The 3D Rasterization Pipeline

COS 426, Spring 2015

Princeton University

3D Rendering Scenarios

- Batch
  - One image generated with as much quality as possible for a particular set of rendering parameters
    - Take as much time as is needed (minutes)
    - Useful for photorealististism, movies, etc.

- Interactive
  - Images generated in fraction of a second (<1/10) with user input, animation, varying camera, etc.
    - Achieve highest quality possible in given time
    - Visualization, games, etc.

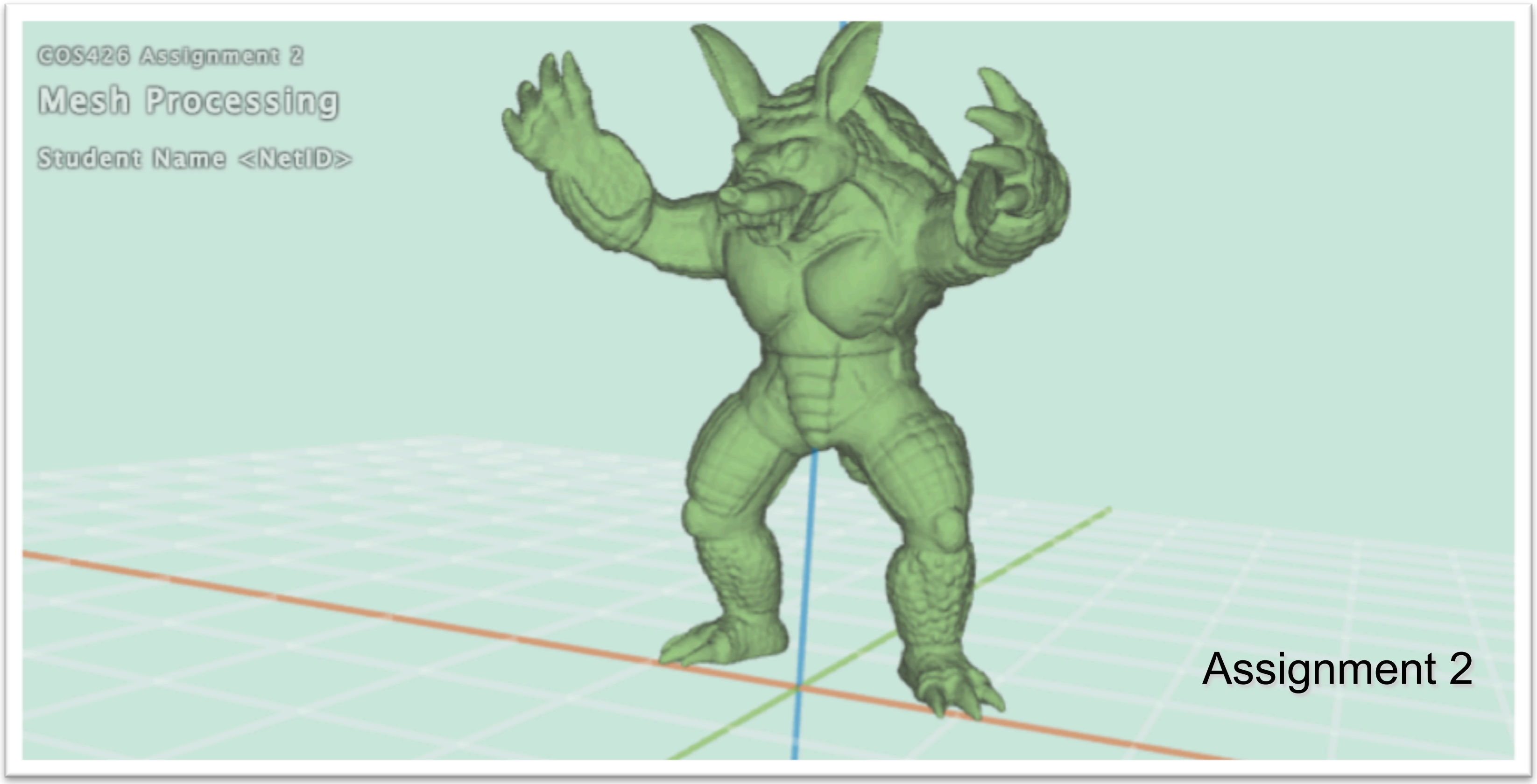
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination

Bungie

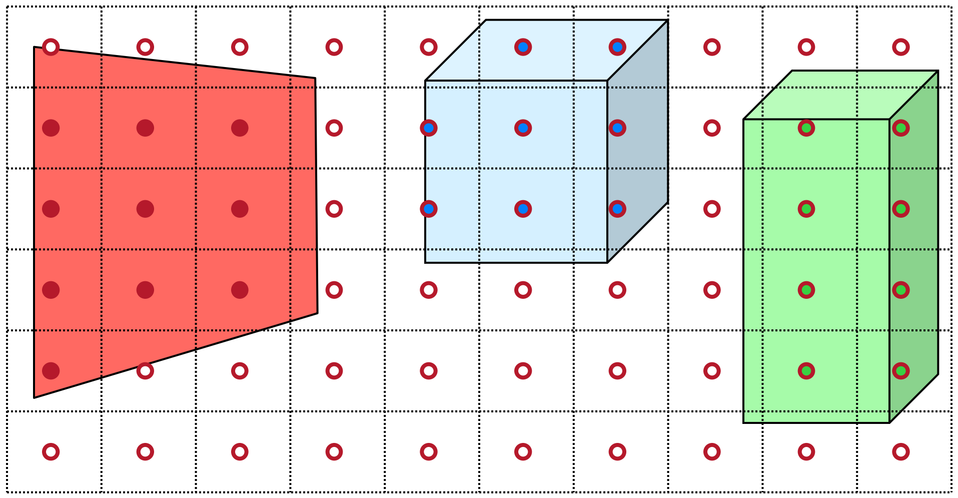
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination



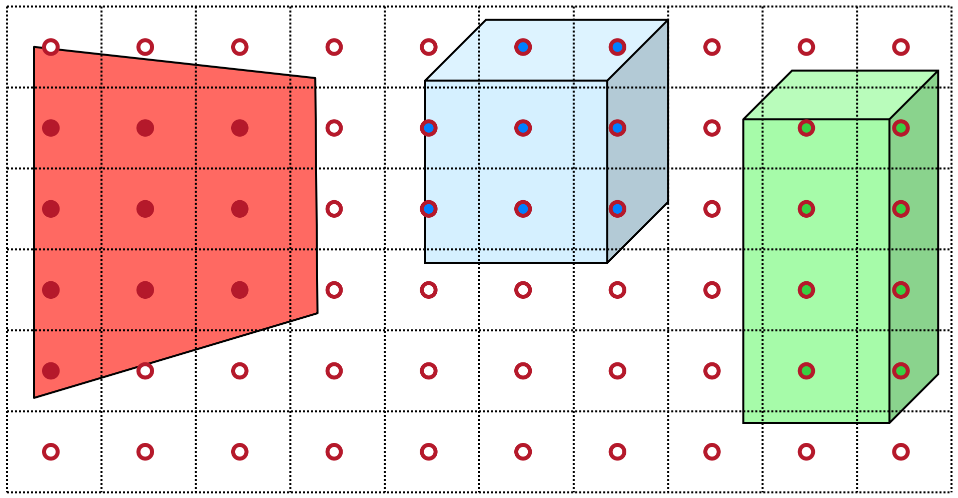
Ray Casting Revisited

- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color of sample based on illumination



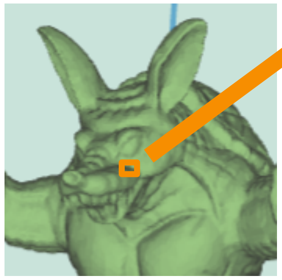
3D Polygon Rendering

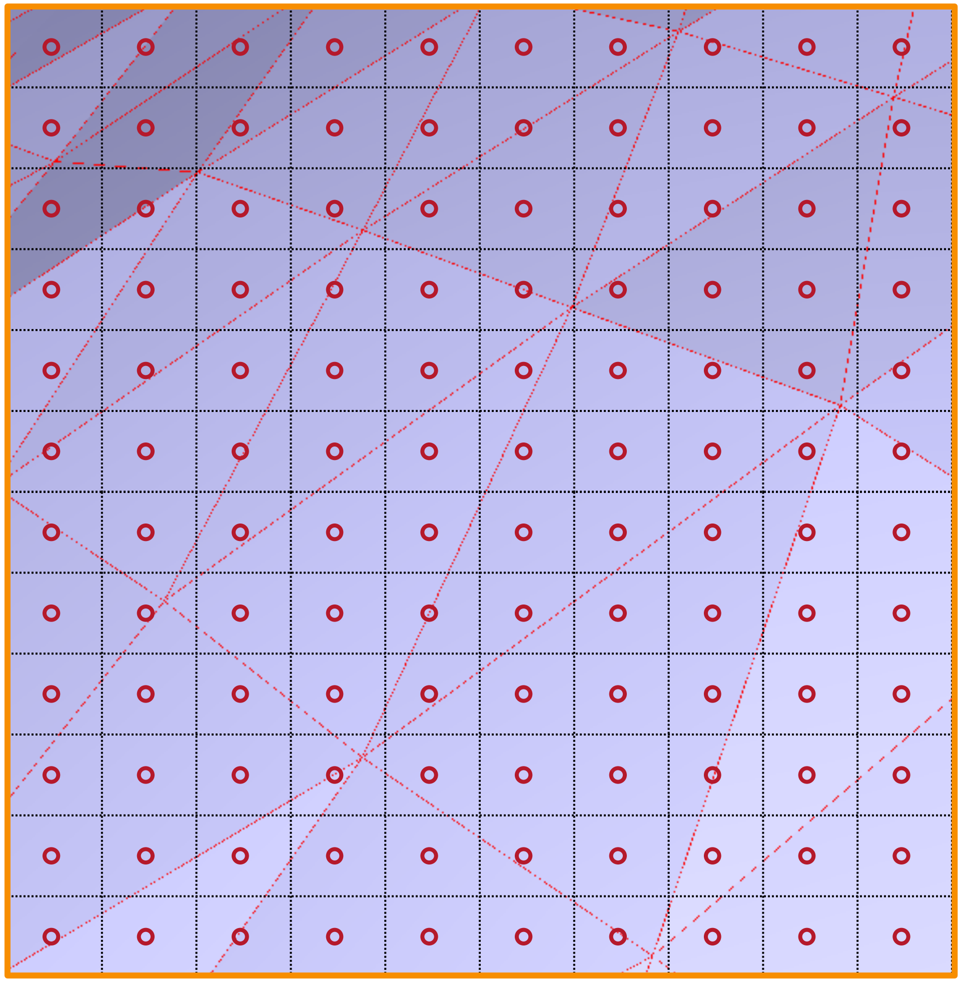
- We can render polygons faster if we take advantage of spatial coherence



3D Polygon Rendering

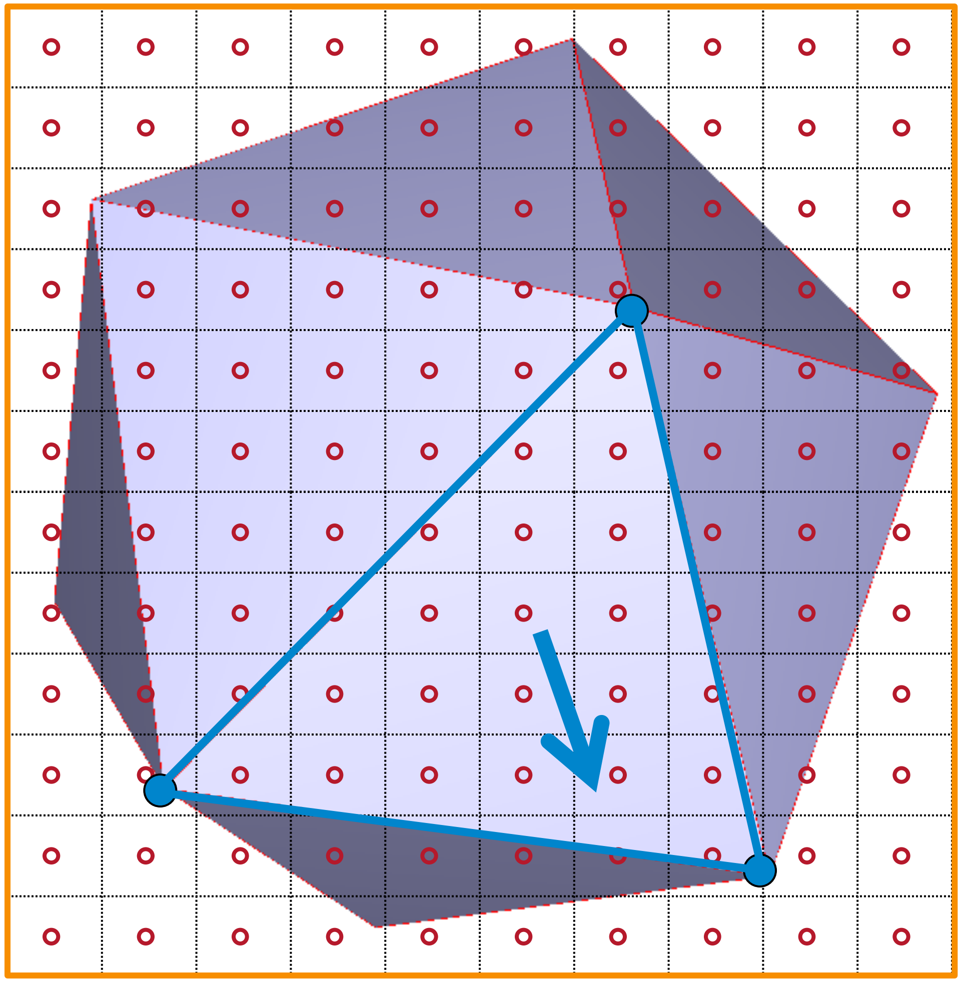
- How?





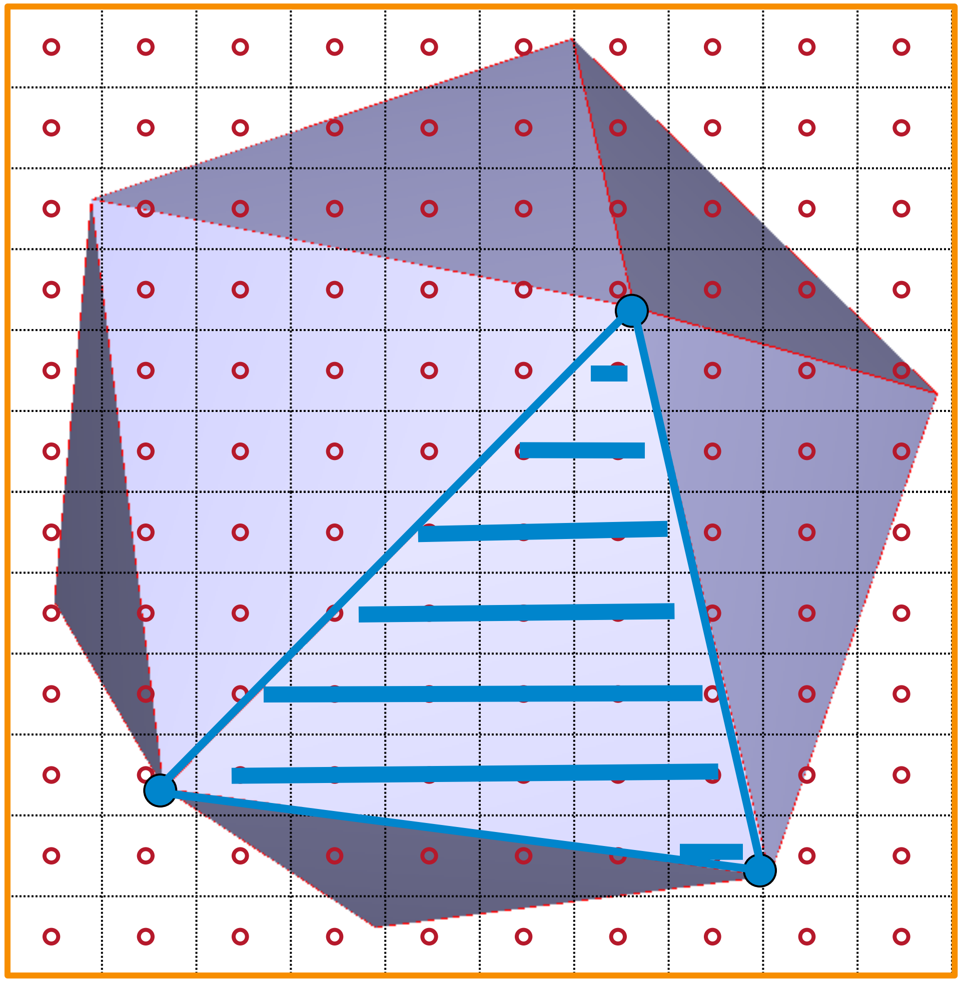
3D Polygon Rendering

- How?

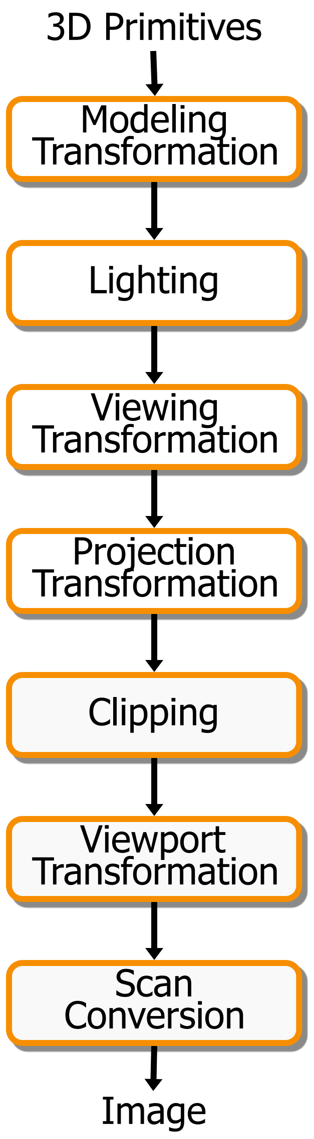


3D Polygon Rendering

- How?

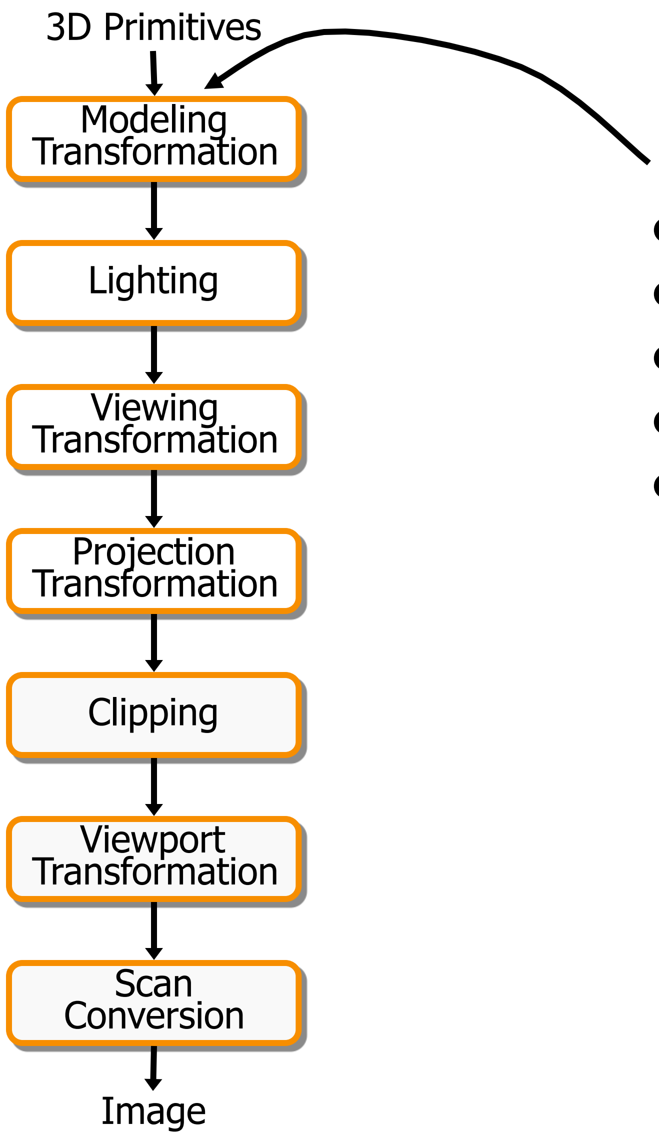


3D Rendering Pipeline (for direct illumination)



This is a pipelined sequence of operations to draw 3D primitives into a 2D image

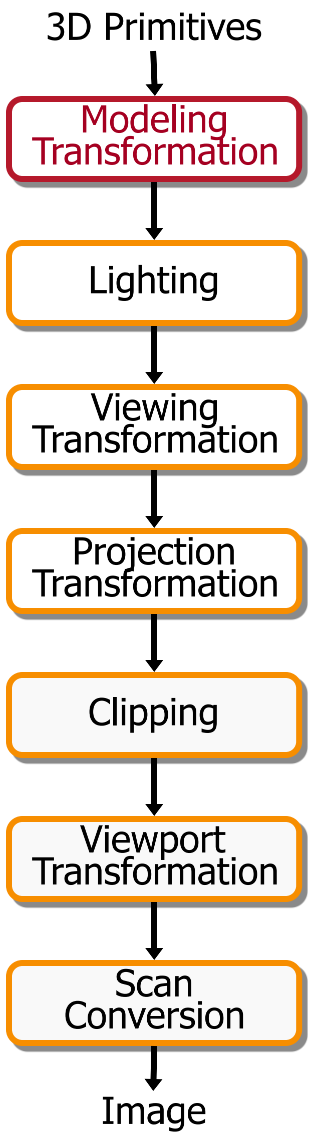
3D Rendering Pipeline (for direct illumination)



```
glBegin(GL_POLYGON);
glVertex3f(0.0, 0.0, 0.0);
glVertex3f(1.0, 0.0, 0.0);
glVertex3f(0.0, 1.0, 0.0);
glEnd();
```

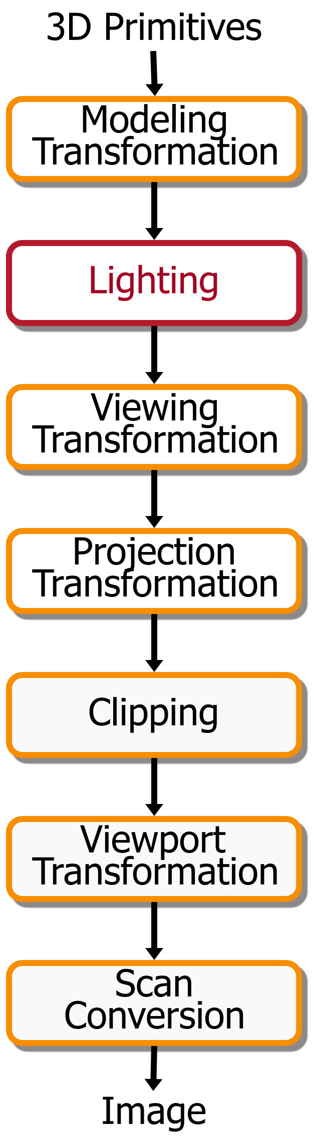
OpenGL executes steps of 3D rendering pipeline for each polygon

3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

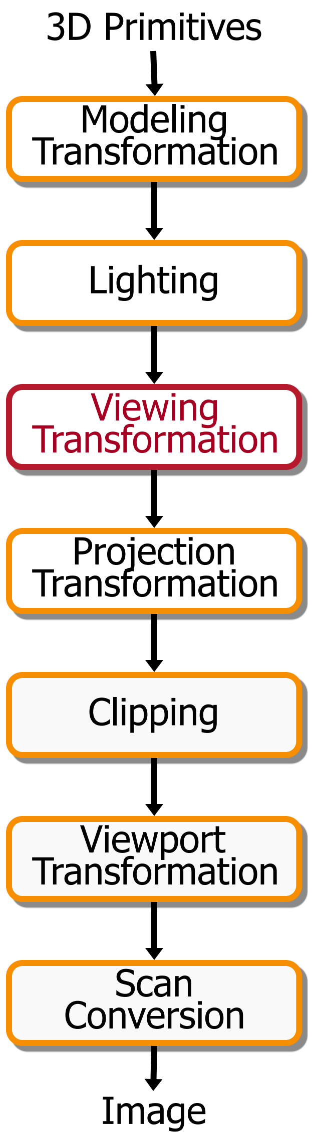
3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

3D Rendering Pipeline (for direct illumination)

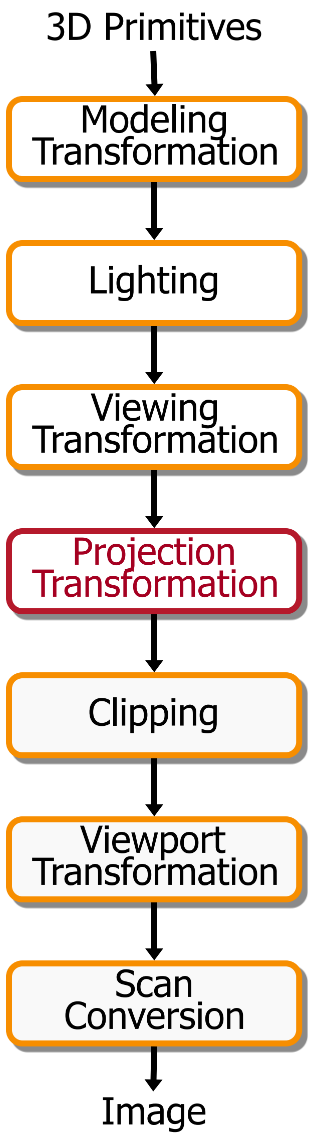


Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

3D Rendering Pipeline (for direct illumination)



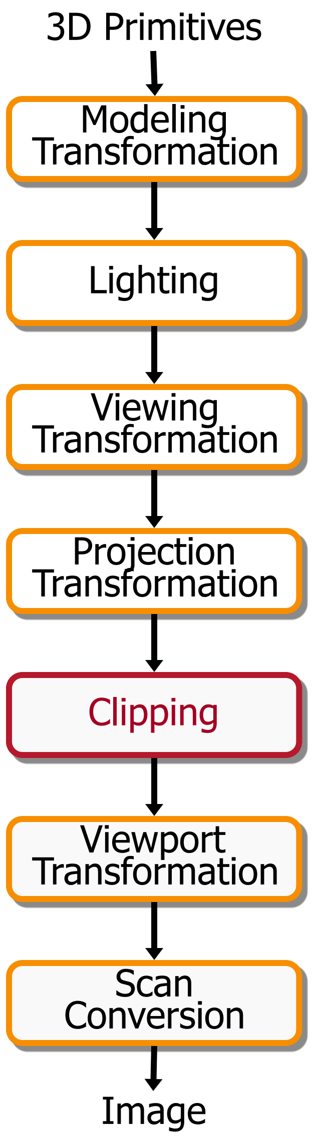
Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

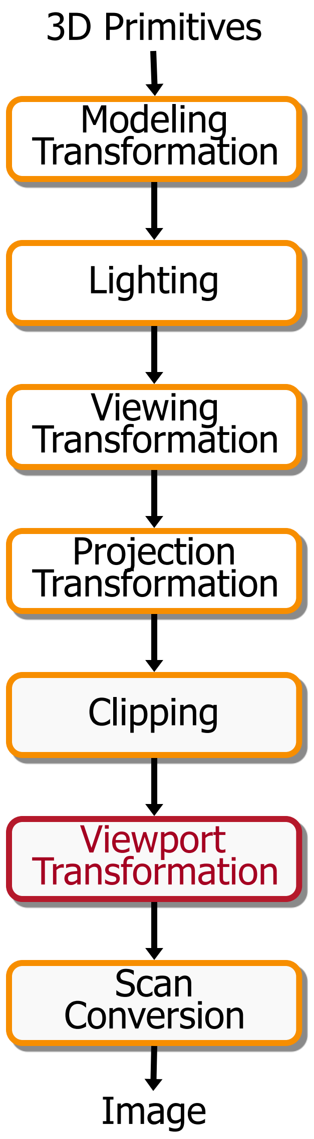
Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

Clip primitives outside camera's view

3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

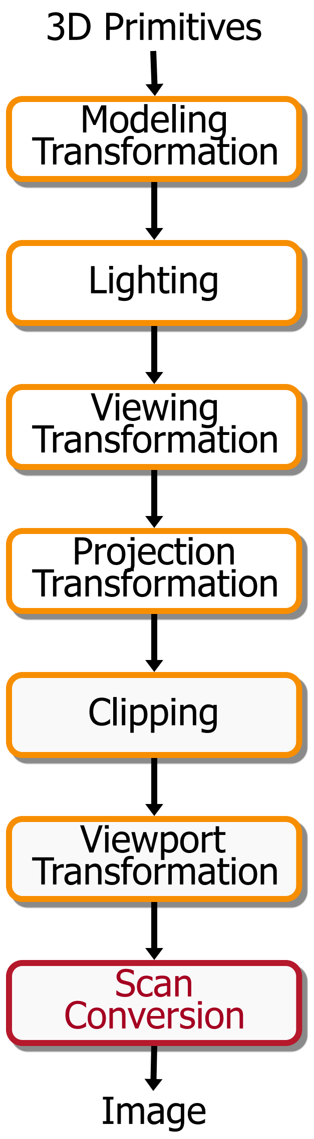
Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

Clip primitives outside camera’s view

Transform into image coordinate system

3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

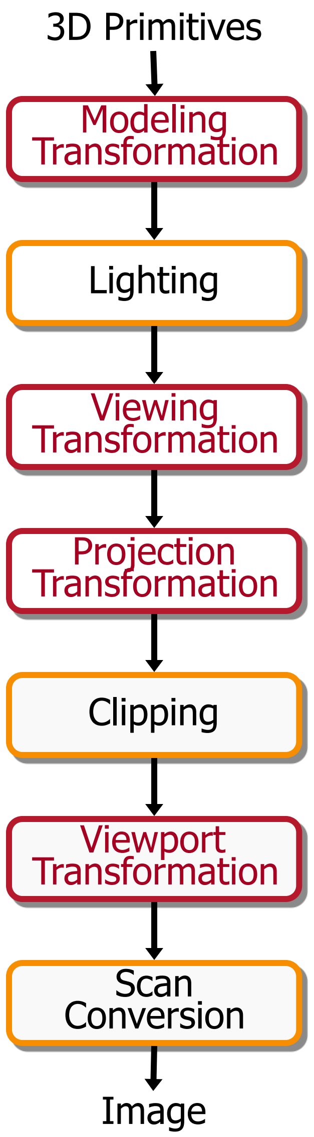
Transform into 2D camera coordinate system

Clip primitives outside camera’s view

Transform into image coordinate system

Draw pixels (includes texturing, hidden surface, ...)

3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

Clip primitives outside camera's view

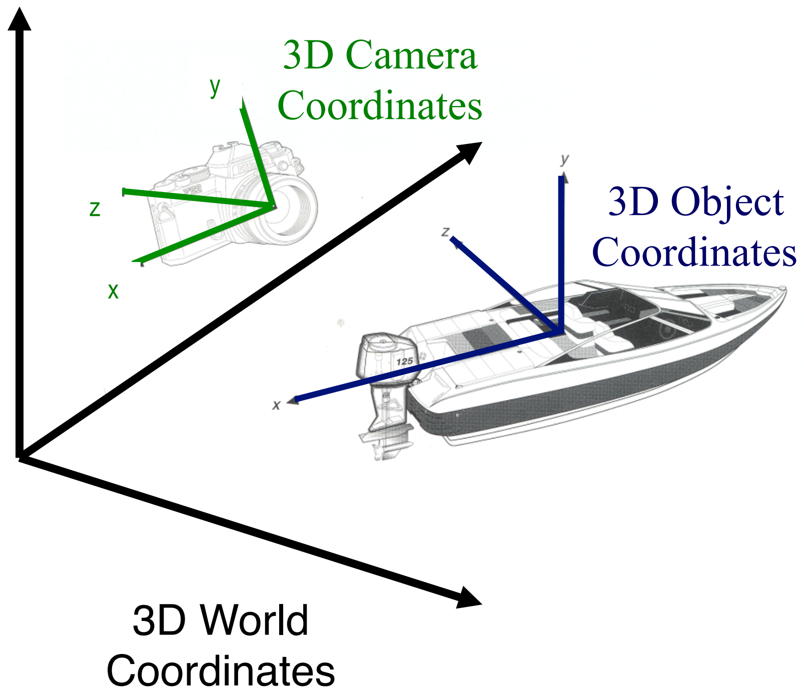
Transform into image coordinate system

Draw pixels (includes texturing, hidden surface, ...)

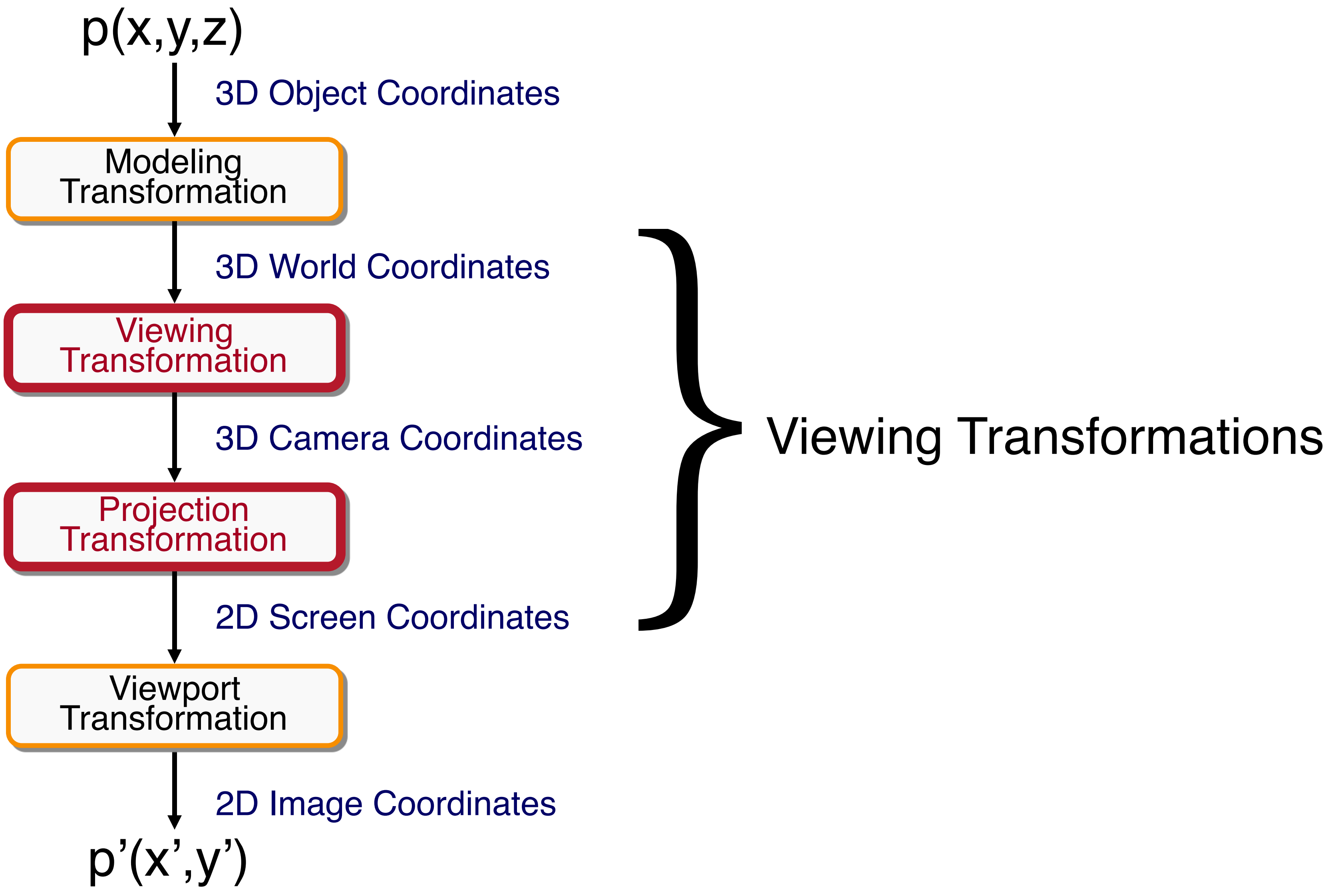
Transformations

$$\begin{tikzcd}
p(x,y,z) \arrow[d, "\text{3D Object Coordinates}"] \\
\text{Modeling Transformation} \arrow[d, "\text{3D World Coordinates}"] \\
\text{Viewing Transformation} \arrow[d, "\text{3D Camera Coordinates}"] \\
\text{Projection Transformation} \arrow[d, "\text{2D Screen Coordinates}"] \\
\text{Viewport Transformation} \arrow[d, "\text{2D Image Coordinates}"] \\
p'(x',y')
\end{tikzcd}$$

Transformations map points from one coordinate system to another

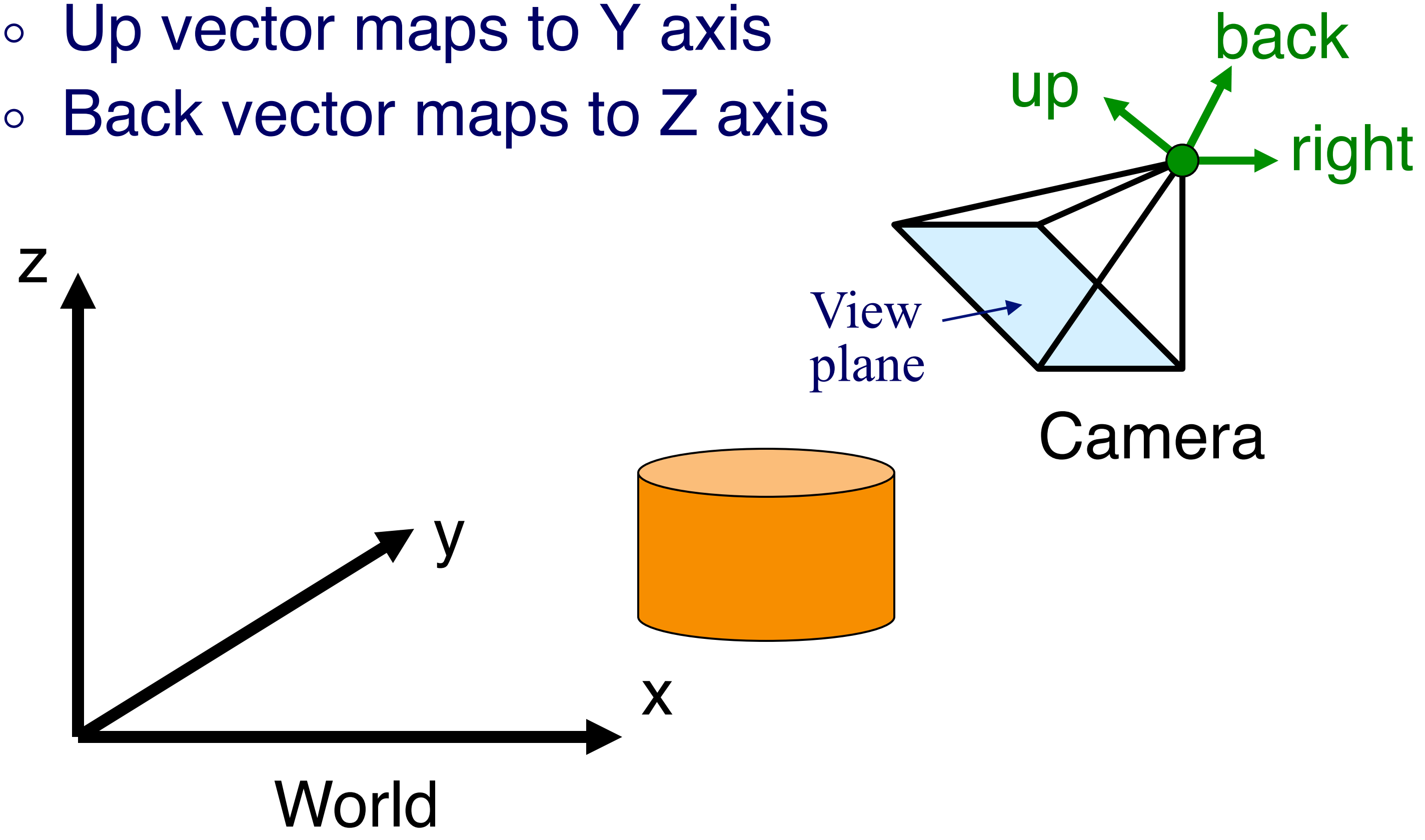


Viewing Transformations



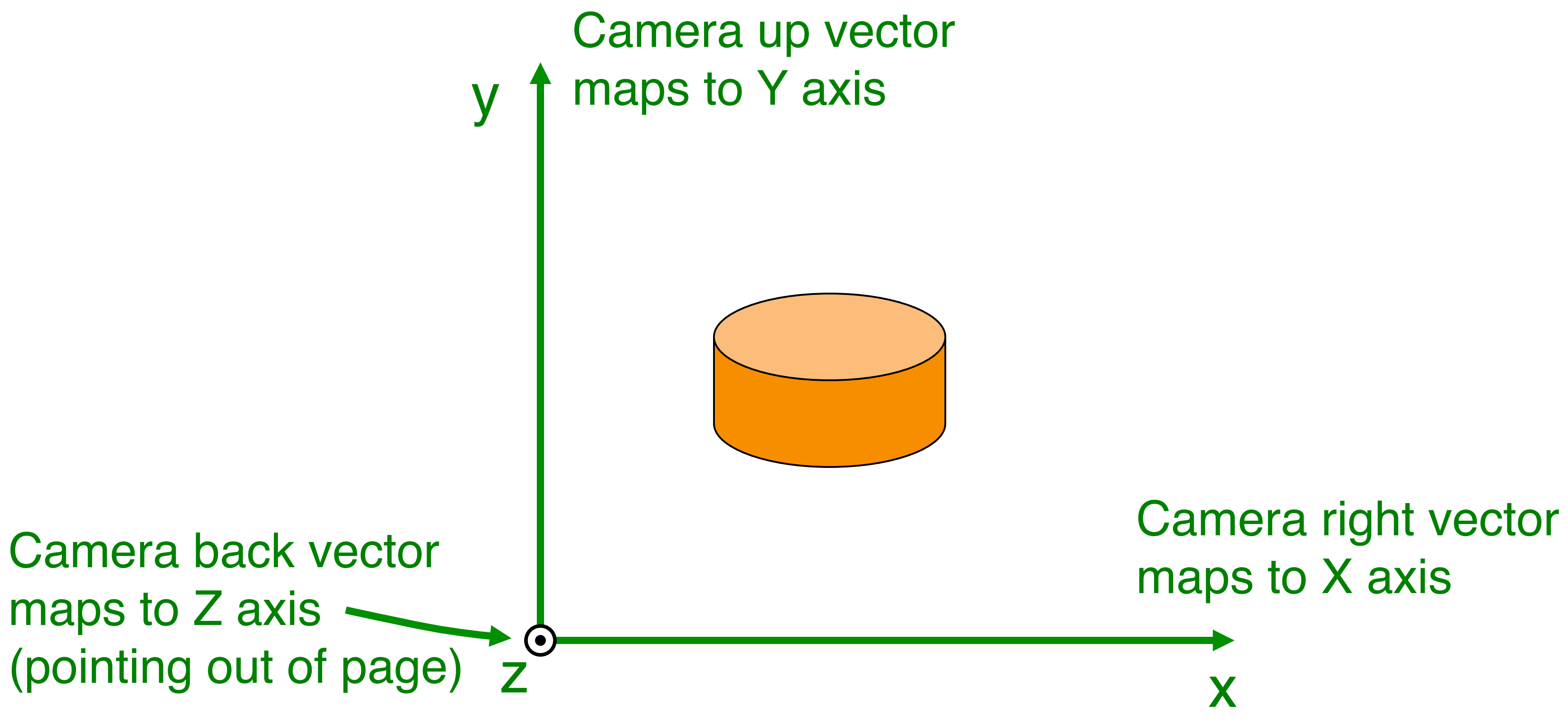
Review: Viewing Transformation

- Mapping from world to camera coordinates
  - Eye position maps to origin
  - Right vector maps to X axis
  - Up vector maps to Y axis
  - Back vector maps to Z axis



Review: Camera Coordinates

- Canonical coordinate system
  - Convention is right-handed (looking down -z axis)
  - Convenient for projection, clipping, etc.



Finding the viewing transformation

- We have the camera (in world coordinates)
- We want T taking objects from world to camera

$$p^c = T\ p^w$$

- Trick: find T^{-1} taking objects in camera to world

$$p^w = T^{-1} p^c$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \underset{\overset{\Uparrow}{?}}{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

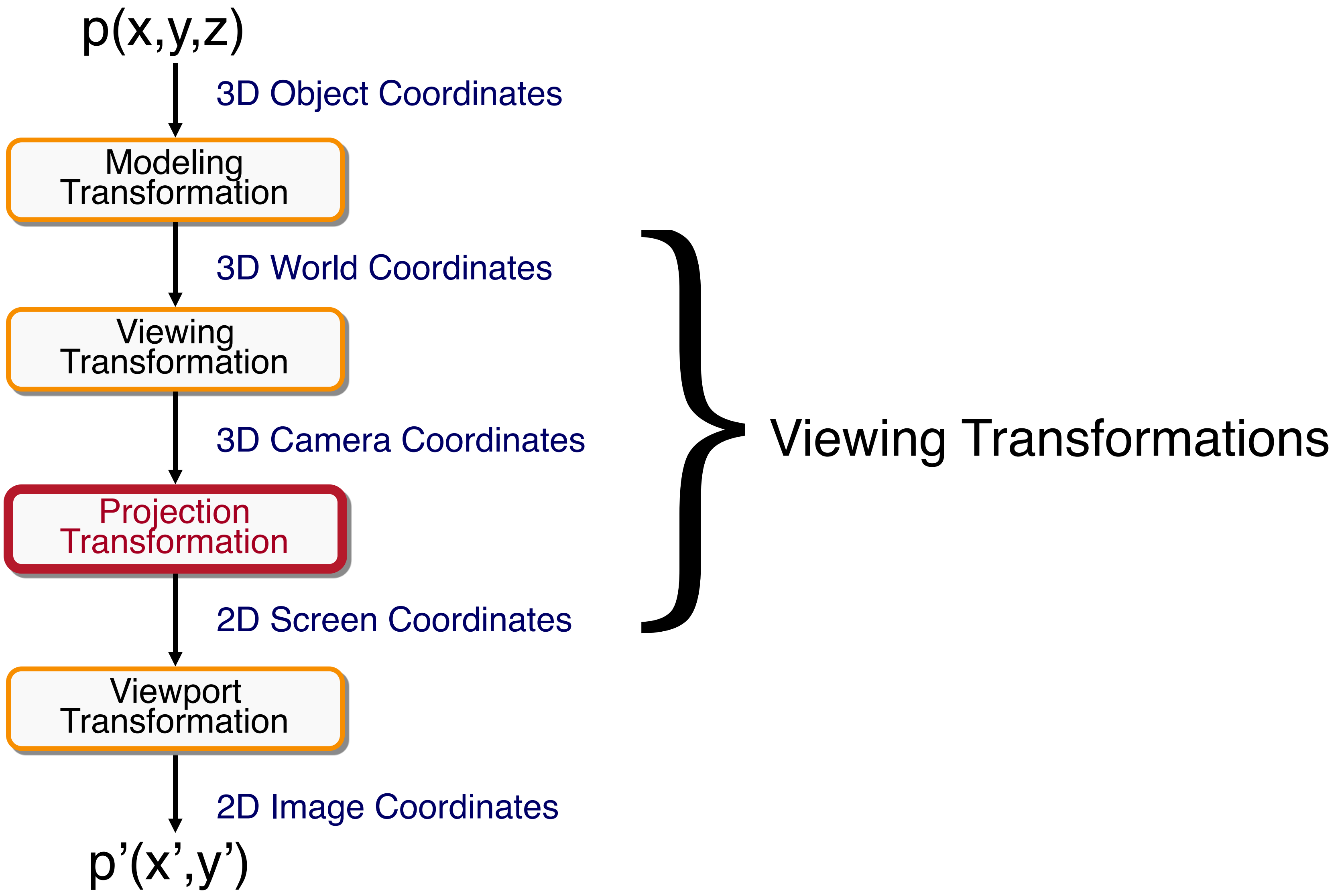
Finding the Viewing Transformation

- Trick: map from camera coordinates to world
  - Origin maps to eye position
  - Z axis maps to Back vector
  - Y axis maps to Up vector
  - X axis maps to Right vector

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_y & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

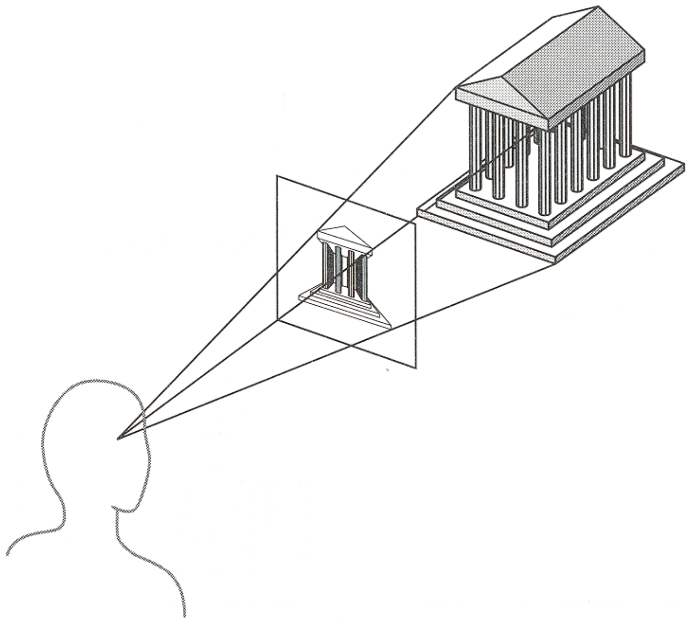
- This matrix is T^{-1} so we invert it to get T … easy!

Viewing Transformations

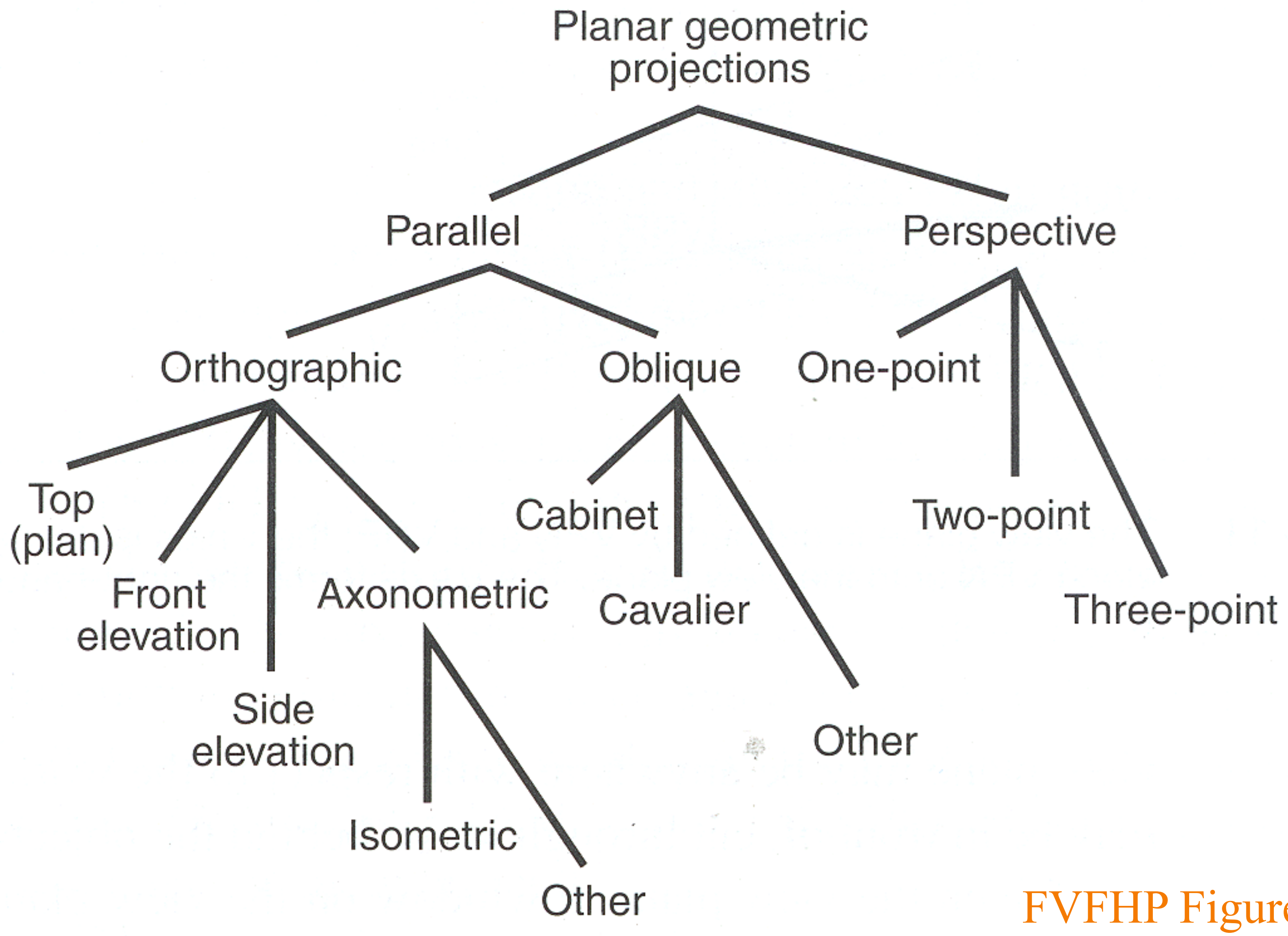


Projection

- General definition:
  - Transform points in *n*-space to *m*-space (*m*<*n*)
- In computer graphics:
  - Map 3D camera coordinates to 2D screen coordinates

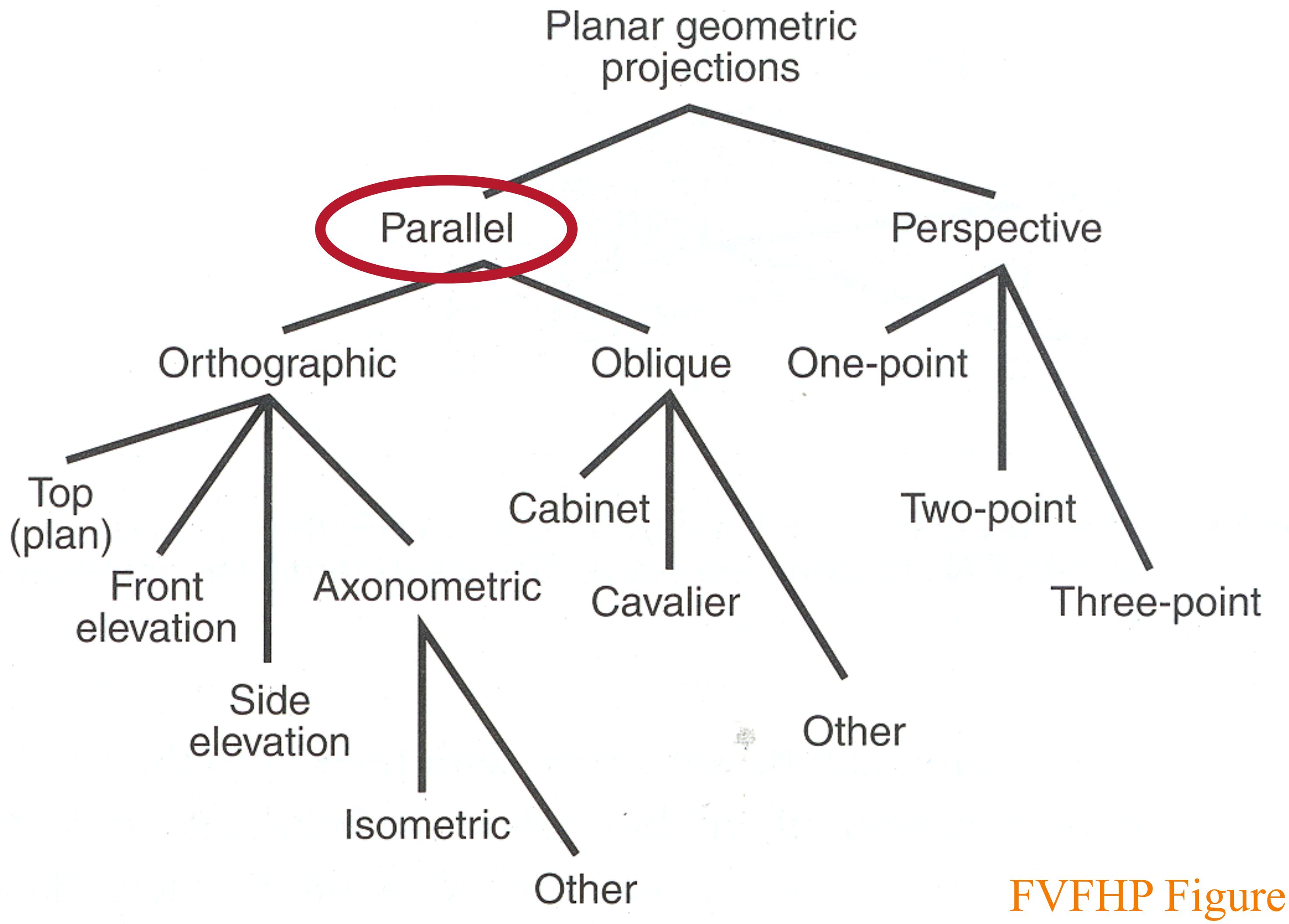


Taxonomy of Projections



FVFHP Figure 6.10

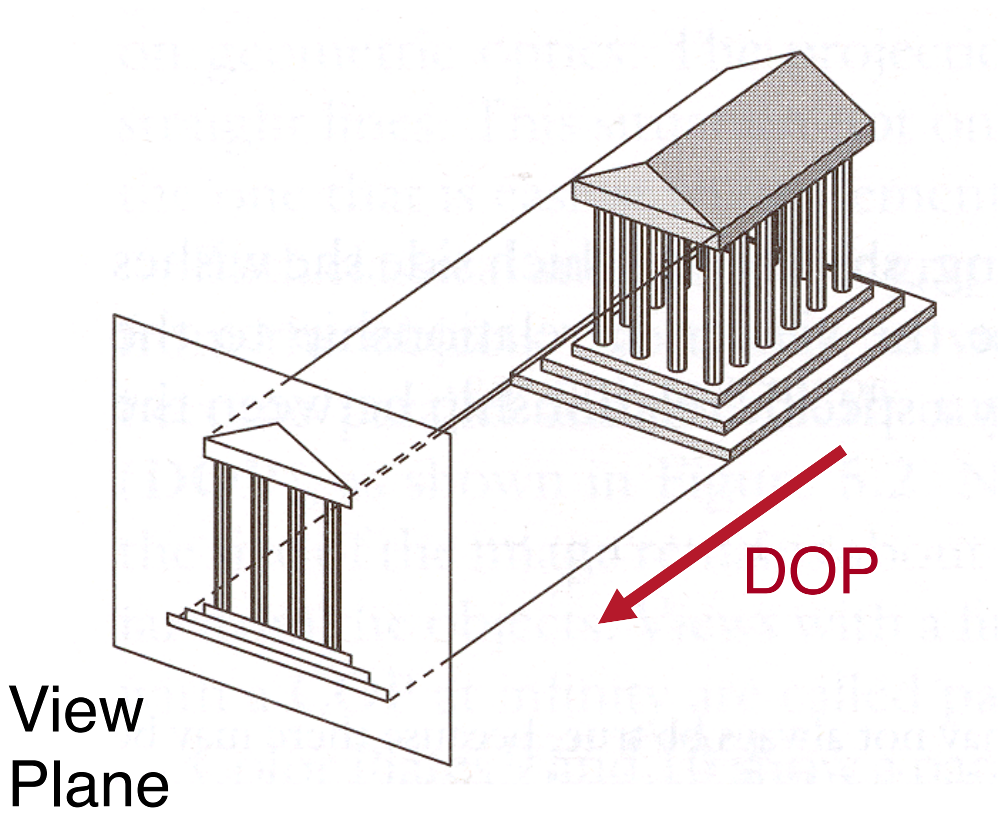
Taxonomy of Projections



FVFHP Figure 6.10

Parallel Projection

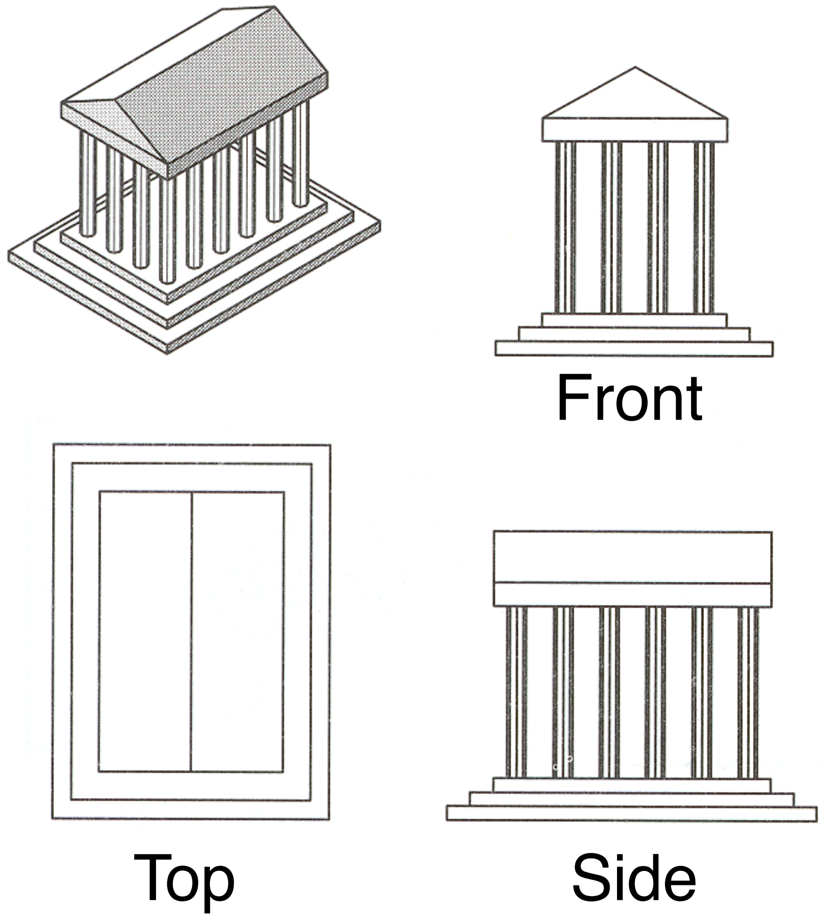
- Center of projection is at infinity
  - Direction of projection (DOP) same for all points



Angel Figure 5.4

Orthographic Projections

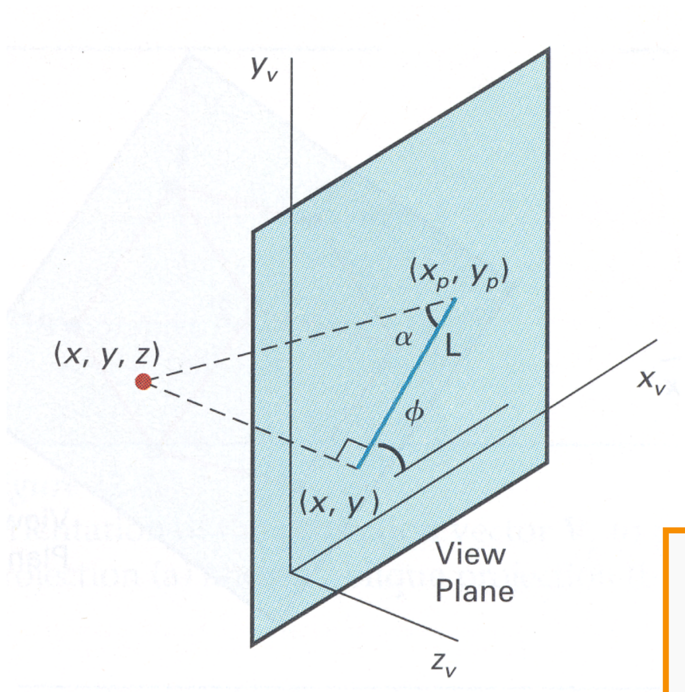
- DOP perpendicular to view plane



Angel Figure 5.5

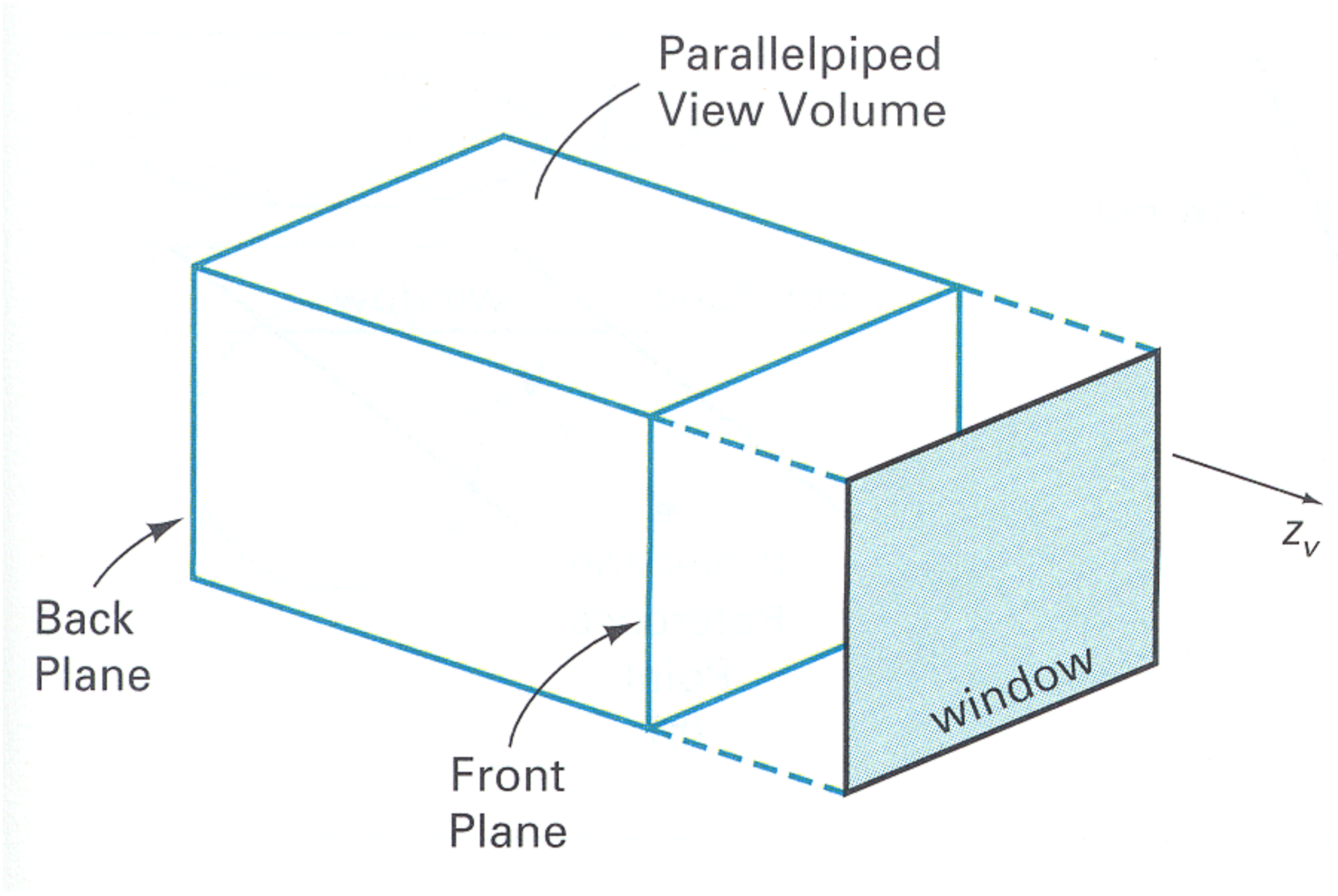
Parallel Projection Matrix

- General parallel projection transformation:



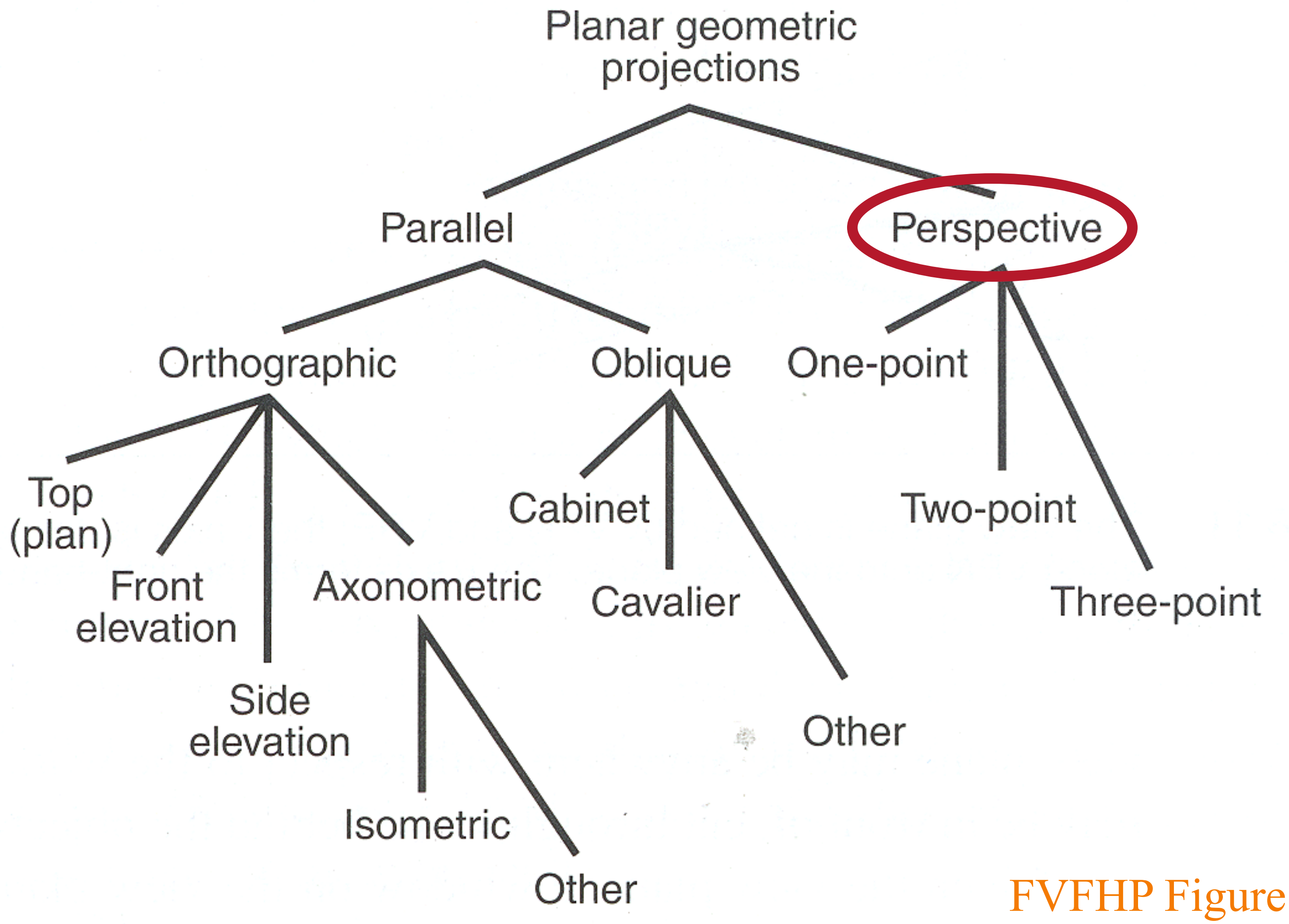
$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & L\cos\phi & 0 \\ 0 & 1 & L\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Parallel Projection View Volume



H&B Figure 12.30

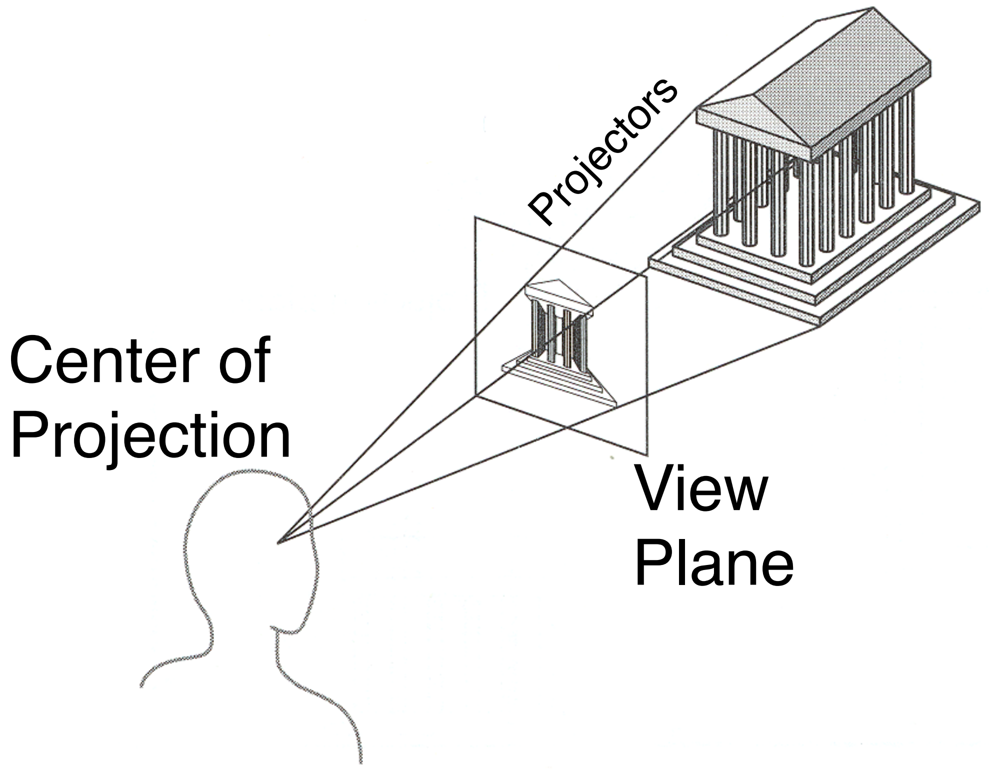
Taxonomy of Projections



FVFHP Figure 6.10

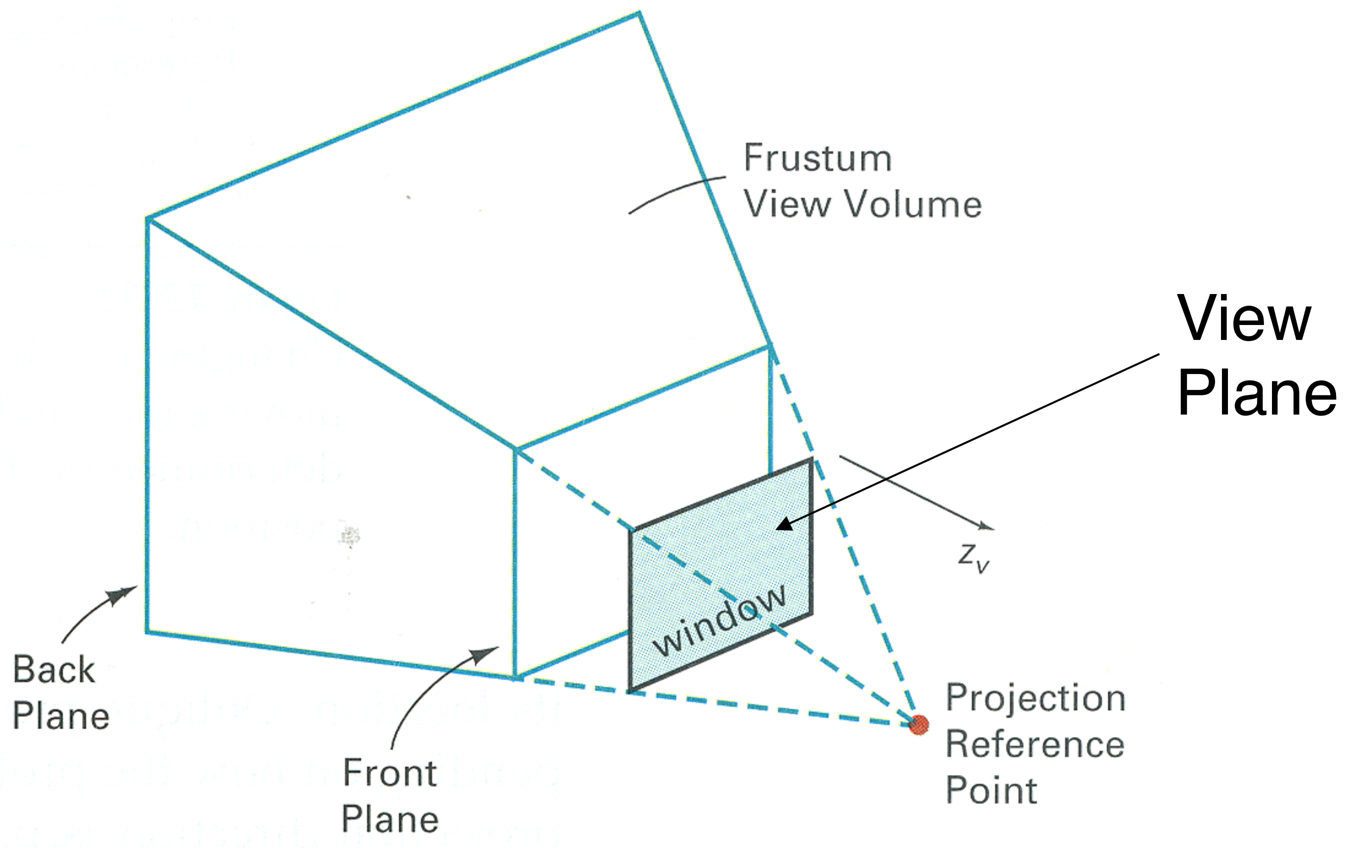
Perspective Projection

- Map points onto "view plane" along "projectors" emanating from "center of projection" (COP)



Angel Figure 5.9

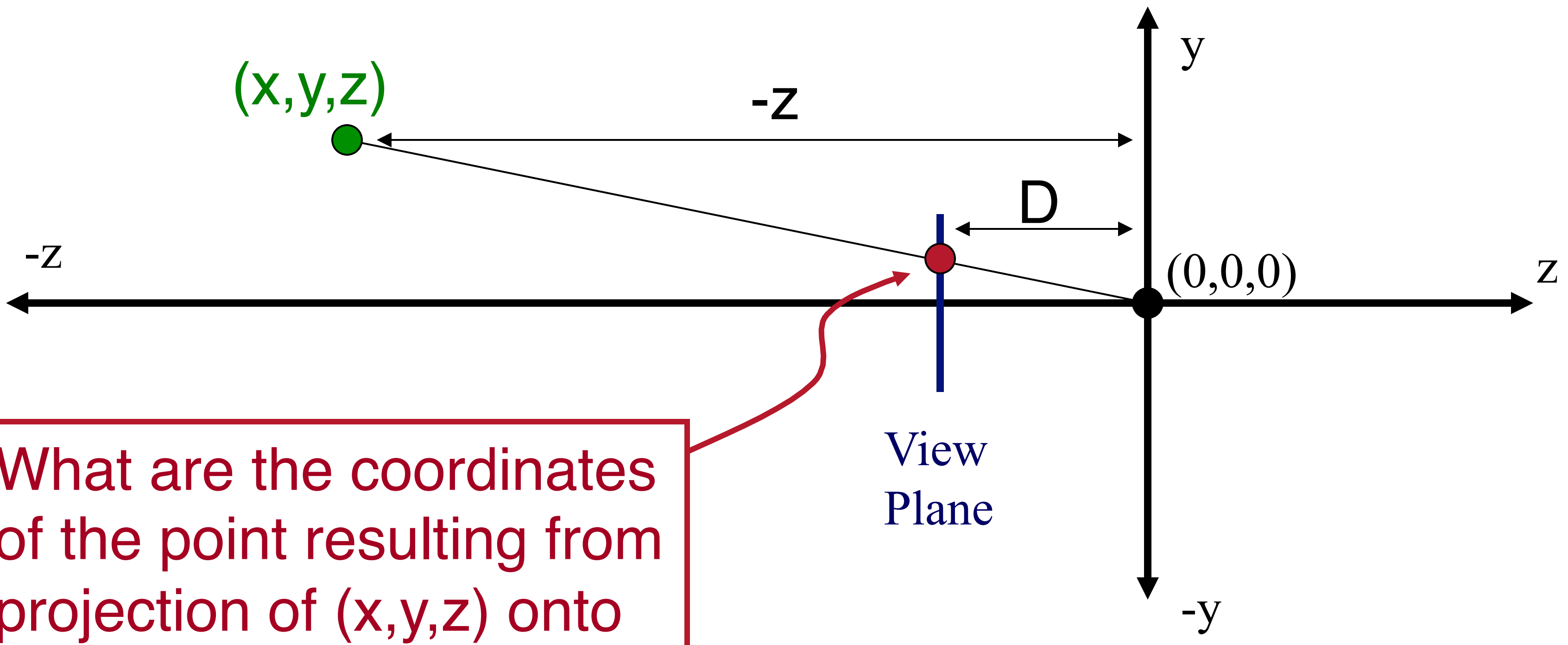
Perspective Projection View Volume



H&B Figure 12.30

Perspective Projection

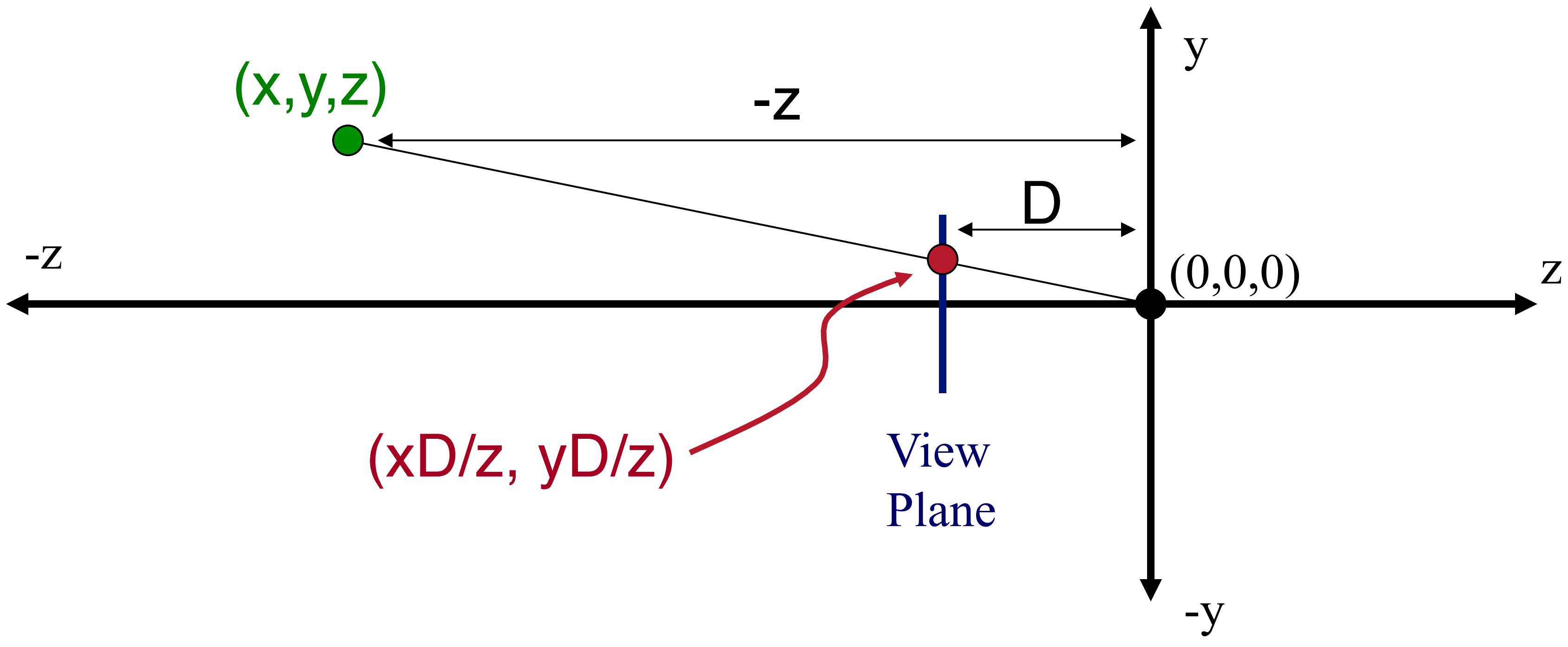
- Compute 2D coordinates from 3D coordinates with similar triangles



What are the coordinates of the point resulting from projection of (x,y,z) onto the view plane?

Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection Matrix

- 4x4 matrix representation?

$$\begin{aligned} x_s &= x_c D / z_c \\ y_s &= y_c D / z_c \\ z_s &= D \\ w_s &= 1 \end{aligned}$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

- 4x4 matrix representation?

$$\begin{aligned} x_s &= x_c D / z_c \\ y_s &= y_c D / z_c \\ z_s &= D \\ w_s &= 1 \end{aligned} \qquad \begin{aligned} x_s &= x' / w' \\ y_s &= y' / w' \\ z_s &= z' / w' \end{aligned} \qquad \begin{aligned} x' &= x_c \\ y' &= y_c \\ z' &= z_c \\ w' &= z_c / D \end{aligned}$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

- 4x4 matrix representation?

$$\begin{aligned} x_s &= x_c D / z_c \\ y_s &= y_c D / z_c \\ z_s &= D \\ w_s &= 1 \end{aligned} \qquad \begin{aligned} x_s &= x' / w' \\ y_s &= y' / w' \\ z_s &= z' / w' \end{aligned} \qquad \begin{aligned} x' &= x_c \\ y' &= y_c \\ z' &= z_c \\ w' &= z_c / D \end{aligned}$$

$$\boxed{\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}}$$

Perspective Projection Matrix

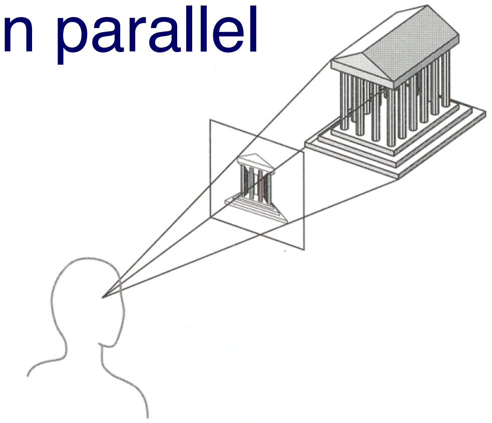
- In practice, want to compute a value related to depth to include in visibility calculations

$$\begin{aligned} x_s &= x_c D / z_c \\ y_s &= y_c D / z_c \\ z_s &= -D / z_c \\ w_s &= 1 \end{aligned} \qquad \begin{aligned} x_s &= x' / w' \\ y_s &= y' / w' \\ z_s &= z' / w' \end{aligned} \qquad \begin{aligned} x' &= x_c \\ y' &= y_c \\ z' &= -1 \\ w' &= z_c / D \end{aligned}$$

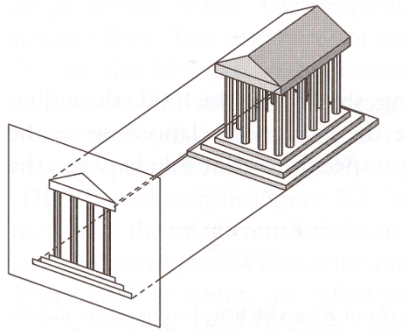
$$\boxed{\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}}$$

Perspective vs. Parallel

- Perspective projection
  - + Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel



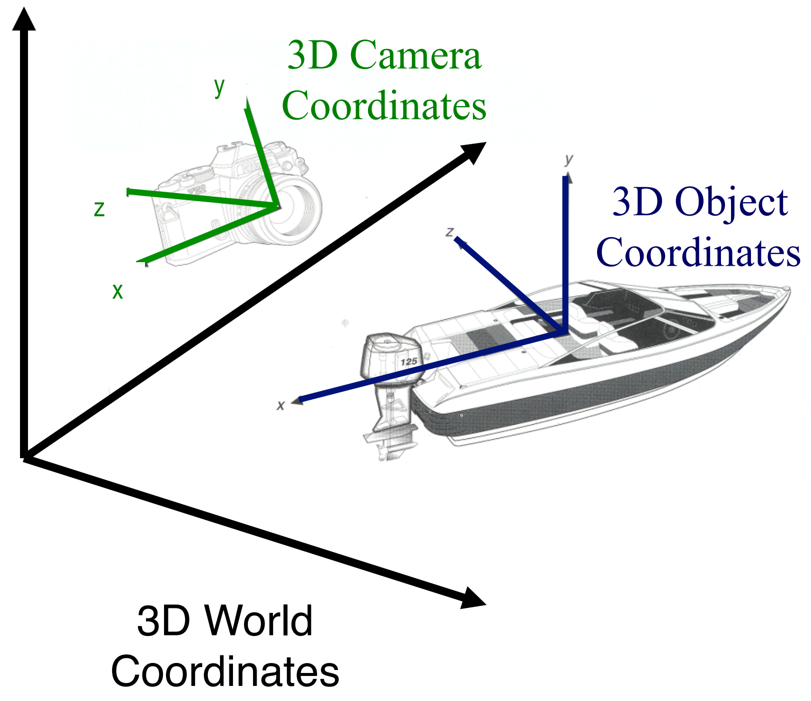
- Parallel projection
  - + Good for exact measurements
  - + Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking



Transformations

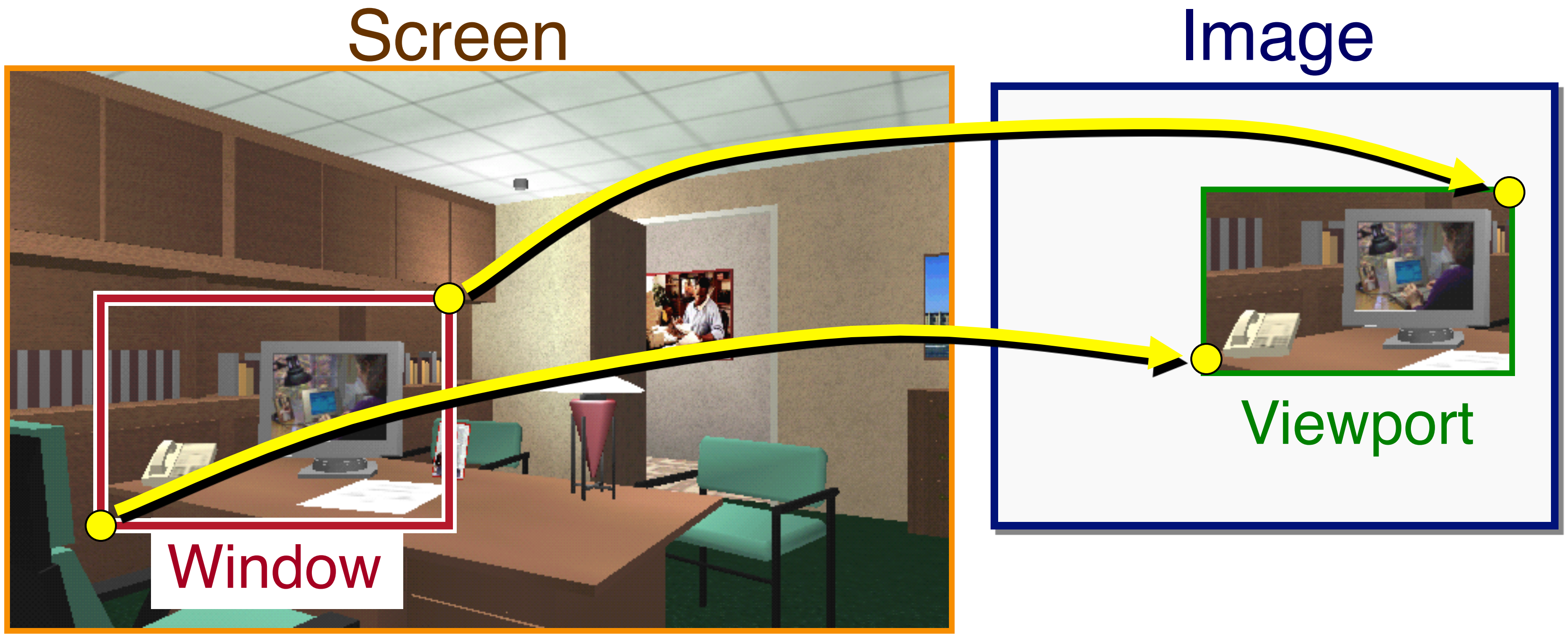
$$\begin{tikzcd}
p(x,y,z) \arrow[d, "\text{3D Object Coordinates}"] \\
\boxed{\text{Modeling Transformation}} \arrow[d, "\text{3D World Coordinates}"] \\
\boxed{\text{Viewing Transformation}} \arrow[d, "\text{3D Camera Coordinates}"] \\
\boxed{\text{Projection Transformation}} \arrow[d, "\text{2D Screen Coordinates}"] \\
\boxed{\text{Viewport Transformation}} \arrow[d, "\text{2D Image Coordinates}"] \\
p'(x',y')
\end{tikzcd}$$

Transformations map points from one coordinate system to another



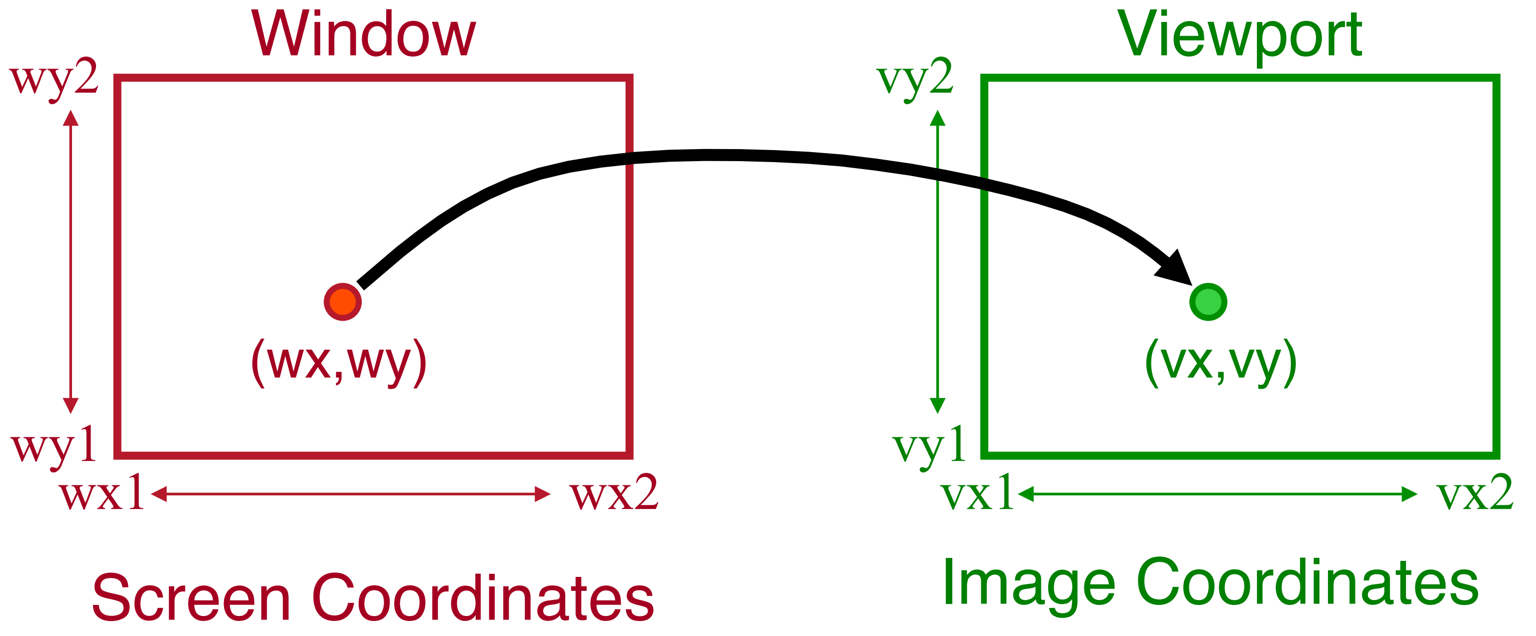
Viewport Transformation

- Transform 2D geometric primitives from screen coordinate system (normalized device coordinates) to image coordinate system (pixels)



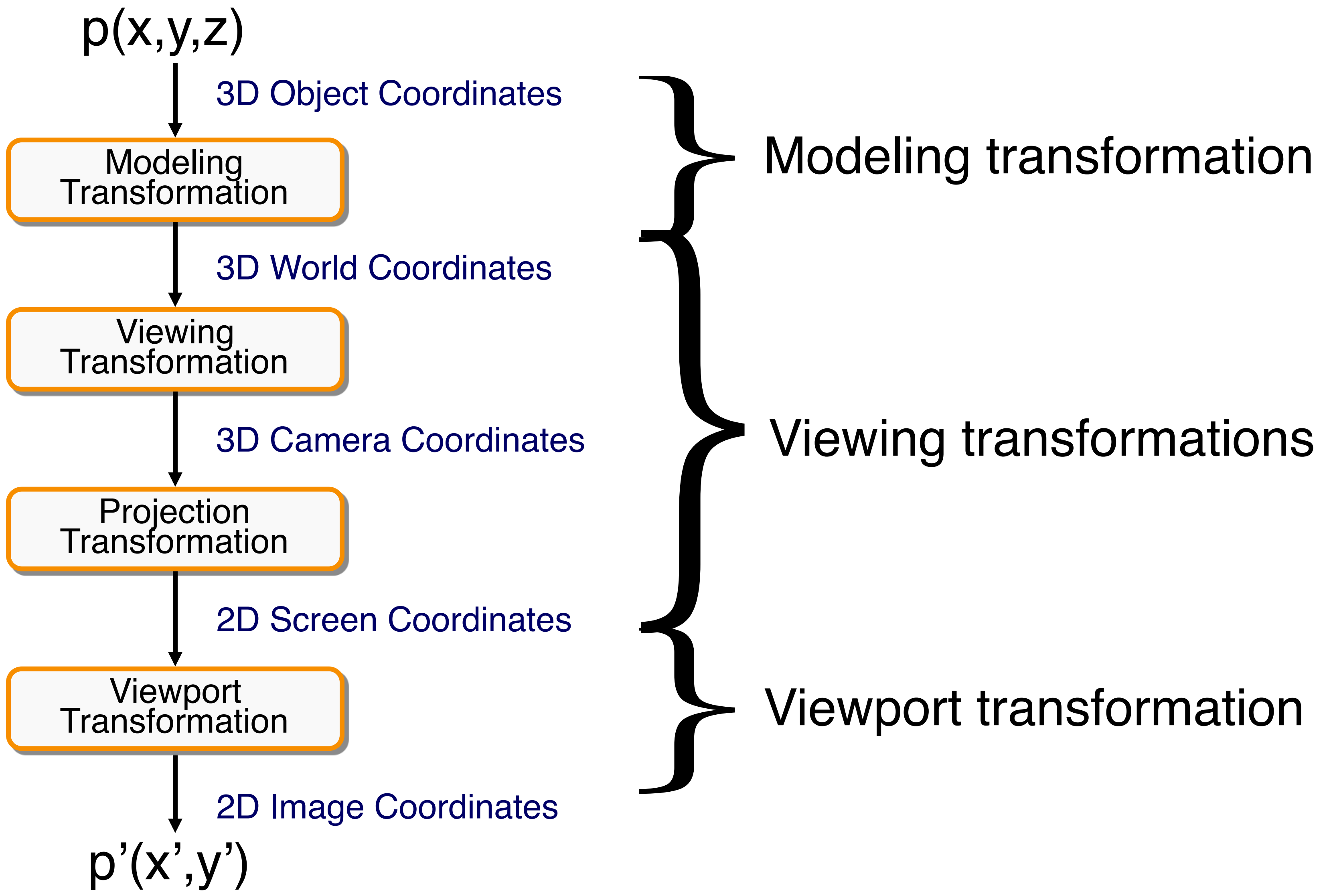
Viewport Transformation

- Window-to-viewport mapping



```
vx = vx1 + (wx - wx1) * (vx2 - vx1) / (wx2 - wx1);
vy = vy1 + (wy - wy1) * (vy2 - vy1) / (wy2 - wy1);
```

Summary of Transformations



3D Rendering Pipeline (for direct illumination)

3D Primitives

↓ 3D Modeling Coordinates

Modeling Transformation

↓ 3D World Coordinates

Lighting

↓ 3D World Coordinates

Viewing Transformation

↓ 3D Camera Coordinates

Projection Transformation

↓ 2D Screen Coordinates

Clipping

↓ 2D Screen Coordinates

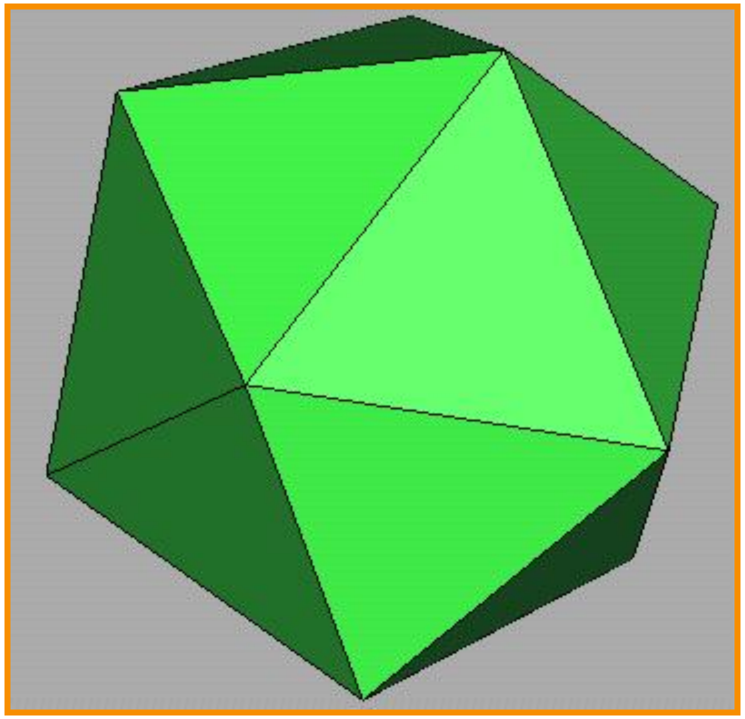
Viewport Transformation

↓ 2D Image Coordinates

Scan Conversion

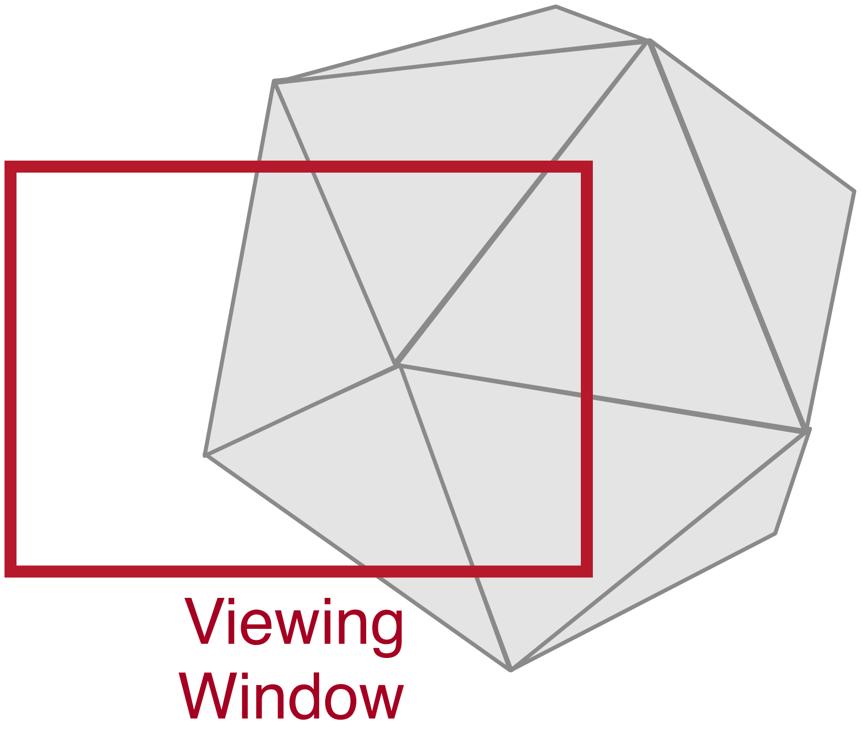
↓ 2D Image Coordinates

Image



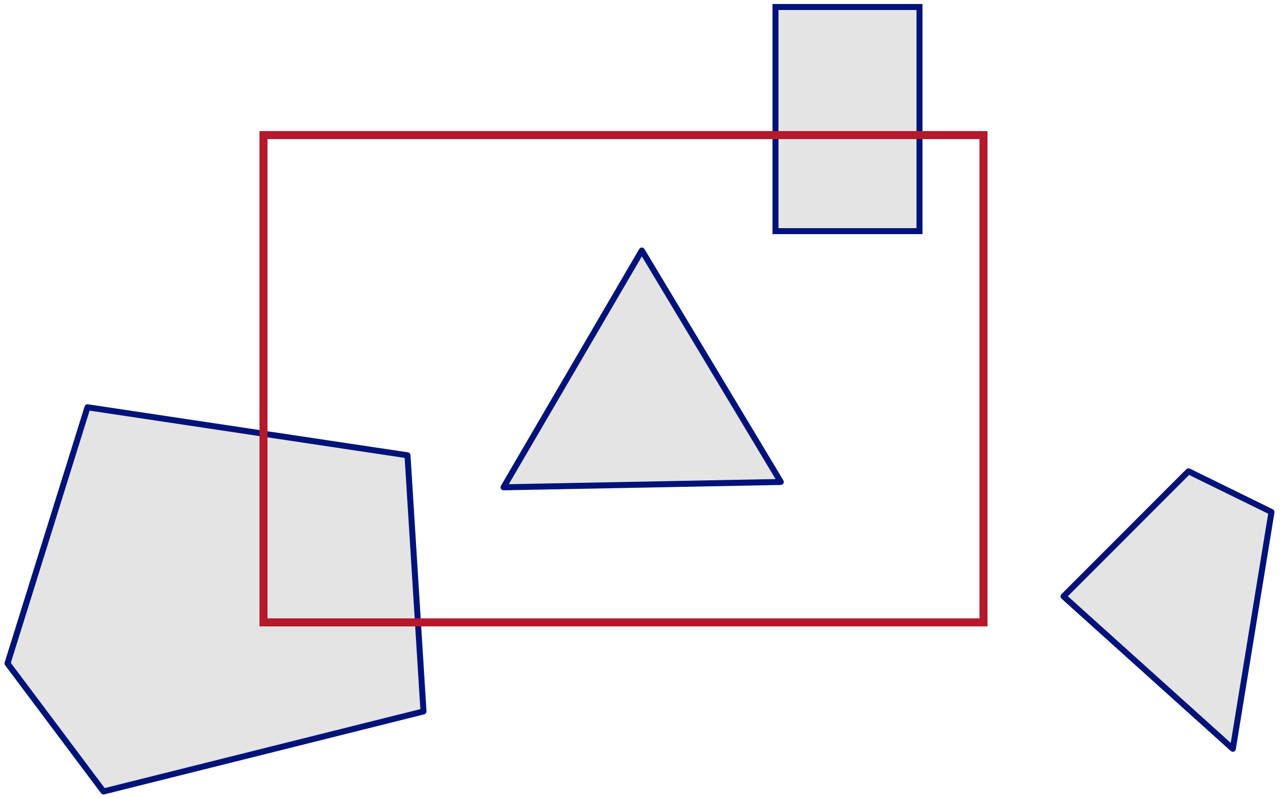
Clipping

- Avoid drawing parts of primitives outside window
  - Window defines part of scene being viewed
  - Must draw geometric primitives only inside window



Polygon Clipping

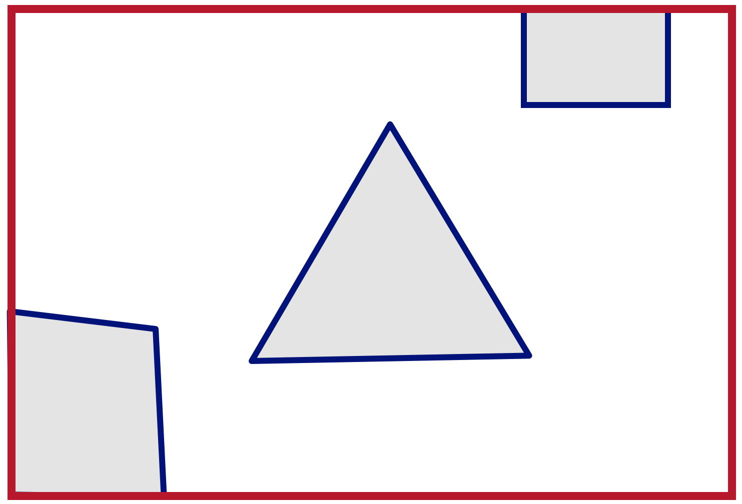
- Find the part of a polygon inside the clip window?



Before Clipping

Polygon Clipping

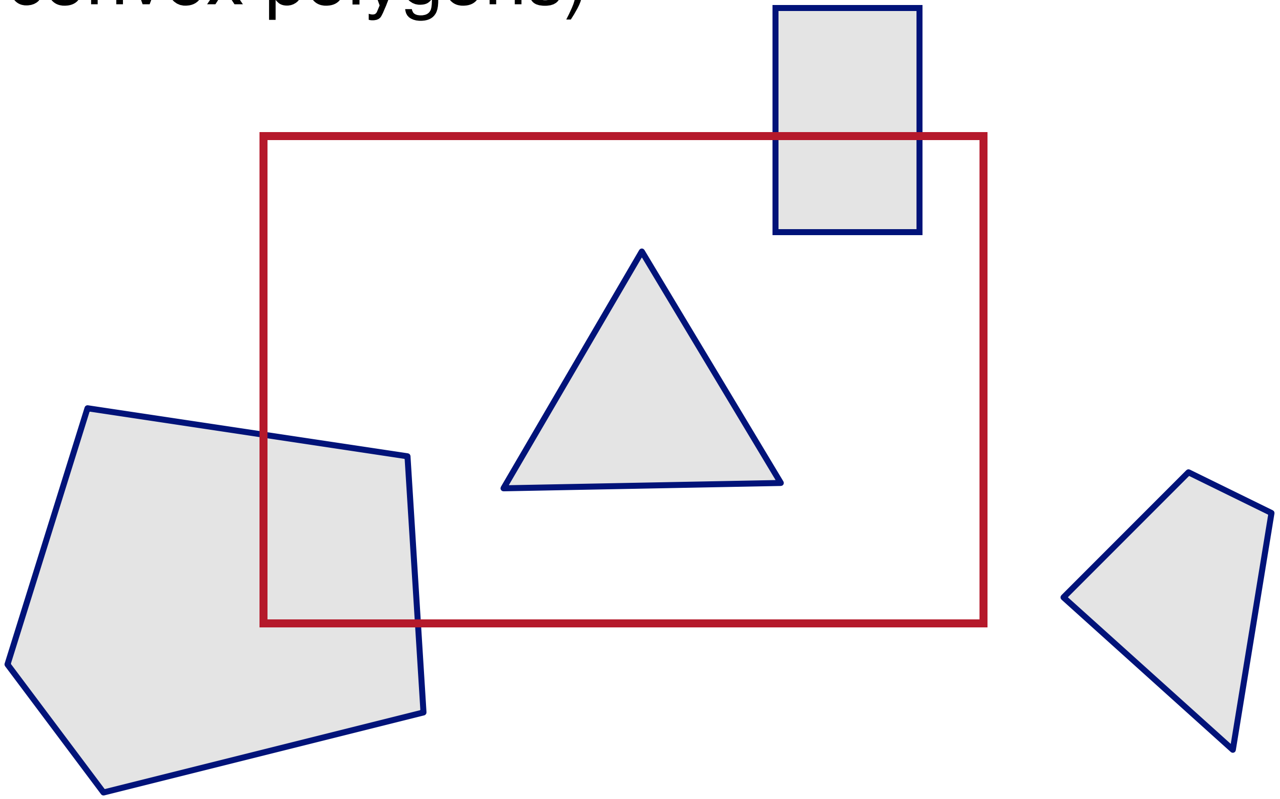
- Find the part of a polygon inside the clip window?



After Clipping

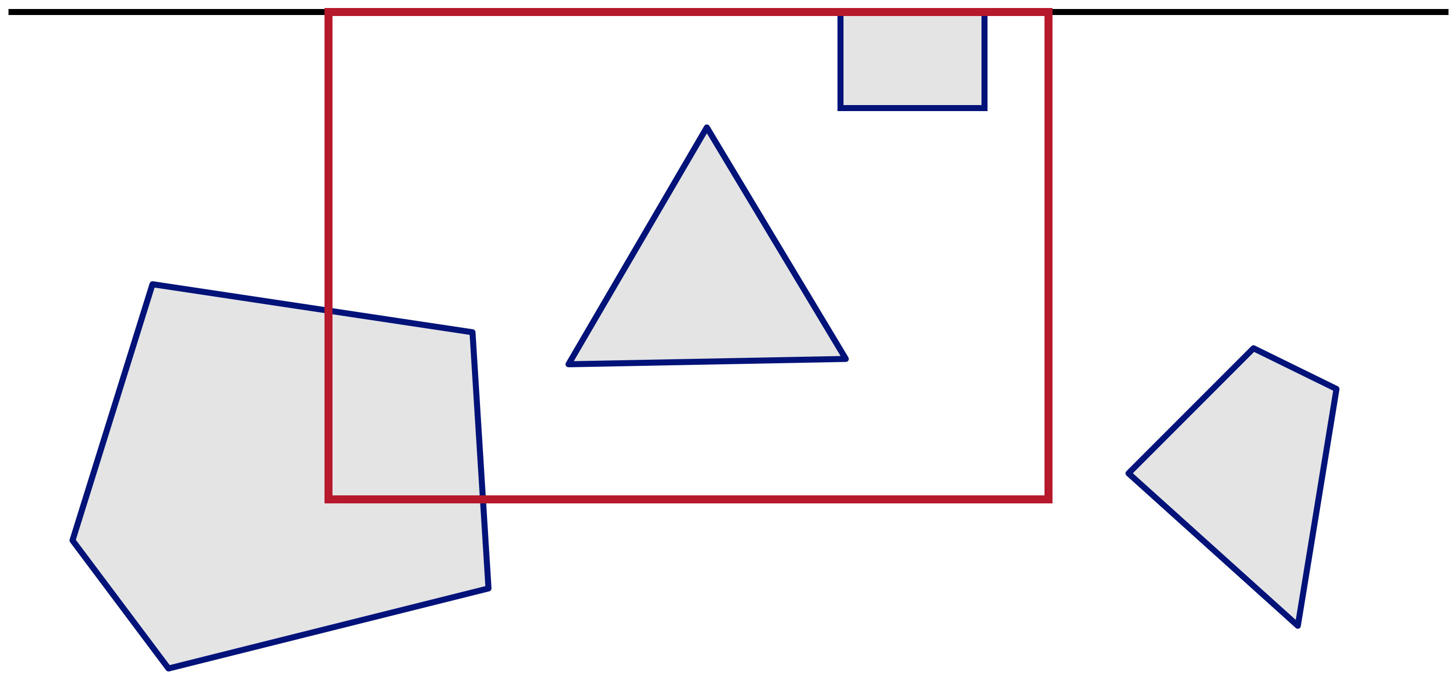
Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time (for convex polygons)



Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time



Sutherland Hodgeman Clipping

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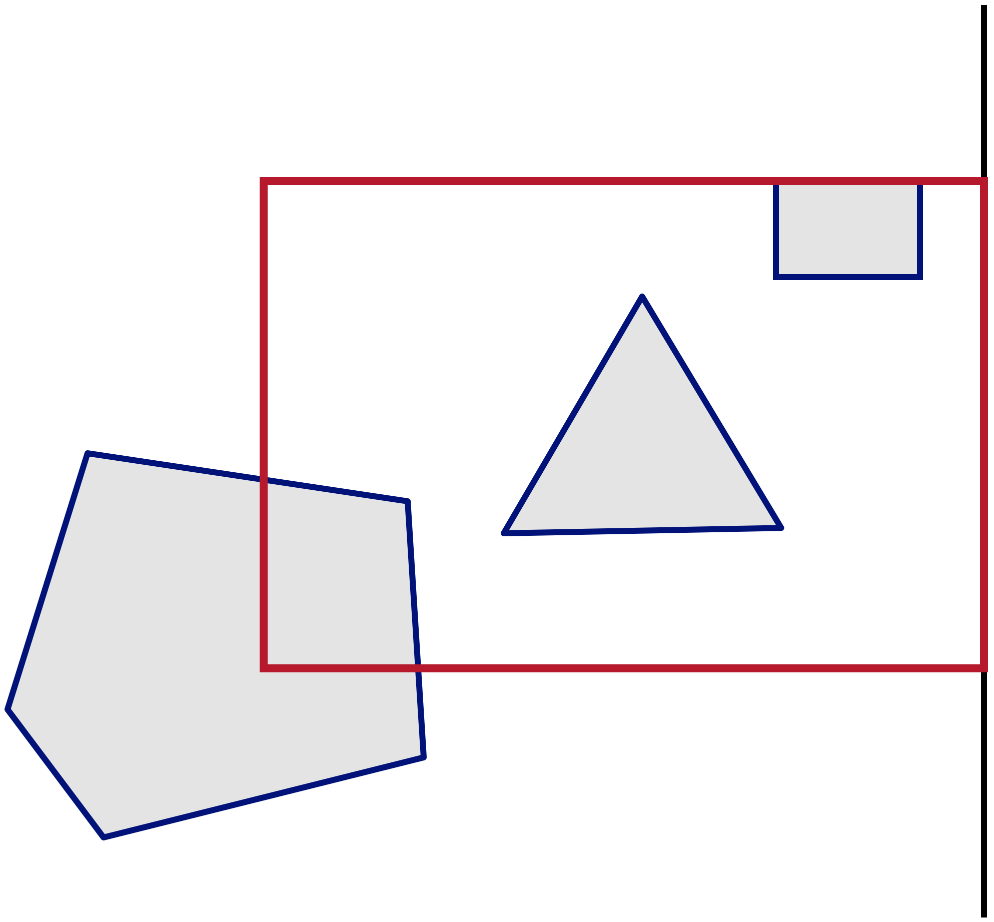

Sutherland Hodgeman Clipping

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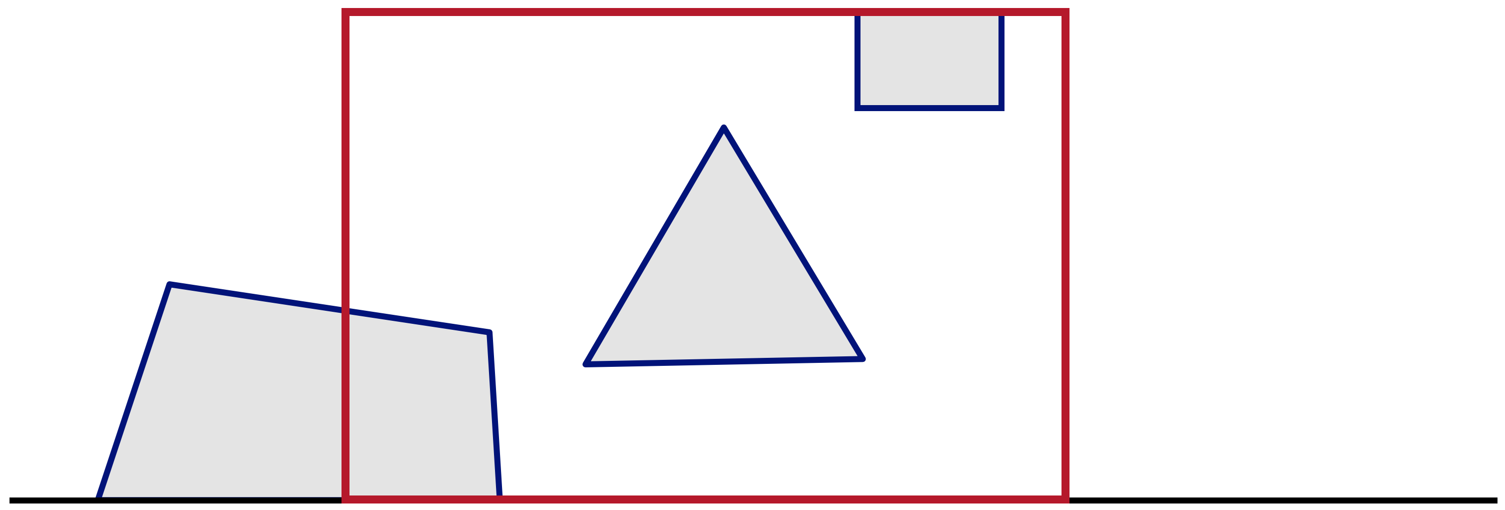

Sutherland Hodgeman Clipping

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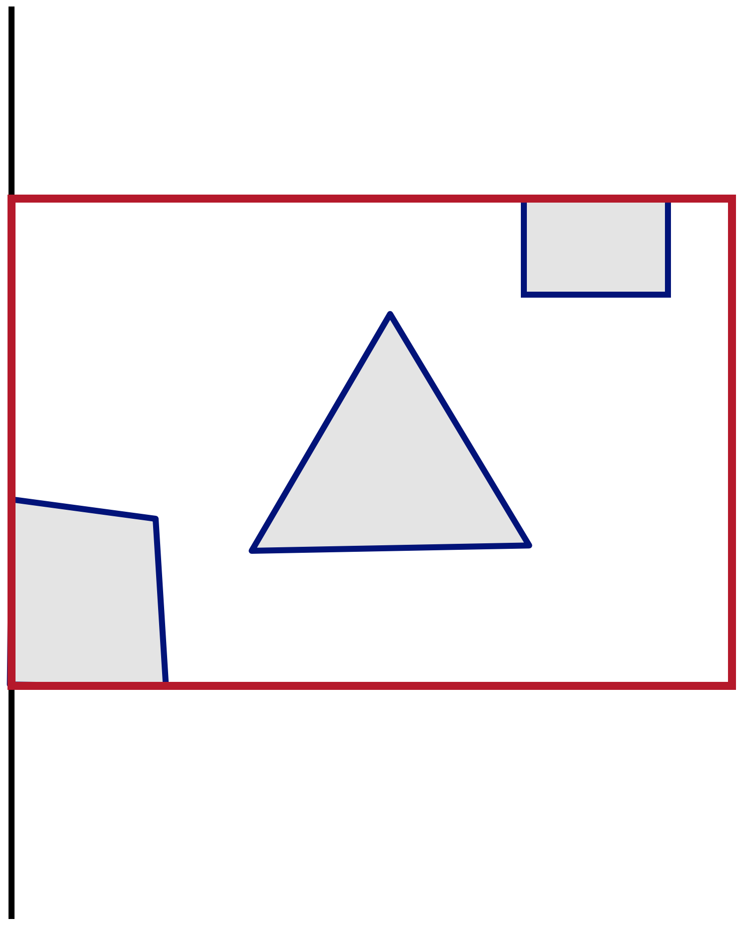
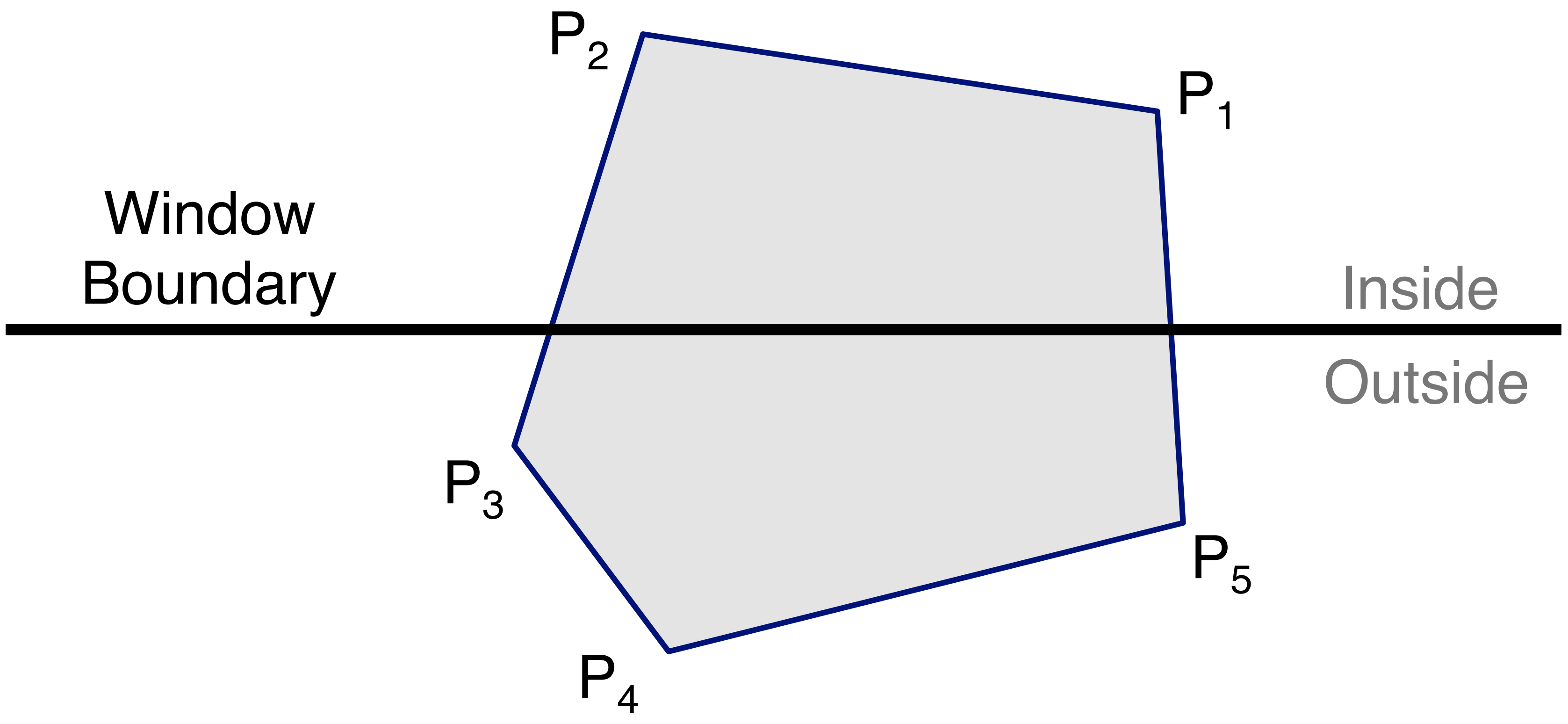


Clipping to a Boundary

- Do inside test for each point in sequence, Insert new points when cross window boundary, Remove points outside window boundary



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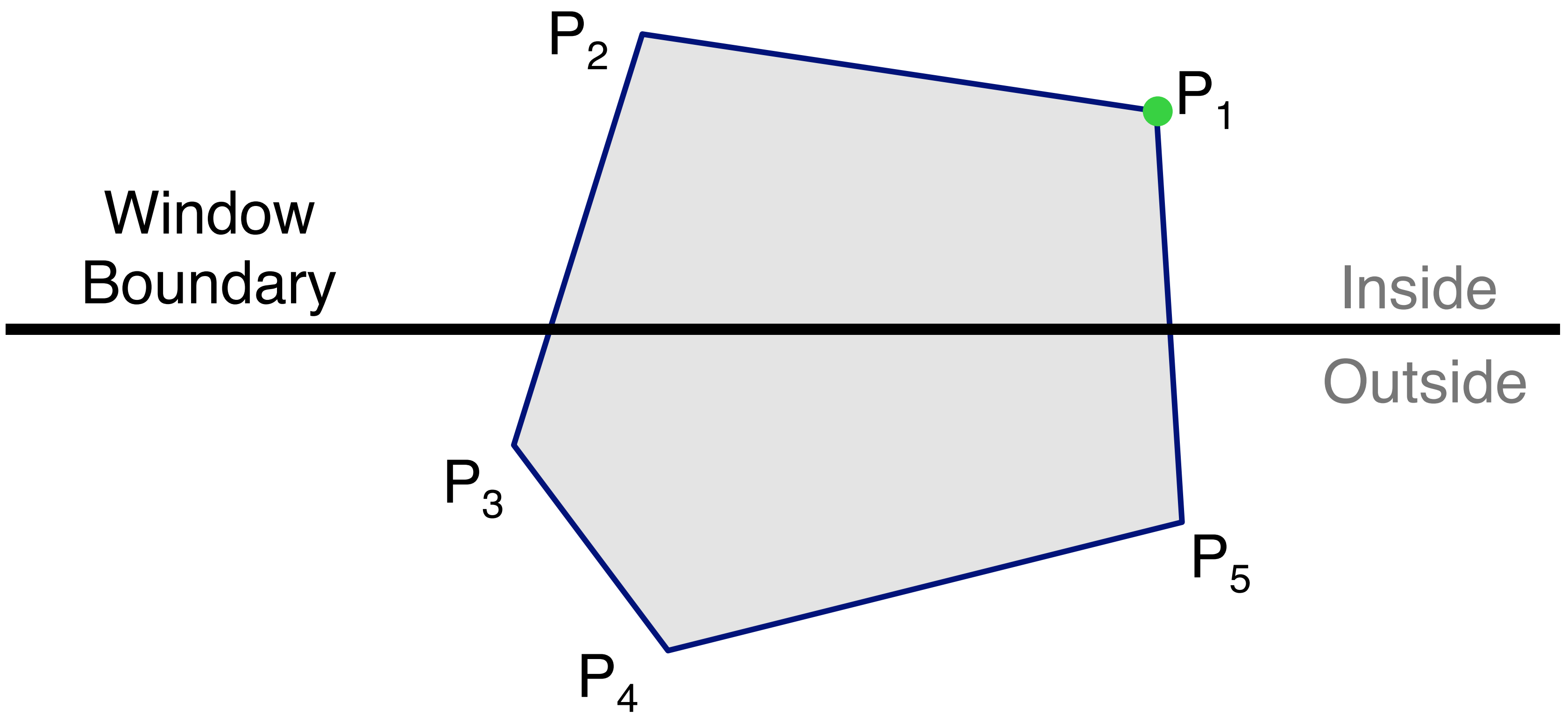

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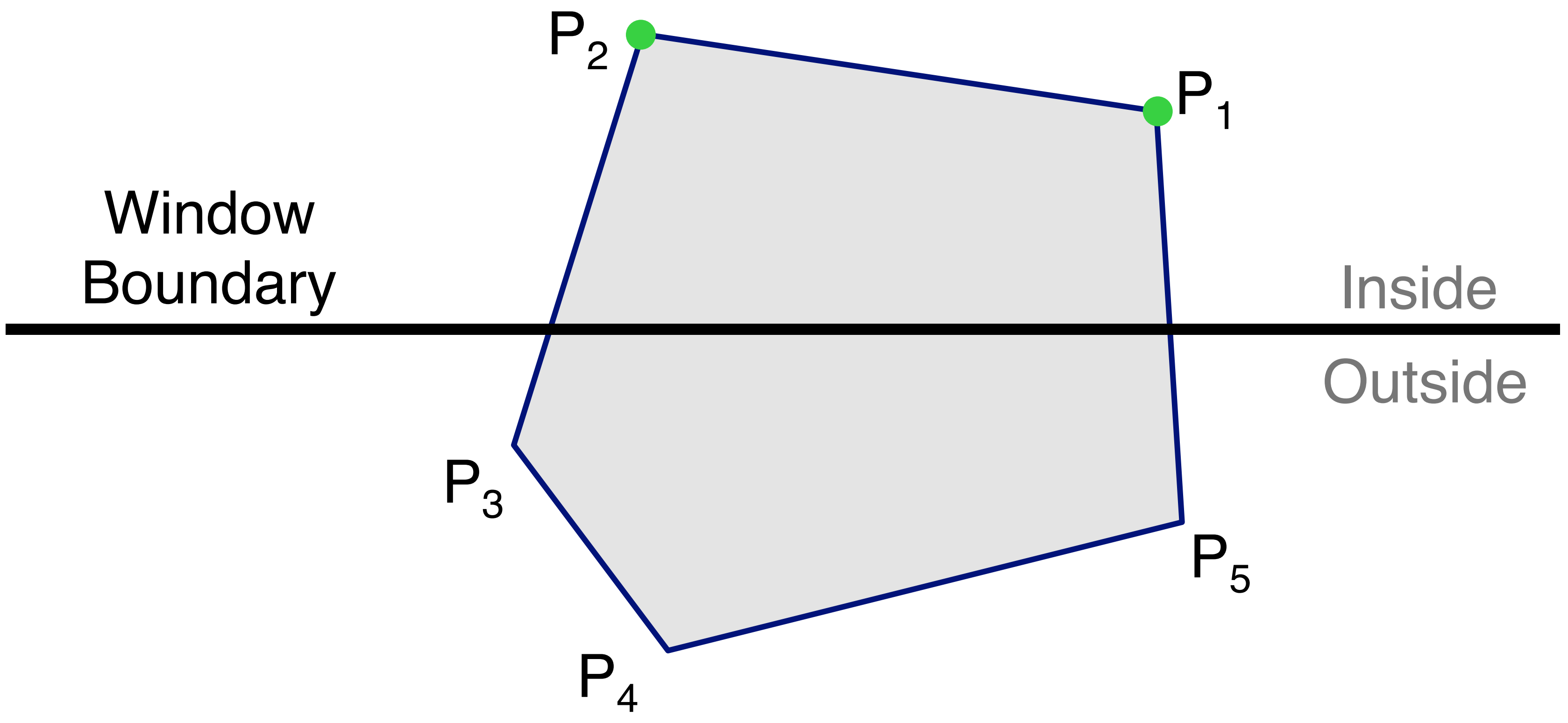

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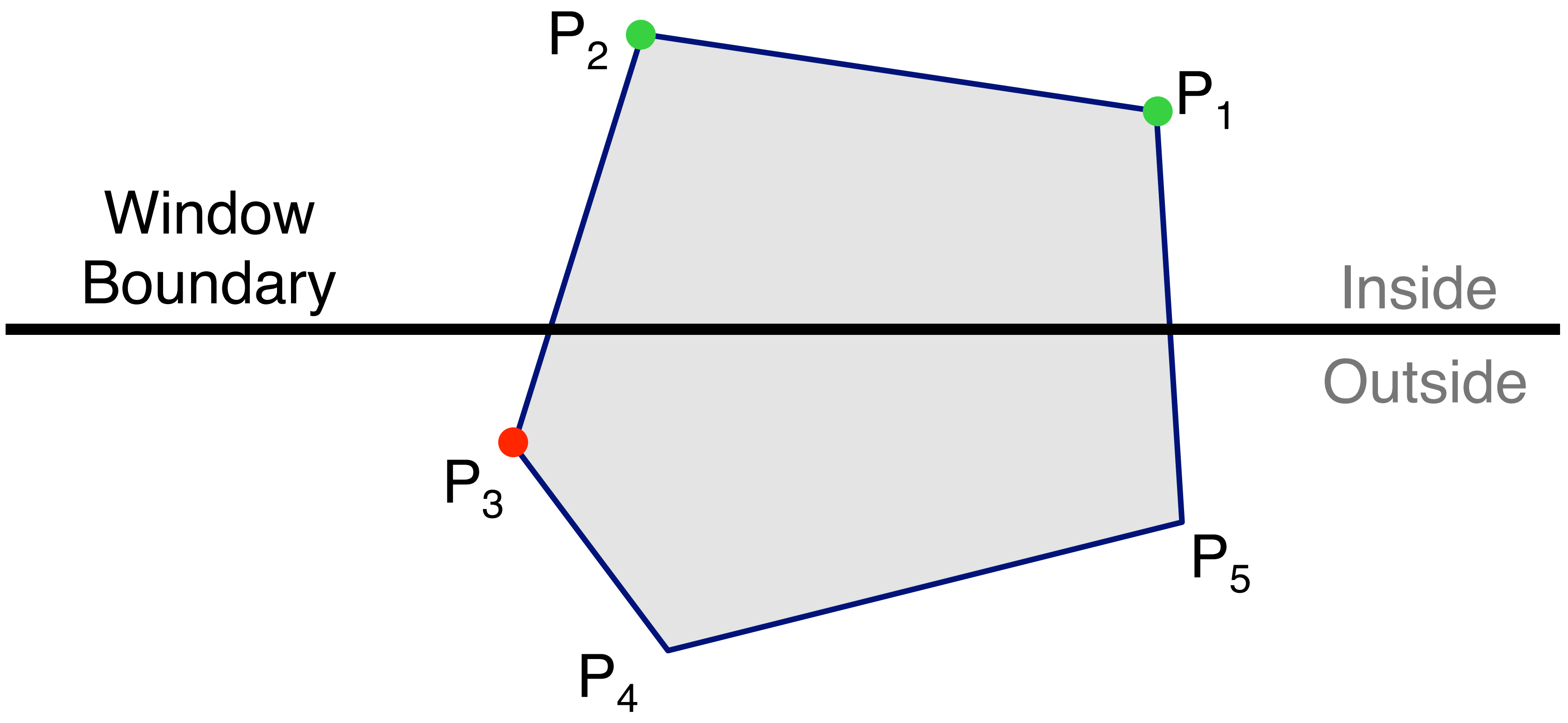

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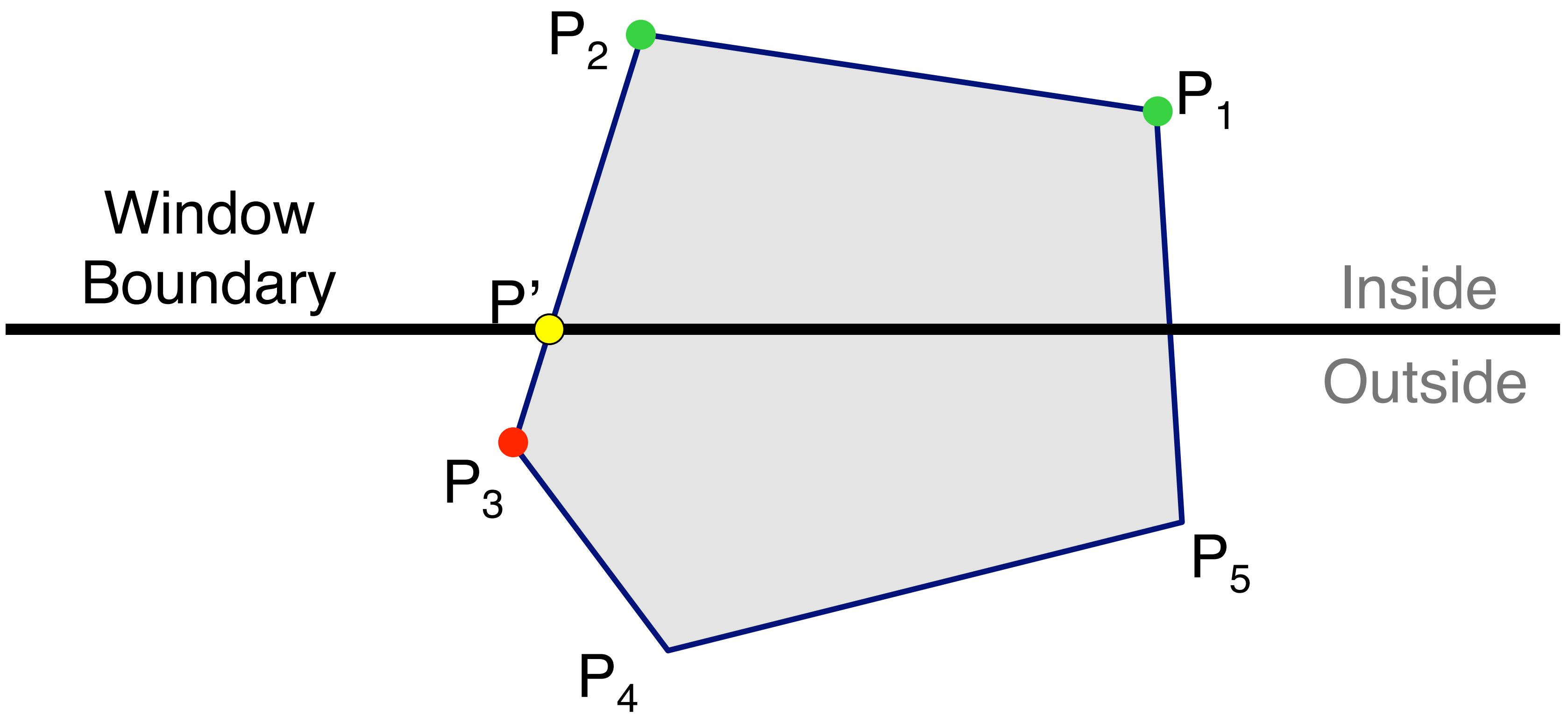

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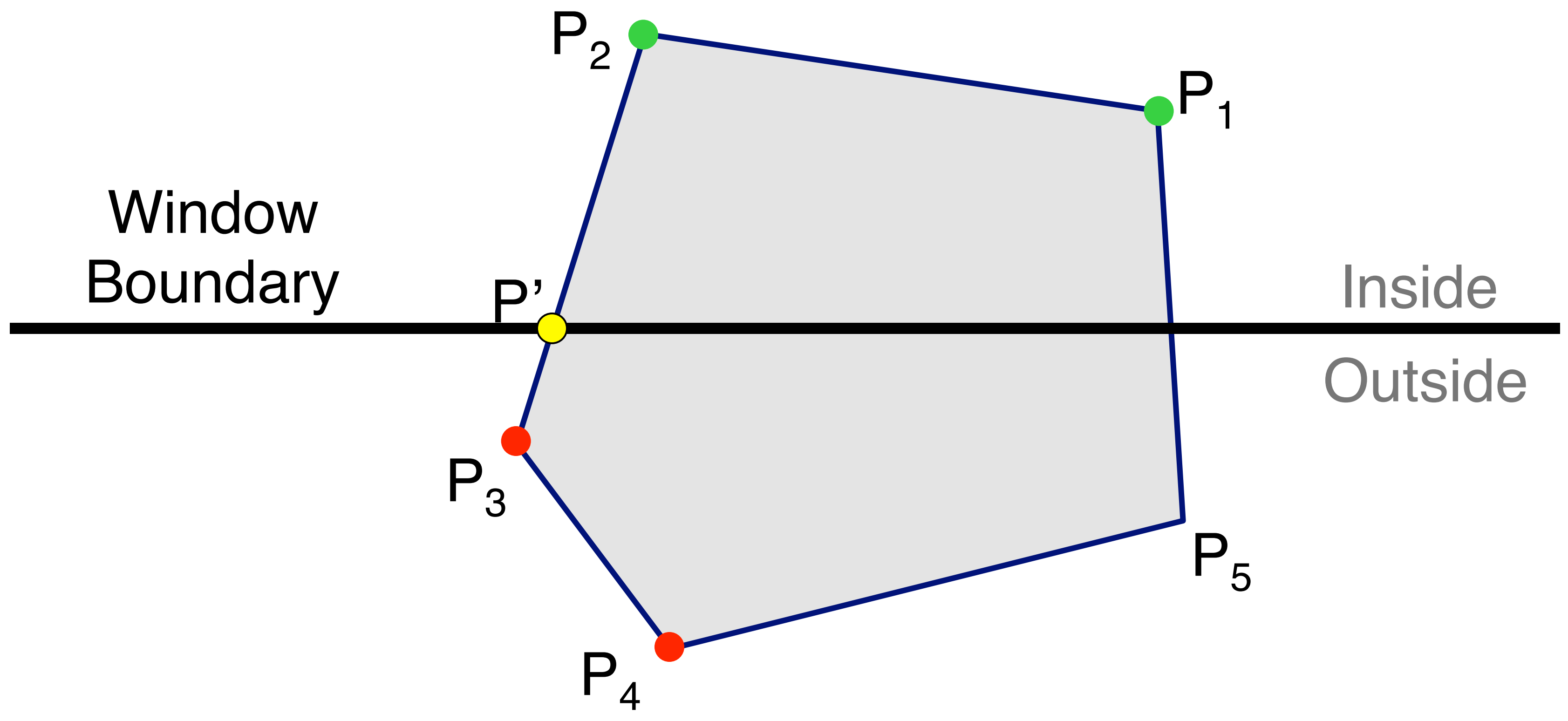

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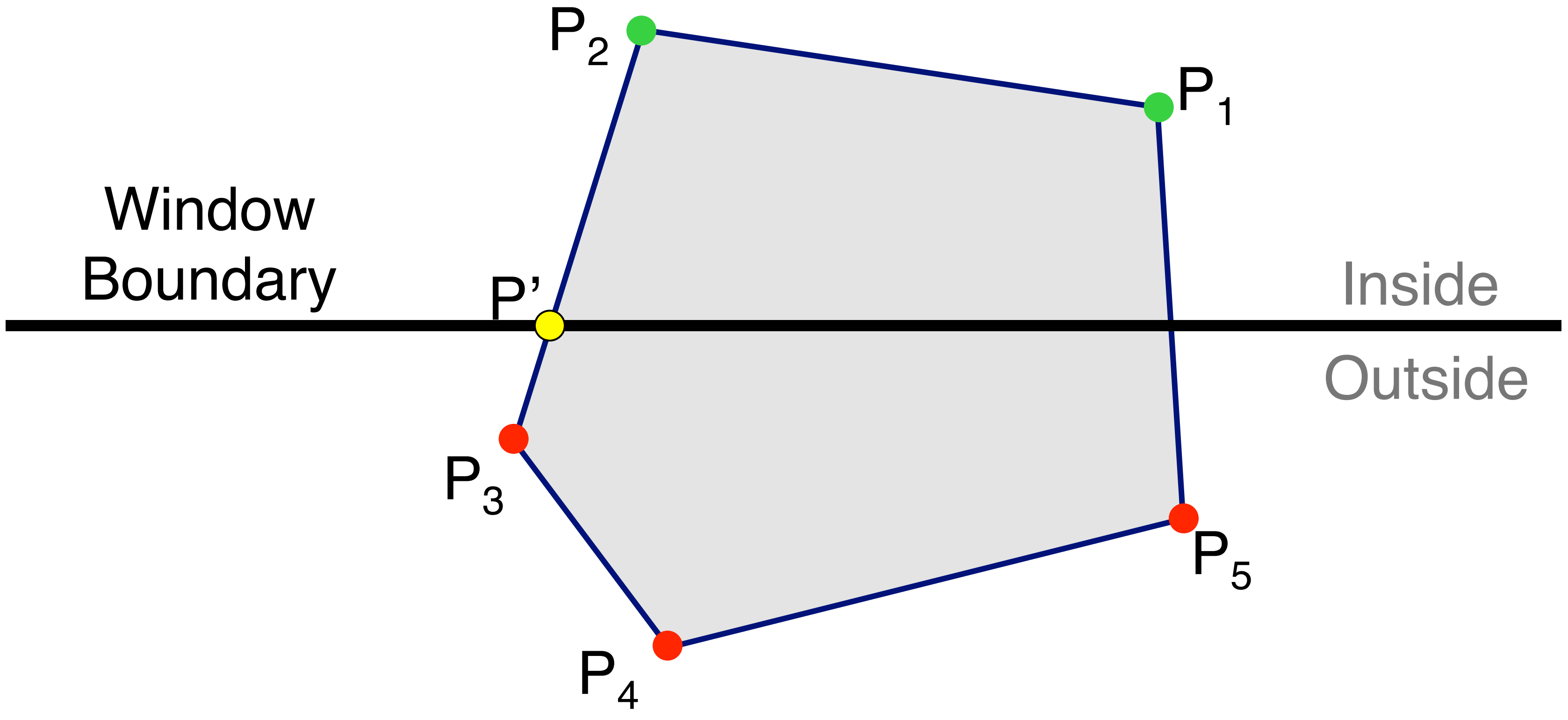

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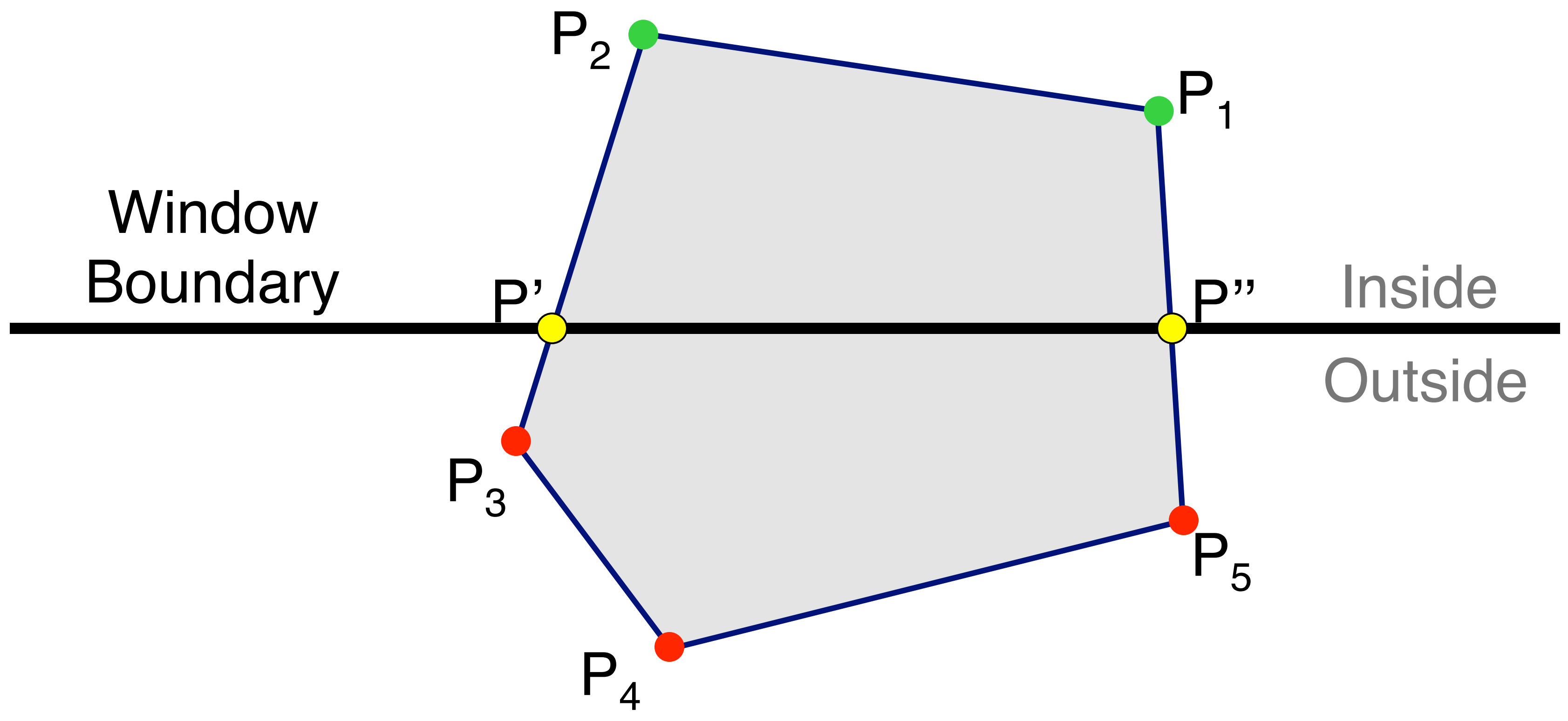

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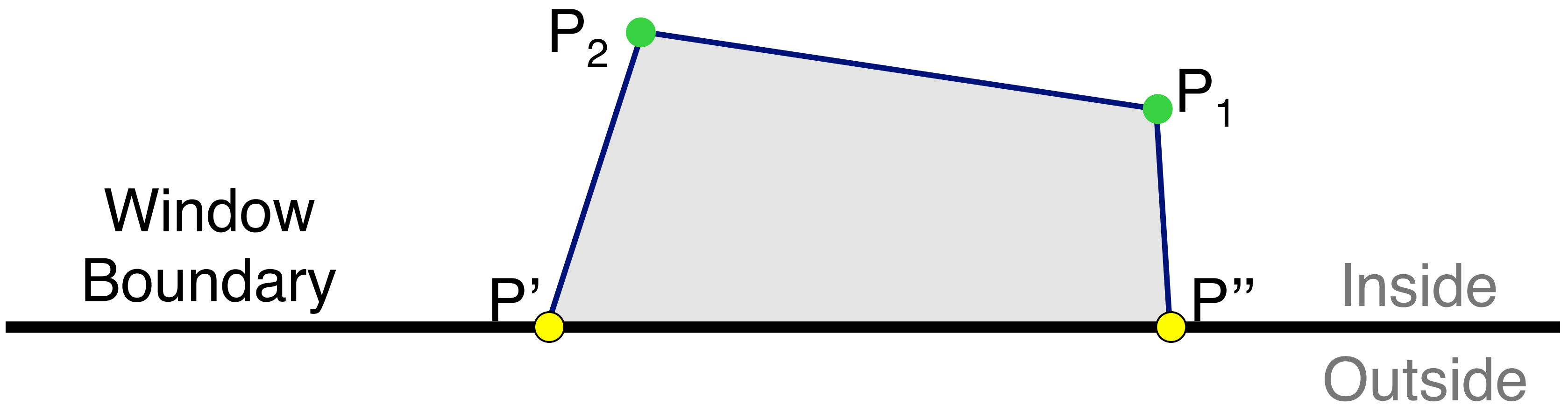
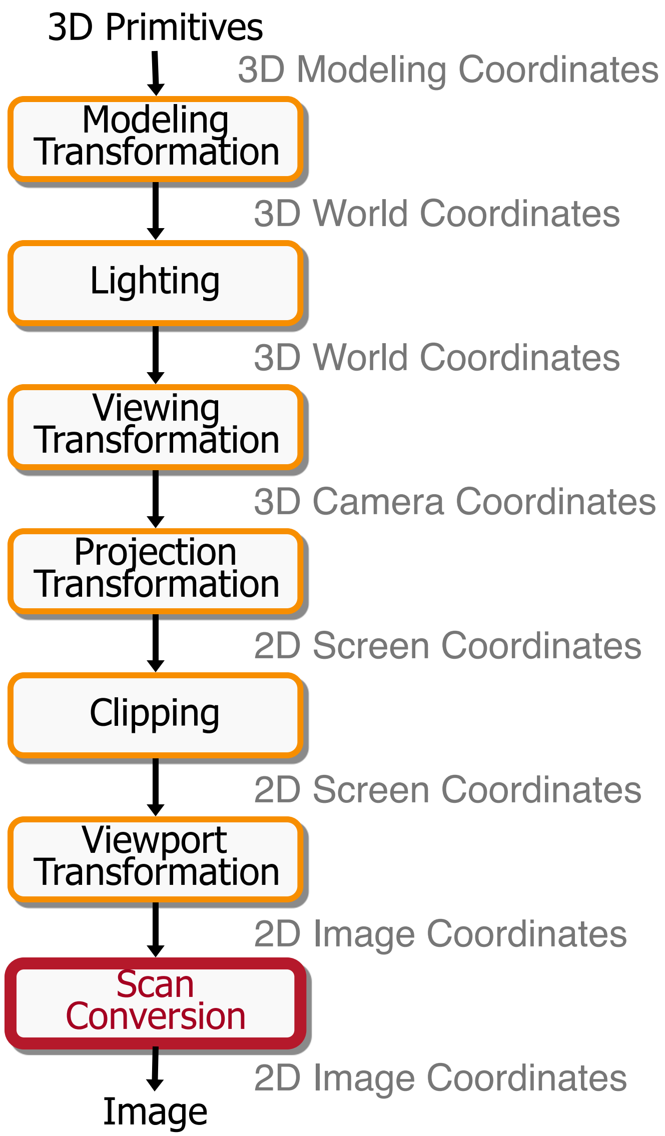
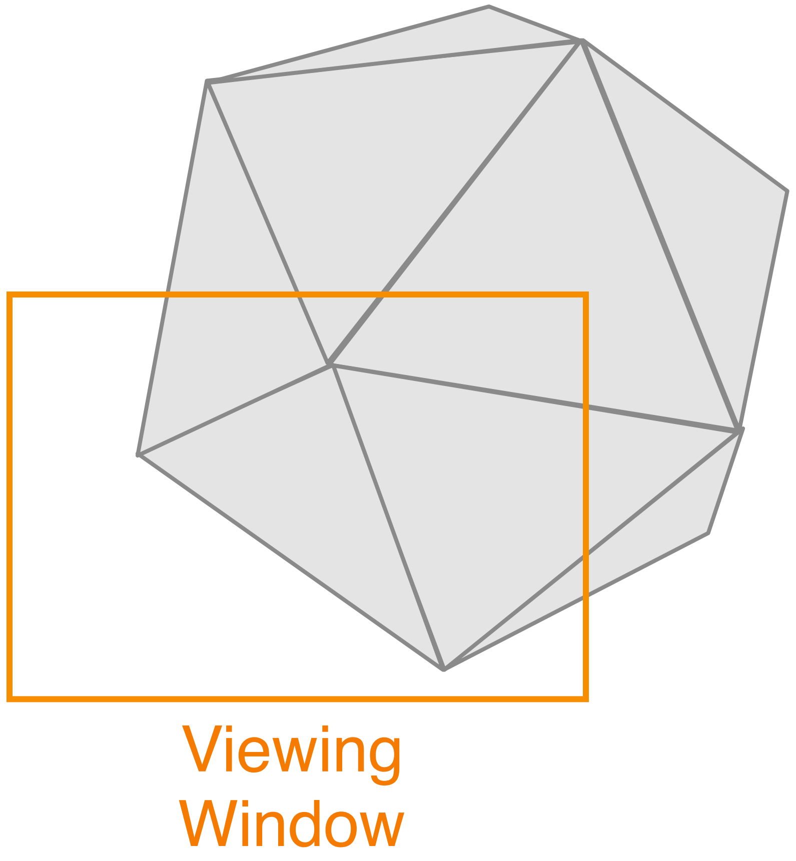
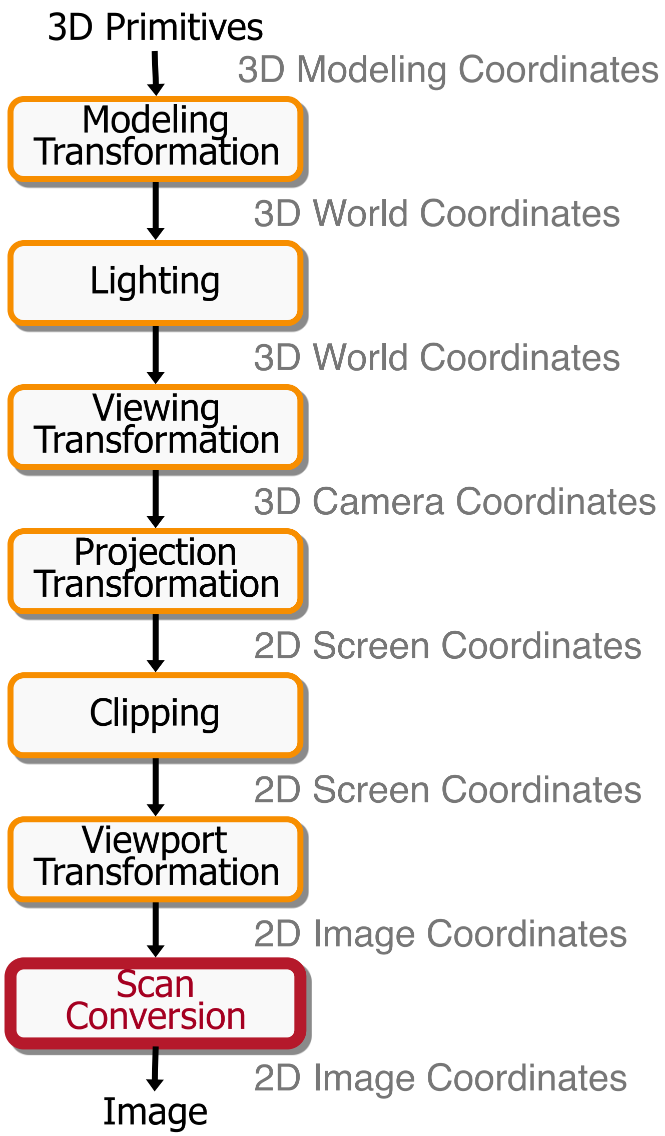


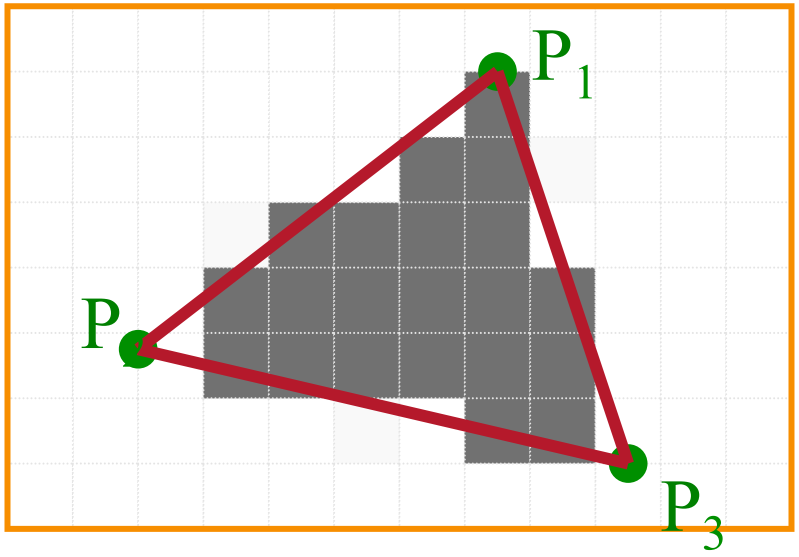
3D Rendering Pipeline (for direct illumination)





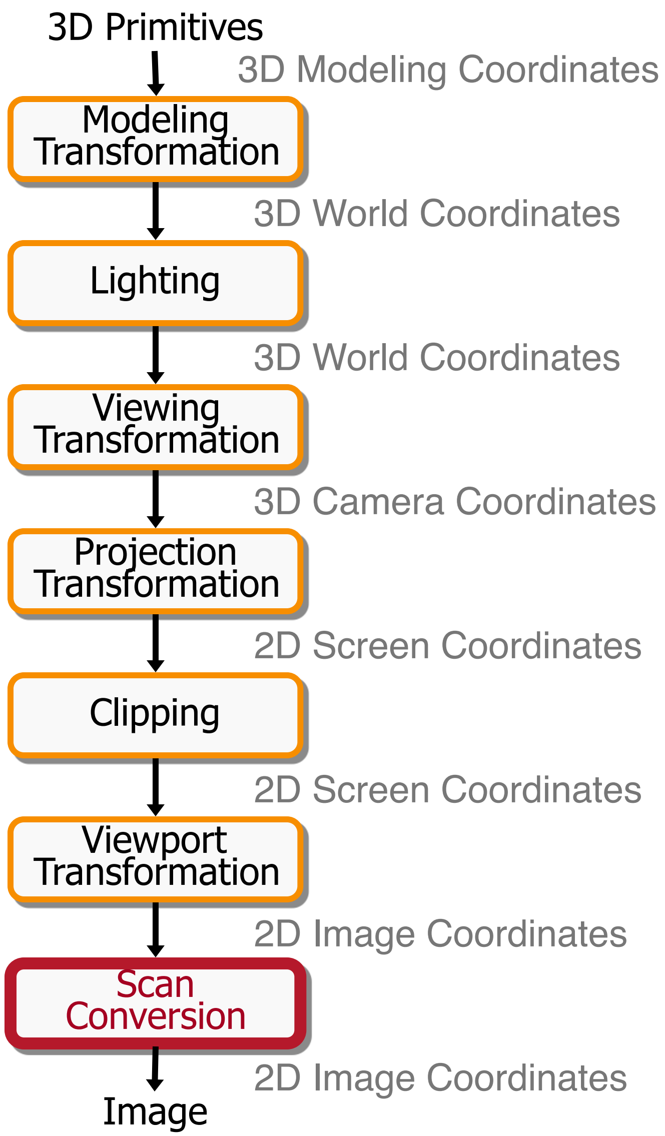
3D Rendering Pipeline (for direct illumination)

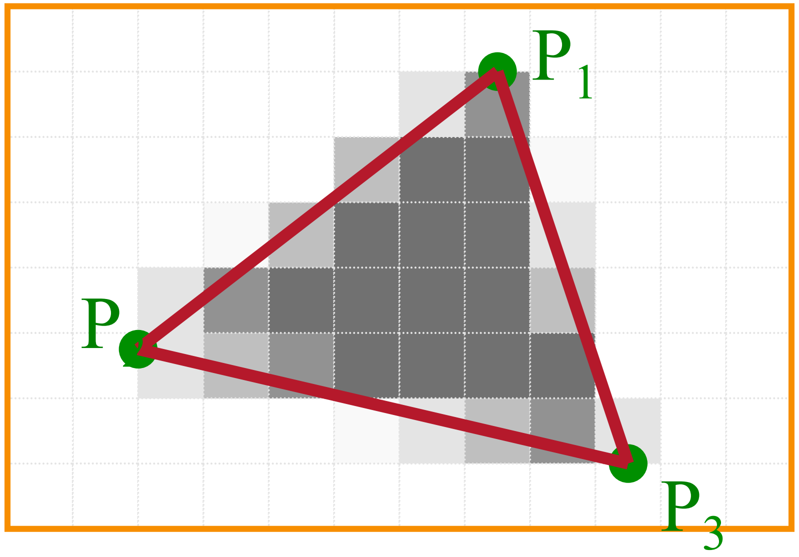




Standard (aliased)
Scan Conversion

3D Rendering Pipeline (for direct illumination)





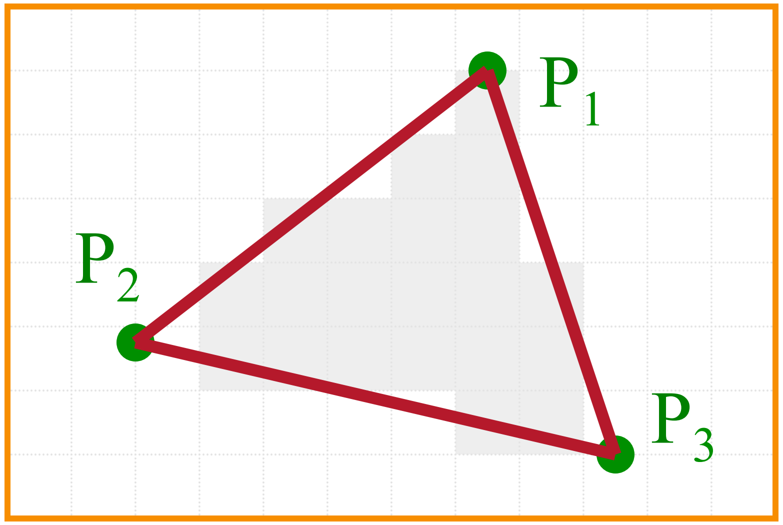
Antialiased
Scan Conversion

Scan Conversion

- Render an image of a geometric primitive by setting pixel colors

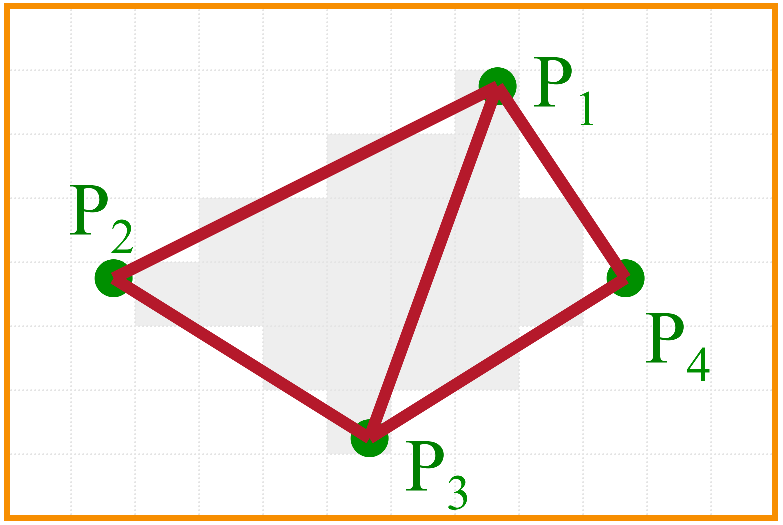
```
void SetPixel(int x, int y, Color rgba)
```

- Example: Filling the inside of a triangle



Triangle Scan Conversion

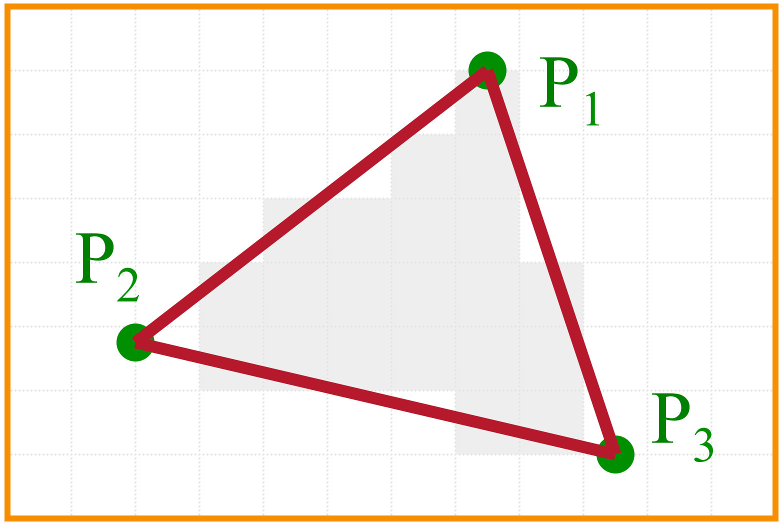
- Properties of a good algorithm
  - Symmetric
  - Straight edges
  - No cracks between adjacent primitives
  - (Antialiased edges)
  - FAST!



Simple Algorithm

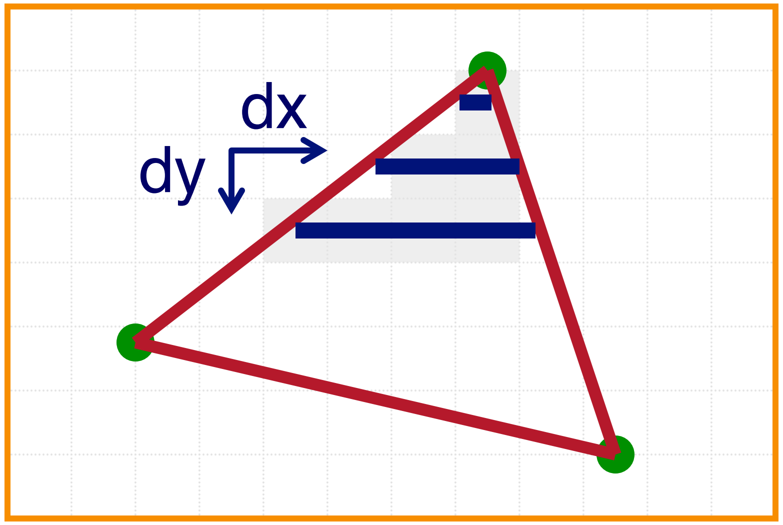
- Color all pixels inside triangle

```
void ScanTriangle(Triangle T, Color rgba){
   for each pixel P in bbox(T){
       if (Inside(T, P))
           SetPixel(P.x, P.y, rgba);
   }
}
```



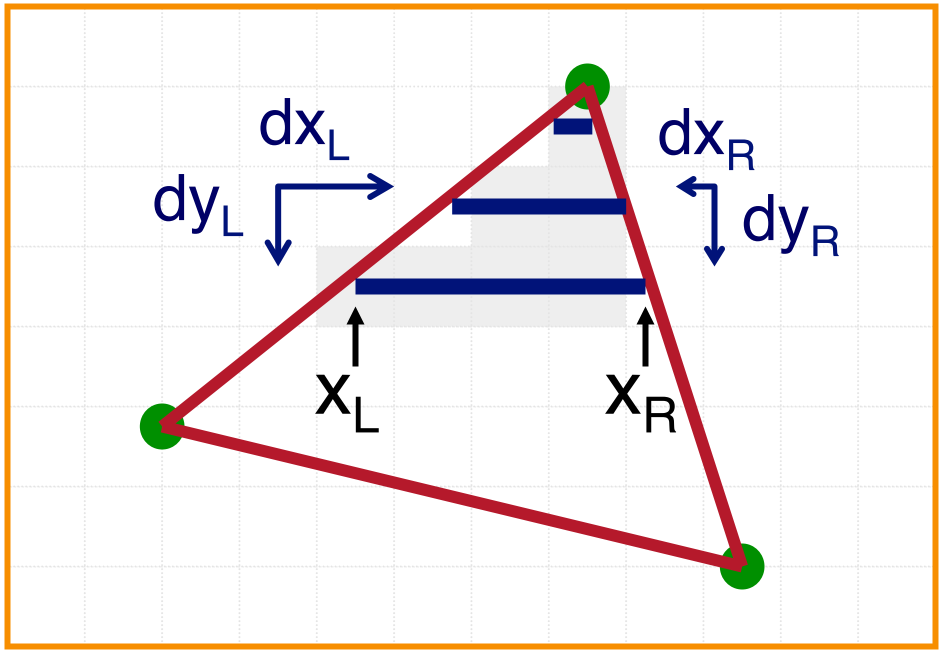
Triangle Sweep-Line Algorithm

- Take advantage of spatial coherence
  - Compute which pixels are inside using horizontal spans
  - Process horizontal spans in scan-line order
- Take advantage of edge linearity
  - Use edge slopes to update coordinates incrementally



Triangle Sweep-Line Algorithm

```
void ScanTriangle(Triangle T, Color rgba){
   for each edge pair {
      initialize $x_L$, $x_R$;
      compute $dx_L/dy_L$ and $dx_R/dy_R$;
      for each scanline at y
         for (int x = $x_L$; x <= $x_R$; x++)
            SetPixel(x, y, rgba);
      $x_L$ += $dx_L/dy_L$;
      $x_R$ += $dx_R/dy_R$;
   }
}
```



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            SetPixel(x, y, rgba);
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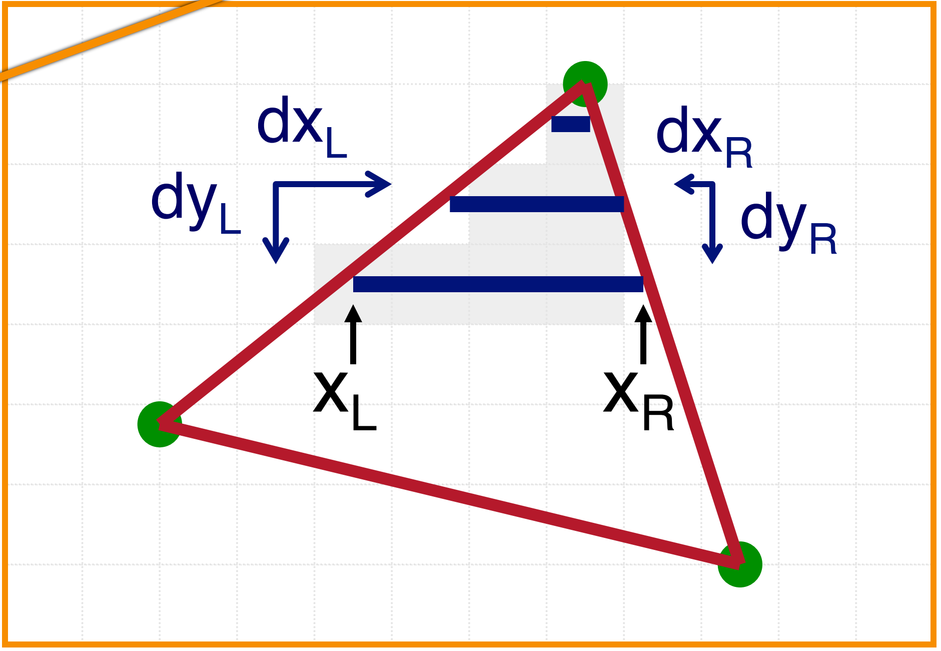
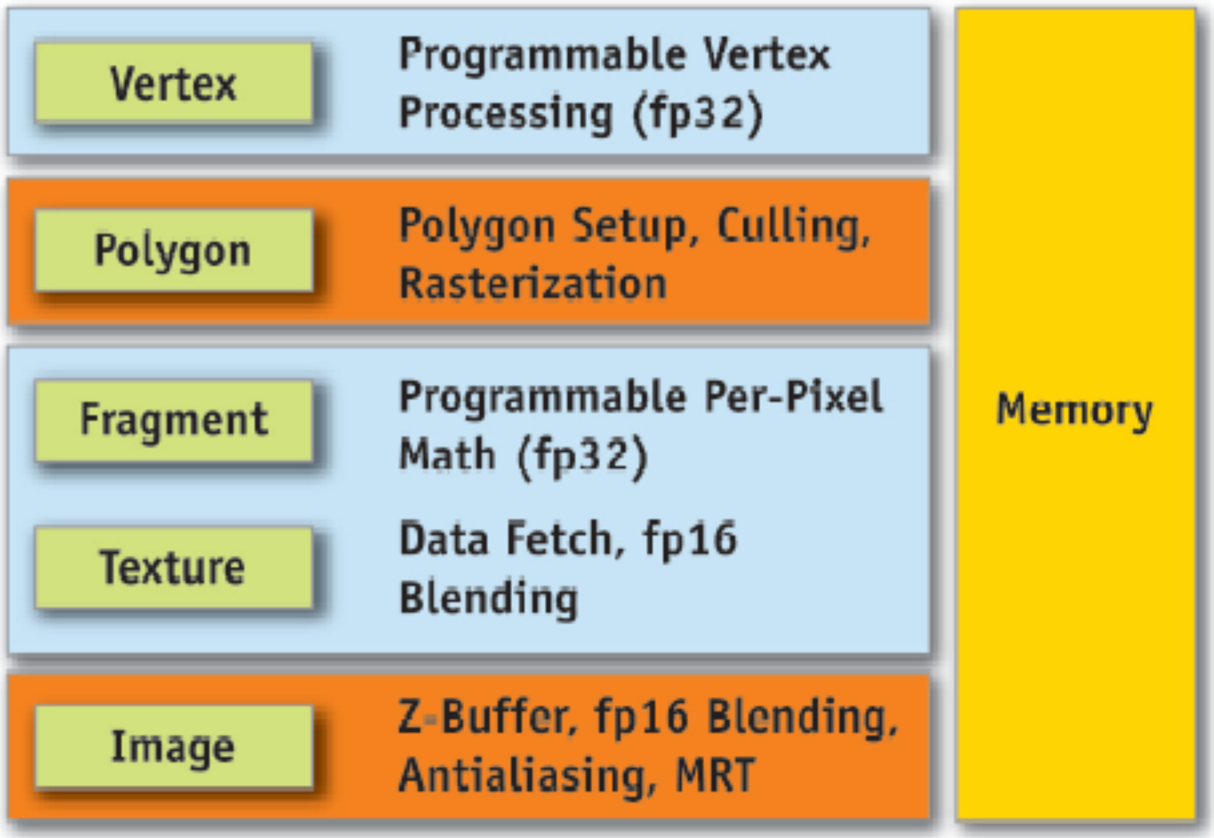
      $x_R$ += $dx_R/dy_R$;
   }
}
```

Minimize computation in inner loops



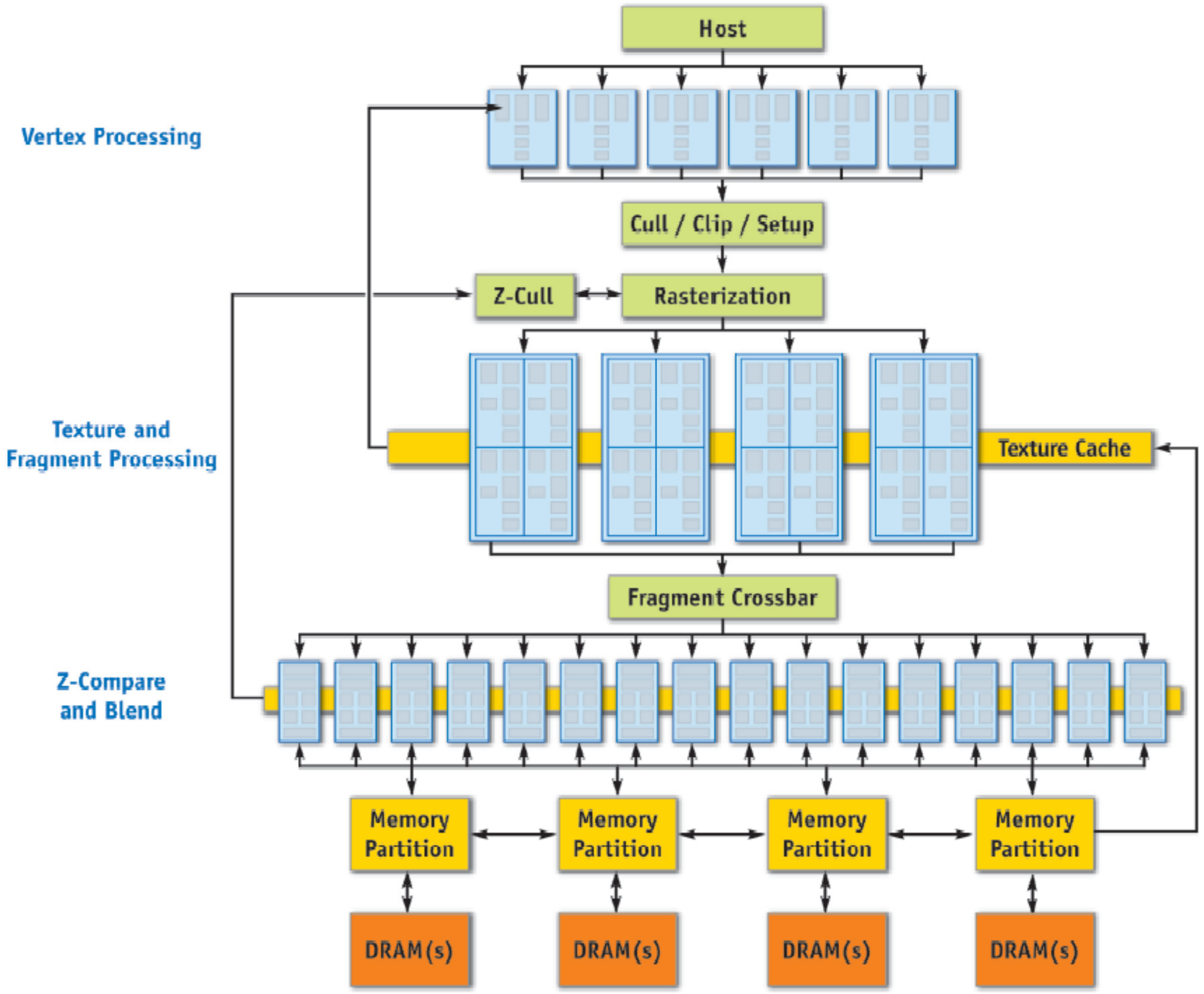
GPU Architecture



Vertex
Programmable Vertex Processing (fp32)
Polygon
Polygon Setup, Culling, Rasterization
Fragment
Programmable Per-Pixel Math (fp32)
Texture
Data Fetch, fp16 Blending
Image
Z-Buffer, fp16 Blending, Antialiasing, MRT
Memory

GeForce 6 Series Architecture

GPU Architecture



GeForce 6 Series Architecture

GPU Gems 2, NVIDIA