
Topic 3: Parsing and Yaccing

COS 320

Compiling Techniques

Princeton University
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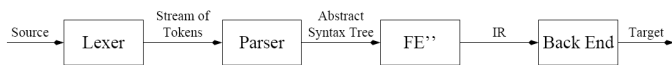
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Syntactical Analysis

Front End:

- Lexical Analysis - Break source into *tokens*.
- Syntax Analysis - Parse phrase structure.
- Semantic Analysis - Calculate meaning.

Our Compiler:



Parser Functions:

- Verify that token stream is valid.
- If it is not valid, report syntax error and recover.
- Build Abstract Syntax Tree (AST).

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Syntactical Analysis

- Every programming language has a set of rules that describe syntax of well-formed programs in language.
- *Syntax Analysis (Parsing)* - Determine if source program satisfies these rules.
- Source program constructs may have recursive structure:

```
digits = [0-9]+  
expr = {digits} | "(" {expr} "+" {expr} ")"
```

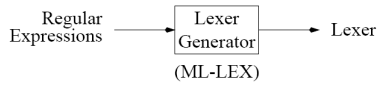
- Finite Automata cannot recognize recursive constructs. (A machine with N states cannot remember a parenthesis-nesting depth greater than N .)

We need a more powerful formalism: *Context-Free Grammar*

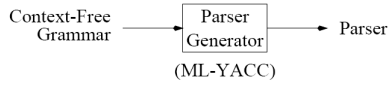
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Context-Free Grammar

Regular Expressions - describe lexical structure of tokens.



Context-Free Grammars - describe syntactic nature of programs.



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Definitions

- *Language* - set of strings
- *String* - finite sequence of *symbols* taken from finite *alphabet*
 - Regular Expressions describe a language.
 - Context-Free Grammar also describes a language.

	Lexical Analysis	Syntax Analysis
language	set of tokens	set of source programs
string	token	source program
symbol	ASCII character	token

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Context-Free Grammar

- Also known as BNF (Backus-Naur Form).
- Context-free grammars are more powerful than regular expressions.
 - Any language that can be generated using regular expressions can be generated by a context-free grammar.
 - There are languages that can be generated by a context-free grammar that cannot be generated by any regular expression.
- Examples:
 - Matching parentheses
 - Nested comments

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Context-Free Grammars

- Context-Free Grammars consist of a set of *productions*.

$$\text{symbol} \rightarrow \text{symbol symbol} \dots \text{symbol}$$

- Symbol types:
 - terminal* that corresponds to a token-type.
 - non-terminal* that denotes a set of strings.
- Left-Hand Side (LHS) - *non-terminal*.
- Right-Hand Side (RHS) - *terminals* or *non-terminals*
- Start Symbol* - A special *non-terminal*.
- Each production specifies how terminals and non-terminals may be combined to form a substring in language.
- Easy to specify recursion:

$$\text{stmt} \rightarrow \text{IF } \text{exp} \text{ THEN } \text{stmt} \text{ ELSE } \text{stmt}$$

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Start Symbol

- String of token-types is in language described by grammar if it can be derived from *start symbol*
- Derivations:
 - begin with start symbol
 - while non-terminals exist, replace any non-terminal with RHS of production
- Multiple derivations exist for given sentence
 - Left-most derivation - replace left-most non-terminal in each step.
 - Right-most derivation - replace right-most non-terminal in each step.

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Example

Non-Terminals:

stmt	: Statement	stmt \rightarrow stmt; stmt
expr	: Expression	stmt \rightarrow ID := expr
expr_list	: Expression List	stmt \rightarrow PRINT (expr_list)

Terminals (tokens):

SEMI	" ; "	expr \rightarrow ID
ID		expr \rightarrow NUM
ASSIGN	" := "	expr \rightarrow expr + expr
LPAREN	" ("	expr \rightarrow (stmt, expr)
RPAREN	") "	
NUM		expr_list \rightarrow expr
PLUS	" + "	expr_list \rightarrow expr_list, expr
PRINT	" print "	
COMMA	" , "	

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Example: Leftmost Derivation

Show that expression can be derived from start symbol.

```
ID := NUM; PRINT (NUM)
```

```
a := 12; print (23)
```

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Example: Rightmost Derivation

Show that expression can be derived from start symbol.

```
ID := NUM; PRINT (NUM)
```

```
a := 12; print (23)
```

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Parse Trees

- *Parse Trees* - Graphical representation of derivation.
- Each internal node is labeled with a non-terminal.
- Each leaf node is labeled with a terminal.
- Parse Tree of the example using right-most derivation production:

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Ambiguous Grammars

A grammar is ambiguous if it can derive a string of tokens with two or more different parse trees.

Non-Terminals:

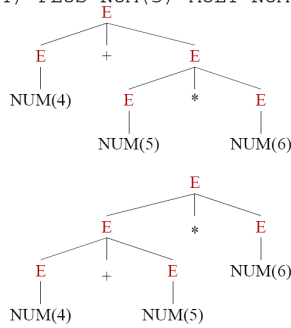
expr : Expression

Terminals (tokens):

ID
 NUM
 PLUS "+"
 MULT "*"

expr → ID
 expr → NUM
 expr → expr + expr
 expr → expr * expr

Consider: 4 + 5 * 6
 NUM(4) PLUS NUM(5) MULT NUM(6)



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Ambiguous Grammars

- *Problem:* compilers use parse trees to interpret meaning of parsed expressions.
 - Different parse trees may have different meanings, resulting in different interpreted results.
 - For example, does 4 + 5 * 6 equal 34 or 54?
- *Solution:* rewrite grammar to eliminate ambiguity.
 - If language doesn't have unambiguous grammar, then you have a bad programming language.
 - Operators have a relative *precedence*. We say some operands *bind tighter* than others. ("*" binds tighter than "+")
 - Operators with the same precedence must be resolved by *associativity*. Some operators have *left associativity*, others have *right associativity*.

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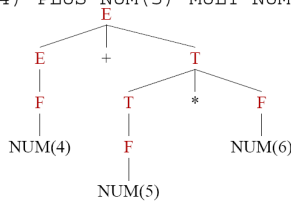
Ambiguous Grammars

Non-Terminals:

expr : Expression
 term : Term (add)
 fact : Factor (mult)

Terminals (tokens):
 expr → expr + term
 expr → term
 term → term * fact
 term → fact
 fact → ID
 fact → NUM

4 + 5 * 6
 NUM(4) PLUS NUM(5) MULT NUM(6)



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End-Of-File Marker

- Parse must also recognize the End-of-File (EOF).
- EOF marker in the grammar is “\$”
- Introduce new start symbol and the production $E' \rightarrow E\$$

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Grammars and Lexical Analysis

- Grammars can also describe token structure:

$(a \mid b)^* abb$

$W \rightarrow aW$

$W \rightarrow bW$

$W \rightarrow aX$

$X \rightarrow bY$

$Y \rightarrow bZ$

$Z \rightarrow \epsilon$

- Can combine lexical analysis and syntax analysis into one module.
- Disadvantages:
 - Regular expression specification is more concise.
 - Separating phases increases compiler modularity.

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Context-Free Grammars and REs

- Context-free grammars are more powerful than regular expressions.
 - Any language that can be generated using regular expressions can be generated by a context-free grammar.
 - There are languages that can be generated by a context-free grammar that cannot be generated by any regular expression.
- As a corollary, CFGs are strictly more powerful than DFAs and NFAs.
- The proof is in two parts:
 - Given a regular expression R , we can generate a CFG G such that $L(R) = L(G)$.
 - We can define a grammar G for which there is no FA F such that $L(F) = L(G)$.

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Context Free Grammars and REs

Base Cases:

- Symbol (a):
 $RE \rightarrow a$
- Epsilon (ϵ):
 $RE \rightarrow \epsilon$

Inductive Cases:

- Alternation ($M|N$):
 $RE \rightarrow M$
 $RE \rightarrow N$
- Concatenation (MN):
 $RE \rightarrow MN$
- Kleen closure (M^*):
 $RE \rightarrow MRE$
 $RE \rightarrow \epsilon$

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Context-Free Grammar with no RE/FA

$$\begin{aligned} S &\rightarrow (S) \\ S &\rightarrow \epsilon \end{aligned}$$

- FAs have a FINITE number of states, N
- FA must “remember” number of “(”s, to generate “)”s
- At or before $N + 1$ “(”s FA will revisit a state.
- That state represents two different counts of “)”s.
- Both counts must now be accepted.
- One count will be invalid.

Representations

- Regular, right-linear, finite-state grammars: FAs
- Context-free grammars: Push-Down Automata

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Further Exploration

We have been talking about Context-Free Grammars.

What is a **context-sensitive grammar**?

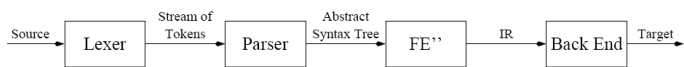
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Parsing

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Our Compiler:



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Outline

- Recursive Descent Parsing
- Shift-Reduce Parsing
- ML-Yacc
- Recursive Descent Parser Generation

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Recursive Descent Parsing

- Recall discussion on Context-Free Grammars: symbols (terminal, non-terminal), productions, derivations, etc.
- Can parse many grammars using algorithm called *recursive descent* parsing.
 - A.K.A.: *predictive parsing*
 - A.K.A.: *top-down parsing*
 - A.K.A.: *LL(1)* - Left-to-right parse, Leftmost-derivation, 1-symbol lookahead.
- One recursive function for each non-terminal.
- Each production becomes clause in function.

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Example

Grammar:

non-terminals: S, L, E

terminals: IF (*if*), THEN (*then*), ELSE (*else*), BEGIN (*begin*),
PRINT (*print*), END (*end*), SEMI (;), NUM, EQ (=)

$S \rightarrow \text{if } E \text{ then } S \text{ else } S$

$S \rightarrow \text{begin } S \text{ L}$

$S \rightarrow \text{print } E$

$L \rightarrow \text{end}$

$L \rightarrow ; S \text{ L}$

$E \rightarrow \text{num} = \text{num}$

```
datatype token = EOF | IF | THEN | ELSE | BEGIN |
              PRINT | END | SEMI | NUM | EQ

fun eat(t) = if (!tok = t) then advance() else error()

fun S() = case !tok of
  IF    => (eat(IF); E(); eat(THEN); S());
        eat(ELSE); S();
  BEGIN => (eat(BEGIN); S(); L());
  PRINT => (eat(PRINT); E())
and L() = case !tok of
  END    => (eat(END))
  SEMI   => (eat(SEMI); S(); L())
and E() = (eat(NUM); eat(EQ); eat(NUM))
```

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Another Example

Grammar:

$A \rightarrow S \text{ EOF}$	$E \rightarrow \text{id}$
$S \rightarrow \text{id} := E$	$E \rightarrow \text{num}$
$S \rightarrow \text{print } (L)$	$L \rightarrow E$
	$L \rightarrow L, E$

```
fun A() = (S(); eat(EOF))
and S() = case !tok of
  ID    => (eat(ID); eat(ASSIGN); E())
  PRINT => (eat(PRINT); eat(LPAREN);
          L(); eat(RPAREN))
and E() = case !tok of
  ID    => (eat(ID))
  NUM   => (eat(NUM))
and L() = case !tok of
  ID    => (?????)
  NUM   => (?????)
```

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The Problem

- If !tok = ID, parser cannot determine which production to use:
 $L \rightarrow E$ (E could be ID)
 $L \rightarrow L, E$ (L could be ID)
- Predictive parsing only works for grammars where first terminal symbol of each subexpression provides enough information to choose which production to use.
- Can write predictive parser by eliminating *left recursion*.

$L \rightarrow E$		$L \rightarrow E M$
$L \rightarrow L, E$	\implies	$M \rightarrow , E M$
		$M \rightarrow \epsilon$

```
and L() = case !tok of
  ID    => (E(); M())
  NUM   => (E(); M())
and M() = case !tok of
  COMMA => (eat(COMMA); E(); M())
  RPAREN => ()
```

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Another Option: Shift-Reduce Parsing

- Given next input token, predictive parser must predict which production to use.
- *Shift-reduce parsing* delays decision until it has seen input token corresponding to entire RHS of production.
 - A.K.A.: *bottom-up parsing*
 - A.K.A.: *LR(k)* - Left-to-right parse, Rightmost derivation, k-token lookahead
- Shift-reduce parsing can parse more grammars than predictive parsing.
- Parser has *stack*.
- Based on stack contents and next input token, one of two action performed:
 1. *Shift* - push next input token onto top of stack.
 2. *Reduce* - choose production ($X \rightarrow ABC$); pop off RHS (C, B, A); push LHS (X).
- Stack is initially empty.
- Parser points to beginning of input stream.
- If \$ is shifted, then input stream has been parsed successfully.

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Shift-Reduce Parsing

How does parser know when to shift or reduce?

- DFA: applied to stack contents, not input stream
- Each state corresponds to contents of stack at some point in time.
- Edges labelled with terms/non-terms that can appear on stack.

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Example

Grammar:

- 1 $A \rightarrow S \text{ EOF}$
- 2 $S \rightarrow (L)$
- 3 $S \rightarrow id = num$
- 4 $L \rightarrow L; S$
- 5 $L \rightarrow S$

Input:

$(a = 4; b = 5) \rightarrow (ID_a = NUM_4; ID_b = NUM_5)$

```
      input: ( ID = NUM ; ID = NUM )
           |
0  stack:
      action: shift
```

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Example

```
      input: ( ID = NUM ; ID = NUM )
1      |
      stack: (
      action: shift
-----
      input: ( ID = NUM ; ID = NUM )
2      |
      stack: ( ID
      action: shift
-----
      input: ( ID = NUM ; ID = NUM )
3      |
      stack: ( ID =
      action: shift
-----
      input: ( ID = NUM ; ID = NUM )
4      |
      stack: ( ID = NUM
      action: reduce 3
```

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Example

```
      input: ( ID = NUM ; ID = NUM )
5      |
      stack: ( S
      action: reduce 5
-----
      input: ( ID = NUM ; ID = NUM )
6      |
      stack: ( L
      action: shift
-----
      input: ( ID = NUM ; ID = NUM )
7      |
      stack: ( L ;
      action: shift
-----
      input: ( ID = NUM ; ID = NUM )
8      |
      stack: ( L ; ID
      action: shift
```

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Example

```
      input: ( ID = NUM ; ID = NUM )
9      |
      stack: ( L ; ID =
      action: shift
-----
      input: ( ID = NUM ; ID = NUM )
10     |
      stack: ( L ; ID = NUM
      action: reduce 3
-----
      input: ( ID = NUM ; ID = NUM )
11     |
      stack: ( L ; S
      action: reduce 4
-----
      input: ( ID = NUM ; ID = NUM )
12     |
      stack: ( L
      action: shift
```

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Example

```
      input: ( ID = NUM ; ID = NUM )
13  stack: ( L )
      action: reduce 2
-----
      input: ( ID = NUM ; ID = NUM )
14  stack: S
      action: ACCEPT
```

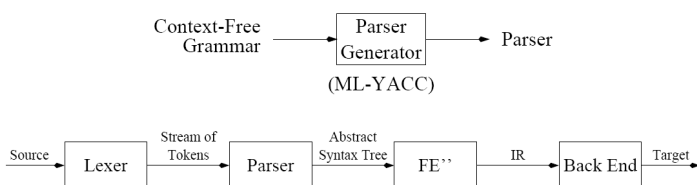
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The Dangling Else Problem

- Valid Program: if a then if b then S1 else S2
 - $S \rightarrow \text{if } E \text{ then } S \text{ else } S$
 - $S \rightarrow \text{if } E \text{ then } S$
 - $S \rightarrow \text{OTHER}$
- 2 interpretations: if a then [if b then S1 else S2]
if a then [if b then S1] else S2
- Want first behaviour, but parse will report *shift-reduce conflict* when S1 is on top stack.
- Eliminate Ambiguity by modifying grammar (matched/unmatched):
 - $S \rightarrow M$
 - $S \rightarrow U$
 - $M \rightarrow \text{if } E \text{ then } M \text{ else } M$
 - $M \rightarrow \text{OTHER}$
 - $U \rightarrow \text{if } E \text{ then } S$
 - $U \rightarrow \text{if } E \text{ then } M \text{ else } U$

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ML-YACC (Yet Another Compiler-Compiler)



- Input to **ml-yacc** is a context-free grammar specification.
- Output from **ml-yacc** is a shift-reduce parser in ML.

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Context-Free Grammar Specification

- CFG specification consists of 3 parts:

```
User Declarations
%%
ML-YACC Definitions
%%
Rules
```

- **User Declarations:** define various values that are available to *rules* section.
- **ML-YACC Definitions:** declare terminal and non-terminal symbols; declare precedences for terminals that help resolve shift-reduce conflicts.
- **Rules:** specify productions of grammar and *semantic actions* associated with productions.

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ML-YACC Declarations

- Need to specify type associated with positions of tokens in input file

```
%pos int
```

- Need to specify terminal and non-terminal symbols (no symbols can be in both lists)

```
%term IF | THEN | ELSE | ...
%nonterm prog | stmt | expr | ...
```

- Optionally specify end-of-parse symbol - terminals which may follow start symbol

```
%eop EOF
```

- Optionally specify start symbol - otherwise, LHS non-terminal of first rule is taken as start symbol

```
%start prog
```

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Attribute Grammar

- ML-YACC employs *attribute grammar* scheme
 - Each terminal or non-terminal symbol may have associated attribute/value.
 - When parser reduces using production $A \rightarrow \alpha$, semantic action associated with production is executed in order to compute value for A based on the values of symbols in α .
 - Parser returns value associated with start symbol. (If no attribute, () is returned.)
- Can specify *types* of attributes associated with symbols.

```
%term      ID of string | NUM of int | IF | THEN | ...
%nonterm    prgm | stmt | expr of int | ...
```

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Rules

$symbol_0 : symbol_1 symbol_2 \dots symbol_n (semantic_action)$

- Semantic action typically builds piece of AST corresponding to derived string
- Can access attribute/value of RHS symbol X using $X\langle n \rangle$, where n specifies a particular occurrence of X on RHS.

```
%term    PLUS | MINUS | NUM of int | ...
%nonterm exp of int | ...
```

```
exp: exp PLUS exp    (exp1 + exp2)
   | exp MINUS exp   (exp1 - exp2)
   | NUM              (NUM)
```

- Type of value computed by semantic action must match type of value associated with LHS non-terminal.

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Example

```
%%
```

```
%term    ID | NUM | PLUS | MINUS | MULT | DIV | EOF
%nonterm expr
%pos int
%start expr
%eop EOF
%verbose
```

```
%%
```

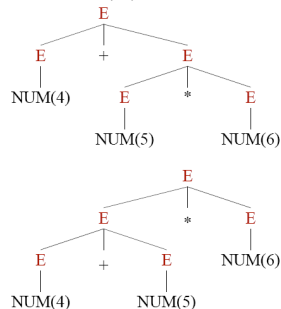
```
expr : ID          ()
     | NUM         ()
     | expr PLUS expr ()
     | expr MINUS expr ()
     | expr MULT expr ()
     | expr DIV expr ()
```

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ML-YACC and Ambiguous Grammars

- A grammar is ambiguous if it can derive a string of tokens with two or more different parse trees.
- Consider: $4 + 5 * 6$, NUM(4) PLUS NUM(5) MULT NUM(6)

```
expr → ID
expr → NUM
expr → expr + expr
expr → expr * expr
```



- We prefer to bind “*” tighter than “+”.

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ML-YACC and Ambiguous Grammars

- Similarly Consider: $4 + 5 + 6$, NUM (4) PLUS NUM (5) PLUS NUM (6)
- We prefer to bind left “+” first.
- ML-YACC will report *shift-reduce* conflicts when parsing strings.
 - $4 + 5 * 6$, NUM (4) PLUS NUM (5) MULT NUM (6)
 - * At some point, $E + E$ will be on top of stack, “*” will be the current token-type in stream.
 - * Parser can reduce by rule $E \rightarrow E + E$, or shift. Prefer *shift*.
 - $4 + 5 + 6$, NUM (4) PLUS NUM (5) PLUS NUM (6)
 - * At some point, $E + E$ will be on top of stack, “+” will be the current token-type in stream.
 - * Parser can reduce by rule $E \rightarrow E + E$, or shift. Prefer *reduce*.

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Directives

Three Solutions:

1. Let YACC complain, but demonstrate that its choice (to shift) was correct.
2. Rewrite grammar to eliminate ambiguity.
3. Keep grammar, but add *precedence directives* which enable conflicts to be resolved.
Use `%left`, `%right`, `%nonassoc`
 - For this grammar:
 - `%left PLUS MINUS`
 - `%left MULT DIV`
 - PLUS, MINUS are left associative, bind equally tightly
 - MULT, DIV are left associative, bind equally tightly
 - MULT, DIV bind tighter than PLUS, MINUS

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Directives

- Given directives, ML-YACC assigns precedence to each *terminal* and *rule*
 - Precedence of terminal based on order in which associativity specified
 - Precedence of rule is the precedence of right-most terminal. For example, precedence($E \rightarrow E + E$) = precedence(PLUS).
- Given shift-reduce conflict, ML-YACC performs the following:
 1. Find precedence of rule to be reduced, terminal to be shifted.
 2. $\text{prec}(\text{terminal}) > \text{prec}(\text{rule}) \Rightarrow$ shift.
 3. $\text{prec}(\text{rule}) > \text{prec}(\text{terminal}) \Rightarrow$ reduce.
 4. $\text{prec}(\text{terminal}) = \text{prec}(\text{rule})$, then:
 - $\text{assoc}(\text{terminal}) = \text{left} \Rightarrow$ reduce.
 - $\text{assoc}(\text{terminal}) = \text{right} \Rightarrow$ shift.
 - $\text{assoc}(\text{terminal}) = \text{nonassoc} \Rightarrow$ report as error.

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Precedence Examples

```
input: 4 + 5 * 6
      |
1  stack: 4 + 5
   action: prec(*) > prec(+) -> shift
-----
input: 4 * 5 + 6
      |
2  stack: 4 * 5
   action: prec(*) > prec(+) -> reduce
-----
input: 4 + 5 + 6
      |
3  stack: 4 + 5
   action: assoc(+) = left -> reduce
```

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Default Behavior

What if directives not specified?

- shift-reduce: report error, *shift* by default.
- reduce-reduce: report error, reduce by rule that occurs first.

What to do:

- shift-reduce: acceptable in well defined cases (dangling else).
- reduce-reduce: unacceptable. Rewrite grammar.

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Direct Rule Precedence Specification

Can assign *specific* precedence to rule, rather than precedence of last terminal.

- Use the %prec directive.
- Commonly used for the *unary minus* problem.
- Consider $-4 * 6$, MINUS NUM(4) MULT NUM(6)
- We prefer to bind left unary minus (“-”) tighter. Here, precedence of MINUS is lower than MULT, so we get $-(4 * 6)$, not $(-4) * 6$.

• Solution:

```
%left PLUS MINUS
%left MULT DIV
%left UMINUS
```

```
exp : MINUS expr %prec UMINUS ()
    | expr PLUS expr ()...
```

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Syntax vs. Semantics

Consider language with two classes of expressions

- *Arithmetic* expressions (ae)

```
ae : ae PLUS ae ()
    | ID           ()
```

- *Boolean* expressions (be)

```
be : be AND be ()
    | be OR be  ()
    | be EQ be  ()
    | ID        ()
```

- Consider: a := b, ID(a) ASSIGN ID(b):

- Reduce-reduce conflict - parser can't choose between $be \rightarrow ID$ or $ae \rightarrow ID$.
- For now ae and be should be aliased - let semantic analysis (next phase) determine that $a \ \& \ b \ + \ c$ is a type error.
- Type checking cannot be done easily in context free grammars.

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Recursive Descent/Predictive/LL(1) Parser Generation

Grammar:

```
A → S EOF           E → id
S → id := E         E → num
S → print ( L )    L → E
                   L → L, E
```

```
fun A () =
  (S (); eat (EOF))
and S () = case !tok of
  ID    => (eat (ID); eat (ASSIGN); E ())
  PRINT => (eat (PRINT); eat (LPAREN);
           L (); eat (RPAREN))
and E () = case !tok of
  ID    => (eat (ID))
  NUM   => (eat (NUM))
and L () = case !tok of
  ID    => (?????)
  NUM   => (?????)
```

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Problem

- Based on current function and next token-type in input stream, parser must predict which production to use.
- If !tok = ID, parser cannot determine which production to use:
L → E (E could be ID)
L → L, E (L could be ID)
- Predictive parsing only works for grammars where first terminal symbol of each subexpression provides enough information to choose which production to use.

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Formal Techniques

Can use formal techniques to determine whether or not a predictive parser can be built for a particular grammar.

- Let γ be a string of terminal and non-terminal symbols
- Need to compute 3 values:
 1. For each γ corresponding to RHS of production, must determine if γ can derive empty string (ϵ) \Rightarrow **nullable**.
 2. For each γ corresponding to RHS of production, must determine set of all terminal symbols that can begin any string derived from $\gamma \Rightarrow$ **first**(γ).
 3. For each non-terminal X in grammar, must determine set of all terminal symbols that can immediately follow X in a derivation \Rightarrow **follow**(X).

Computation of Nullable:

- γ is nullable if every symbol $S \in \gamma$ is nullable.
- Check if every S can derive ϵ .

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Computation of First

- If T is a terminal symbol, then $\text{first}(T) = \{T\}$.
- If X is a non-terminal and $X \rightarrow Y_1Y_2Y_3\dots Y_n$, then
$$\begin{aligned} \text{first}(Y_1) &\in \text{first}(X) \\ \text{first}(Y_2) &\in \text{first}(X), \text{ if } Y_1 \text{ is nullable} \\ \text{first}(Y_3) &\in \text{first}(X), \text{ if } Y_1, Y_2 \text{ is nullable} \\ &\vdots \\ \text{first}(Y_n) &\in \text{first}(X), \text{ if } Y_1, Y_2, \dots, Y_{n-1} \text{ is} \\ &\text{nullable} \end{aligned}$$

- Let $\gamma = S_1S_2\dots S_k$. Then,

$$\text{first}(\gamma) = \begin{cases} \text{first}(S_1) \\ \text{first}(S_2), \text{ if } S_1 \text{ is nullable} \\ \text{first}(S_3), \text{ if } S_1, S_2 \text{ is nullable} \\ \vdots \\ \text{first}(S_k), \text{ if } S_1, S_2, \dots, S_{k-1} \text{ is nullable} \end{cases}$$

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Computation of Follow

Let X, Y be non-terminals; γ, γ_1 , and γ_2 be strings of terminals and non-terminals

- if grammar includes production: $X \rightarrow \gamma Y$
 $\Rightarrow \text{follow}(X) \in \text{follow}(Y)$.
- if grammar includes production: $X \rightarrow \gamma_1 Y \gamma_2$
 $\Rightarrow \text{first}(\gamma_2) \in \text{follow}(Y)$
 $\Rightarrow \text{follow}(X) \in \text{follow}(Y)$, if γ_2 is nullable.

Perform *iterative* technique in order to compute nullable, first, and follow sets for each non-terminal in grammar.

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Building a Predictive Parser

$Z \rightarrow XYZ$ $Y \rightarrow c$ $X \rightarrow a$
 $Z \rightarrow d$ $Y \rightarrow \epsilon$ $X \rightarrow b Y e$

Initial:		
	nullable	first follow
Z	no	
Y	no	
X	no	

Examine each production in grammar, modifying nullable and adding to first and follow sets, until no more changes can be made.

Iteration 1:		
	nullable	first follow
Z	no	
Y	no	
X	no	

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Building a Predictive Parser

$Z \rightarrow XYZ$ $Y \rightarrow c$ $X \rightarrow a$
 $Z \rightarrow d$ $Y \rightarrow \epsilon$ $X \rightarrow b Y e$

Iteration 1:		
	nullable	first follow
Z	no	
Y	yes	
X	no	

Iteration 2:		
	nullable	first follow
Z	no	
Y	yes	
X	no	

Iteration 3:		
	nullable	first follow
Z	no	d,a,b
Y	yes	c e,d,a,b
X	no	a,b c,d,a,b

No Changes

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Predictive Parsing Table

	nullable	first	follow
Z	no	d,a,b	
Y	yes	c	e,d,a,b
X	no	a,b	c,d,a,b

Build *predictive parsing table* from nullable, first, and follow sets.

	a	b	c	d	e
Z	$Z \rightarrow XYZ$	$Z \rightarrow XYZ$		$Z \rightarrow d$	
Y	$Y \rightarrow \epsilon$	$Y \rightarrow \epsilon$	$Y \rightarrow c$	$Y \rightarrow \epsilon$	$Y \rightarrow \epsilon$
X	$X \rightarrow a$	$X \rightarrow b Y e$			

- Enter $S \rightarrow \gamma$ in row S , column T : for each $T \in \text{first}(\gamma)$.
- If γ is nullable, enter $S \rightarrow \gamma$ in row S , column T : for each $T \in \text{follow}(S)$.
- Entry in row S , column T tells parser which clause to execute if current function is $S()$ and next token-type is T
- Blank entries are syntax errors.

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Predictive Parsing Table

If the predictive parsing table contains *no* duplicate entries, can build predictive parser for grammar.

- Grammar is LL(1) (left-to-right parse, left-most derivation, 1 symbol lookahead).
- Grammar is LL(k) if its LL(k) predictive parsing table has no duplicate entries.
 - Rows correspond to non-terminals, columns correspond to every possible sequence of k terminals.
 - The $\text{first}(\gamma) =$ set of all k-length terminal sequences that can begin any string derived from γ .
 - LL(k) parsing tables can be too large.
 - Ambiguous grammars are not LL(k), $\forall k$.

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Example

$$\begin{array}{lll}
 S' \rightarrow S\$ & S \rightarrow \text{IF } E \text{ THEN } A \text{ ELSE } A & T \rightarrow \text{NUM} \\
 S \rightarrow E & E \rightarrow E + T & A \rightarrow \text{ID} = \text{NUM} \\
 S \rightarrow \text{IF } E \text{ THEN } A & E \rightarrow T &
 \end{array}$$

Iteration 1:

	nullable	first	follow
S'	no		
S	no	IF	\$
E	no	NUM	\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

Iteration 2:

	nullable	first	follow
S'	no	IF	
S	no	IF	\$
E	no	NUM	\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

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Example

$$\begin{array}{lll}
 S' \rightarrow S\$ & S \rightarrow \text{IF } E \text{ THEN } A \text{ ELSE } A & T \rightarrow \text{NUM} \\
 S \rightarrow E & E \rightarrow E + T & A \rightarrow \text{ID} = \text{NUM} \\
 S \rightarrow \text{IF } E \text{ THEN } A & E \rightarrow T &
 \end{array}$$

Iteration 3:

	nullable	first	follow
S'	no	IF	
S	no	IF, NUM	\$
E	no	NUM	\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

Iteration 4:

	nullable	first	follow
S'	no	IF, NUM	
S	no	IF, NUM	\$
E	no	NUM	\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

No further changes

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Predictive Parsing Table

	nullable	first	follow
S'	no	IF, NUM	
S	no	IF, NUM, \$	
E	no	NUM, \$, THEN, +	
T	no	NUM, \$, THEN, +	
A	no	ID, \$, ELSE	

Build *predictive parsing table* from nullable, first, and follow sets.

	IF	THEN	ELSE	+	NUM	ID	=	\$
S'	$S' \rightarrow S$				$S' \rightarrow S$			
S	$S \rightarrow \text{IF } E \text{ THEN } A$ $S \rightarrow \text{IF } E \text{ THEN } A \text{ ELSE } A$				$S \rightarrow E$			
E					$E \rightarrow E + T$ $E \rightarrow T$			
T					$T \rightarrow \text{NUM}$			
A						$A \rightarrow \text{ID} = \text{NUM}$		

Table has duplicate entries \Rightarrow grammar is not LL(1)!

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Problems

- $E \rightarrow E + T$
 $E \rightarrow T$
 - $\text{first}(E+T) = \text{first}(T)$
 - When in function $E()$, if next token is NUM, parser will get stuck.
 - Grammar is *left-recursive* - left-recursive grammars cannot be LL(1).
 - Solution: rewrite grammar so that it is *right-recursive*.
 $E \rightarrow TE'$
 $E' \rightarrow \epsilon$
 $E' \rightarrow +TE'$
 - In general, $X \rightarrow X\gamma$
 $X \rightarrow \alpha$ derives strings of form $\alpha\gamma^*$ (α doesn't start with X).
 These two productions can be rewritten as follows:
 $X \rightarrow \alpha X'$
 $X' \rightarrow \epsilon$
 $X' \rightarrow \gamma X'$

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Problems

- $S \rightarrow \text{IF } E \text{ THEN } A$
 $S \rightarrow \text{IF } E \text{ THEN } A \text{ ELSE } A$
 - Two productions begin with same symbol.
 - $\text{first}(\text{IF } E \text{ THEN } A) = \text{first}(\text{IF } E \text{ THEN } A \text{ ELSE } A)$
 - Solution: use *left-factoring*
 $S \rightarrow \text{IF } E \text{ THEN } A V$
 $V \rightarrow \epsilon$
 $V \rightarrow \text{ELSE } A$

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Example

Show that modified grammar is LL(1).

$$\begin{array}{lll}
 S' \rightarrow S\$ & V \rightarrow \text{ELSE } A & T \rightarrow \text{NUM} \\
 S \rightarrow E & E \rightarrow TE' & A \rightarrow \text{ID} = \text{NUM} \\
 S \rightarrow \text{IF } E \text{ THEN } A V & E' \rightarrow \epsilon & \\
 V \rightarrow \epsilon & E' \rightarrow +TE &
 \end{array}$$

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Example

Show that the grammar is LL(1).

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Example

Show that modified grammar is LL(1). Build predictive parsing table.

	nullable	first	follow
S'	no	IF, NUM	\$
S	no	IF, NUM	\$
V	yes	ELSE	\$
E	no	NUM	\$, THEN
E'	yes	+	\$, THEN
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

	IF	THEN	ELSE	+	NUM	ID	=	\$
S'	$S' \rightarrow S$				$S' \rightarrow S$			
S	$S \rightarrow \text{IF } E \text{ THEN } A V$				$S \rightarrow E$			
V			$V \rightarrow \text{ELSE } A$					$V \rightarrow \epsilon$
E					$E \rightarrow TE'$			
E'		$E' \rightarrow \epsilon$		$E' \rightarrow +TE'$				$E' \rightarrow \epsilon$
T					$T \rightarrow \text{NUM}$			
A						$A \rightarrow \text{ID} = \text{NUM}$		

Table does not have duplicate entries \Rightarrow modified grammar is LL(1)!

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- LR(0)
- SLR
- LR(1)
- LALR(1)

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Shift-Reduce, Bottom Up, LR(1) Parsing

- Shift-reduce parsing can parse more grammars than predictive parsing.
- *Shift-reduce parsing* has stack and input.
- Based on stack contents and next input token, one of two action performed:
 1. *Shift* - push next input token onto top of stack.
 2. *Reduce* - choose production ($X \rightarrow ABC$); pop off RHS (C, B, A); push LHS (X).
- If S is shifted, then input stream has been parsed successfully.

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LR(k)

Can generalize to case where parser makes decision based on stack contents and next k tokens. LR(k):

- Left-to-right parse
- right-most derivation
- k -symbol lookahead

LR(k) parsing, $k > 1$, rarely used in compilation:

- DFA too large: need transition for every sequence of k terminals.
- Most programming languages can be described by LR(1) grammars.

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Shift Reduce Parsing DFA

Parser uses DFA to make shift/reduce decisions:

- Each state corresponds to contents of stack at some point in time.
- Edges labeled with terminals/non-terminals.

Rather than scanning entire stack to determine current DFA state, parser can remember state reached for each stack element.

- Transition table for LR(1) or LR(0) DFA:

	Terminals (T_1, T_2, \dots, T_n)	Non-Terminals (N_1, N_2, \dots, N_n)
1	<i>actions</i>	<i>actions</i>
2	sn \rightarrow shift n	gz \rightarrow goto z
3	rk \rightarrow reduce k	
:	a \rightarrow accept	
n	\rightarrow error	

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Parsing Algorithm

Look up DFA state on top of stack, next terminal in input:

- shift(n):
 - Advance input by one.
 - Push input token on stack with n (the new state).
- reduce(k):
 - Pop stack as many times as number of symbols on RHS of rule k .
 - Let X be LHS of rule k
 - In state now on top of stack, look up X to get goto(z)
 - Push X on stack with z (the new state).
- accept \rightarrow stop, report success.
- error \rightarrow stop, report syntax error.

To understand LR(k) parsing, first focus on LR(0) parser construction using an example.

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LR(0) Parsing

$$\begin{array}{lll}
 1 \ S' \rightarrow S \ \$ & 3 \ S \rightarrow x & 5 \ L \rightarrow L, S \\
 2 \ S \rightarrow (L) & 4 \ L \rightarrow S &
 \end{array}$$

Initially, stack empty, input contains 'S' string followed by a '\$':

$$\begin{array}{l}
 1 \ S' \rightarrow .SS \\
 S \rightarrow .(L) \\
 S \rightarrow .x
 \end{array}$$

- Combination of production and '.' called LR(0) *item*.
- '.' specifies parser position.
- Three items represent *closure* of: $S' \rightarrow .S \ \$$
- Closure adds more items to a set when dot exists to left of a non-terminal.

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LR(0) States

LR(0) Parsing

- 1 $S' \rightarrow S \$$ 3 $S \rightarrow x$ 5 $L \rightarrow L, S$
- 2 $S \rightarrow (L)$ 4 $L \rightarrow S$

LR(0) states:

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DFA Table Entry Computation

To compute transition table from state diagram perform the following:

- $[S' \rightarrow S.\$] \Rightarrow \text{table}[i, \$] = a.$
- $[\square \xrightarrow{T} \square]$, Terminal $T \Rightarrow \text{table}[i, T] = sj.$
- $[\square \xrightarrow{N} \square]$, Non-Terminal $N \Rightarrow \text{table}[i, N] = gj.$
- $[A \rightarrow \gamma.] \Rightarrow \text{table}[i, T] = rk$, for all terminals $T.$

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Transition Table

	()	x	,	\$	\$	S'	S	L
1									
2									
3									
4									
5									
6									
7									
8									
9									

No duplicate entries \Rightarrow grammar is LR(0)

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Using The Transition Table

1 $S' \rightarrow S \$$
2 $S \rightarrow (L)$

3 $S \rightarrow x$
4 $L \rightarrow S$

5 $L \rightarrow L, S$

	()	x	,	\$	S'	S	L	STACK	INPUT	ACTION	
1	s3		s2						1	(x , x) \$	shift 3	
2	r3	r3	r3	r3	r3				1 (3	x , x) \$	shift 2	
3	s3		s2						1 (3 x2	, x) \$	reduce 3	
4		s6		s7					1 (3 S5	, x) \$	reduce 4	
5	r4	r4	r4	r4	r4				1 (3 L4	, x) \$	shift 7	
6	r2	r2	r2	r2	r2				1 (3 L4 ,7	x) \$	shift 2	
7	s3		s2						1 (3 L4 ,7 x2) \$	reduce 4	
8	r5	r5	r5	r5	r5				1 (3 L4 ,7 S8) \$	reduce 5	
9					a				1 (3 L4)6	\$	reduce 2
									1 S9		\$	accept

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Another Example

1 $S' \rightarrow E \$$
2 $E \rightarrow T + E$

3 $E \rightarrow T$

4 $T \rightarrow x$

LR(0) states:

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Another Example - SLR

Transition Table:

	+	x	\$	S'	E	T
1			s3			g2 g4
2			a			
3	r4	r4	r4			
4	s5/r3	r3	r3			
5			s3			g6 g4
6	r2	r2	r2			

Duplicate entries \Rightarrow grammar is NOT LR(0)

Can make grammar bottom-up parsable using more powerful parsing techniques: **SLR** (Simple LR)

- Use same LR(0) states.
- $[A \rightarrow \gamma] \Rightarrow \text{table}[i, T] = \text{reduce}(k)$, for all terminals $T \in \text{follow}(A)$

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Another Example – SLR

Transition Table:

	+	x	\$	S'	E	T
1			s3		g2	g4
2			a			
3	r4	r4	r4			
4	s5/r3	r3	r3			
5			s3		g6	g4
6	r2	r2	r2			

Follow Set Computation:

	nullable	first	follow
S'	no	x	
E	no	x	\$
T	no	x	+, \$

SLR Transition Table:

	+	x	\$	S'	E	T
1			s3		g2	g4
2			a			
3	r4	r4	r4			
4	s5	r3	r3			
5			s3		g6	g4
6			r2			

- 1 $S' \rightarrow E \$$ 3 $E \rightarrow T$
- 2 $E \rightarrow T + E$ 4 $T \rightarrow x$

No duplicate entries \Rightarrow grammar is SLR.

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Yet Another Example

Sometimes grammar can't be parsed using SLR techniques.

- 1 $S' \rightarrow S \$$ 3 $S \rightarrow E$ 5 $V \rightarrow x$
- 2 $S \rightarrow V = E$ 4 $E \rightarrow V$ 6 $V \rightarrow * E$

This grammar is not SLR. Need more powerful parsing algorithm \Rightarrow LR(1)

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LR(1) Parsing

- LR(1) item consists of two components: $(A \rightarrow \alpha \cdot \beta, x)$
 1. Production
 2. Lookahead symbol (x)
- α is on top of stack, head of input is string derivable from βx .

LR(0) closure computation

- Initial: $A \rightarrow \alpha \cdot X$
- Add all items $X \rightarrow \cdot \gamma$
- Repeat closure computation

LR(1) closure computation

- Initial: $A \rightarrow \alpha \cdot X \beta, z$
- Add all items $(X \rightarrow \cdot \gamma, \omega)$ for each $\omega \in \text{first}(\beta z)$
- Repeat closure computation

- shift, goto, accept table entries computed same way as LR(0)/SLR.
- reduce entries computed differently:

$$\boxed{A \rightarrow \gamma \cdot z} \Rightarrow \text{table}[i, z] = \text{reduce}(k)$$

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Yet Another Example – LR(1)

$$\begin{array}{lll}
 1 \ S' \rightarrow S \$ & 3 \ S \rightarrow E & 5 \ V \rightarrow x \\
 2 \ S \rightarrow V = E & 4 \ E \rightarrow V & 6 \ V \rightarrow * E
 \end{array}$$

LR(1) states:

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Yet Another Example – LR(1)

	=	x	*	\$	S'	S	L	V
1		s11	s12	.	g2	g10	g3	
2				a				
3	s4			r4				
4		s7	s8			g5	g6	
5				r2				
6				r4				
7				r5				
8		s7	s8			g9	g6	
9				r6				
10				r3				
11	r5			r5				
12		s11	s12			g13	g14	
13	r6			r6				
14	r4			r4				

No duplicate entries \Rightarrow grammar is LR(1)

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LALR(1)

- Problem with LR(1) parsers: tables too large!
 - Can make smaller table by merging states whose items are identical except for look-ahead sets \Rightarrow LALR(1) (Look-Ahead LR(1)).
 - LALR(1) transition table may contain shift-reduce/reduce-reduce conflicts where LR(1) table has none.

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LALR(1)

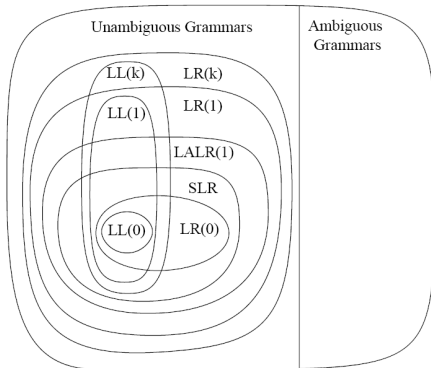
Can make smaller table by merging states whose items are identical except for look-ahead sets \Rightarrow LALR(1) (Look-Ahead LR(1)).

	=	x	*	\$	S'	S	L	V
1		s11	s12		g2	g10	g3	
2				a				
3	s4			r4				
4		s7	s8			g5	g6	
5				r2				
6/14	r4			r4				
7/11	r5			r5				
8/12		s7/11	s8/12			g9/13	g6/14	
9/13	r6			r6				
10				r3				

No conflicts \Rightarrow grammar is LALR(1).

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Parsing Power



ML-YACC uses LALR(1) parsing because reasonable programming languages can be specified by an LALR(1) grammar. (Figure from MCI in ML.)

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Parsing Error Recovery

Syntax Errors:

- A *Syntax Error* occurs when stream of tokens is an invalid string.
- In LL(k) or LR(k) parsing tables, blank entries refer to syntax errors.

How should syntax errors be handled?

1. Report error, terminate compilation \Rightarrow not user friendly
2. Report error, *recover* from error, search for more errors \Rightarrow better

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Error Recovery

Error Recovery: process of adjusting input stream so that parsing may resume after syntax error reported.

- Deletion of token types from input stream
- Insertion of token types
- Substitution of token types

Two classes of recovery:

1. *Local Recovery:* adjust input at point where error was detected.
2. *Global Recovery:* adjust input *before* point where error was detected.

These may be applied to both LL and LR parsing techniques.

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LL Local Error Recovery

Consider LL(1) parsing context:

$Z \rightarrow XYZ$	$Y \rightarrow c$	$X \rightarrow a$
$Z \rightarrow d$	$Y \rightarrow \epsilon$	$X \rightarrow bYe$

	nullable	first	follow
Z	no	a,b,d	
Y	yes	c	a,b,d,e
X	no	a,b	a,b,c,d

	a	b	c	d	e
Z	$Z \rightarrow XYZ$	$Z \rightarrow XYZ$		$Z \rightarrow d$	
Y	$Y \rightarrow \epsilon$	$Y \rightarrow \epsilon$	$Y \rightarrow c$	$Y \rightarrow \epsilon$	$Y \rightarrow \epsilon$
X	$X \rightarrow a$	$X \rightarrow bYe$			

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LL Local Error Recovery

Local Recovery Technique: in function $A()$, delete token types from input stream until token type in $\text{follow}(A)$ found \Rightarrow *synchronizing* token types.

```
datatype token = a | b | c | d | e;  
val tok = ref(getToken());  
fun advance() = tok := getToken();  
fun eat(t) = if(!tok = t) then advance() else error();  
...  
and X() = case !tok of  
  a => (eat(a))  
  | b => (eat(b); Y(); eat(e))  
  | c => (print "error!"; skipTo[a,b,c,d])  
  | d => (print "error!"; skipTo[a,b,c,d])  
  | e => (print "error!"; skipTo[a,b,c,d])  
  
and skipTo(synchTokens) =  
  if member(!tok, synchTokens) then ()  
  else (eat(!tok); skipTo(synchTokens))
```

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LR Local Error Recovery

Consider:

1 $E \rightarrow ID$ 3 $E \rightarrow (E)$ 5 $ES \rightarrow ES ; E$
2 $E \rightarrow E + E$ 4 $ES \rightarrow E$

- Match a sequence of erroneous input tokens using the *error* token (a terminal).

6 $E \rightarrow (error)$ 7 $ES \rightarrow error ; E$

- In general, follow *error* with synchronizing lookahead token.
 1. Pop stack (if necessary) until a state is reached in which the action for the *error* token is *shift*.
 2. Shift the *error* token.
 3. Discard input symbols (if necessary) until a state is reached that has a non-error action in the current state.
 4. Resume normal parsing.

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Global Error Recovery

Consider LR(1) parsing:

```
let type a := intArray[10] of 0 in ... end
```

Local Recovery Techniques would:

1. report syntax error at ‘:=’
2. substitute ‘=’ for ‘:=’
3. report syntax error at ‘[’
4. delete token types from input stream, synchronizing on ‘in’

Global Recovery Techniques would substitute ‘var’ for ‘type’:

- Actual syntax error occurs *before* point where error was detected.
- ML-Yacc uses global error recovery technique \Rightarrow *Burke-Fisher*
- Other Yacc versions employ local recovery techniques.

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Burke-Fisher

Suppose parser gets stuck at n^{th} token in input stream.

- Burke-Fisher repairer tries every *single-token-type* insertion, deletion, and substitution at all points between $(n - k)^{th}$ and n^{th} token.

n-k n-k+1 ----- n-1 n

- Best repair: one that allows parser to parse furthest past n^{th} token.
- If languages has N token types, then:
total # of repairs = deletions + insertions + substitutions
total # of repairs = $(k) + (k + 1)N + (k)(N - 1)$

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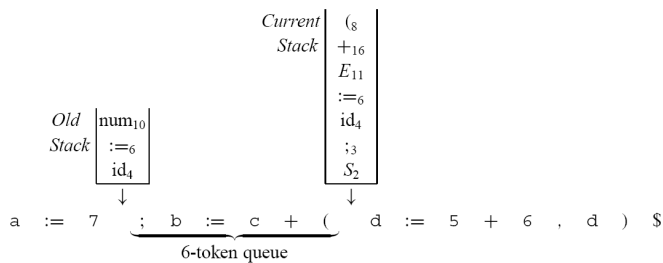
Burke-Fisher

In order to backup K tokens and reparse repaired input, 2 structures needed:

1. *k-length buffer/queue* - if parser currently processing n^{th} token, queue contains tokens $(n - k) \rightarrow (n - 1)$. (ML-Yacc $k = 15$)
2. *old parse stack* - if parser currently processing n^{th} token, old stack represents stack state when parser was processing $(n - k)^{th}$ token.
 - Whenever token shifted onto current stack, also put onto queue tail.
 - Simultaneously, queue head removed, shifted onto old stack.
 - Whenever token shifted onto either stack, appropriate reductions performed.

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Burke-Fisher Example



- Semantic actions are only applied to old stack.
 - Not desirable if semantic actions affect lexical analysis.
 - Example: `typedef` in C.

(Figure from MCI/ML.)

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Burke-Fisher

For each repair R that can be applied to token $(n - k) \rightarrow n$:

1. copy queue, copy n^{th} token
2. copy old parse stack
3. apply R to copy of queue or copy of n^{th} token
4. reparse queue copy (and copy of n^{th} token) from old stack copy
5. evaluate R

Choose best repair R , and apply.

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Semantic Values

- Insertions need semantic values

```
%value ID {"bogus"}
%value INT {1}
%value STRING {"STRING"}
```

Programmer-Specified Substitutions

- Some single token insertions and deletions are common.
- Some multiple token insertions and deletions are common.

```
%change EQ -> ASSIGN | SEMICOLON ELSE -> ELSE
| -> IN INT END
```