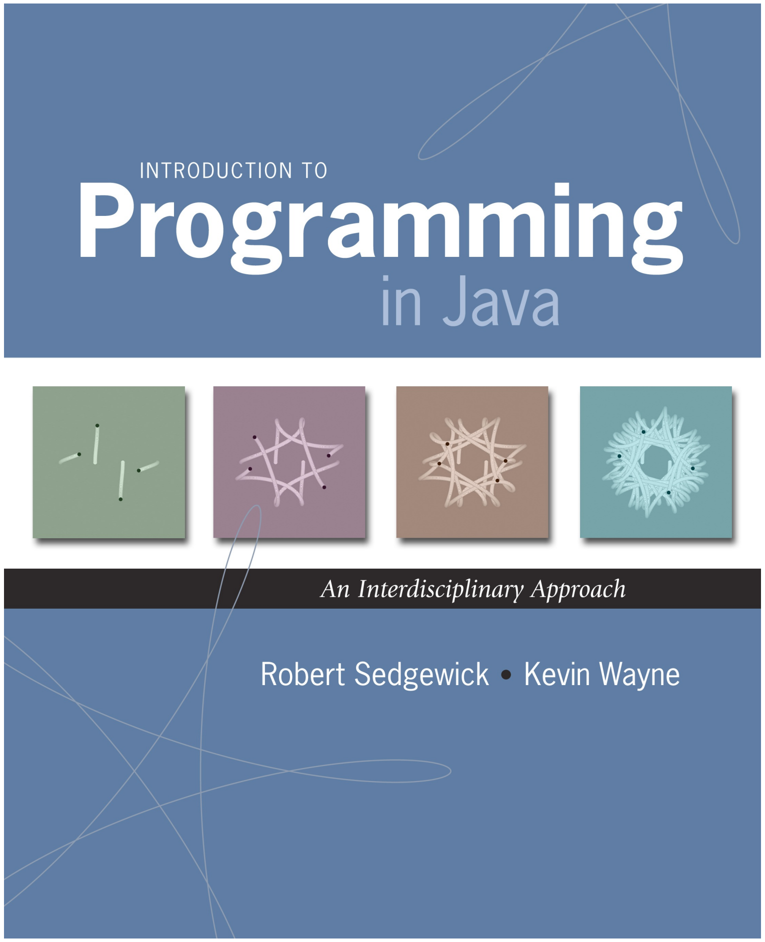
4.5 Small World Phenomenon



Small World Phenomenon

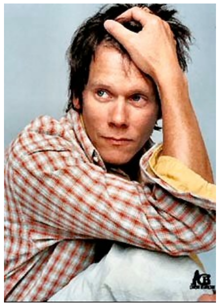
Small world phenomenon. Six handshakes away from anyone.

An experiment to quantify effect. [Stanley Milgram, 1960s]

- You are given personal info of another person.
- Goal: deliver message.
- Restriction: can only forward to someone you know by first name.
- Outcome: message delivered with average of 5 intermediaries.

e.g., occupation and age

Stanley Milgram



Kevin Bacon

Applications of Small World Phenomenon

Sociology applications.

- Looking for a job.
- Marketing products or ideas.
- Formation and spread of fame and fads.
- Train of thought followed in a conversation.
- Defining representative-ness of political bodies.
- Kevin Bacon game (movies, rock groups, facebook, etc.).

Other applications.

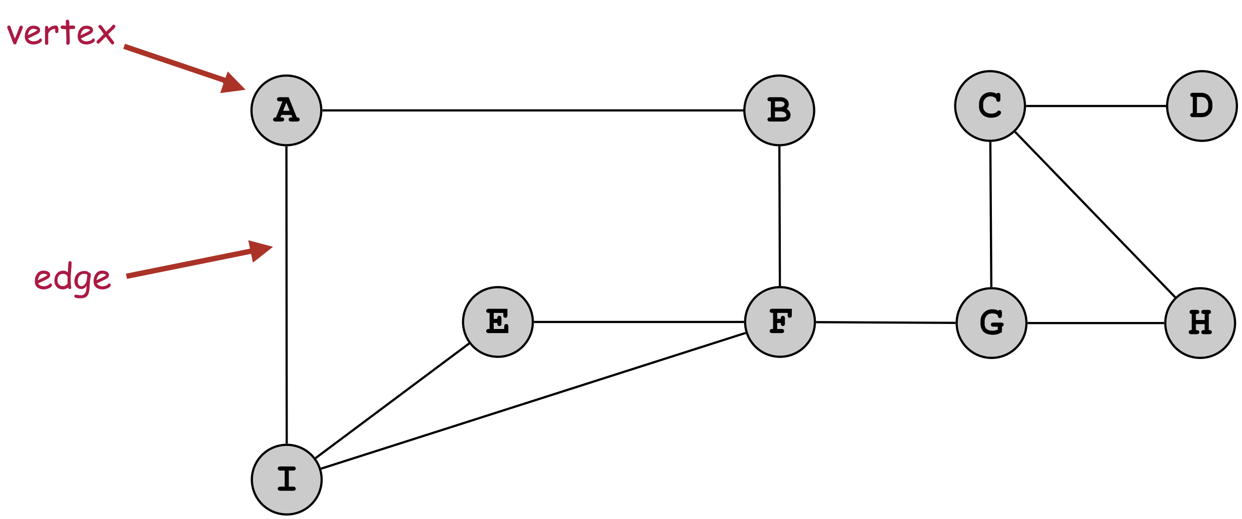
- Electronic circuits.
- Synchronization of neurons.
- Analysis of World Wide Web.
- Design of electrical power grids.
- Modeling of protein interaction networks.
- Phase transitions in coupled Kuramoto oscillators.
- Spread of infectious diseases and computer viruses.
- Evolution of cooperation in multi-player iterated Prisoner's Dilemma.

Reference. Duncan J. Watts, Small Worlds: The Dynamics of Networks between Order and Randomness, Princeton University Press, 1999.

Graph Data Type

Application demands a new data type.

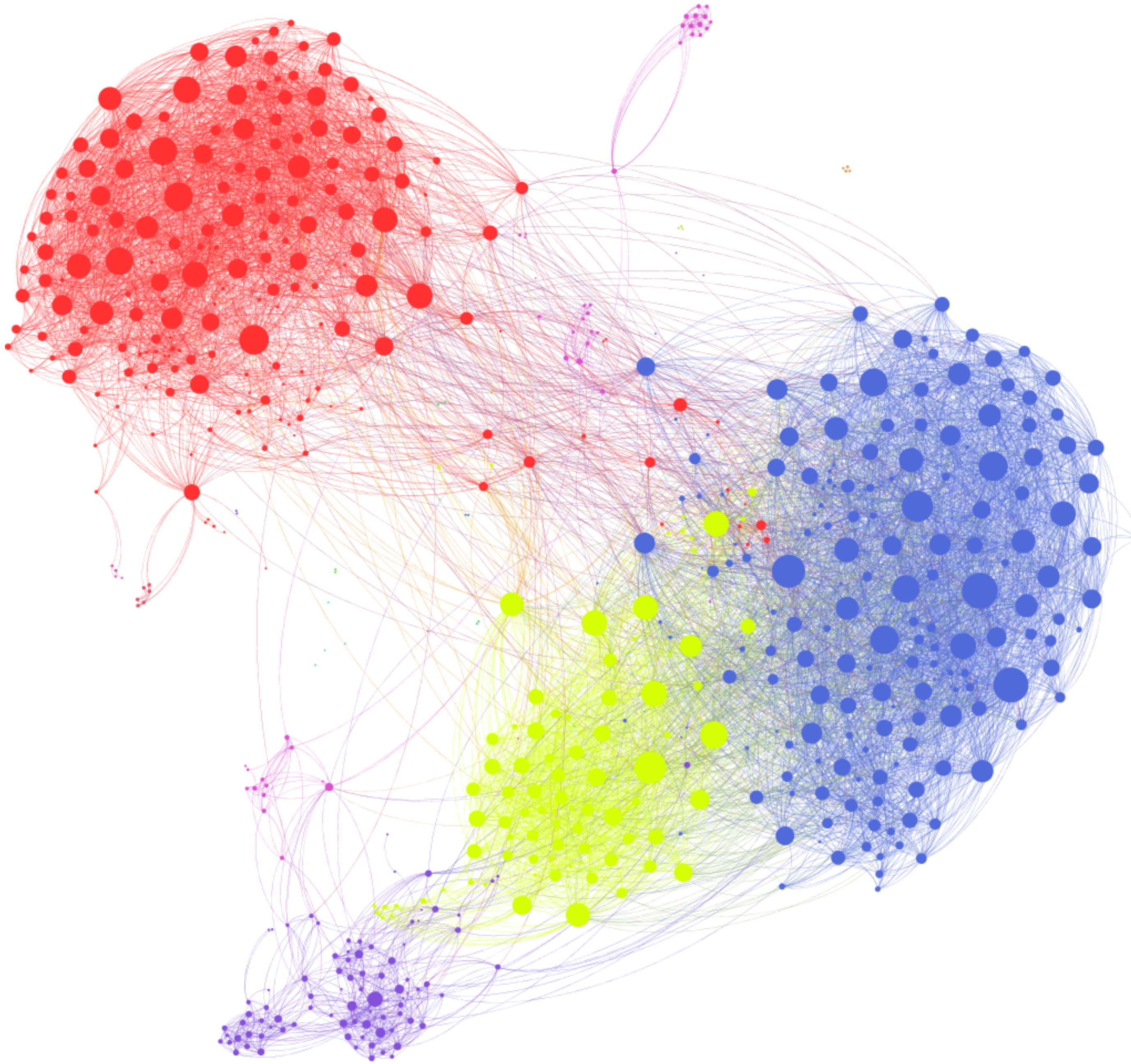
- Graph = data type that represents pairwise connections.
- Vertex = element.
- Edge = connection between two vertices.



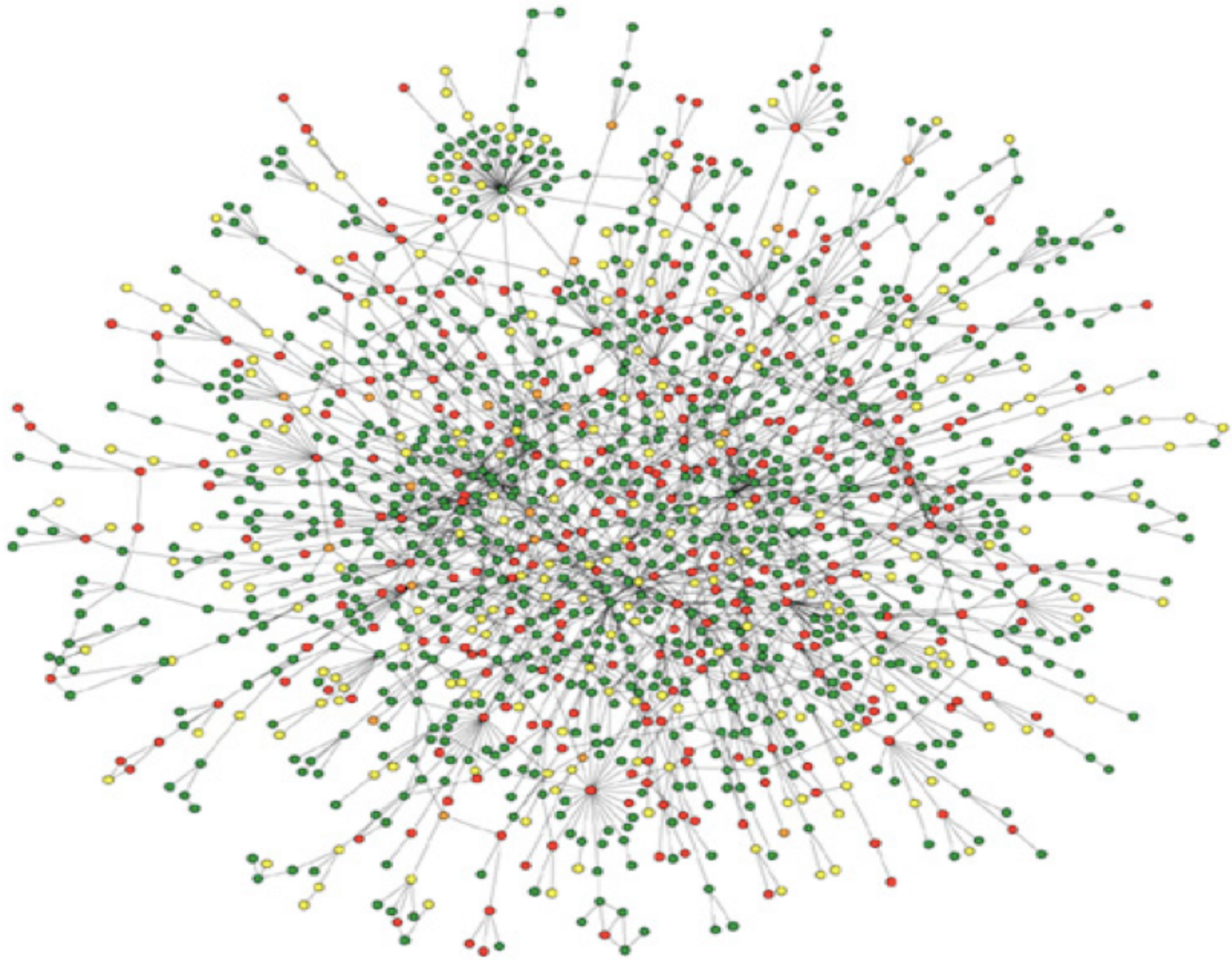
Graph Applications

graph	vertices	edges
communication	telephones, computers	fiber optic cables
circuits	gates, registers, processors	wires
mechanical	joints	rods, beams, springs
hydraulic	reservoirs, pumping stations	pipelines
financial	stocks, currency	transactions
transportation	street intersections, airports	highways, airway routes
scheduling	tasks	precedence constraints
software systems	functions	function calls
internet	web pages	hyperlinks
games	board positions	legal moves
social relationship	people, actors	friendships, movie casts
neural networks	neurons	synapses
protein networks	proteins	protein-protein interactions
chemical compounds	molecules	bonds

Facebook Friends

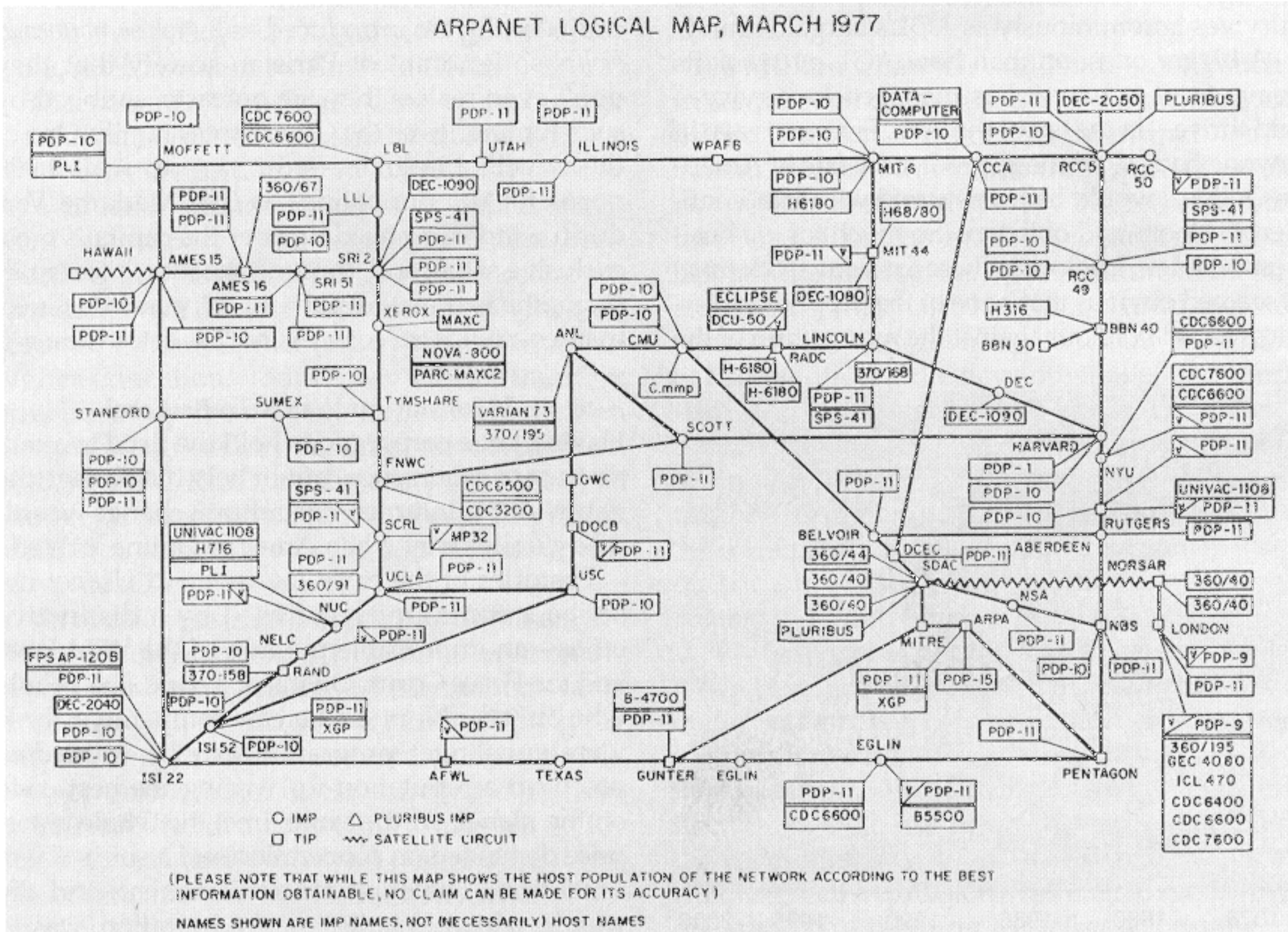


Protein Interaction Network

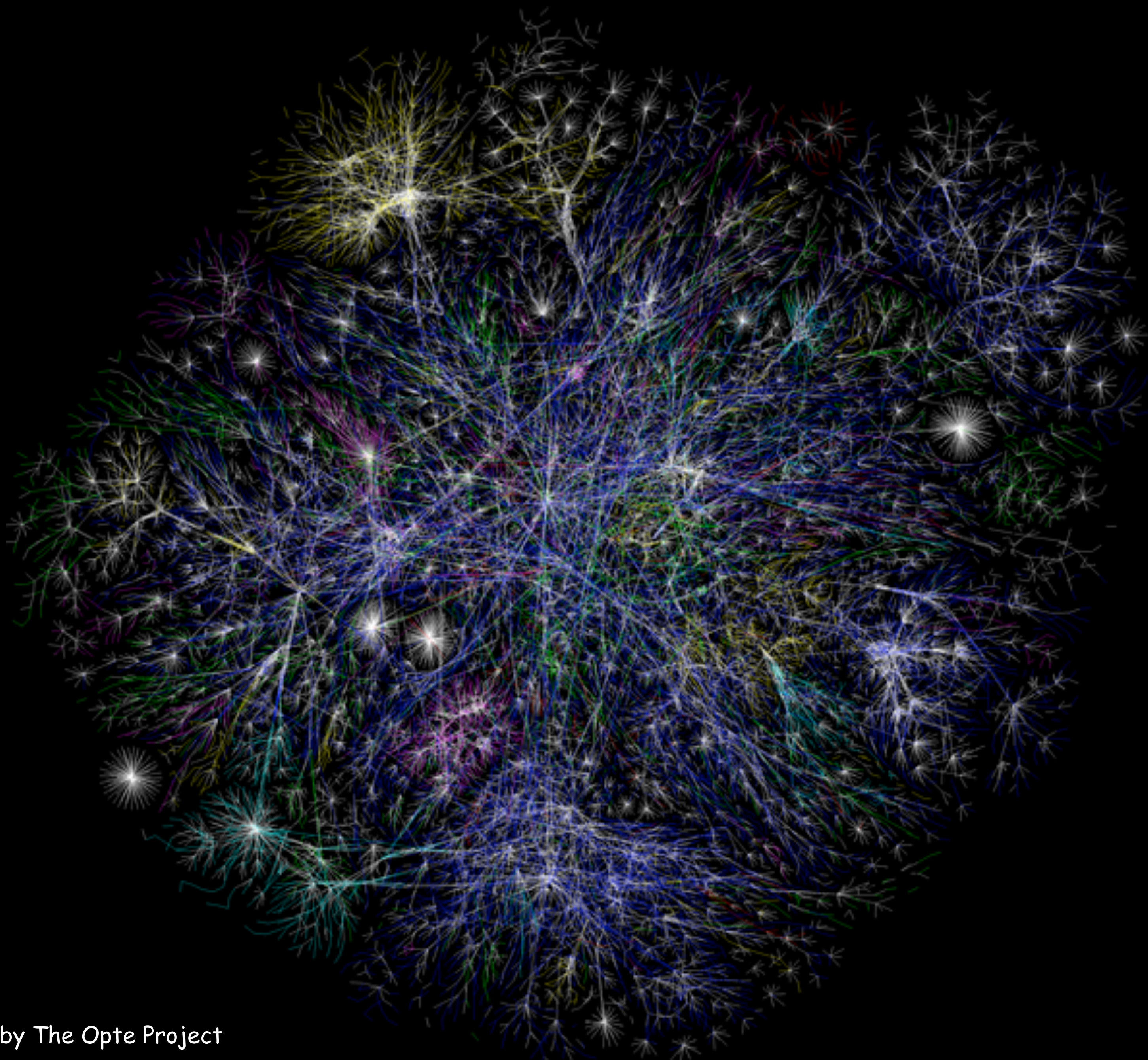


Reference: Jeong et al, Nature Review | Genetics

ARPANET


ARPANET LOGICAL MAP, MARCH 1977
O IMP
△ PLURIBUS IMP
□ TIP
SATELLITE CIRCUIT
(PLEASE NOTE THAT WHILE THIS MAP SHOWS THE HOST POPULATION OF THE NETWORK ACCORDING TO THE BEST INFORMATION OBTAINABLE, NO CLAIM CAN BE MADE FOR ITS ACCURACY)
NAMES SHOWN ARE IMP NAMES, NOT (NECESSARILY) HOST NAMES

The Internet



The Internet as mapped by The Opte Project
http://www.opte.org

Internet Movie Database

Input format. Movie followed by list of performers, separated by slashes.

```
% more movies.txt
...
Tin Men (1987)/DeBoy, David/Blumenfeld, Alan/... /Geppi, Cindy/Hershey, Barbara
Tirez sur le pianiste (1960)/Heymann, Claude/.../Berger, Nicole (I)
Titanic (1997)Paxton, Bill/DiCaprio, Leonardo/.../Winslet, Kate
Titus (1999)/Weisskopf, Hermann/Rhys, Matthew/.../McEwan, Geraldine
To All a Good Night (1980)/George, Michael (II)/.../Gentile, Linda
To Be or Not to Be (1942)/Verebes, Ernö (I)/.../Lombard, Carole (I)
To Be or Not to Be (1983)/Brooks, Mel (I)/.../Bancroft, Anne
To Catch a Thief (1955)/París, Manuel/Grant, Cary/.../Kelly, Grace
To Die For (1989)/Bond, Steve (I)/Jones, Duane (I)/.../Maddalena, Julie
To Die For (1995)/Smith, Kurtwood/Kidman, Nicole/.../Tucci, Maria
To Die Standing (1990)/Sacha, Orlando/Anthony, Gerald/.../Rose, Jamie
To End All Wars (2001)/Kimura, Sakae/Ellis, Greg (II)/.../Sutherland, Kiefer
To Kill a Clown (1972)/Alda, Alan/Clavering, Eric/Lamberts, Heath/Danner, Blythe
To Live and Die in L.A. (1985)/McGroarty, Pat/Williams, Donnie/.../Dafoe, Willem
...
```

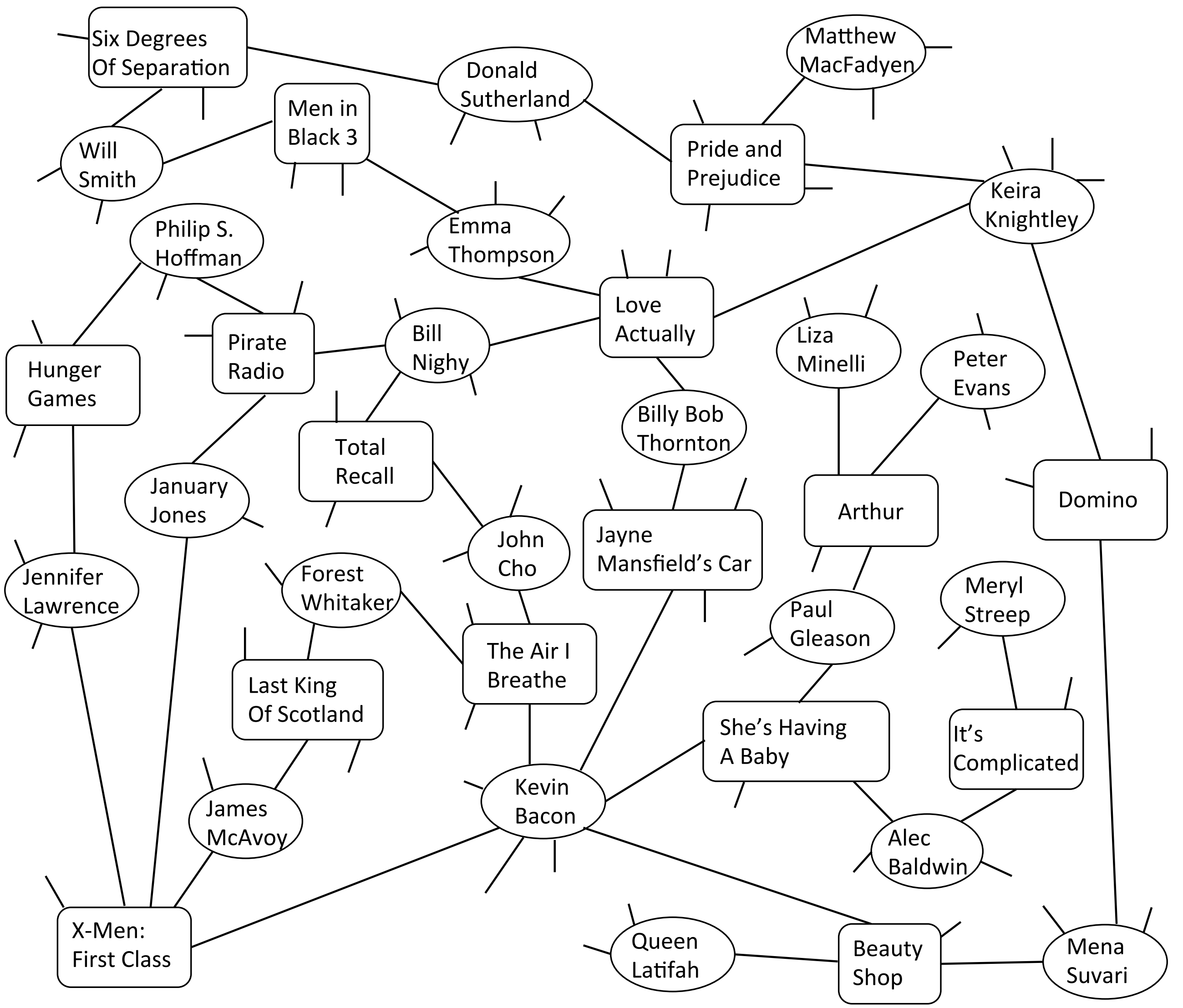
http://www.imdb.com/interfaces

Internet Movie Database

Q. How to represent the movie-performer relationships?

A. Use a graph.

- Vertex: performer or movie.
- Edge: connect performer to movie.

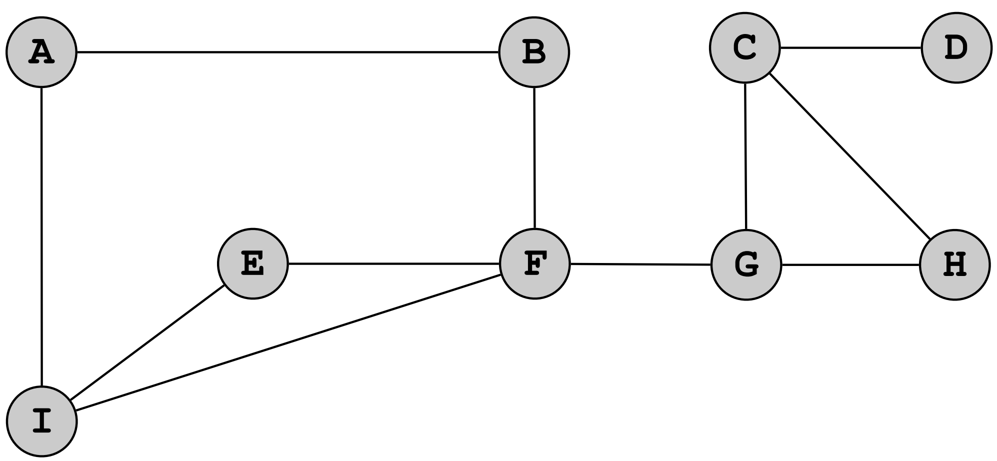


Graph API

Graph data type.

public class Graph (graph with String vertices)		
	Graph()	*create an empty graph*
	Graph(In in)	*read graph from input stream*
void	addEdge(String v, String w)	*add edge v-w*
Iterable<String>	adjacentTo(String v)	*neighbors of v*

to support use with foreach

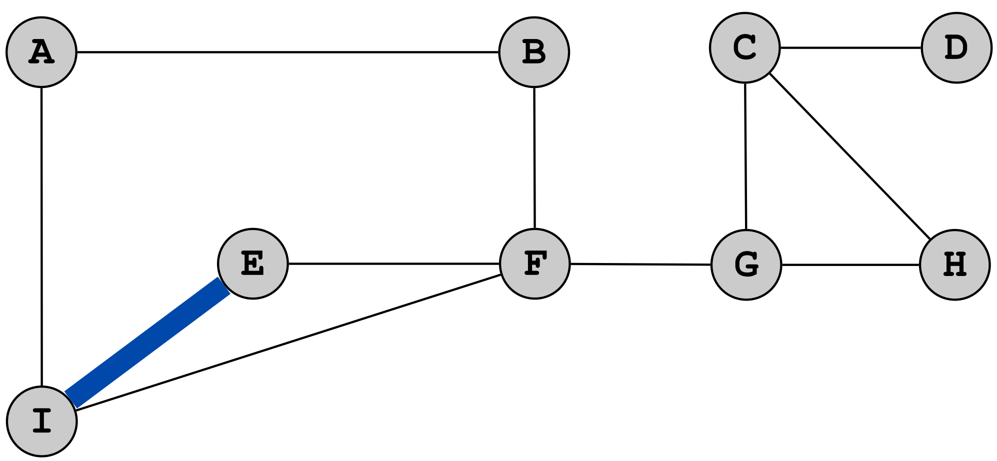



```
% more tiny.txt

A/B/I
B/A/F
C/D/G/H
D/C
E/F/I
F/B/E/G/I
G/C/F/H
H/C/G
I/A/E/F
```

Graph Representation

Graph representation: use a symbol table.

- Key = name of vertex.
- Value = set of neighbors.



String	SET<String>
key	value
A	B I
B	A F
C	D G H
D	C
E	F I
F	B E G I
G	C F H
H	C G
I	A E F

symbol table

Set Data Type

Set data type. Unordered collection of distinct keys.

```
public class SET<Key extends Comparable<Key>>
```

	`SET()`	*create a set*
`boolean`	`isEmpty()`	*is the set empty?*
`void`	`add(Key key)`	*add* `key` *to the set*
`boolean`	`contains(Key key)`	*is* `key` *in the set?*

Note: Implementations should also implement the `Iterable<Key>` *interface to enable clients to access keys in sorted order with foreach loops*

Q. How to implement?

A. Identical to symbol table, but ignore values.

Graph Implementation

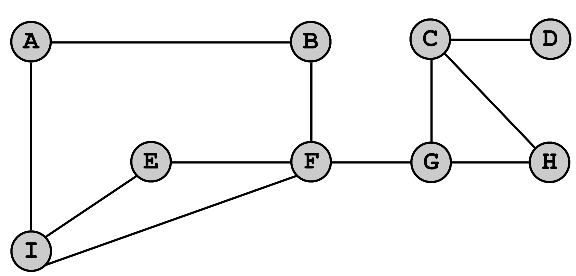
```
public class Graph {
   private ST<String, SET<String>> st;

   public Graph() {
      st = new ST<String, SET<String>>();
   }

   public void addEdge(String v, String w) {
      if (!st.contains(v)) addVertex(v);
      if (!st.contains(w)) addVertex(w);
      st.get(v).add(w);     ←  add w to v's set of neighbors
      st.get(w).add(v);     ←  add v to w's set of neighbors
   }

   private void addVertex(String v) {      add new vertex v
      st.put(v, new SET<String>());  ←   with no neighbors
   }

   public Iterable<String> adjacentTo(String v) {
      return st.get(v);
   }
}
```

Graph Implementation (continued)

Second constructor. To read graph from input stream.

```
public Graph(In in) {
    st = new ST<String, SET<String>>();
    while (!in.isEmpty()) {
        String line = in.readLine();
        String[] names = line.split("/");
        for (int i = 1; i < names.length; i++)
            addEdge(names[0], names[i]);
    }
}
```

```
In in = new In("tiny.txt");
Graph G = new Graph(G, in);
```




```
% more tiny.txt
A/B/I
B/A/F
C/D/G/H
D/C
E/F/I
F/B/E/G/I
G/C/F/H
H/C/G
I/A/E/F
```

Graph Client: Movie Finder

Performer and movie queries.

- Given a performer, find all movies in which they appeared.
- Given a movie, find all performers.

```
public class MovieFinder {
   public static void main(String[] args) {

      In in    = new In(args[0]);
      Graph G = new Graph(in);

      while (!StdIn.isEmpty()) {
         String v = StdIn.readLine();
         for (String w : G.adjacentTo(v))
            StdOut.println(w);
      }

   }
}
```

read in graph from a file

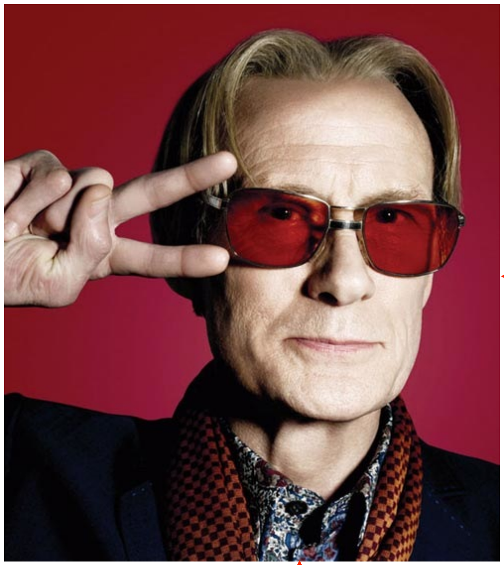
process queries

Graph Client: Movie Finder

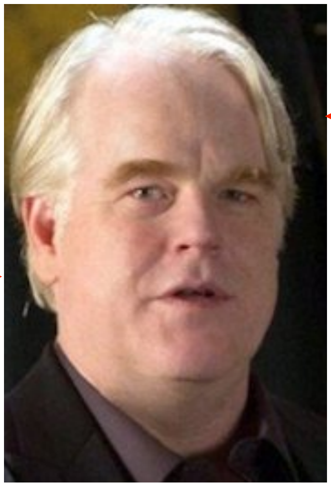
```
% java MovieFinder action.txt
Bacon, Kevin

Death Sentence (2007)
River Wild, The (1994)
Tremors (1990)


Roberts, Julia

Blood Red (1989)
I Love Trouble (1994)
Mexican, The (2001)
Ocean's Eleven (2001)


Eisgruber, Christopher
```

```
% java MovieFinder mpaa.txt

Bacon, Kevin
Air I Breathe, The (2007)
Air Up There, The (1994)
Animal House (1978)
Apollo 13 (1995)
Balto (1995)
Beauty Shop (2005)
Big Picture, The (1989)
…
Sleepers (1996)
Starting Over (1979)
Stir of Echoes (1999)
Telling Lies in America (1997)
Trapped (2002)
Tremors (1990)
We Married Margo (2000)
Where the Truth Lies (2005)
White Water Summer (1987)
Wild Things (1998)
Woodsman, The (2004)
X-Men: First Class (2011)
```

Kevin Bacon Numbers



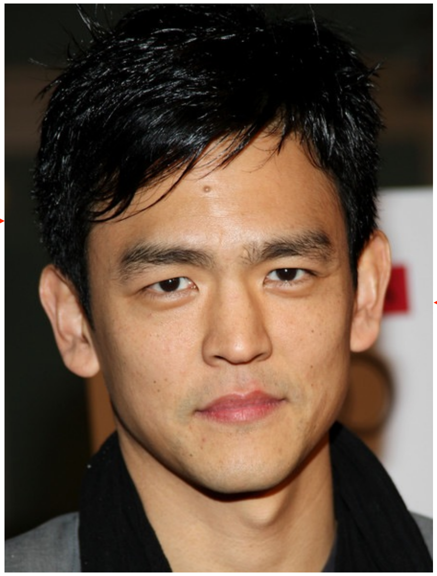
Bill Nighy was in "Pirate Radio" with Philip Seymour Hoffman



Philip Seymour Hoffman was in "Hunger Games" with Jennifer Lawrence

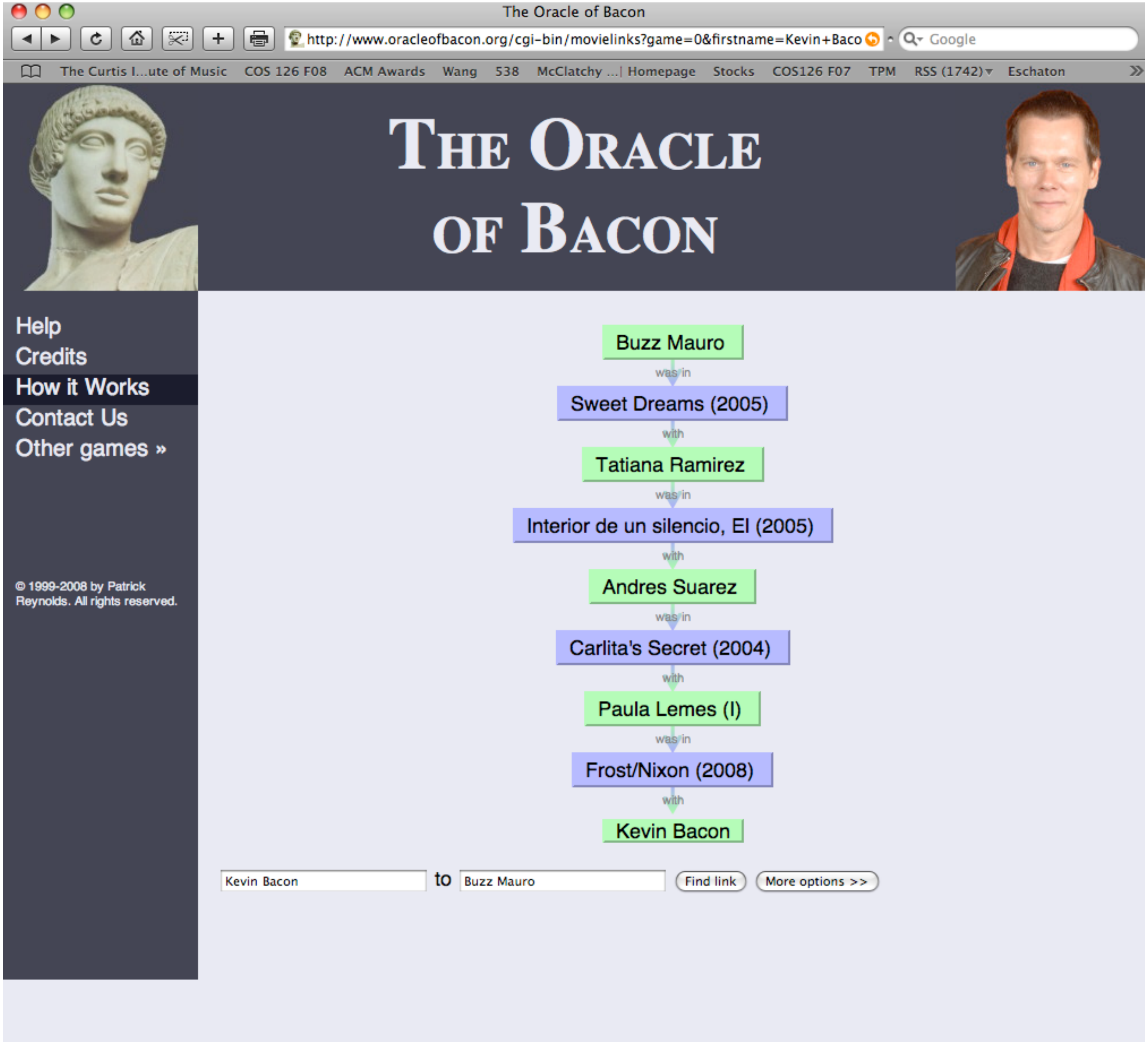
Jennifer Lawrence was in "X-Men: First Class" with Kevin Bacon

John Cho was in "Total Recall" with Kevin Bacon



Bill Nighy was in "Total Recall" with John Cho

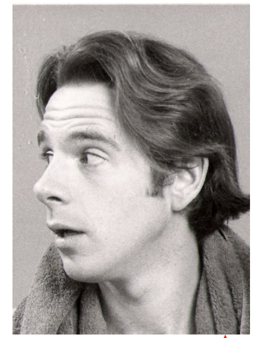
Oracle of Kevin Bacon



Kevin Bacon Game

Game. Given an actor or actress, find shortest chain of movies connecting them to Kevin Bacon.

Actor	Was in	With
Matthew MacFadyen	Pride and Prejudice	Keira Knightley
Keira Knightley	Love Actually	Bill Nighy
Bill Nighy	Pirate Radio	Philip S. Hoffman
Philip S. Hoffman	Hunger Games	Jennifer Lawrence
Jennifer Lawrence	X-Men: First Class	James McAvoy
James McAvoy	Last King of Scotland	Forest Whitaker
Forest Whitaker	The Air I Breathe	Kevin Bacon
Kevin Bacon		



Peter Evans was in "Arthur" with Paul Gleason

Paul Gleason was in "She's Having a Baby" with Kevin Bacon

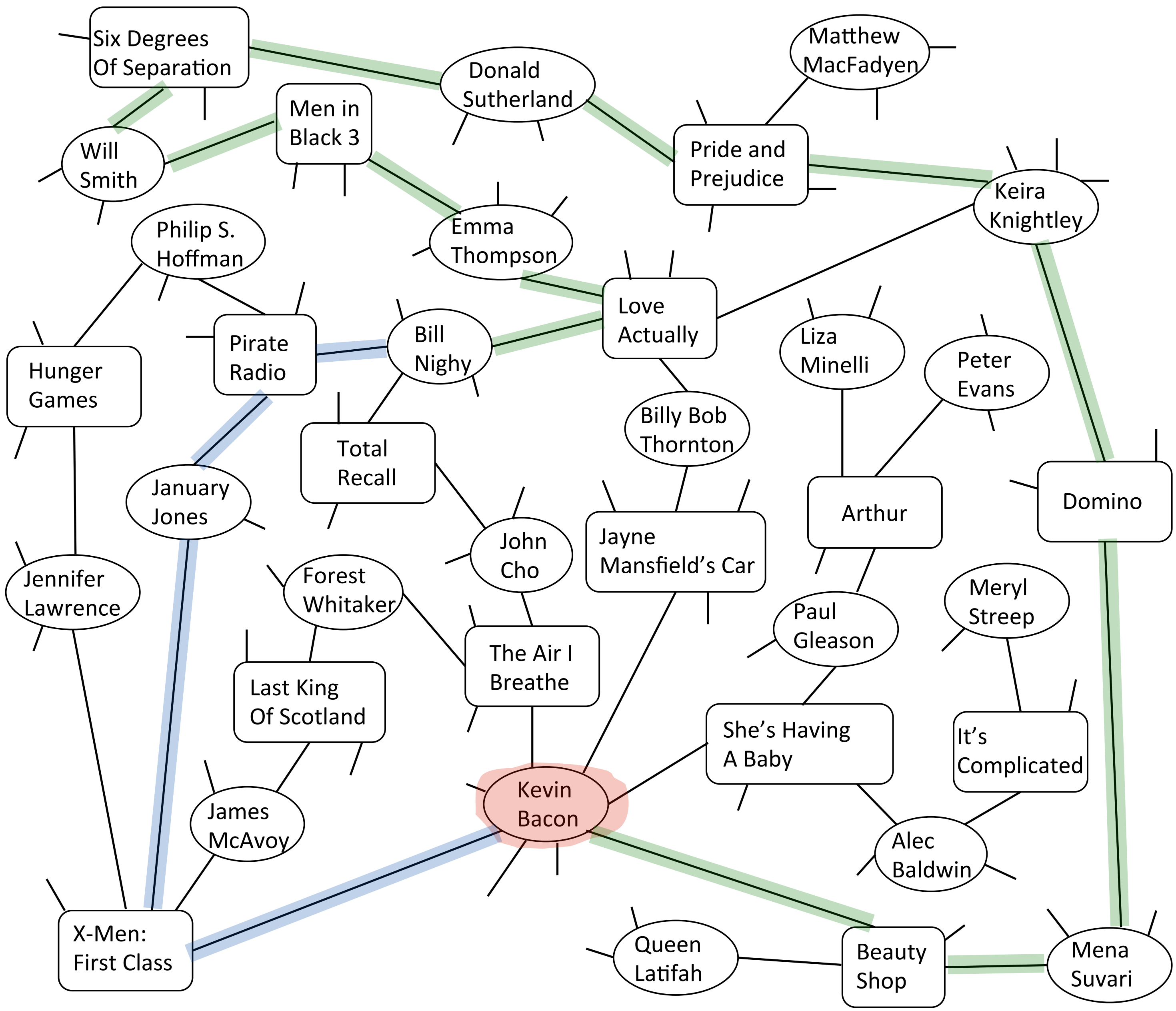
Will Smith was in "Men in Black 3" with Emma Thompson

Emma Thompson was in "Love Actually" with Billy Bob Thornton

Billy Bob Thornton was in "Jayne Mansfield's Car" with Kevin Bacon

Computing Bacon Numbers

How to compute. Find shortest path in performer-movie graph.



PathFinder API.

public class PathFinder		
	`PathFinder(Graph G, String s)`	*constructor*
`int`	`distanceTo(String v)`	*length of shortest path from* s *to* v *in* G
`Iterable<String>`	`pathTo(String v)`	*shortest path from* s *to* v *in* G

Design principles.

- Decouple graph algorithm from graph data type.
- Avoid feature creep: don't encrust Graph with search features; instead make a new datatype.

Computing Bacon Numbers: Java Implementation

```
public class Bacon {
   public static void main(String[] args) {

      In in = new In(args[0]);
      Graph G = new Graph(in);

      String s = "Bacon, Kevin";
      PathFinder finder = new PathFinder(G, s);

      while (!StdIn.isEmpty()) {
         String actor = StdIn.readLine();
         for (String v : finder.pathTo(actor))
            StdOut.println(v);
      }
   }
}
```

read in the graph from a file

create object to return shortest paths

process queries

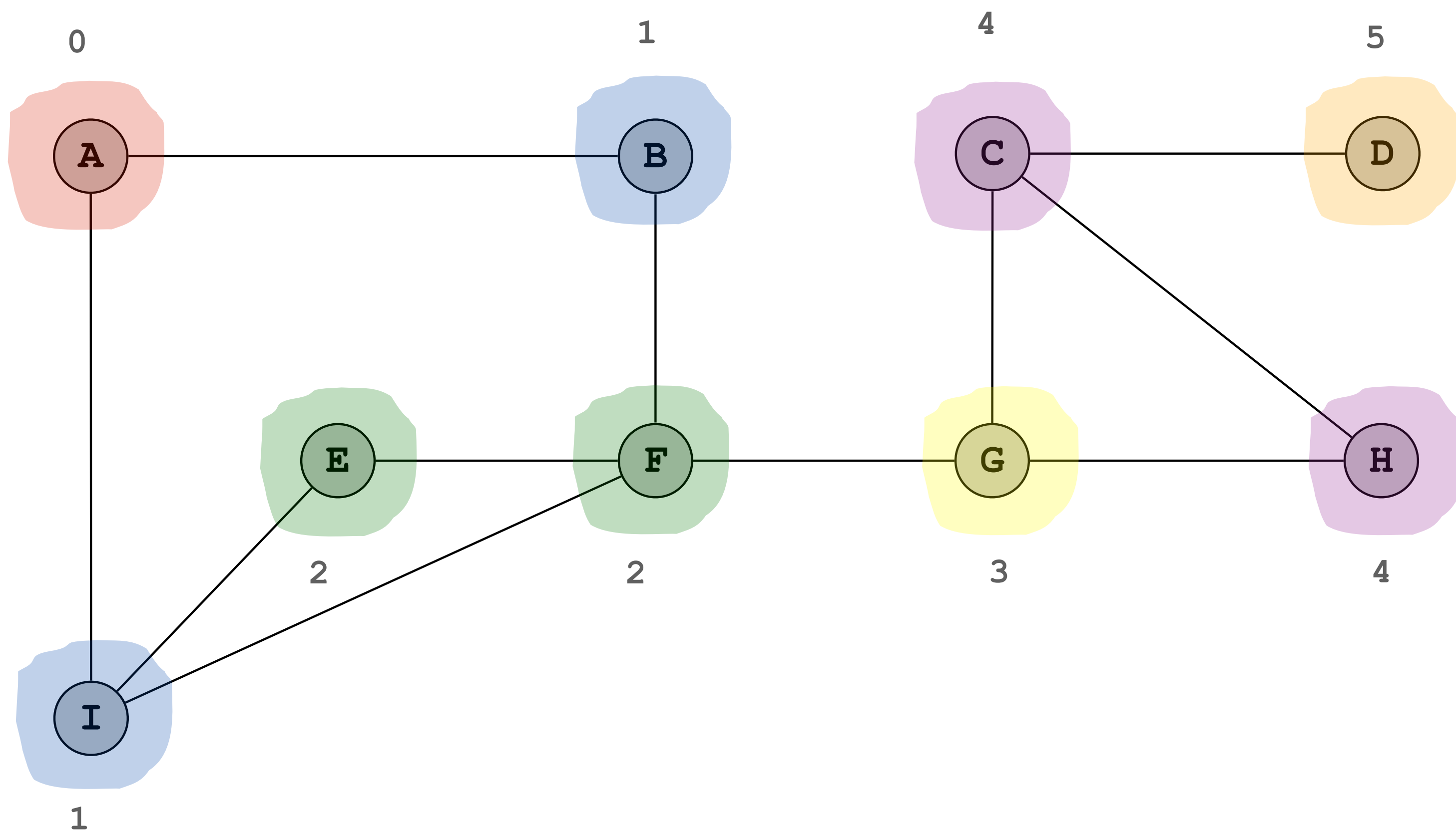
```
% java Bacon top-grossing.txt
Stallone, Sylvester
Rocky III (1982)
Tamburro, Charles A.
Terminator 2: Judgment Day (1991)
Berkeley, Xander
Apollo 13 (1995)
Bacon, Kevin
```

```
% java Bacon top-grossing.txt
Goldberg, Whoopi
Sister Act (1992)
Grodénchik, Max
Apollo 13 (1995)
Bacon, Kevin

Eisgruber, Christopher
```

Computing Shortest Paths

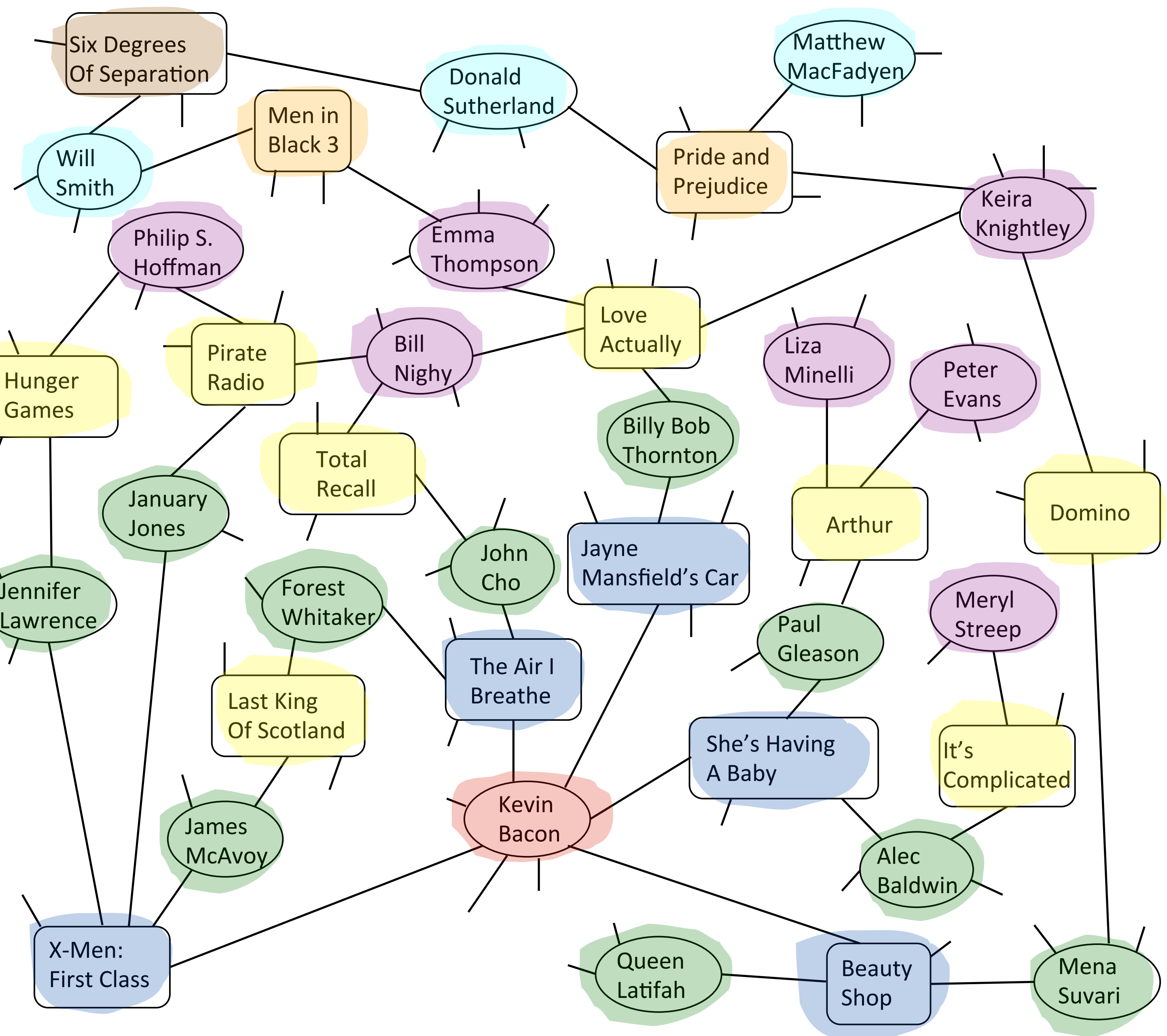
To compute shortest paths:

- Source vertex is at distance 0.
- Its neighbors are at distance 1.
- Their remaining neighbors are at distance 2.
- Their remaining neighbors are at distance 3.
- ...



Computing Shortest Paths

distance	Bacon number
0	0
1	
2	1
3	
4	2
5	
6	3
7	



Breadth First Search

Goal. Given a vertex **s**, find shortest path to every other vertex **v**.

BFS from source vertex **s**

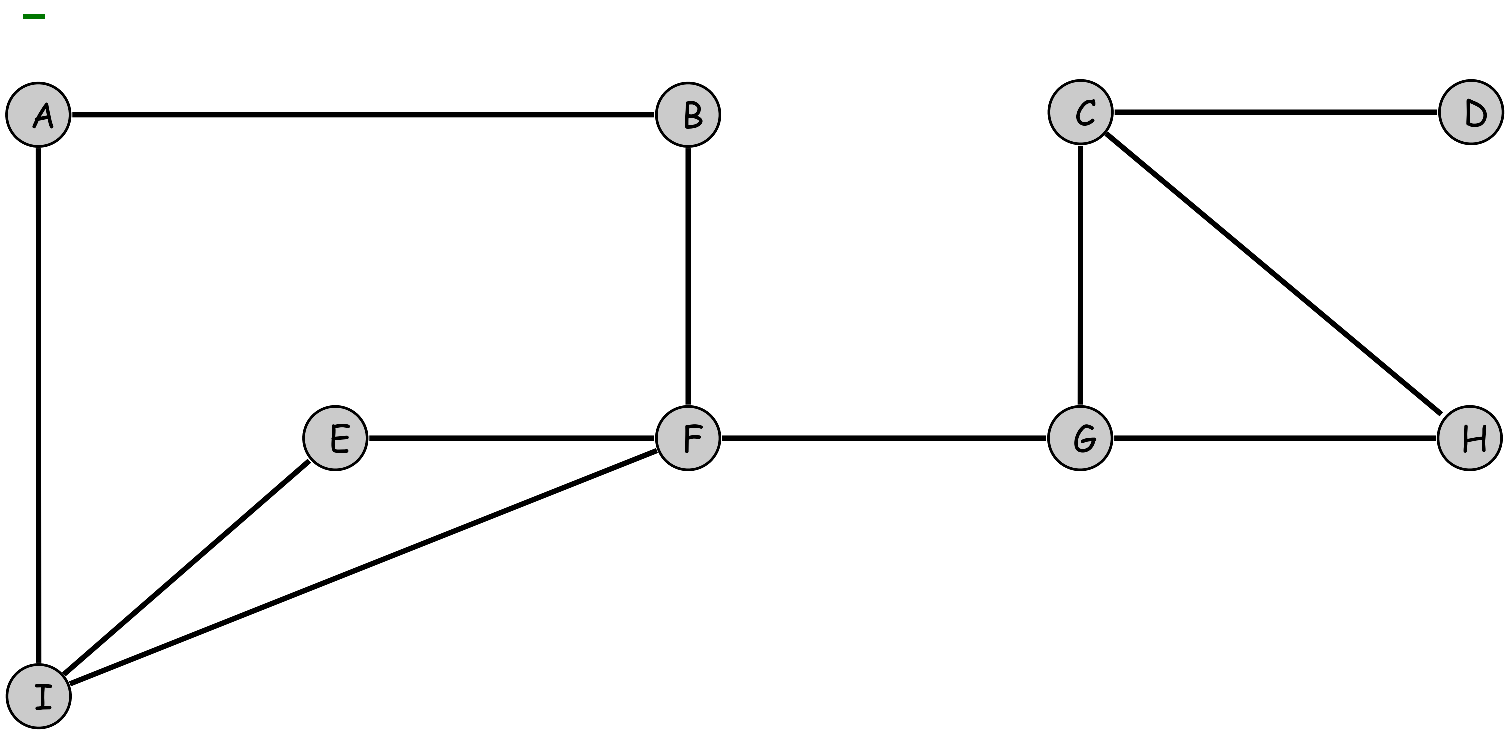
Put **s** onto a FIFO queue.

Repeat until the queue is empty:

- dequeue the least recently added vertex **v**
- add each of **v**'s unvisited neighbors to the queue, and mark them as visited.

Key observation. Vertices are visited in increasing order of distance from `s` because we use a FIFO queue.

Breadth First Search



front

FIFO Queue

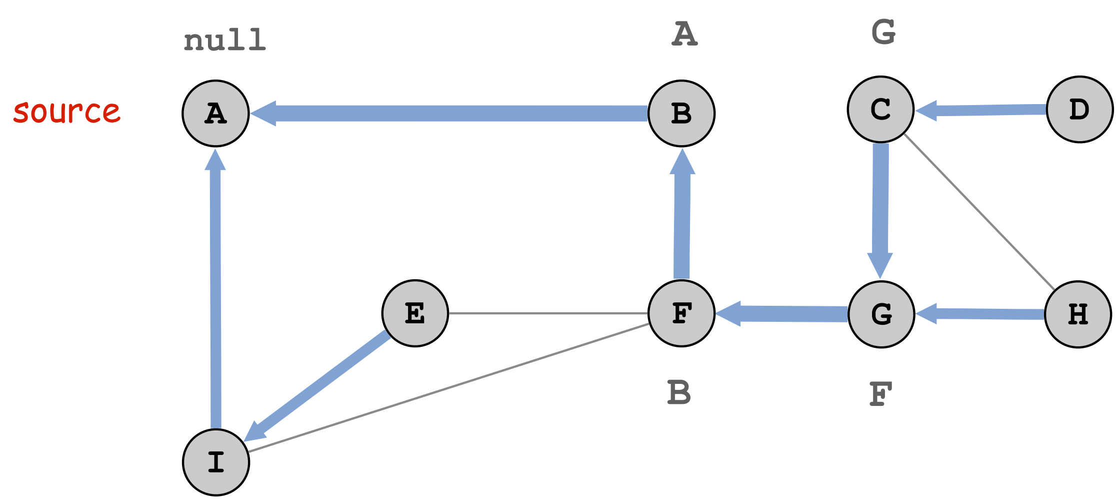
Breadth First Searcher: Preprocessing

```java
public class PathFinder {
   private ST<String, String>  prev = new ST<String, String>();
   private ST<String, Integer> dist = new ST<String, Integer>();

   public PathFinder(Graph G, String s) {
      Queue<String> q = new Queue<String>();
      q.enqueue(s);
      dist.put(s, 0);
      while (!q.isEmpty()) {
         String v = q.dequeue();
         for (String w : G.adjacentTo(v)) {
            if (!dist.contains(w)) {
               q.enqueue(w);
               dist.put(w, 1 + dist.get(v));
               prev.put(w, v);
            }
         }
      }
   }
   // other PathFinder methods go here
}
```

Breadth First Searcher: Finding the Path

To find shortest path: follow `prev[]` from vertex `v` back to source `s`.

- Consider vertices: `v`, `prev[v]`, `prev[prev[v]]`, …, `s`.
- Ex: shortest path from `C` to `A`: `C - G - F - B - A`
- Use stack to reverse order



key	prev	dist
A	-	0
B	A	1
C	G	4
D	C	5
E	I	2
F	B	2
G	F	3
H	G	4
I	A	1

symbol tables

```
public Iterable<String> pathTo(String v) {
   Stack<String> path = new Stack<String>();
   while (dist.contains(v)) {
      path.push(v);
      v = prev.get(v);
   }
   return path;
}
```

Running Time Analysis

Analysis. BFS scales to solve huge problems.

data File	movies	performers	edges	read input	build graph	BFS	pathTo
`G.txt`	1,288	21,177	28K	0.26 sec	0.52 sec	0.32 sec	0 sec
`PG13.txt`	2,538	70,325	100K	0.31 sec	0.99 sec	0.72 sec	0 sec
`action.txt`	14,938	139,861	270K	0.72 sec	2.8 sec	2.0 sec	0 sec
`mpaa.txt`	21,861	280,624	610K	2.1 sec	7.5 sec	5.5 sec	0 sec
`all.txt`	285,462	933,864	3.3M	15 sec	56 sec	39 sec	0 sec

60MB

data as of April 9, 2007

Data Analysis

Exercise. Compute histogram of Kevin Bacon numbers.
Input. ~2.6 million movies, ~5.7 million actors.

Bacon #	Frequency
0	1
1	2,769
2	305,215
3	1,021,901
4	253,177
5	20,060
6	2,033
7	297
8	25
9	7
∞	>32,000

These are *very* hard to find!

Fred Ott, solo actor in
Fred Ott Holding a Bird (1894)

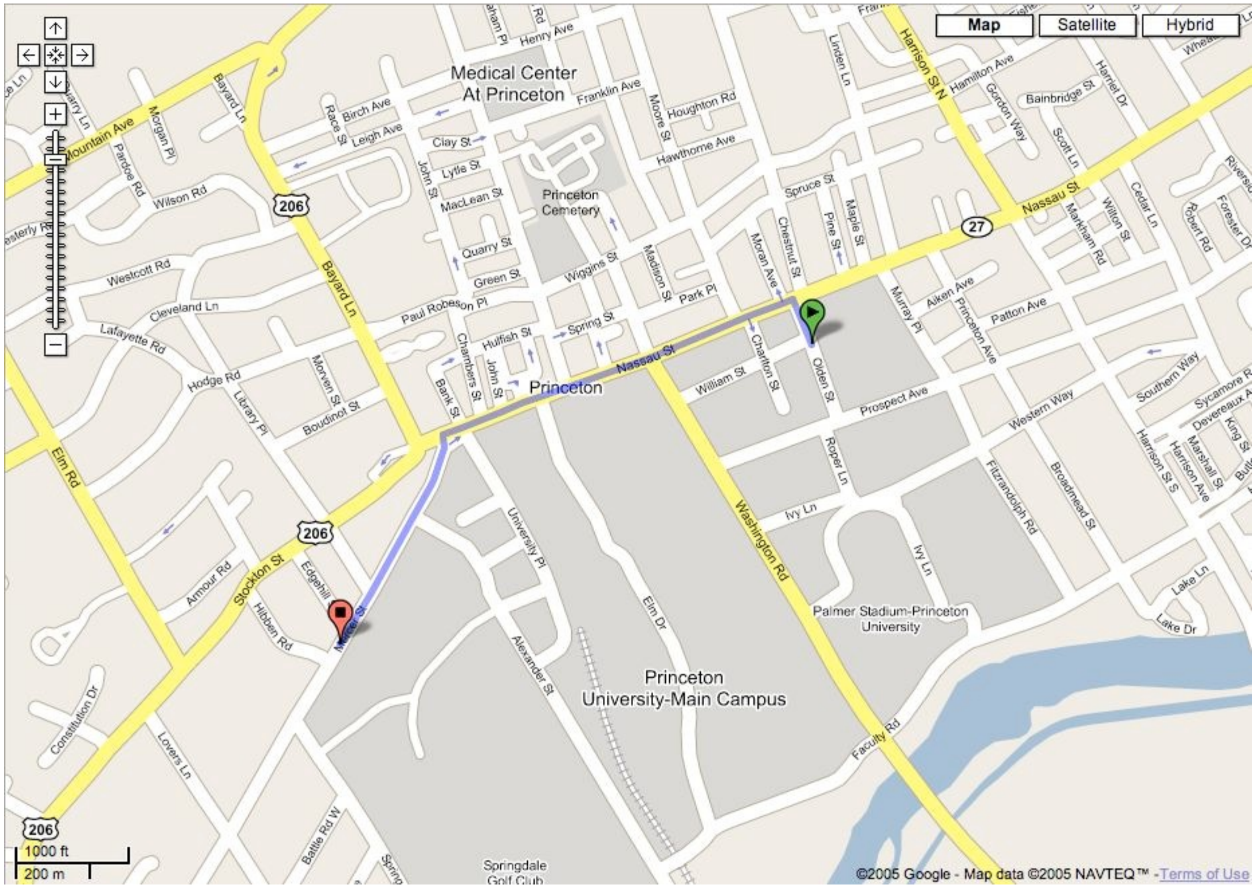
data as of April 28, 2013

Applications of Breadth First Search

More BFS applications.

- Particle tracking.
- Image processing.
- Crawling the Web.
- Routing Internet packets.
- ...

Extensions. Google maps.



Erdös Numbers

Erdös Numbers

Paul Erdös. Legendary, brilliant, prolific mathematician who wrote over 1500 papers!

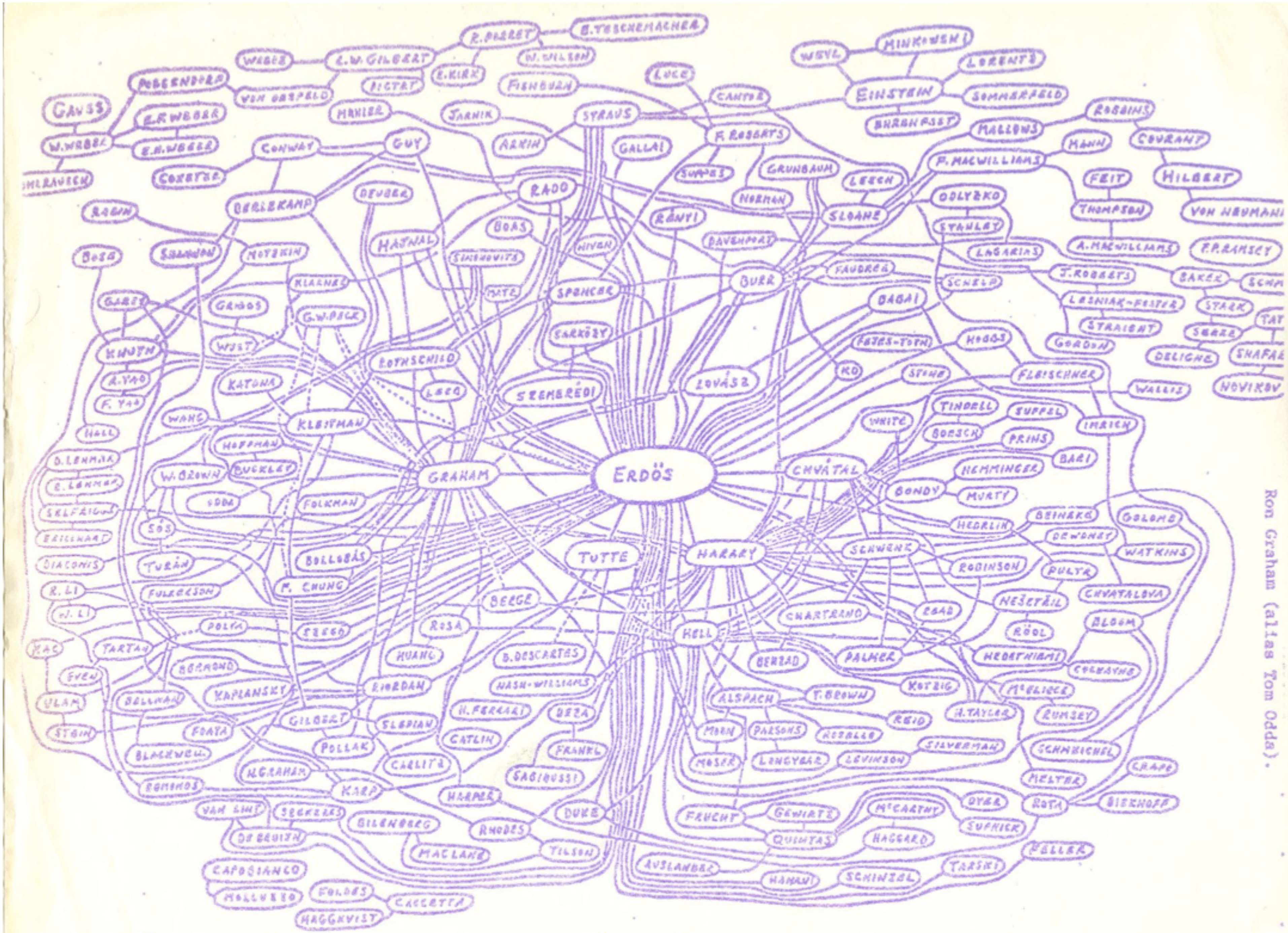
What's your Erdös number?

- Co-authors of a paper with Erdös: 1.
- Co-authors of those co-authors: 2.
- And so on …

Paul Erdös (1913-1996)

Erdös #	Frequency
0	1
1	502
2	5,713
3	26,422
4	62,136
5	66,157
6	32,280
7	10,431
8	3,214
9	953
10	262
11	94
12	23
13	4
14	7
15	1
∞	4 billion +

Erdös Graph



Ron Graham (alias Tom Odda).

Conclusions

Linked list. Ordering of elements.
Binary tree. Hierarchical structure of elements.
Graph. Pairwise connections between elements.

Data structures.

- Queue: linked list.
- Set: binary tree.
- Symbol table: binary tree.
- Graph: symbol table of sets.
- Breadth first searcher: graph + queue + symbol table.

Importance of data structures.

- Enables us to build and debug large programs.
- Enables us to solve large problems efficiently.