LETTER}	Use abbreviation LETTER defined in ML-LEX Definitions
a*	0 or more a's
a+	1 or more a's
a?	0 or 1 a
alb	a or b
if iff :	=> (print("Found token IF or IFF"));
[0-9]+ =	<pre>=> (print("Found token NUM"));</pre>

- Actions can use various values defined in User Declarations section.
- Two values always available:

type lexresult
 -type of the value returned by each rule action.
fun eof()
 -called by lexer when end of input stream reached.

- Several special variables also available to action fragments.
 - yytext input substring matched by regular expression.
 - yypos file position of beginning of matched string.
 - continue() recursively calls lexing engine.

- Start states permit multiple lexical analyzers to run together.
- Rules prefixed with a start state is matched only when lexer is in that state.
- States are entered with YYBEGIN.
- Example:

```
%%
%s COMMENT
%%
<INITIAL> if => (print("Token IF"));
<INITIAL> [a-z] + => (print("Token ID"));
<INITIAL> "(*" => (YYBEGIN COMMENT; continue());
<COMMENT> "*)" => (YYBEGIN INITIAL; continue());
<COMMENT> "\n"|. => (continue());
```

Rule Matching and Start States

<start_state_list> regular_expression => (action_code);

- Regular expression matched only if lexer is in one of the start states in start state list.
- If no start state list specified, the rule matches in all states.
- Lexer begins in predefined start state: INITIAL

If multiple rules match in current start state, use Rule Disambiguation.

Rule Disambiguation

• *Longest match* - longest initial substring of input that matches regular expression is taken as next token.

```
if8 matches ID(``if8''), not IF() and NUM(8).
```

• *Rule priority* - for a particular substring which matches more than one regular expression with equal length, choose first regular expression in rules section.

If we want if to match IF(), not ID(``if''), put keyword regular expression before identifier regular expression.

```
(* -*- ml -*- *)
type lexresult = string
fun eof() = (print("End-of-file\n"); "EOF")
00
INT=[1-9][0-9]*;
%s COMMENT;
000
<INITIAL>"/*"
                         => (YYBEGIN COMMENT; continue());
<COMMENT>"*/"
                         => (YYBEGIN INITIAL; continue());
<COMMENT>"n".
                         => (continue());
<INITIAL>if
                         => (print("Token IF\n");"IF");
                         => (print("Token THEN\n");"THEN");
<INITIAL>then
<INITIAL>{INT}
                         => (print("Token INT(" ^ yytext ^ ")\n");"INT");
<INITIAL>" "|"\n"|"\t" => (continue());
                         => (print("ERR: '" ^ yytext ^ "'.\n");"ERR");
<INITIAL>.
```

```
% cat x.txt
if 999 then 0999
/* This is a comment 099 if */
if 12 then 12
% sml
Standard ML of New Jersey, Version 109.33, November 21, 1997 [CM; ...]
- CM.make();
[....]
val it = () : unit
- MyLexer.tokenize("x.txt");
Token IF
Token INT(999)
Token THEN
ERR: '0'.
Token INT(999)
Token IF
Token INT(12)
Token THEN
Token INT(12)
End-of-file
val it = () : unit
```