Polygonal Meshes

COS 426, Spring 2014

Princeton University

3D Object Representations

Points
- Range image
- Point cloud

Surfaces
- Polygonal mesh
- Subdivision
- Parametric
- Implicit

Solids
- Voxels
- BSP tree
- CSG
- Sweep

High-level structures
- Scene graph
- Application specific

3D Polygonal Mesh

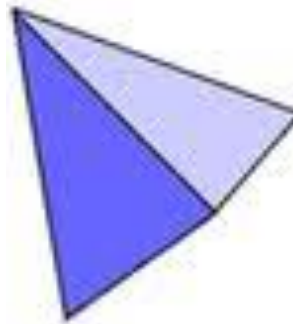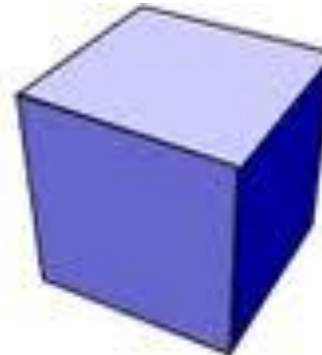Set of polygons representing a 2D surface embedded in 3D

Platonic Solids
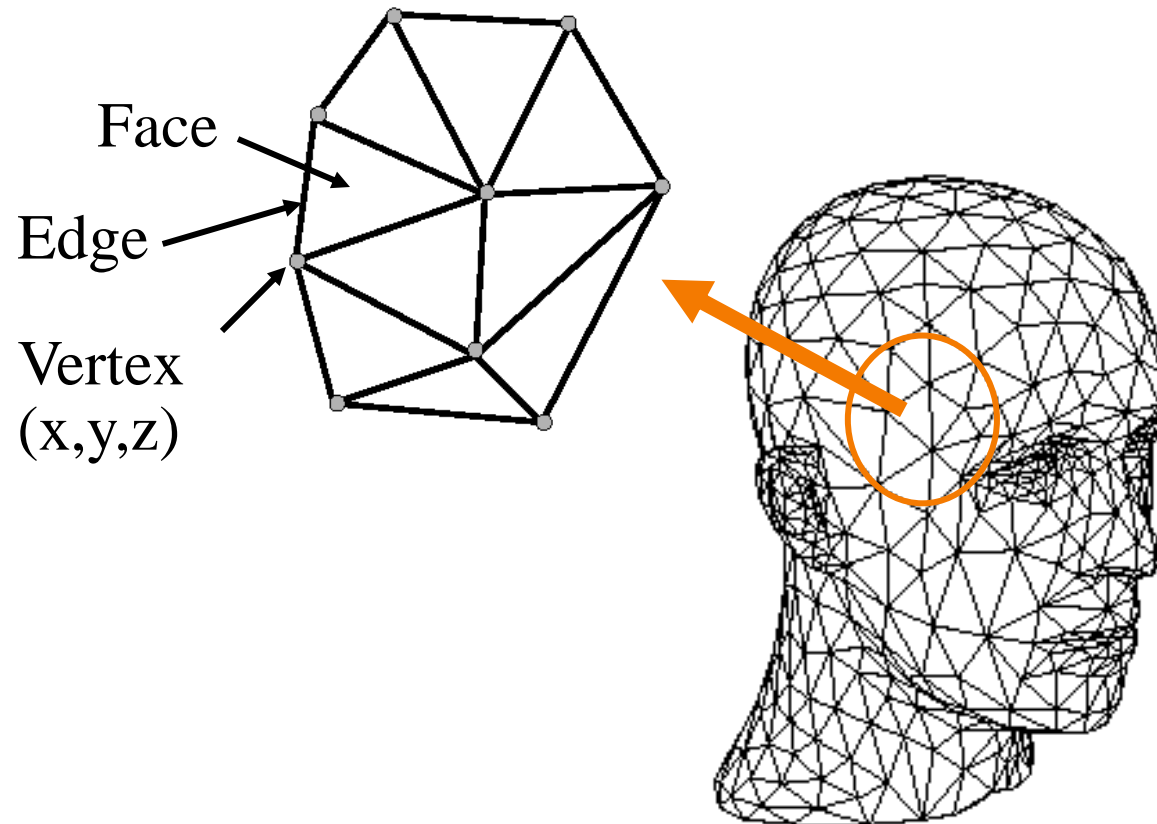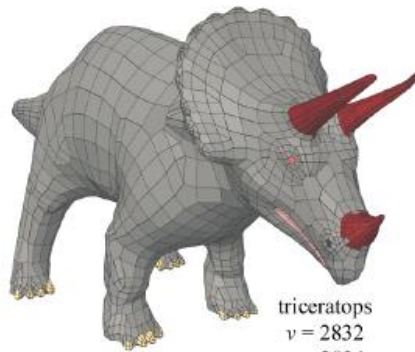
Dodecahedron

Icosahedron

Tetrahedron

Cube

Octahedron

3D Polygonal Mesh

Set of polygons representing a 2D surface embedded in 3D
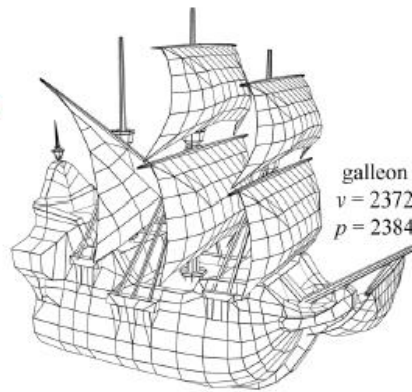
Face

Edge

Vertex
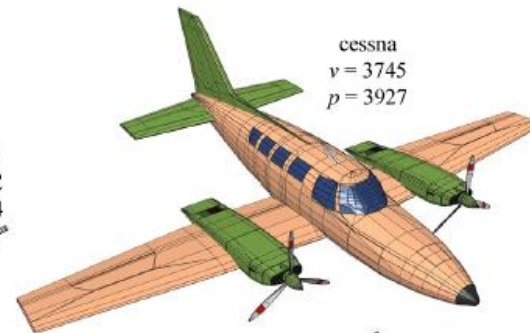(x,y,z)

3D Polygonal Mesh

Set of polygons representing a 2D surface embedded in 3D
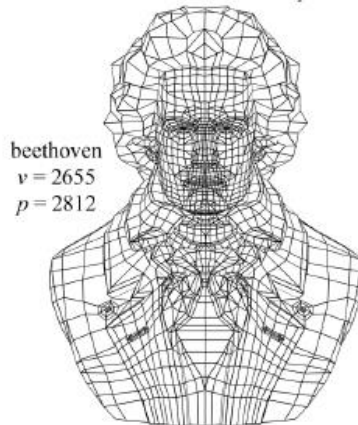

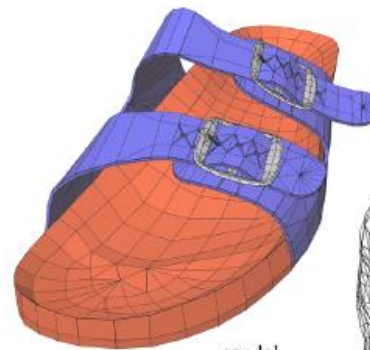triceratops
$v = 2832$
$p = 2834$
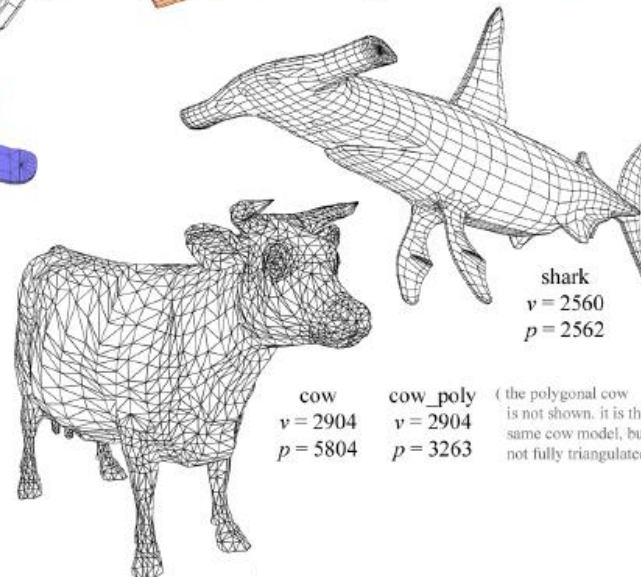
galleon
$v = 2372$
$p = 2384$

cessna
$v = 3745$
$p = 3927$

beethoven
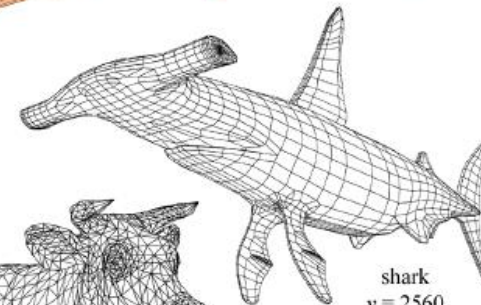$v = 2655$
$p = 2812$

sandal
$v = 2636$
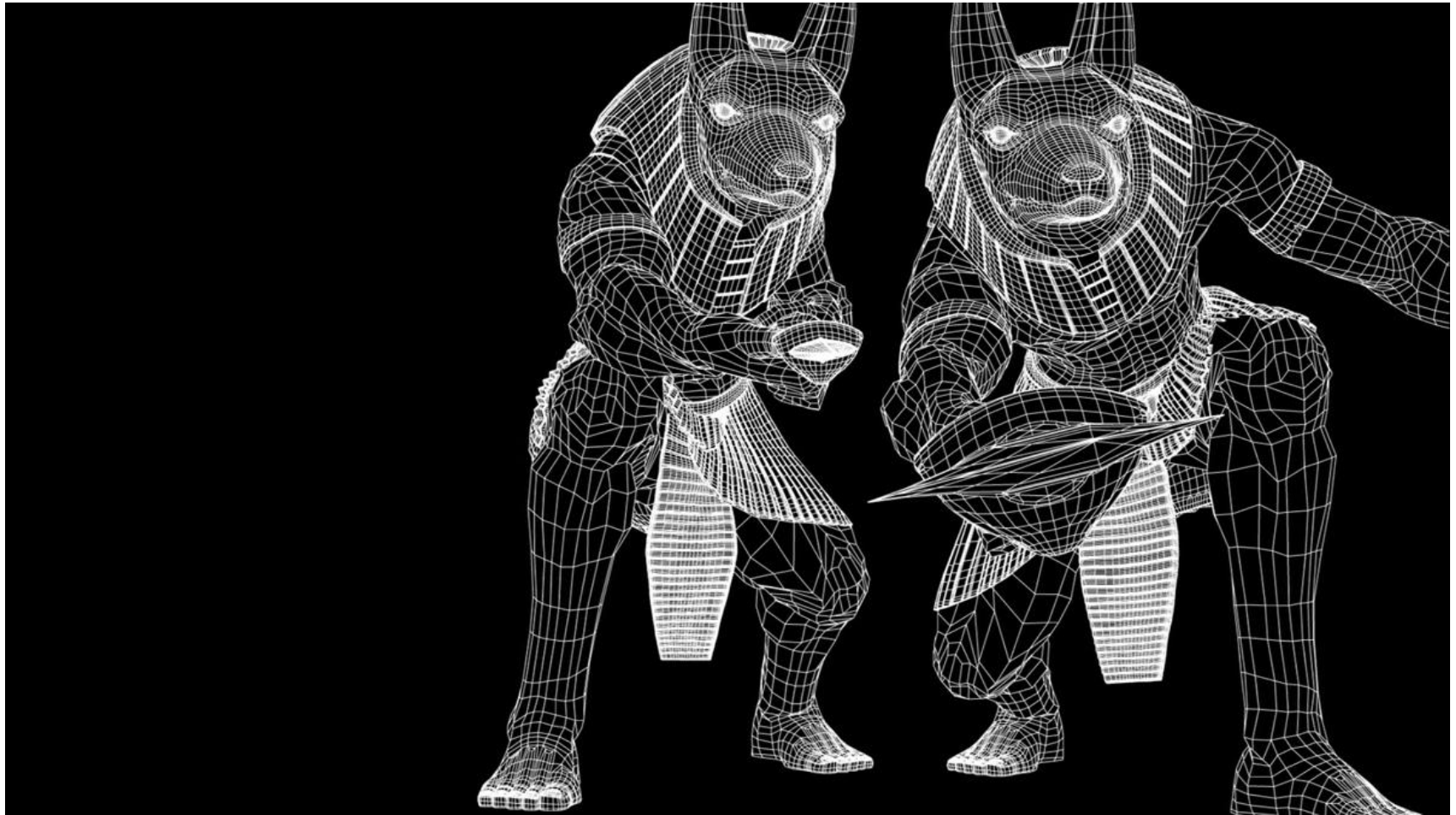$p = 2953$

cow   cow_poly
$v = 2904$ $v = 2904$
$p = 5804$ $p = 3263$

(the polygonal cow
is not shown. it is the
same cow model, but
not fully triangulated)

shark
$v = 2560$
$p = 2562$
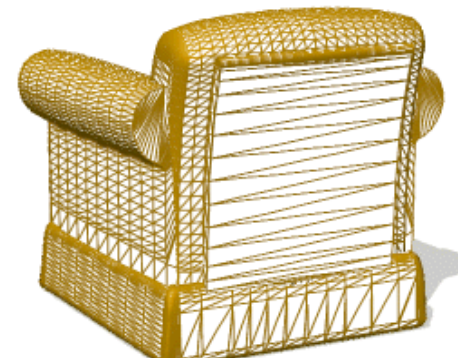
3D Polygonal Meshes

http://www.fxguide.com/featured/Comic_Horrors_Rocks_Statues_and_VanDyke/

3D Polygonal Meshes

Why are they of interest?
- Simple, common representation
- Rendering with hardware support
- Output of many acquisition tools
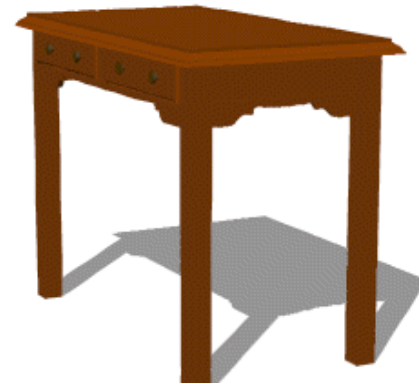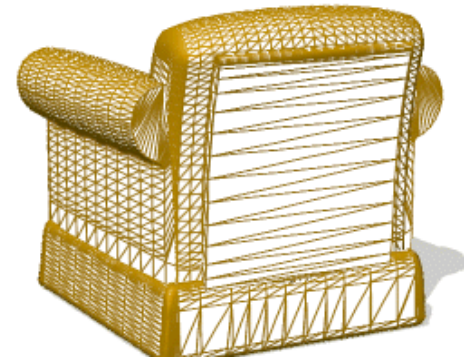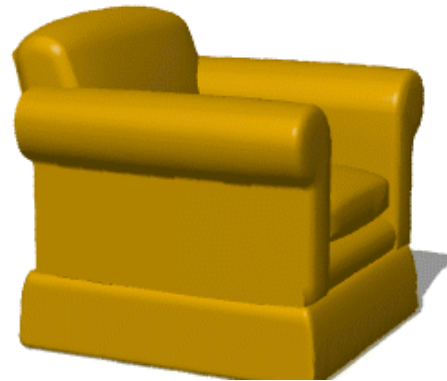- Input to many simulation/analysis tools

Viewpoint

3D Polygonal Meshes

Properties
- ? Efficient display
- ? Easy acquisition
- ? Accurate
- ? Concise
- ? Intuitive editing
- ? Efficient editing
- ? Efficient intersections
- ? Guaranteed validity
- ? Guaranteed smoothness
- ? etc.

Viewpoint

Outline

Acquisition ⬅

Processing

Representation

Polygonal Mesh Acquisition

Interactive modeling
- Polygon editors
- Interchange formats

Scanners
- Laser range scanners
- Geological survey
- CAT, MRI, etc. (isosurfaces)

Simulations
- Physical processes
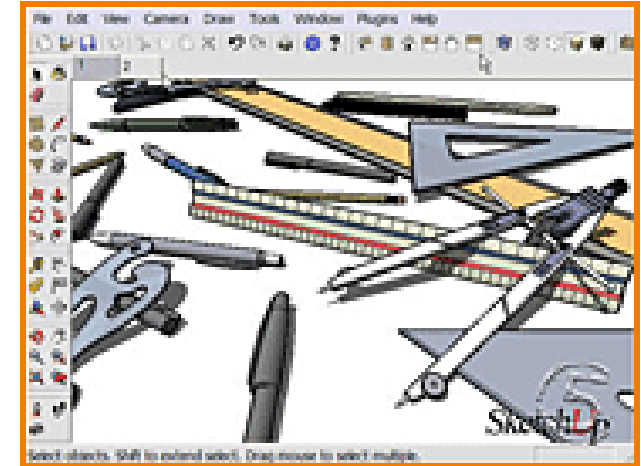
Polygonal Mesh Acquisition

Interactive modeling
- ➢ Polygon editors
- ○ Interchange formats

Scanners
- ○ Laser range scanners
- ○ Geological survey

Simulations
- ○ Physical processes

Sketchup

Blender

Polygonal Mesh Acquisition

Interactive modeling
- Polygon editors
- ➤ Interchange formats

Scanners
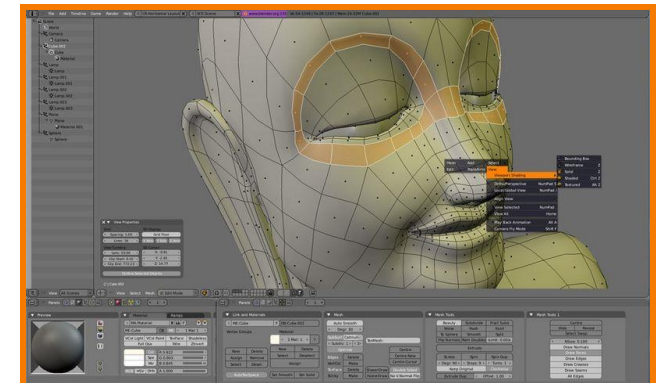- Laser range scanners
- Geological survey

Simulations
- Physical processes

Polygonal Mesh Acquisition

Interactive modeling
- Polygon editors
- Interchange formats

Scanners
- Laser range scanners
- Geological survey

Simulations
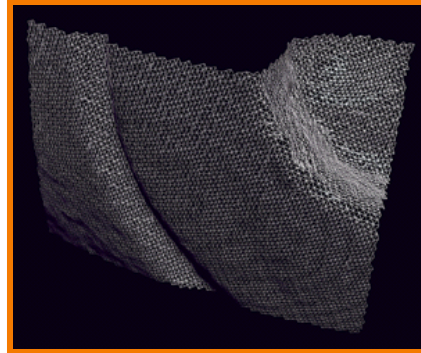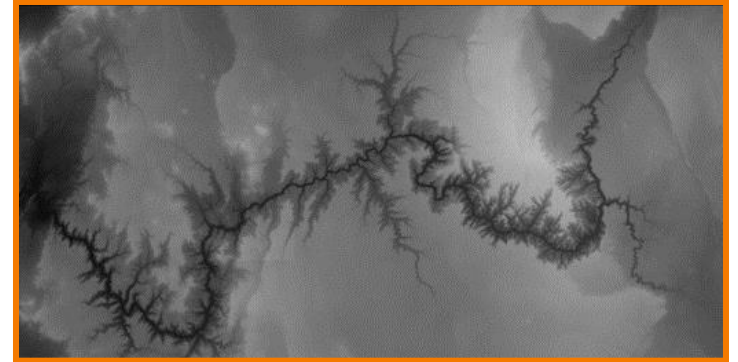- Physical processes

Digital Michelangelo Project
Stanford

Polygonal Mesh Acquisition

Interactive modeling
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Scanners
- Laser range scanners
- ➤ Geological survey

Simulations
- Physical processes

Large Geometric
Model Repository
Georgia Tech

Polygonal Mesh Acquisition

Interactive modeling

- Polygon editors
- Interchange formats

Scanners

- Laser range scanners
- ➢ Geological survey

Simulations

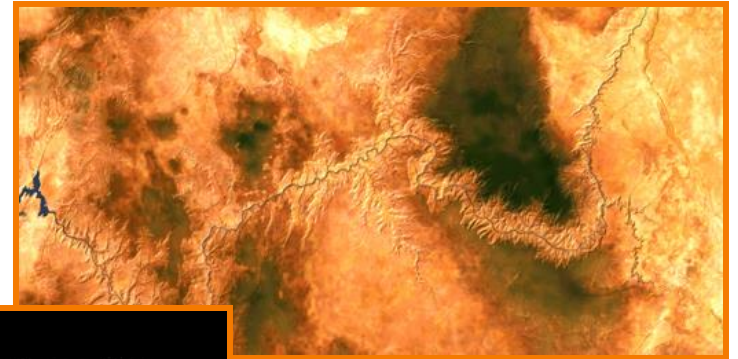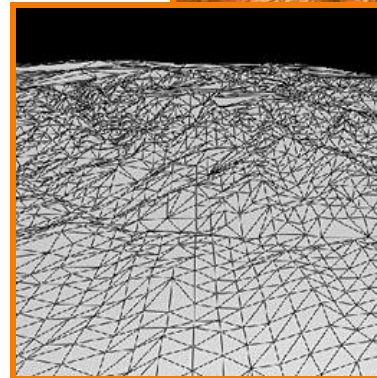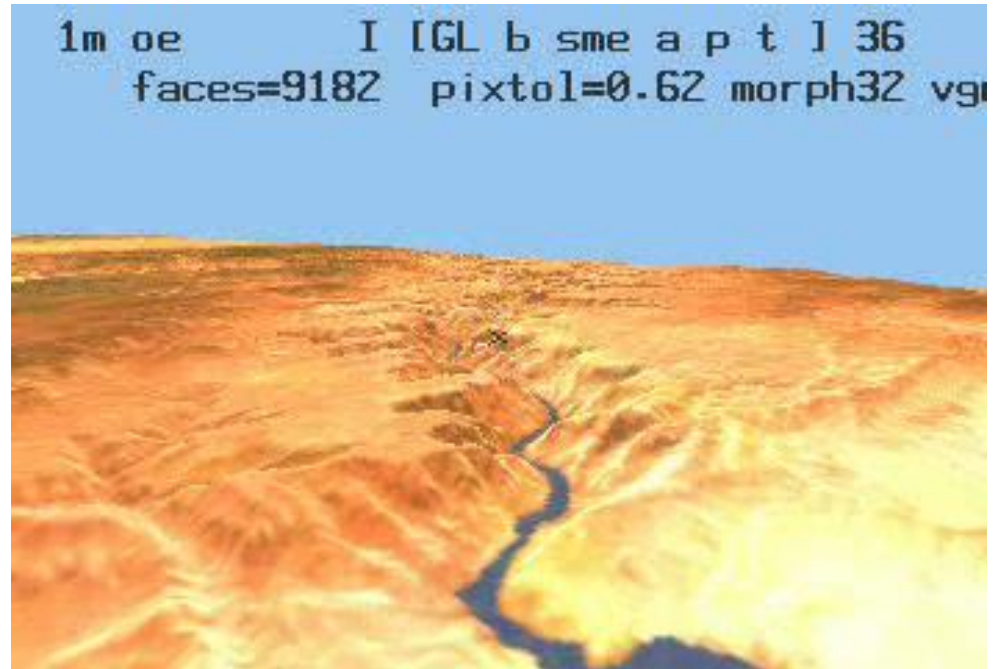- Physical processes

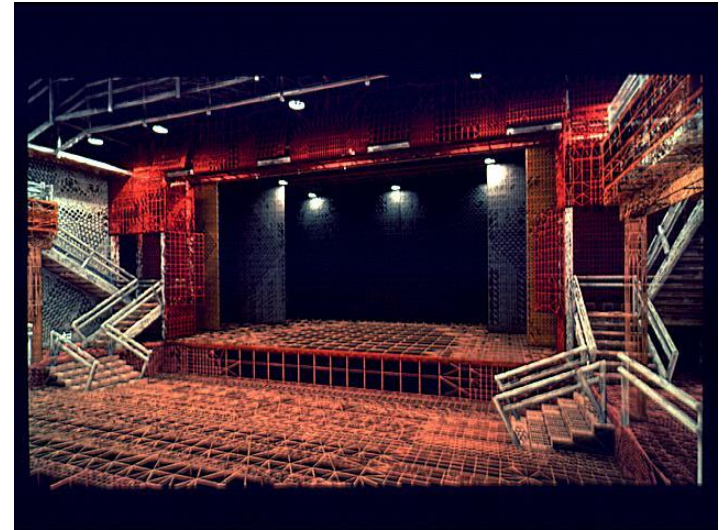# **Polygonal Mesh Acquisition**

Interactive modeling
- Polygon editors
- Interchange formats
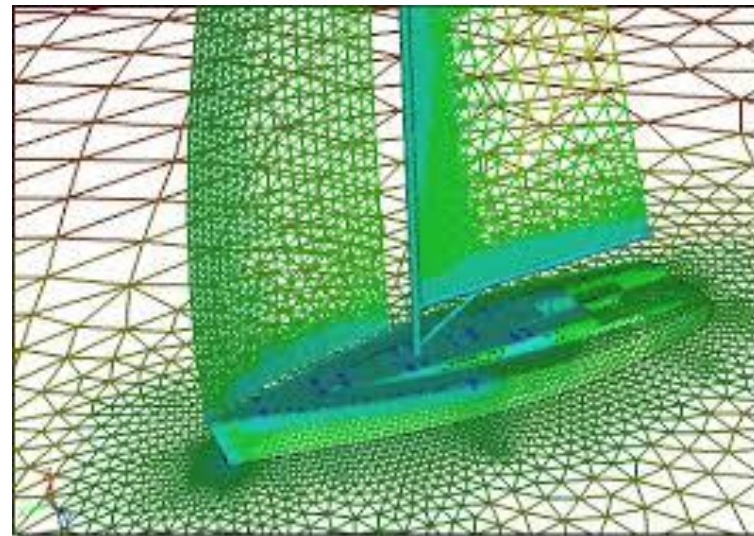
Scanners
- Laser range scanners
- Geological survey

Simulations
- Physical processes

SGI

symscape

Outline

Acquisition

Processing ←

Representation

Polygonal Mesh Processing

Analysis
- Normals
- Curvature

Warps
- Rotate
- Deform

Filters
- Smooth
- Sharpen
- Truncate
- Bevel

Polygonal Mesh Processing

Analysis ⬅

- Normals
- Curvature

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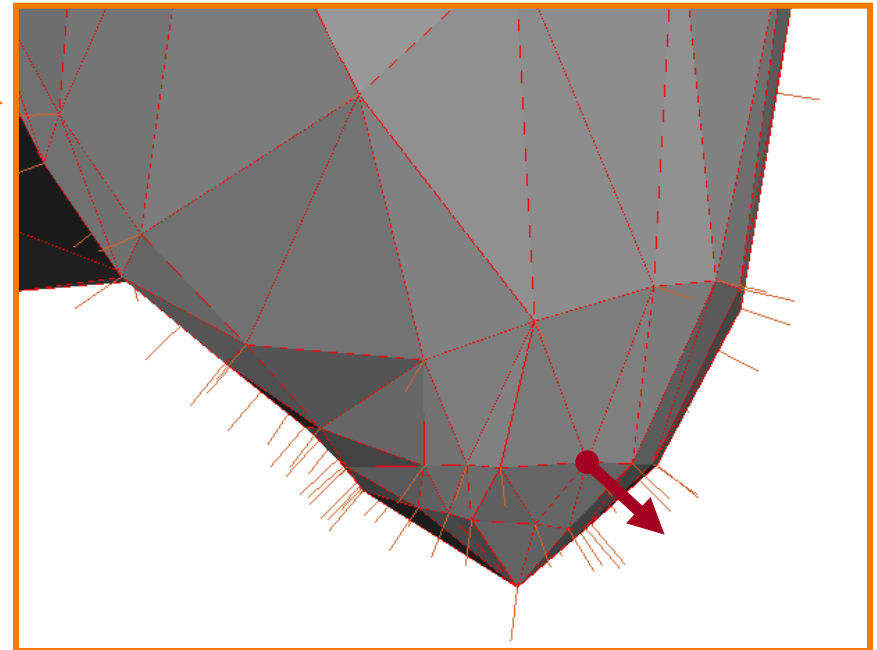- Bevel

Polygonal Mesh Processing

Analysis
- ➢ Normals
  - ○ Curvature

Warps
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Filters
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- ○ Truncate
- ○ Bevel

Polygonal Mesh Processing

Analysis
- ○ Normals
- ➤ Curvature

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normal at P

P

k

"best fit" circle at P

Figure 32: curvature of curve at P is $1/k$

Polygonal Mesh Processing

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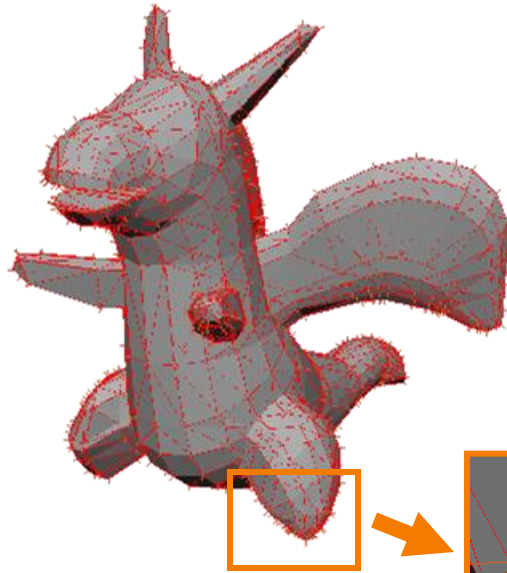# **Polygonal Mesh Processing**

Analysis
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Polygonal Mesh Processing

Analysis
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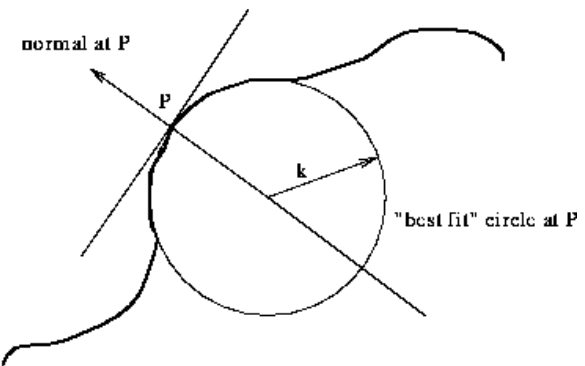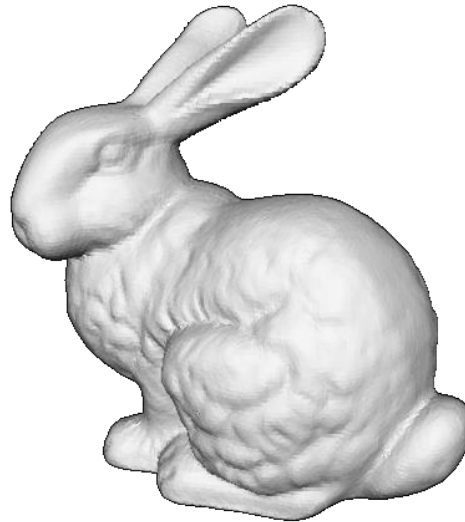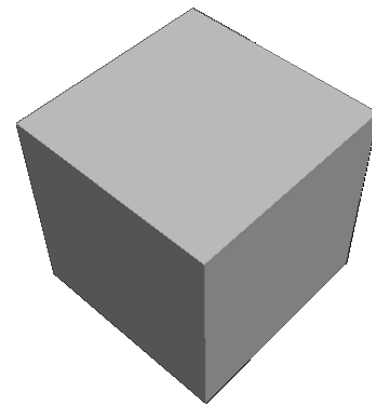- ➢ Deform

Filters
- Smooth
- Sharpen
- Truncate
- Bevel

Sheffer

Polygonal Mesh Processing

Analysis
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Warps
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Sheffer

Polygonal Mesh Processing

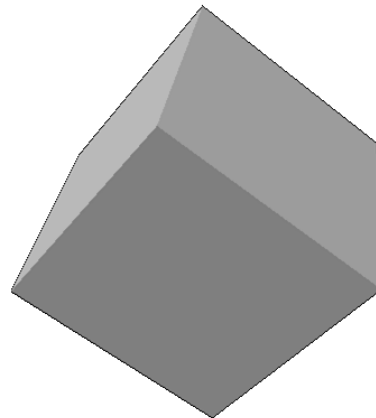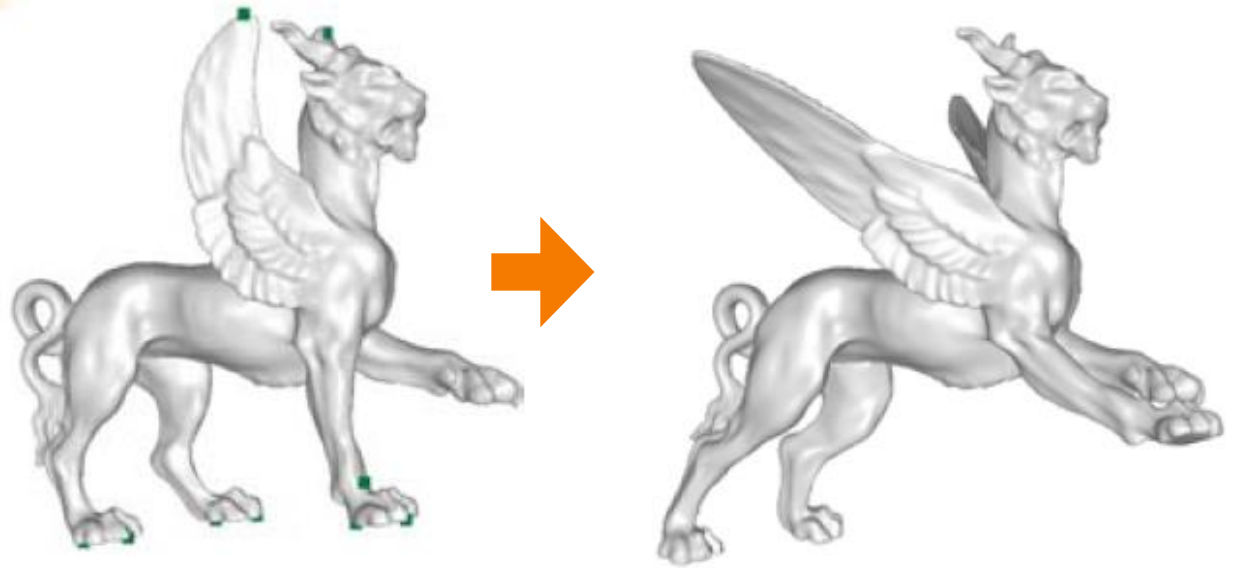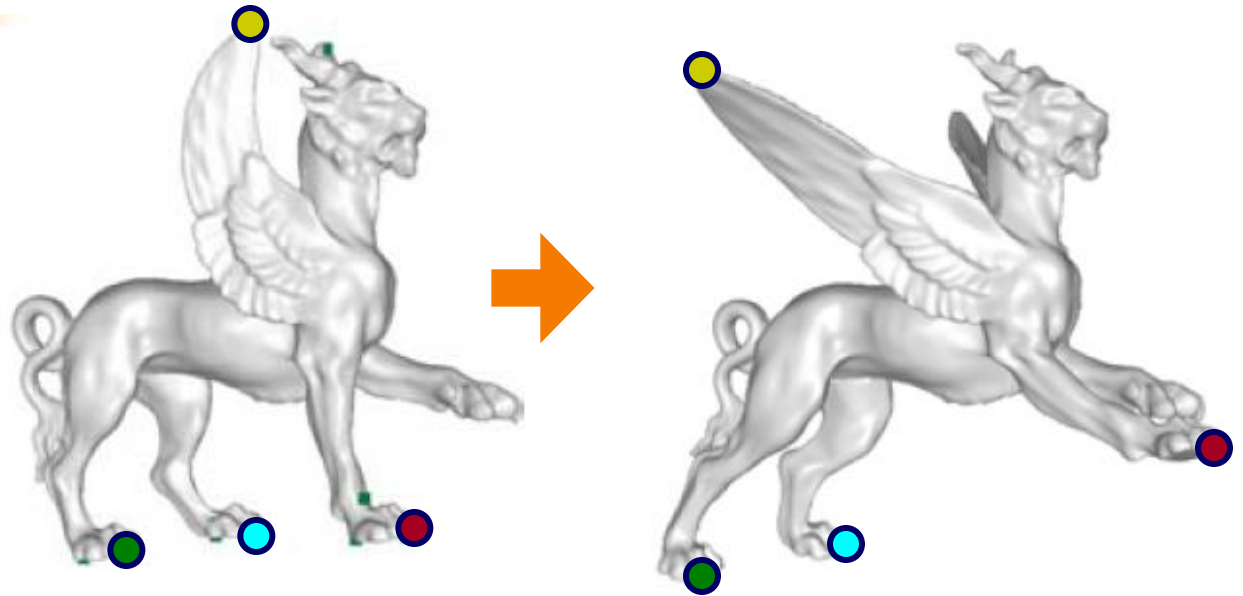Analysis
- Normals
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Polygonal Mesh Processing
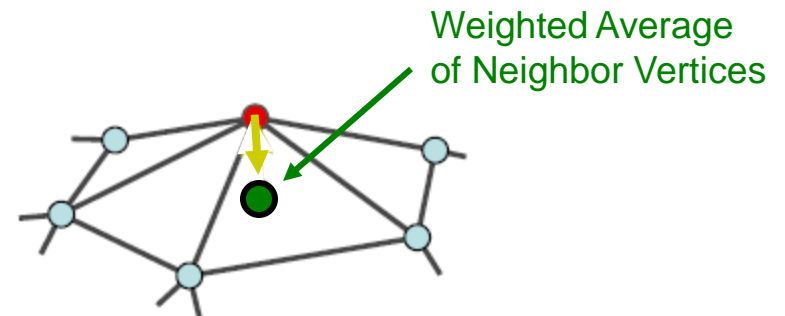
Analysis
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- Sharpen
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Thouis "Ray" Jones

Weighted Average
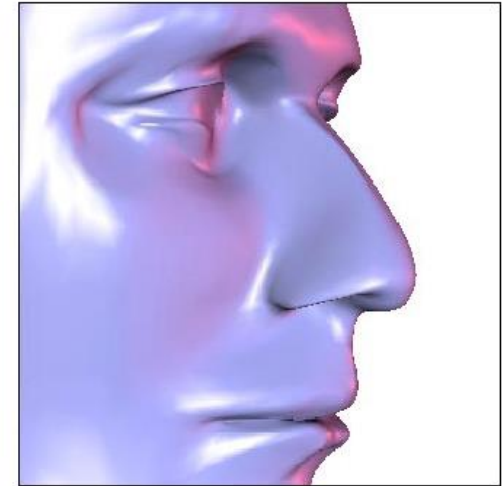of Neighbor Vertices

Olga Sorkine

Polygonal Mesh Processing

Analysis
- Normals
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Desbrun

Weighted Average
of Neighbor Vertices

Olga Sorkine

Polygonal Mesh Processing

Analysis
- ○ Normals
- ○ Curvature

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- ○ Sharpen
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- ○ Bevel
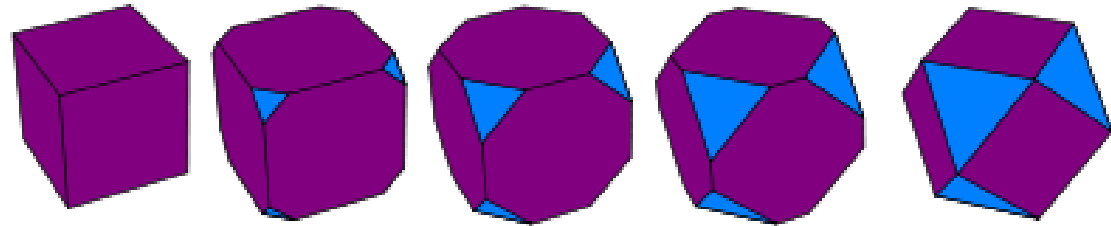
Cube ¼ truncated uniform truncated ¾ truncated Rectified

0.35

Conway

Polygonal Mesh Processing
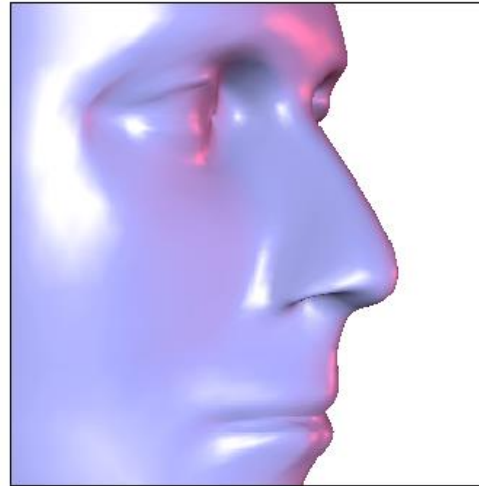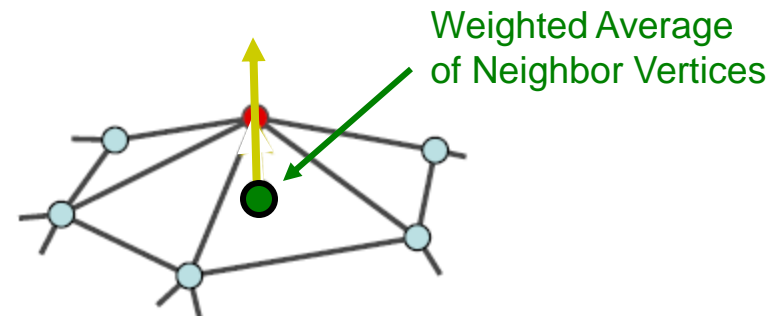
Analysis
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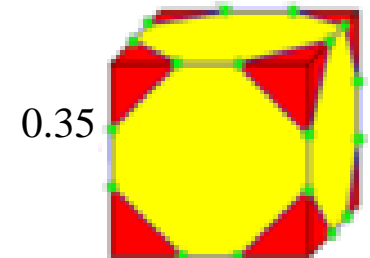# Polygonal Mesh Processing

Analysis
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- Curvature

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- Rotate
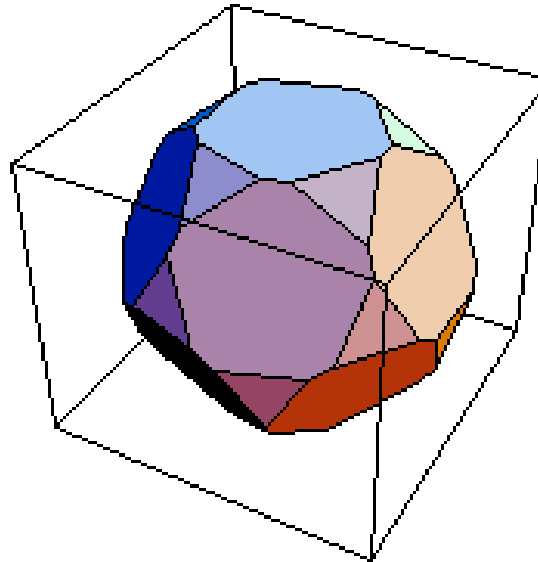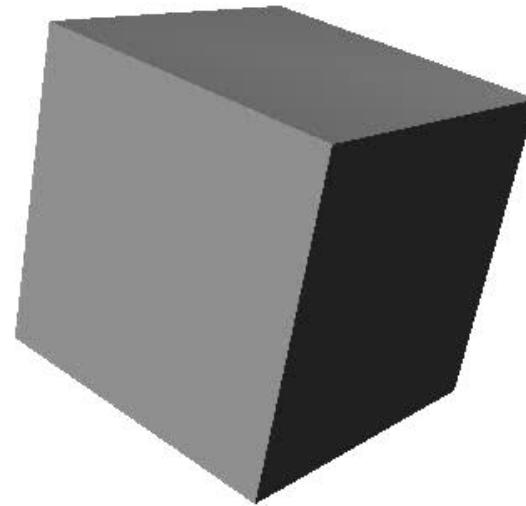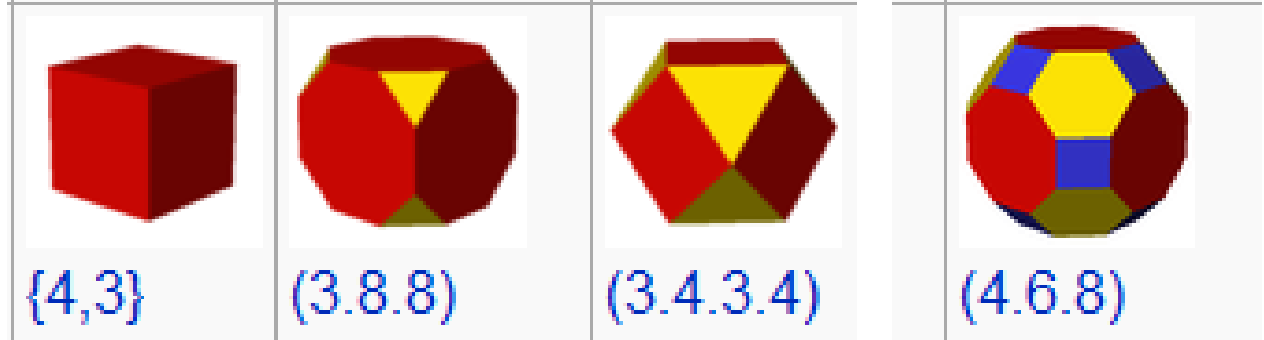- Deform

Filters
- Smooth
- Sharpen
- ➤ Truncate
- Bevel

{3,3}   (3.6.6)   (3.3.3.3)   (4.6.6)

{4,3}   (3.8.8)   (3.4.3.4)   (4.6.8)

{5,3}   (3.10.10)   (3.5.3.5)   (4.6.10)
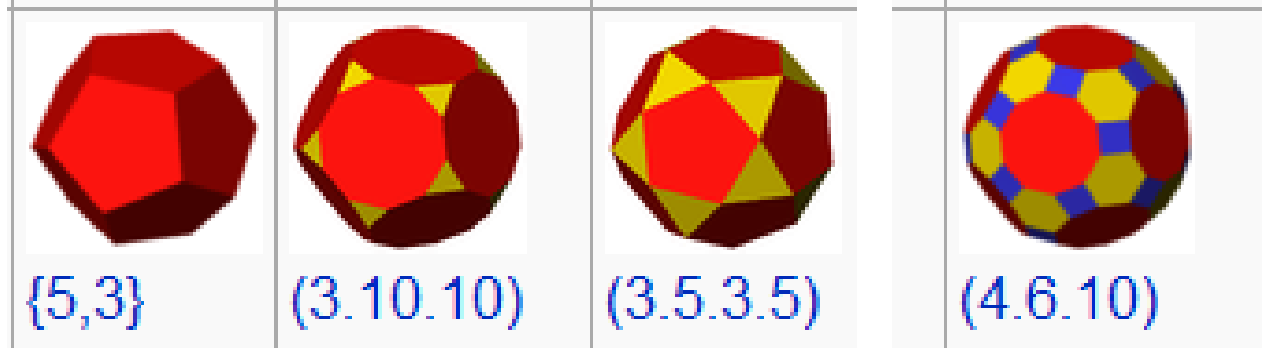
Polygonal Mesh Processing

Analysis
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Archimedean Polyhedra
http://www.uwgb.edu/dutchs/symmetry/archpol.htm

Polygonal Mesh Processing

Analysis
- Ne
- Cu

Warp
- Re
- De

Filter
- Sn
- Sh
- Tr
- Bevel

Carlo Sėquin

Polygonal Mesh Processing

Analysis
- Normals
- Curvature

Warps
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- Deform

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- Truncate
- ➢ Bevel
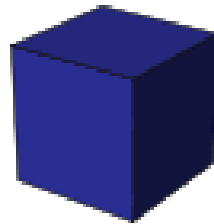
(regular polyhedron)
Cube
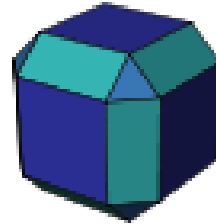
⅓ cantellated
(beveled cube)
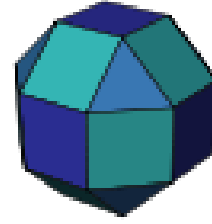
Uniform cantellation
Rhombicuboctahedron
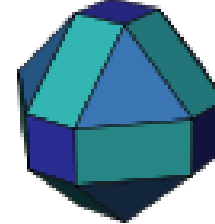
⅓ cantellated
(beveled octahedron)
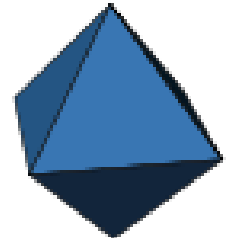
(regular dual)
Octahedron

0.40

Jarek Rossignac

Polygonal Mesh Processing

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www.srcf.ucam.org

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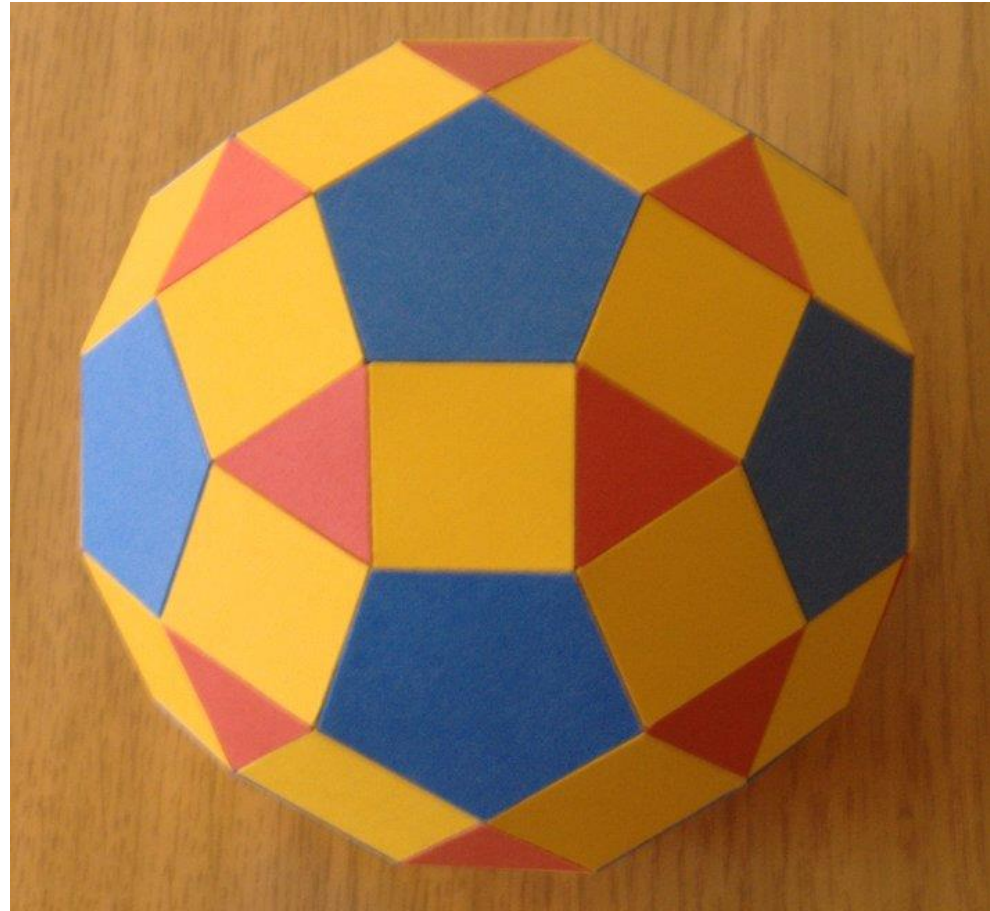
Polygonal Mesh Processing

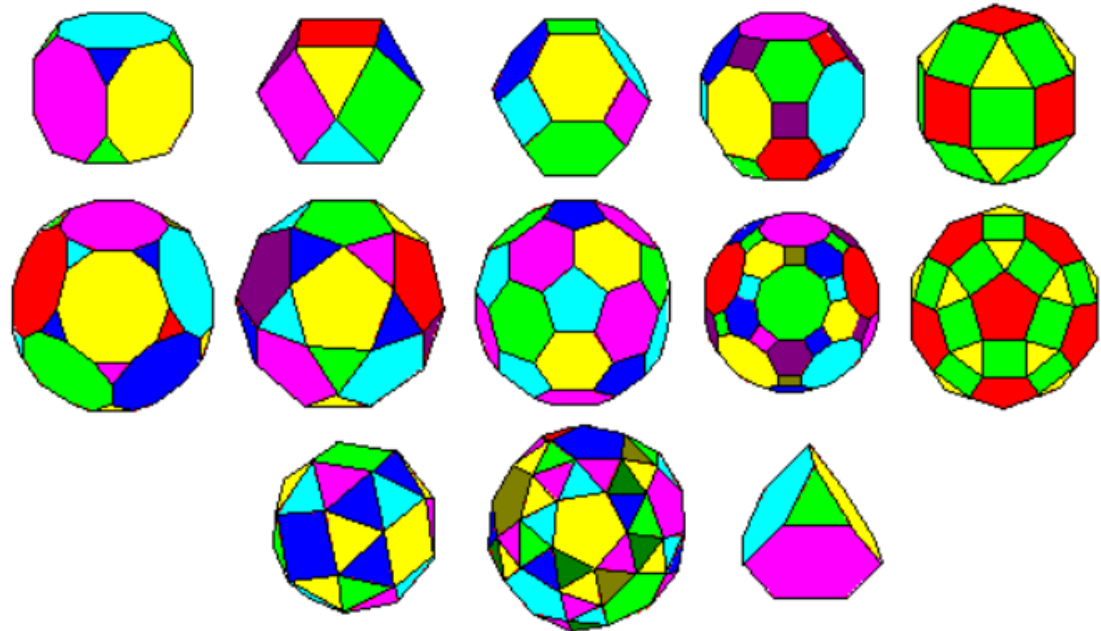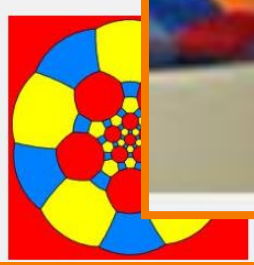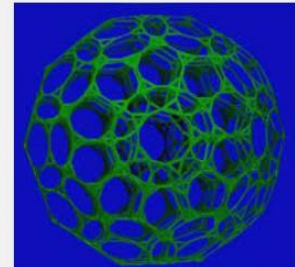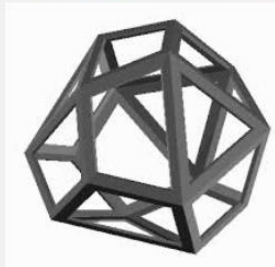Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
- Fill holes
- Fix self-intersections

Boolean operations
- Crop
- Subtract

Polygonal Mesh Processing

Remeshing ←
- Subdivide
- Resample
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Remove Vertex

Collapse edge

Subdivide face

Polygonal Mesh Processing

Remeshing
- ➢ Subdivide
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- ○ Simplify

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Zorin & Schroeder

Polygonal Mesh Processing

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Fractal Landscape

Dirk Balfanz, Igor Guskov,
Sanjeev Kumar, & Rudro Samanta,

Polygonal Mesh Processing

Remeshing
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Original   Resampled

- more uniform distribution
- triangles with nicer aspect

Sorkine

Polygonal Mesh Processing

Remeshing
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Garland

Polygonal Mesh Processing

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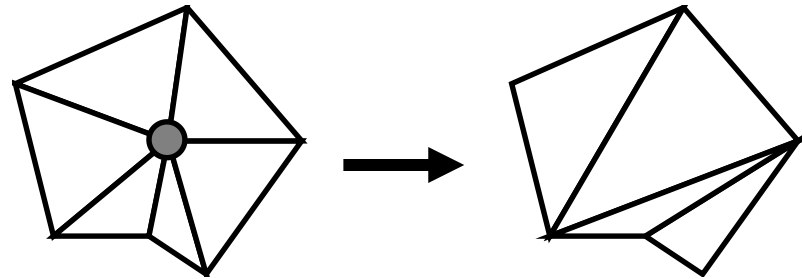
Polygonal Mesh Processing

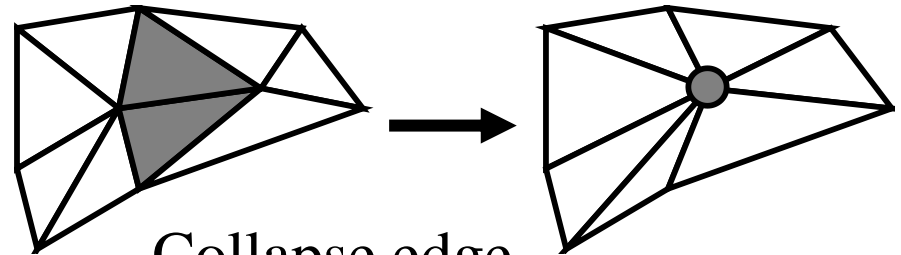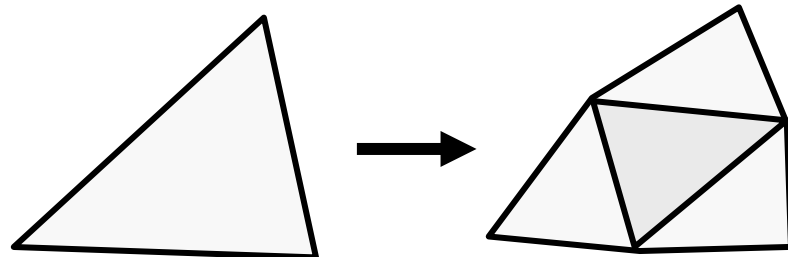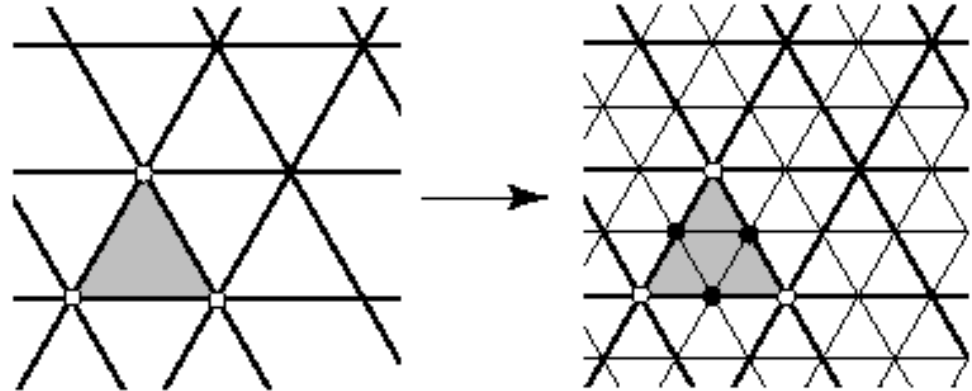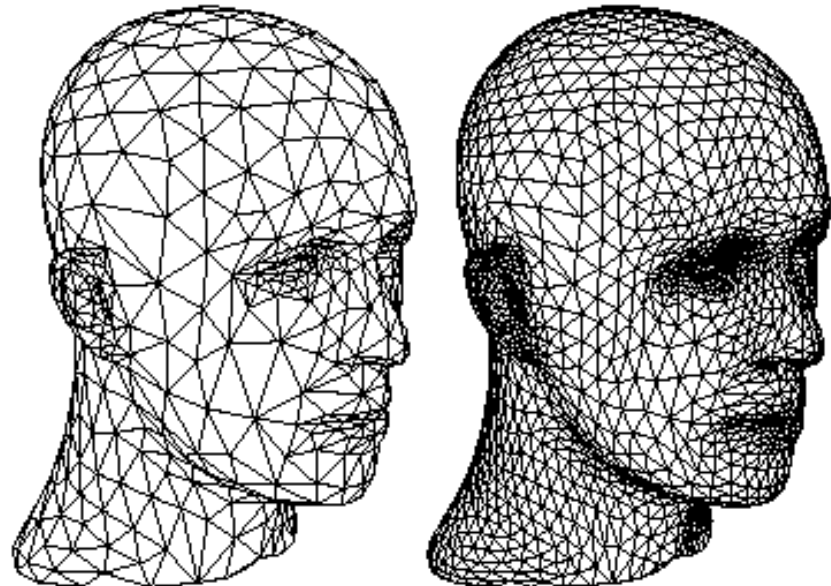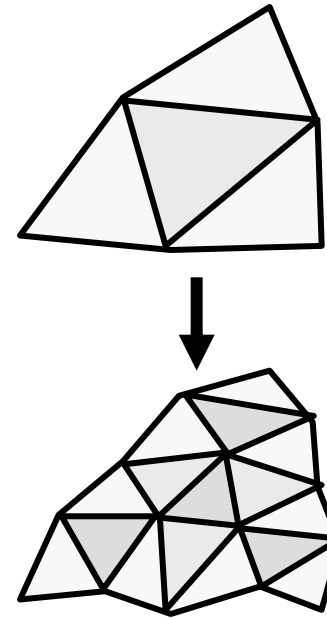Remeshing
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Podolak

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Boolean operations
- Crop
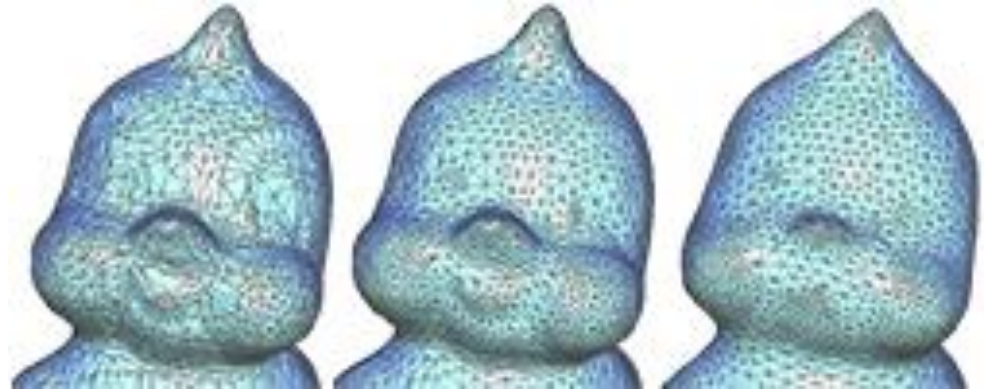- Subtract

Borodin

Polygonal Mesh Processing

Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
- Fill holes
- Fix self-intersections

Boolean operations ←
- Crop
- Subtract

Polygonal Mesh Processing

Remeshing
- Subdivide
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Boolean operations
➤ Crop
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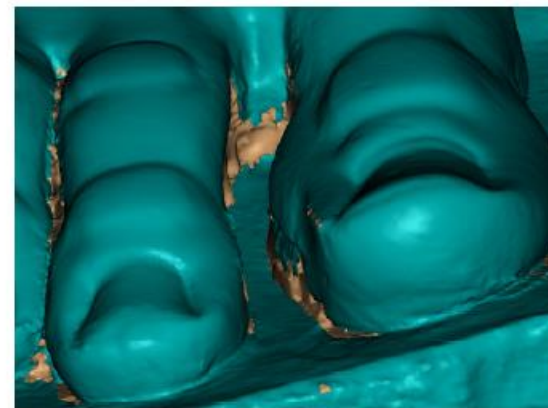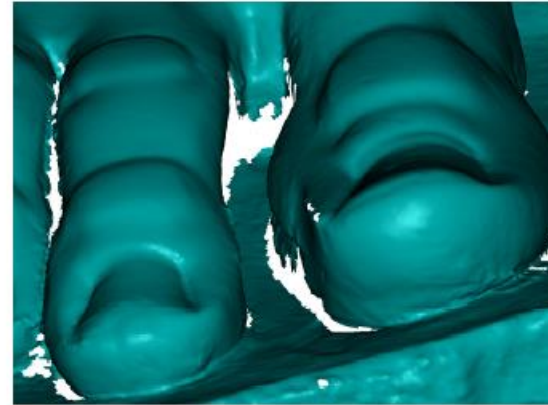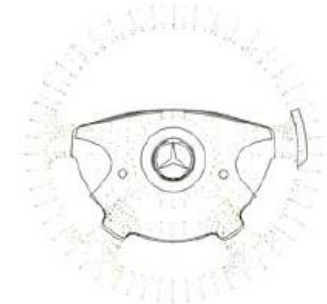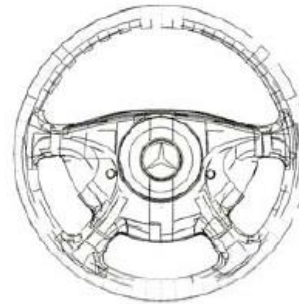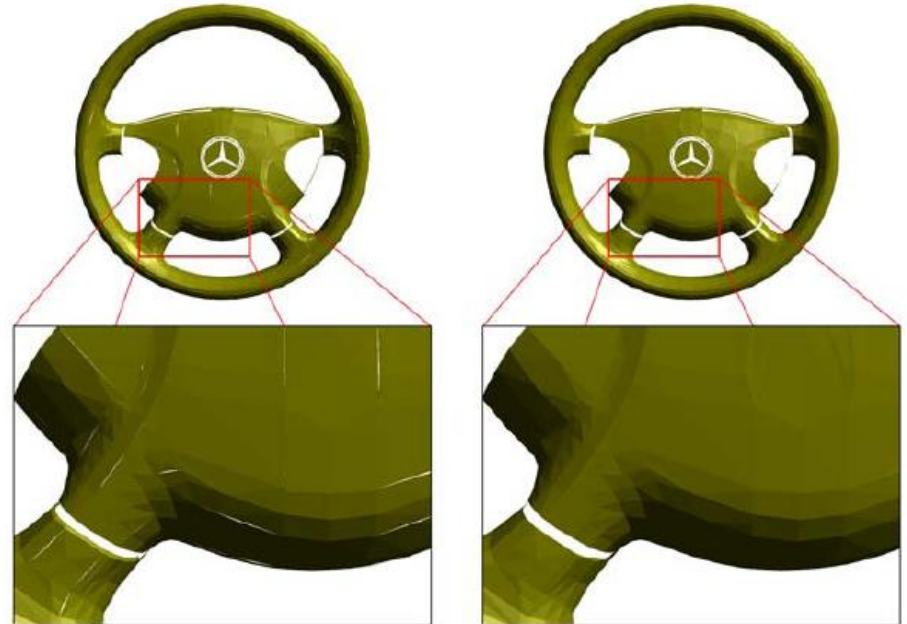# Polygonal Mesh Processing

Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
- Fill holes
- Fix self-intersections

Boolean operations
- ➤ Crop
- Subtract

P_2 P_1

Boundary   Inside

Outside

P_3

P_5

P_4

Polygonal Mesh Processing

Remeshing
- Subdivide
- Resample
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Topological fixup
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Boolean operations
- ➤ Crop
- Subtract

P_2 P_1

Boundary   P'   P" Inside

Outside

P_3

P_5

P_4

Polygonal Mesh Processing

Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
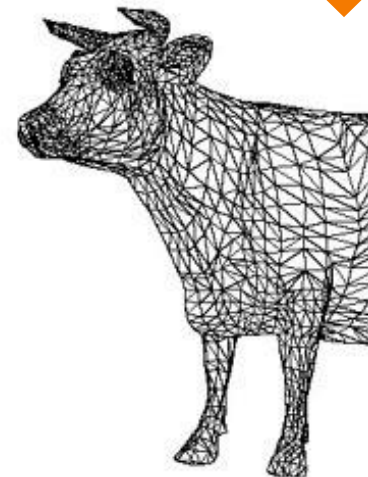- Fill holes
- Fix self-intersections

Boolean operations
- ➢ Crop
- Subtract

P_2 P_1

Boundary P' P" Inside

Outside

Polygonal Mesh Processing

Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
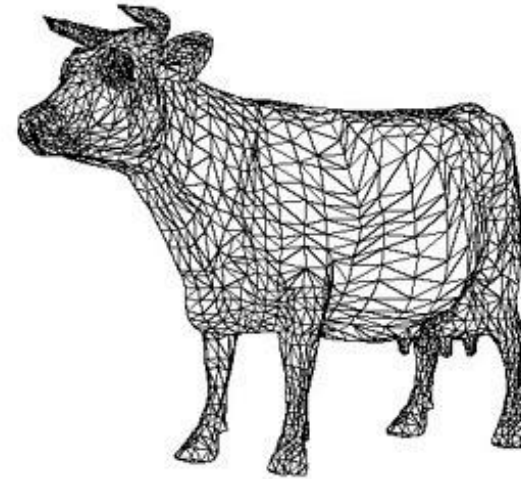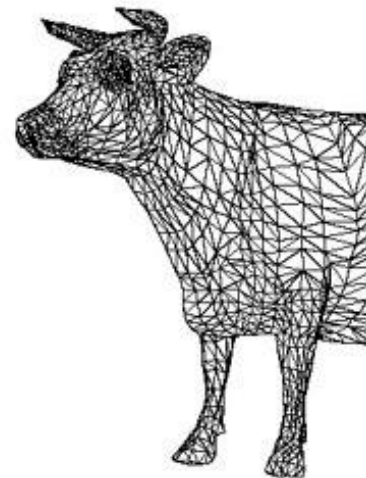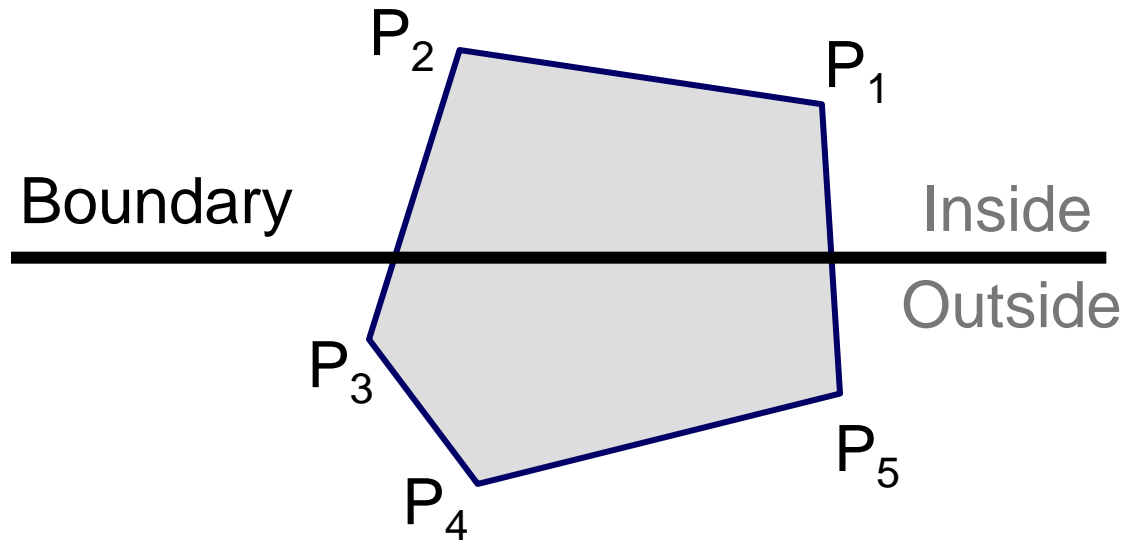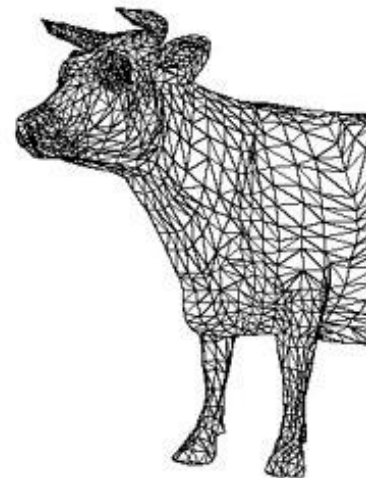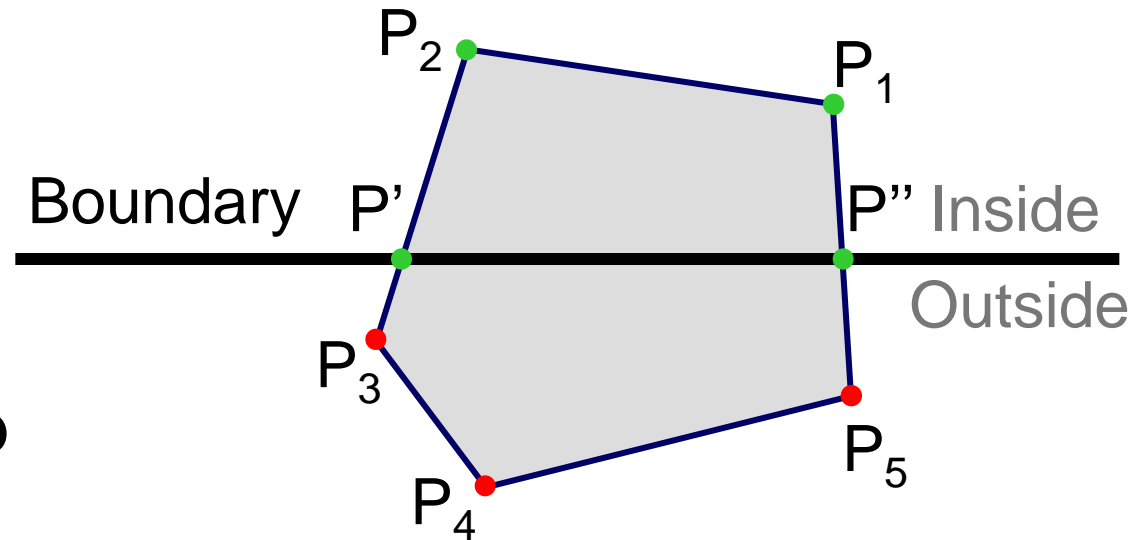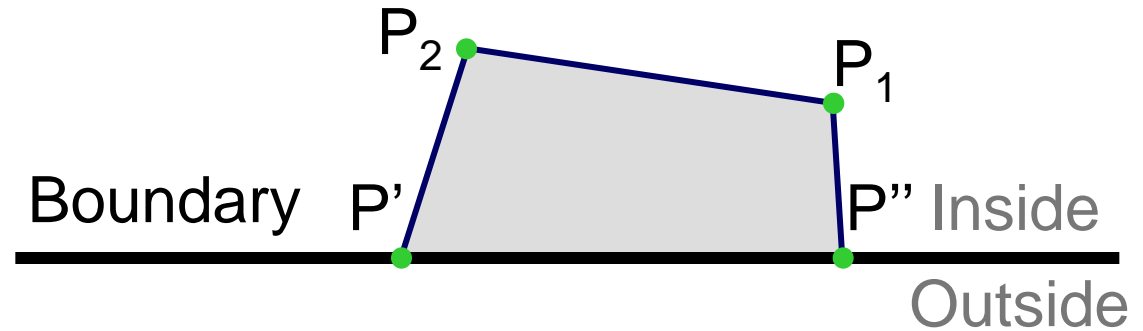- Fill holes
- Fix self-intersections

Boolean operations
- Crop
- ➢ Subtract

Polygonal Mesh Processing

Procedural generation

- Surface of revolution
- Sweep

Polygonal Mesh Processing
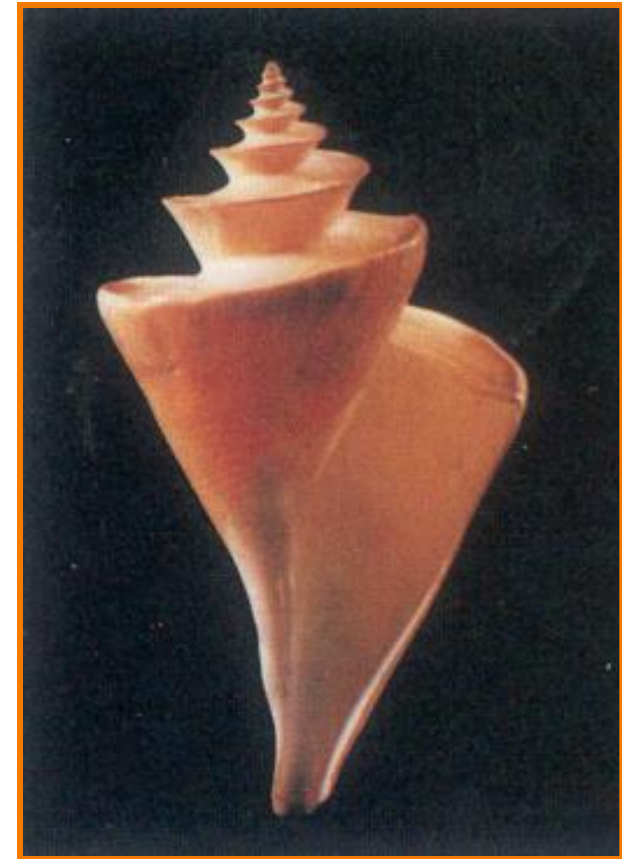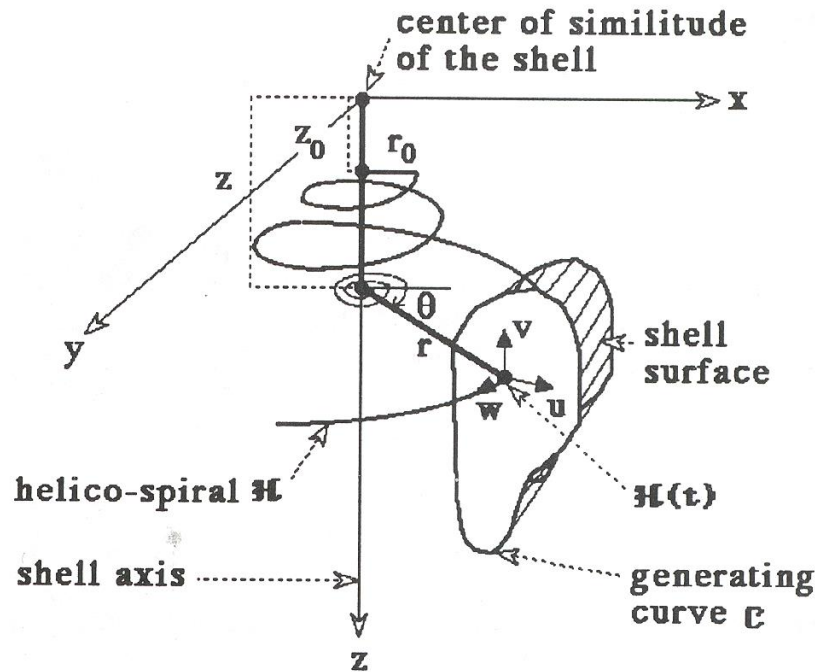
Procedural generation
- ➢ Surface of revolution
- ○ Sweep

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Polygonal Mesh Processing

Procedural generation
- Surface of revolution
- ➢ Sweep

Source curve   Extrusion on z-axis

Source curve   Generator curve   Extruded surface

Polygonal Mesh Processing

Procedural generation

- ○ Surface of revolution
- ➢ Sweep

Fowler et al., 1992

Polygonal Mesh Processing

Procedural generation
- Surface of revolution
- Sweep

Fowler et al., 1992

Outline

Acquisition

Processing
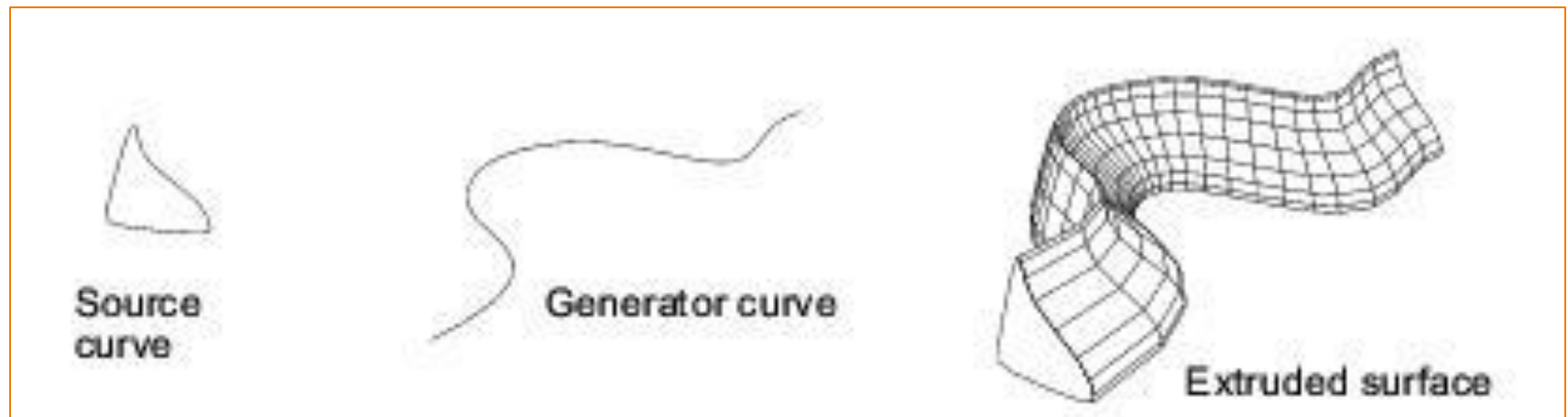
Representation ←

Polygon Mesh Representation
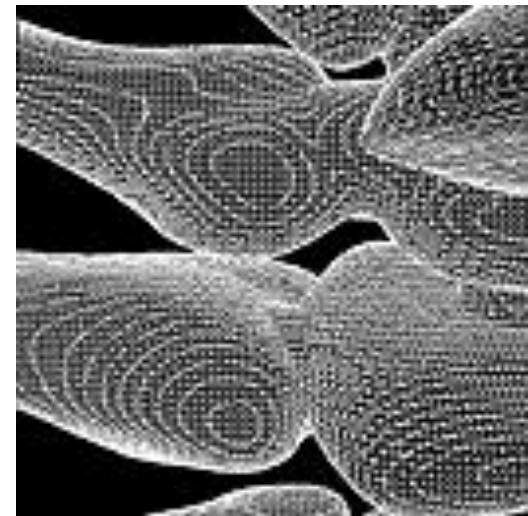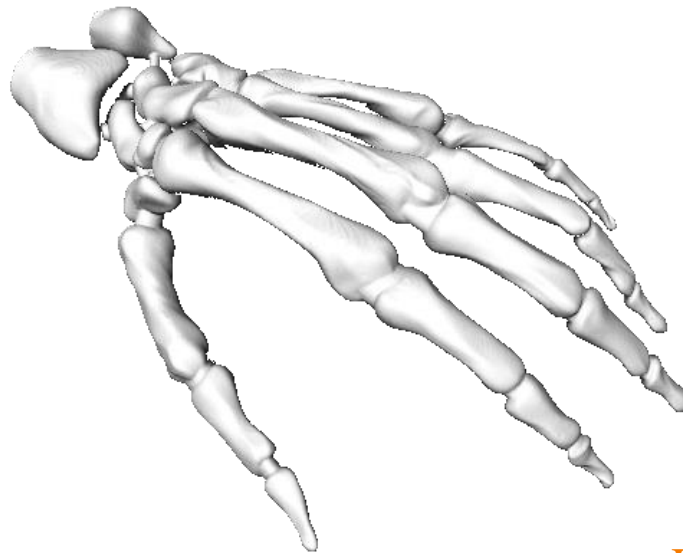
Important properties of mesh representation?

Polygon Mesh Representation

Important properties of mesh representation?

- Efficient traversal of topology
- Efficient use of memory
- Efficient updates
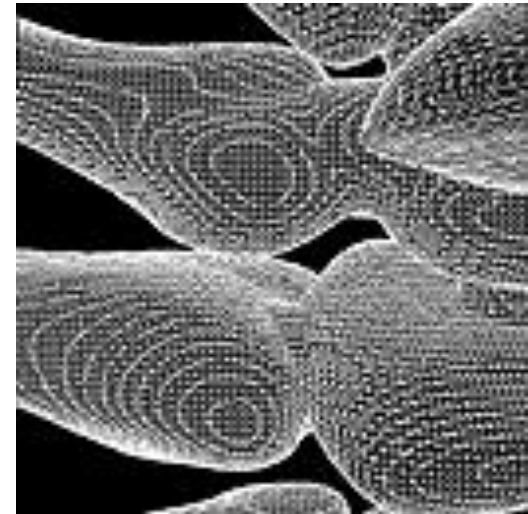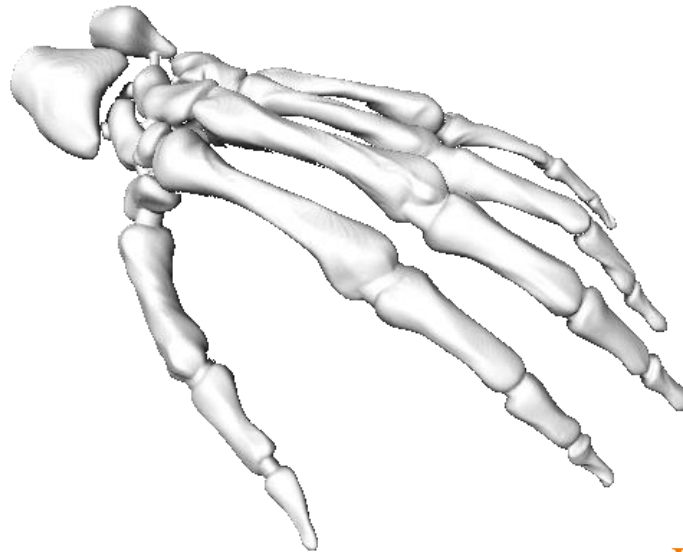
Polygon Mesh Representation

Possible data structures

- List of independent faces
- Vertex and face tables
- Adjacency lists
- Winged edge
- Half edge
- etc.
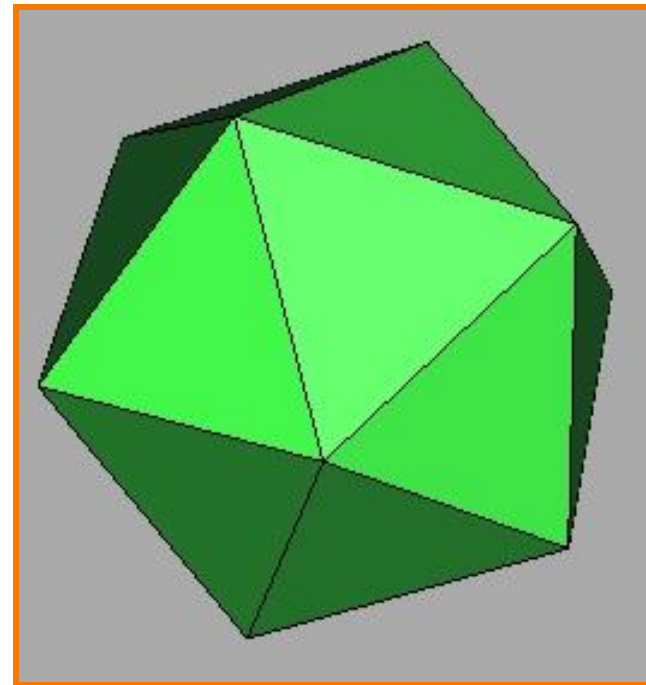
Independent Faces

Each face lists vertex coordinates
- Redundant vertices
- No adjacency information

(x_3, y_3, z_3)

(x_4, y_4, z_4)

F_2

F_1

F_3

(x_1, y_1, z_1)
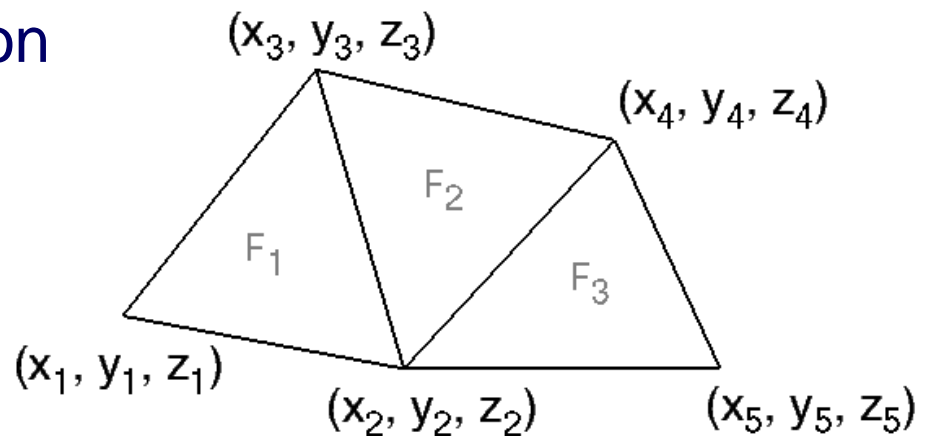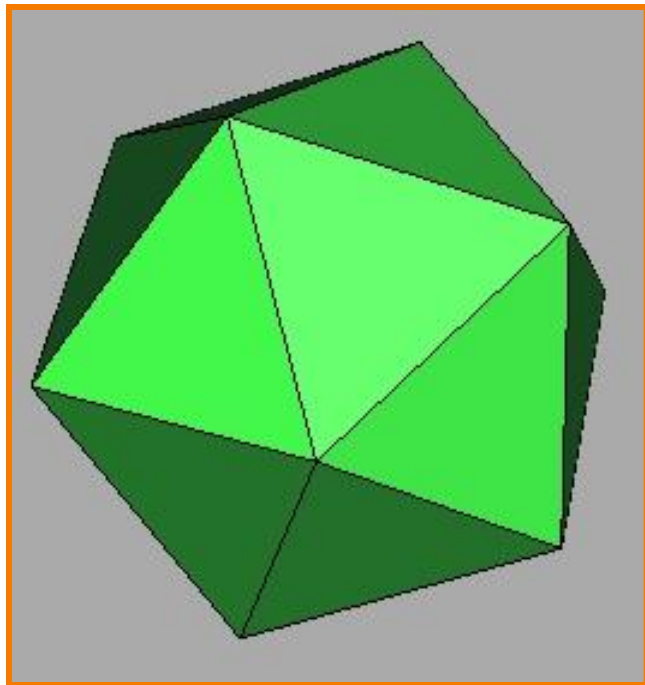
(x_2, y_2, z_2)

(x_5, y_5, z_5)

| FACE TABLE	
F_1	(x_1, y_1, z_1) (x_2, y_2, z_2) (x_3, y_3, z_3)
F_2	(x_2, y_2, z_2) (x_4, y_4, z_4) (x_3, y_3, z_3)
F_3	(x_2, y_2, z_2) (x_5, y_5, z_5) (x_4, y_4, z_4)

Vertex and Face Tables

Each face lists vertex references
- Shared vertices
- Still no adjacency information

(x_3, y_3, z_3)

(x_4, y_4, z_4)

F_2

F_1

F_3

(x_1, y_1, z_1)
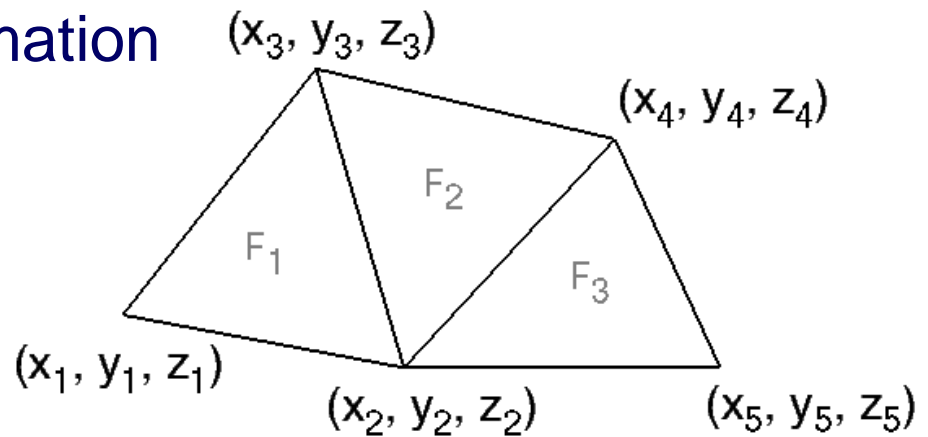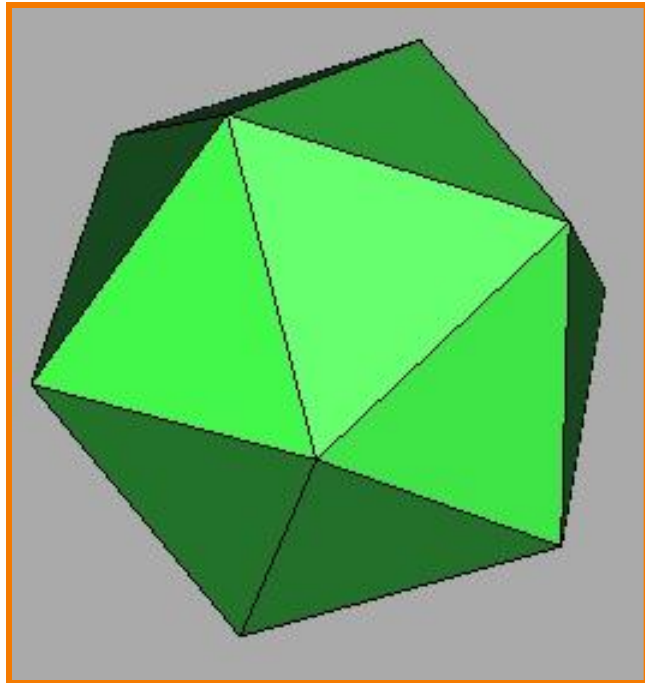
(x_2, y_2, z_2)

(x_5, y_5, z_5)

VERTEX TABLE			
V_1	X_1	Y_1	Z_1
V_2	X_2	Y_2	Z_2
V_3	X_3	Y_3	Z_3
V_4	X_4	Y_4	Z_4
V_5	X_5	Y_5	Z_5

FACE TABLE			
F_1	V_1	V_2	V_3
F_2	V_2	V_4	V_3
F_3	V_2	V_5	V_4

Adjacency Lists

Store all vertex, edge, and face adjacencies

- Efficient adjacency traversal
- Extra storage
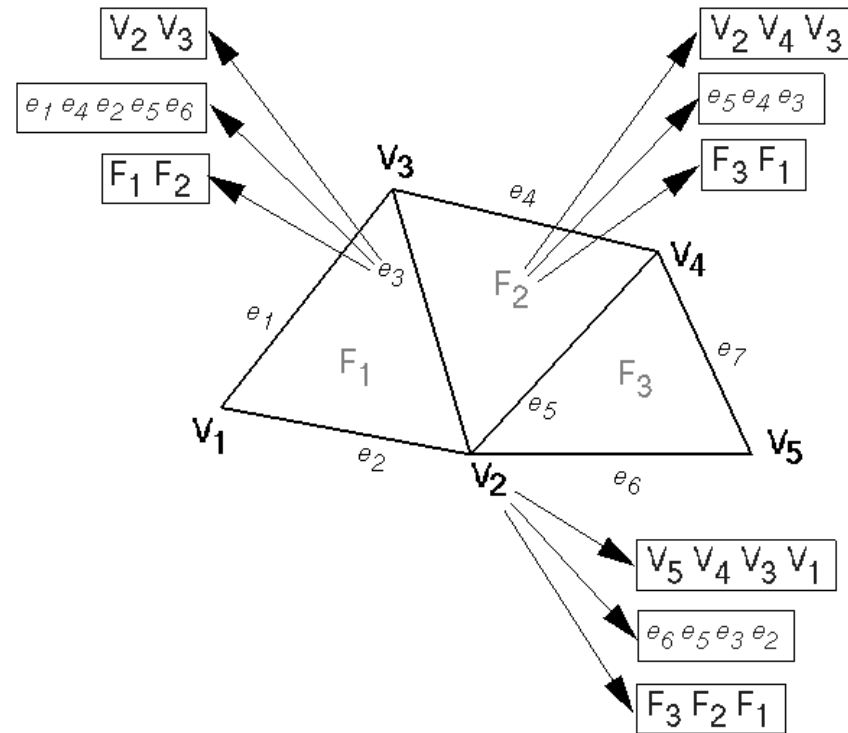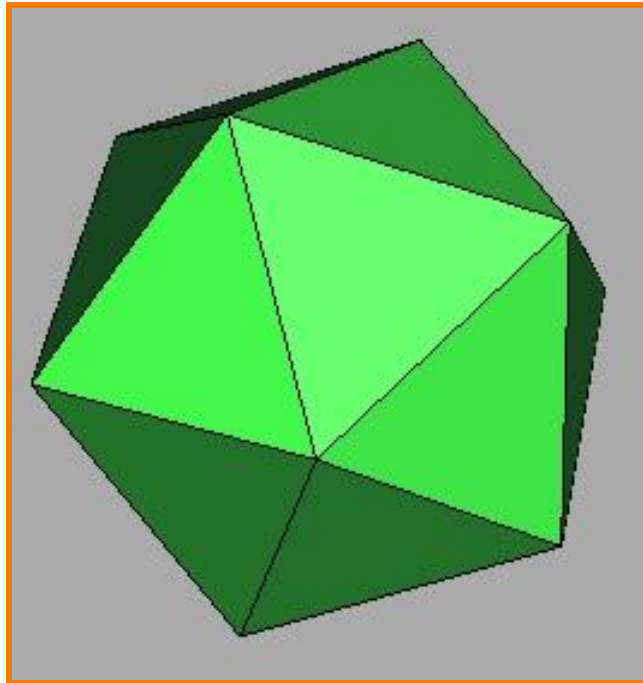
Partial Adjacency Lists

Can we store only some adjacency relationships and derive others?

Winged Edge

Adjacency encoded in edges
- All adjacencies in O(1) time
- Little extra storage (fixed records)
- Arbitrary polygons

Winged Edge

Example:

VERTEX TABLE

V_1	X_1	Y_1	Z_1	e_1
V_2	X_2	Y_2	Z_2	e_6
V_3	X_3	Y_3	Z_3	e_3
V_4	X_4	Y_4	Z_4	e_5
V_5	X_5	Y_5	Z_5	e_6

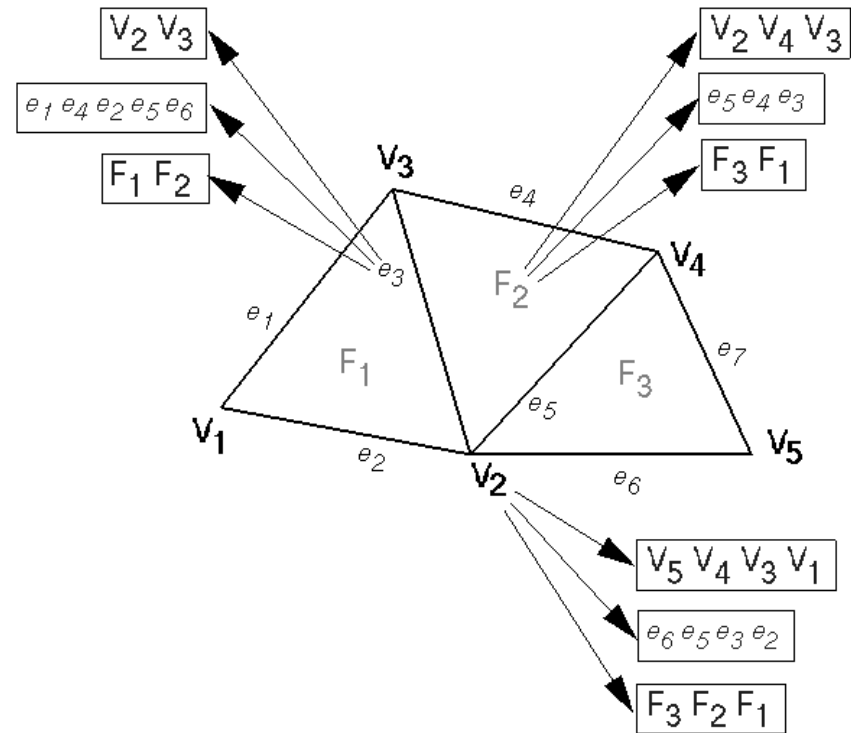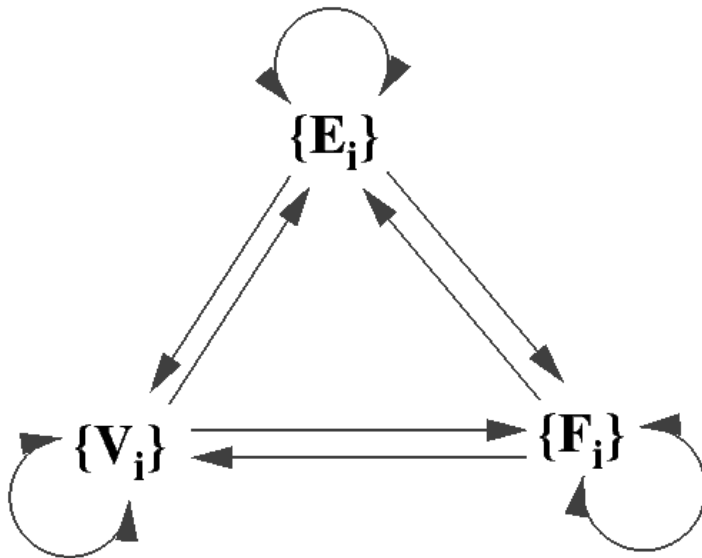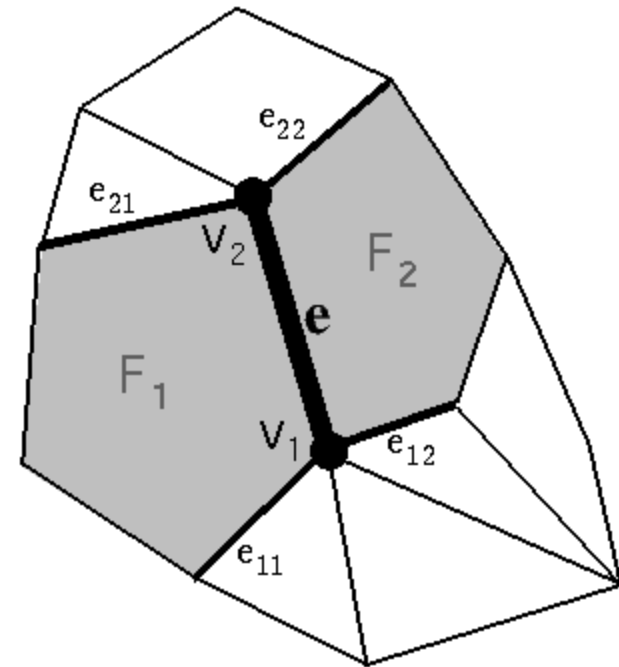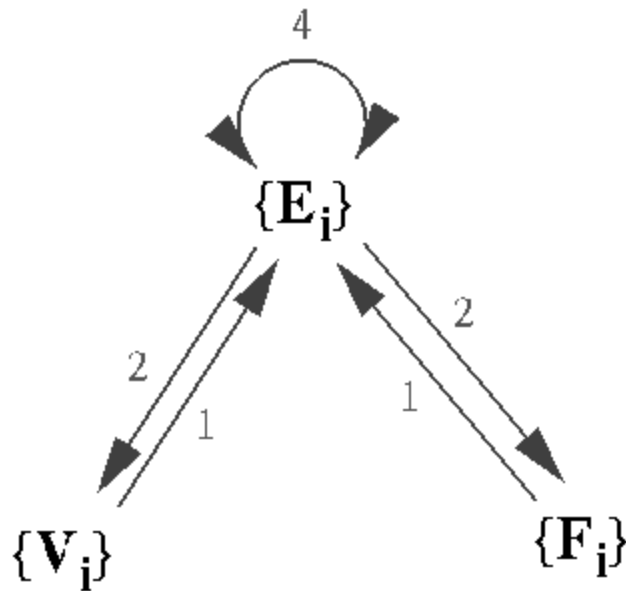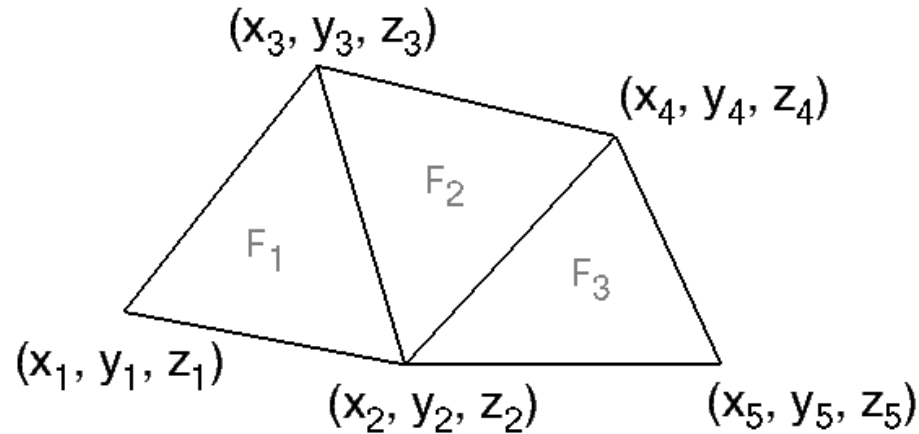**EDGE TABLE**

						11	12	21	22
e_1	V_1	V_3		F_1		e_2	e_2	e_4	e_3
e_2	V_1	V_2	F_1			e_1	e_1	e_3	e_6
e_3	V_2	V_3	F_1	F_2		e_2	e_5	e_1	e_4
e_4	$V3$	V_4		F_2		e_1	e_3	e_7	e_5
e_5	V_2	V_4	F_2	F_3		e_3	e_6	e_4	e_7
e_6	V_2	V_5	F_3			e_5	e_2	e_7	e_7
e_7	V_4	V_5		F_3		e_4	e_5	e_6	e_6

FACE TABLE

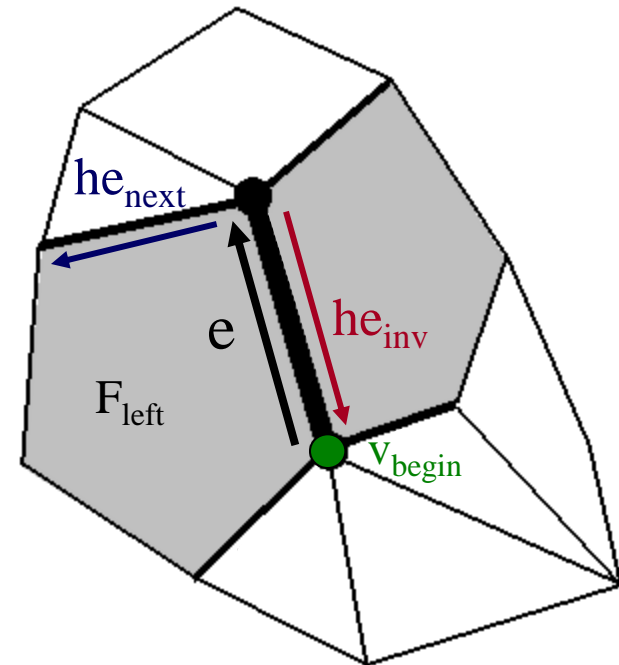|	
F_1	e_1
F_2	e_3
F_3	e_5

Half Edge

Adjacency encoded in edges
- All adjacencies in O(1) time
- Little extra storage (fixed records)
- Arbitrary polygons

Similar to winged-edge,
except adjacency
encoded in half-edges

Summary

Polygonal meshes
- Most common surface representation
- Fast rendering

Processing operations
- Must consider irregular vertex sampling
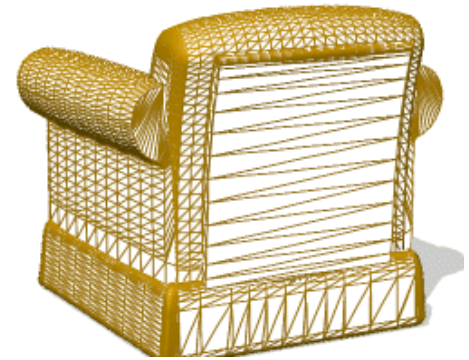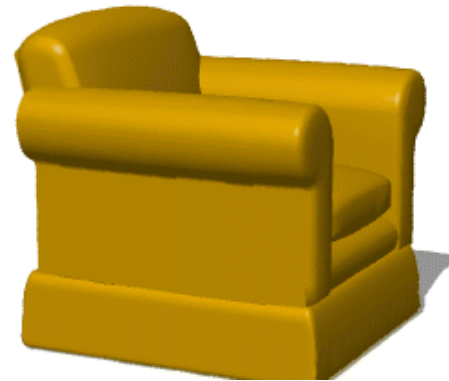- Must handle/avoid topological degeneracies

Representation
- Which adjacency relationships to store depend on which operations must be efficient

3D Polygonal Meshes

Properties
- ? Efficient display
- ? Easy acquisition
- ? Accurate
- ? Concise
- ? Intuitive editing
- ? Efficient editing
- ? Efficient intersections
- ? Guaranteed validity
- ? Guaranteed smoothness
- ? etc.

3D Polygonal Meshes

Properties
- ☺Efficient display
- ☺Easy acquisition
- ☹Accurate
- ☹Concise
- ☹Intuitive editing
- ☹Efficient editing
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