



The 3D Rasterization Pipeline

COS 426

3D Rendering Scenarios



- Batch
 - One image generated with as much quality as possible for a particular set of rendering parameters
 - Take as much time as is needed (minutes)
 - Useful for photorealism, movies, etc.
- **Interactive**
 - Images generated in fraction of a second ($<1/10$) with user input, animation, varying camera, etc.
 - Achieve highest quality possible in given time
 - Visualization, games, etc.

3D Polygon Rendering



- Many applications use rendering of 3D polygons with direct illumination

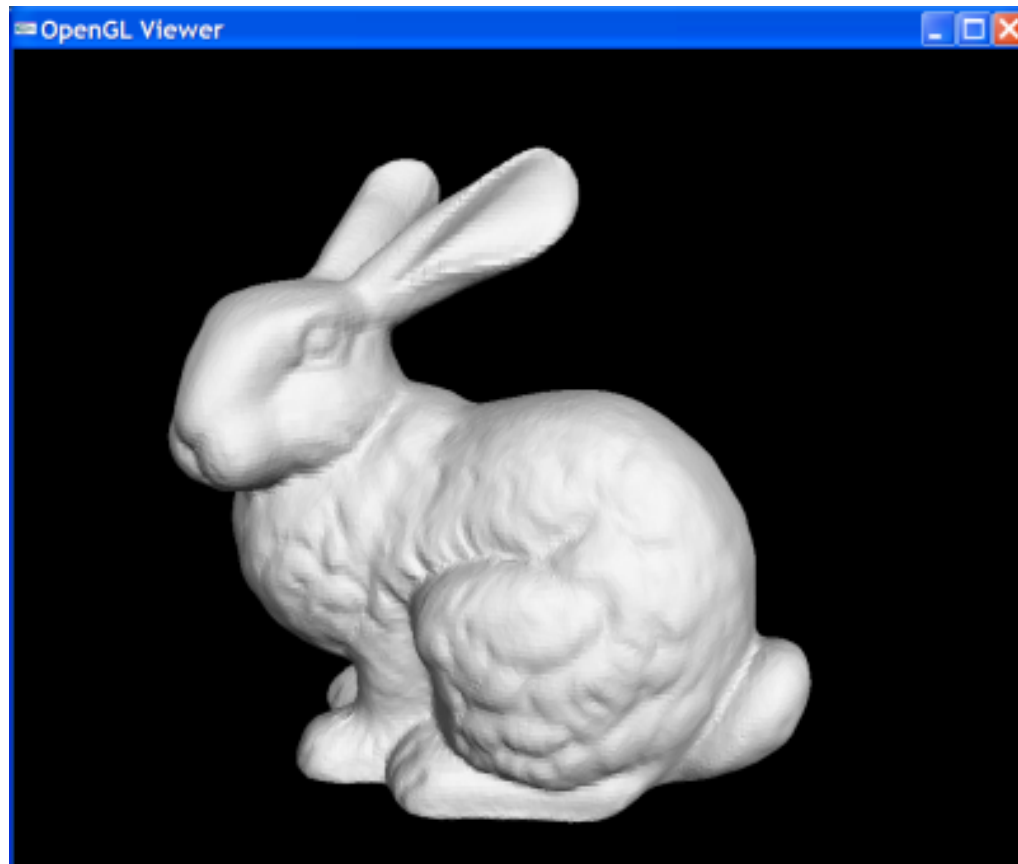


Bungie

3D Polygon Rendering



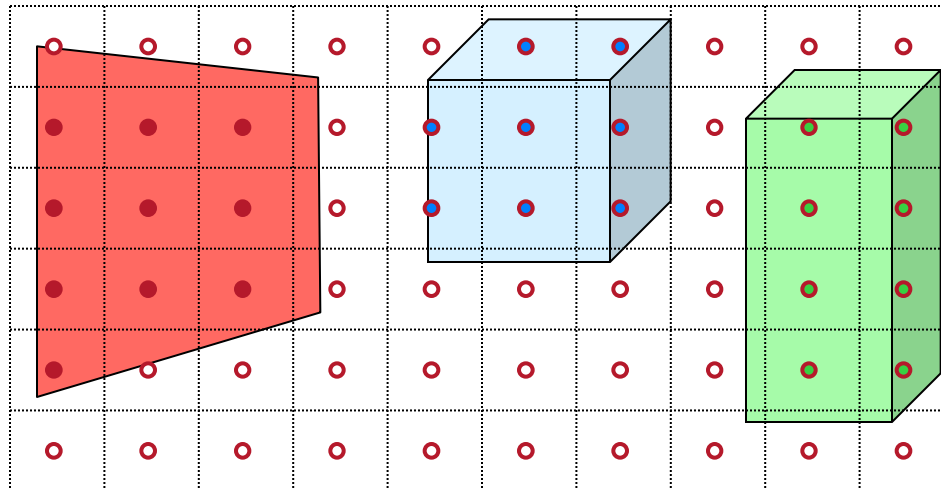
- Many applications use rendering of 3D polygons with direct illumination



meshview

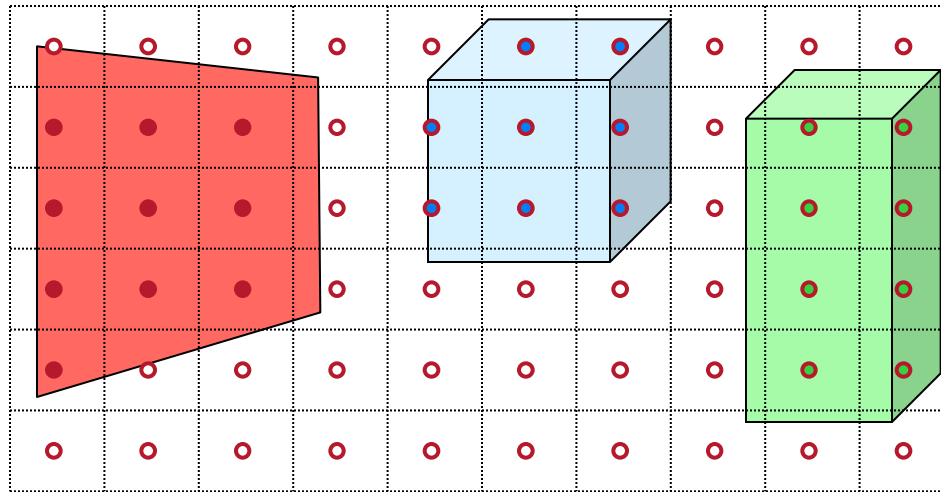
Ray Casting Revisited

- For each sample ...
 - Construct ray from eye position through view plane
 - Find first surface intersected by ray through pixel
 - Compute color of sample based on illumination



3D Polygon Rendering

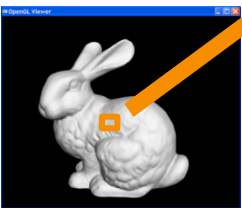
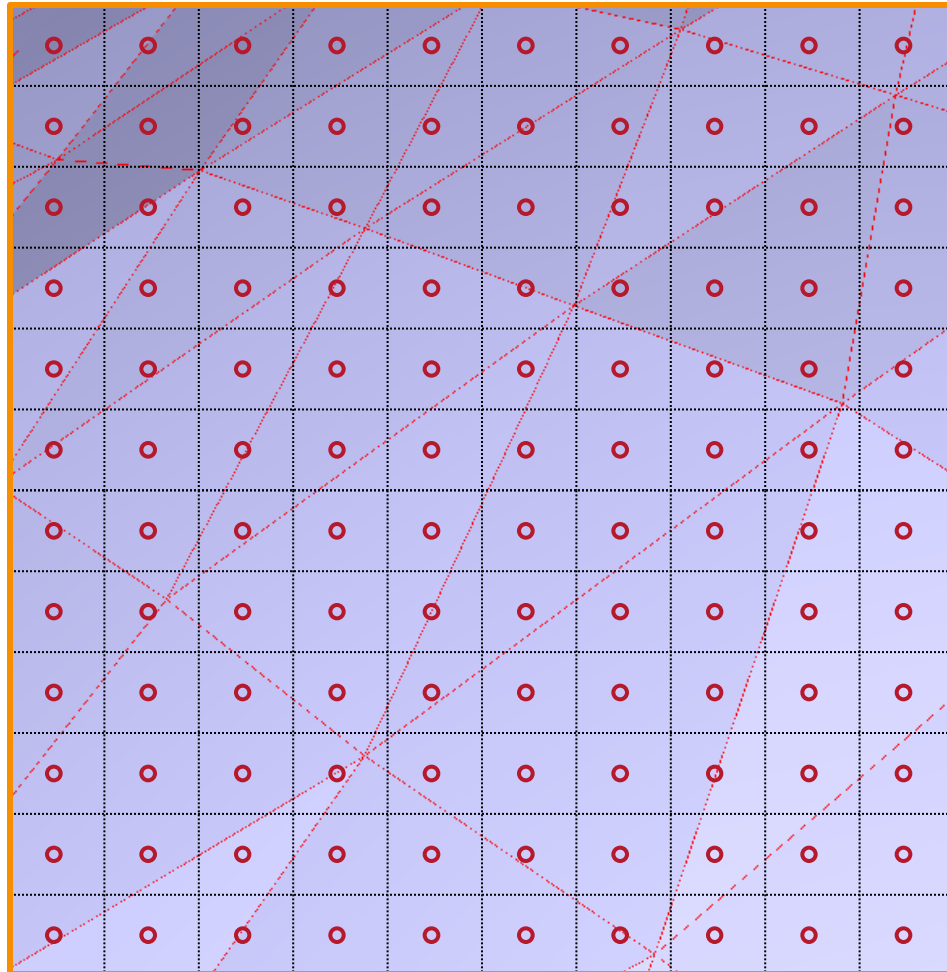
- We can render polygons faster if we take advantage of **spatial coherence**



3D Polygon Rendering



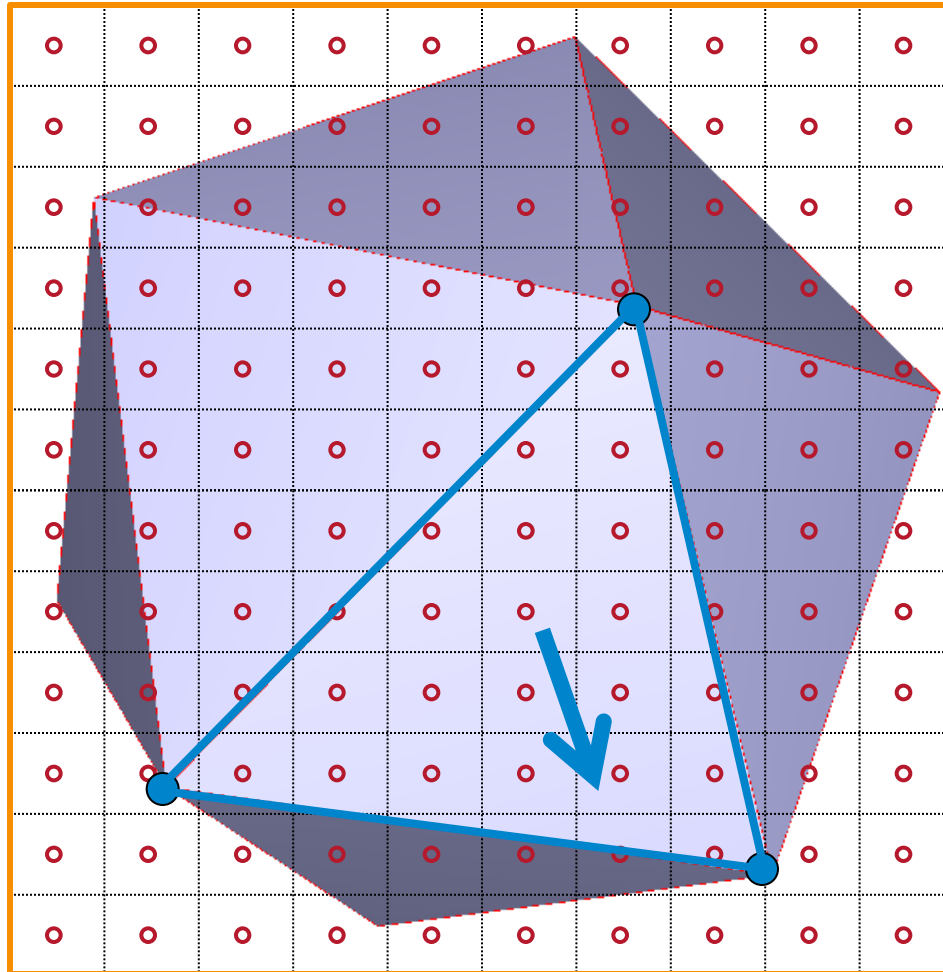
- How?



3D Polygon Rendering



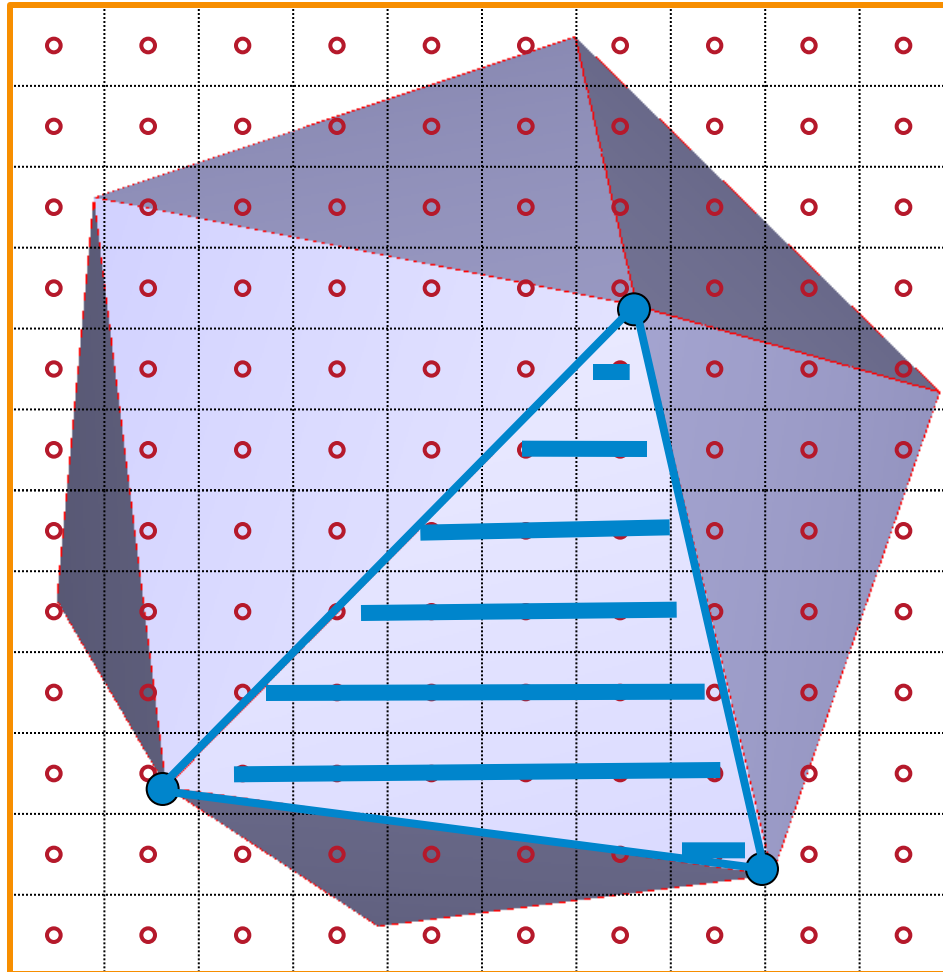
- How?



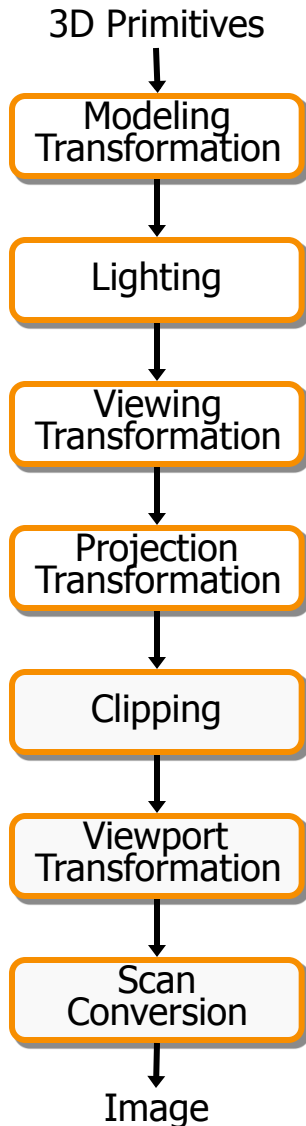
3D Polygon Rendering



- How?

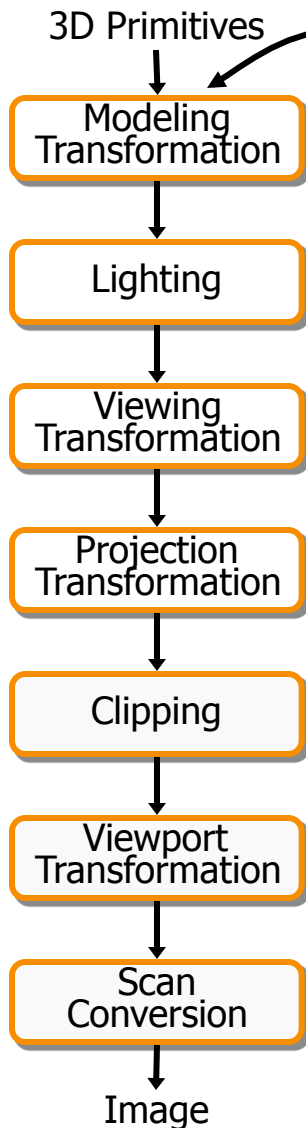


3D Rendering Pipeline (for direct illumination)



This is a pipelined sequence of operations to draw 3D primitives into a 2D image

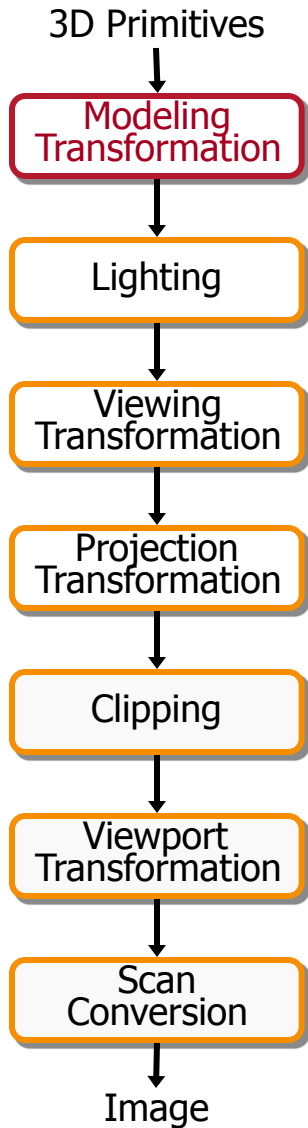
3D Rendering Pipeline (for direct illumination)



```
glBegin(GL_POLYGON);  
glVertex3f(0.0, 0.0, 0.0);  
glVertex3f(1.0, 0.0, 0.0);  
glVertex3f(0.0, 1.0, 0.0);  
glEnd();
```

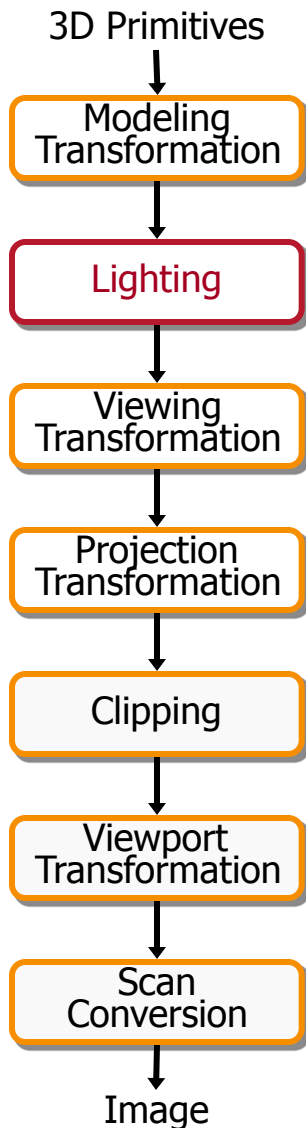
OpenGL executes steps
of 3D rendering pipeline
for each polygon

3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

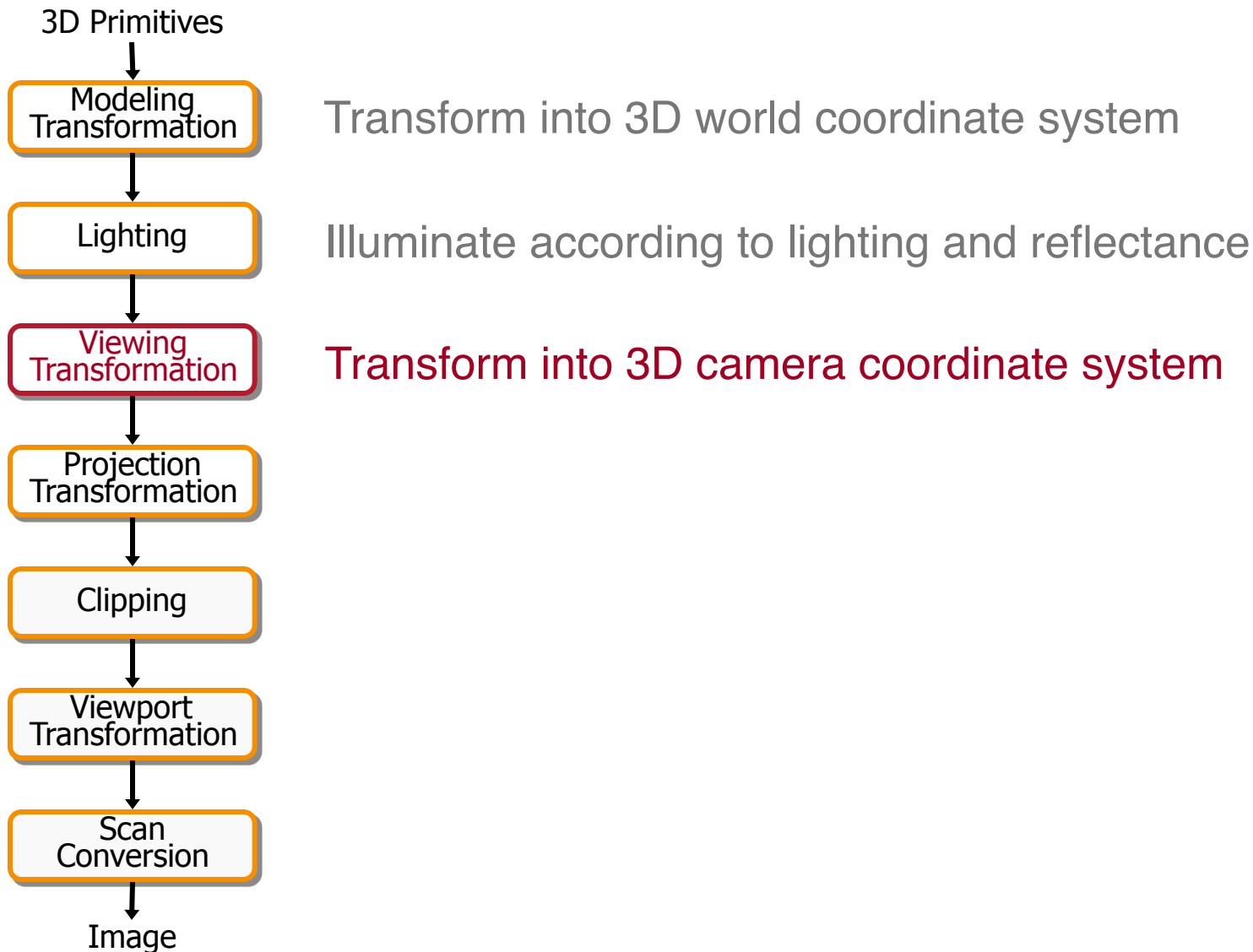
3D Rendering Pipeline (for direct illumination)



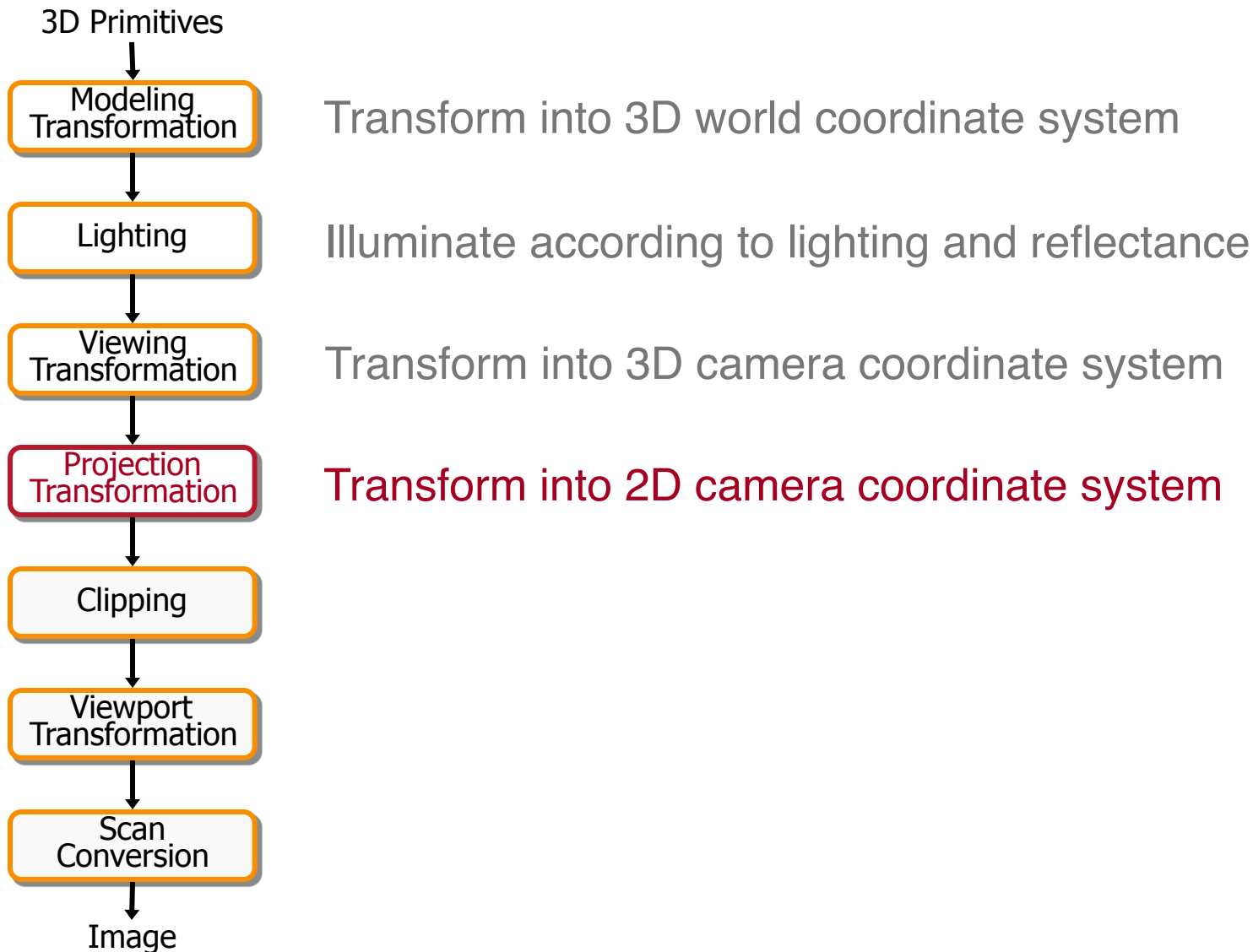
Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

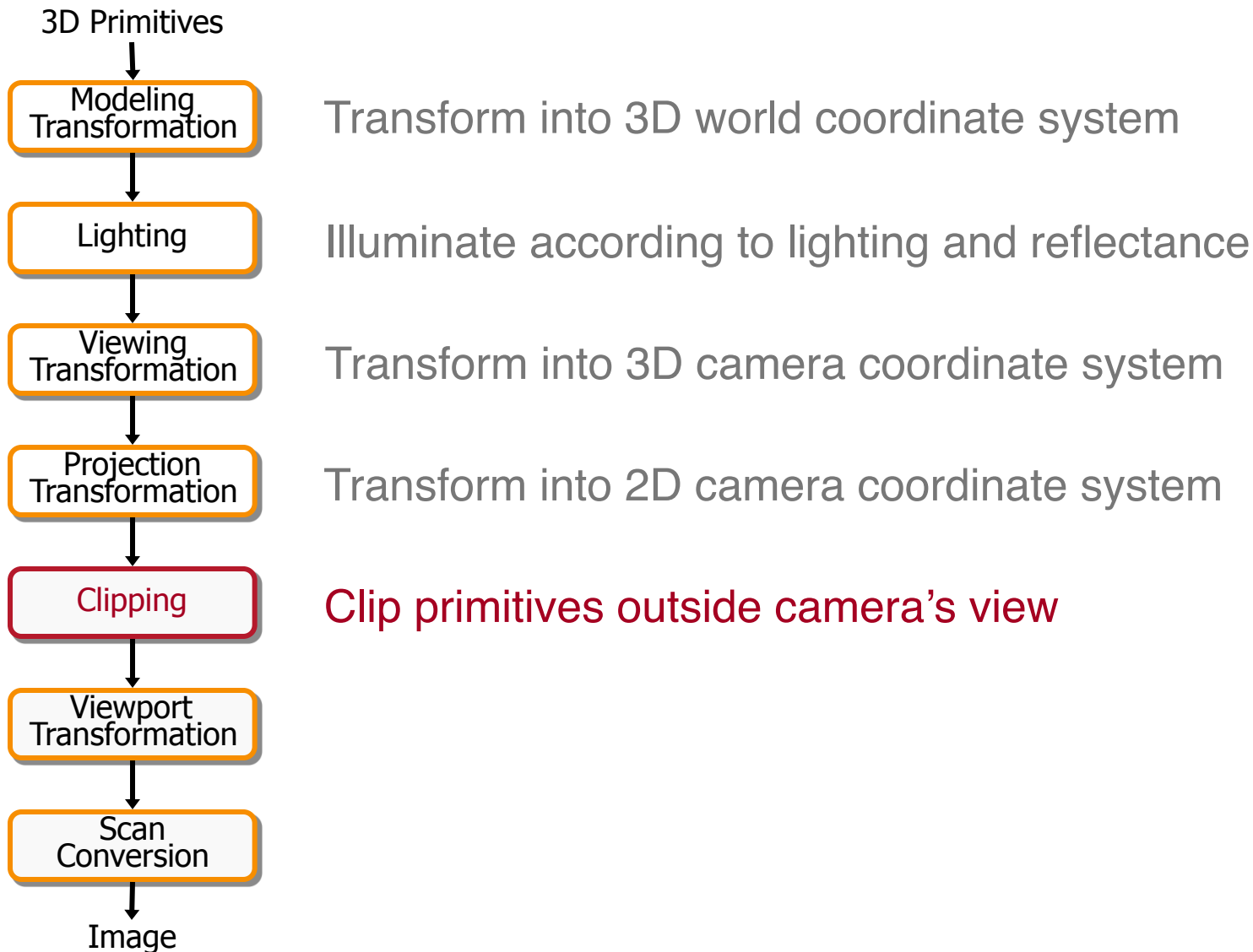
3D Rendering Pipeline (for direct illumination)



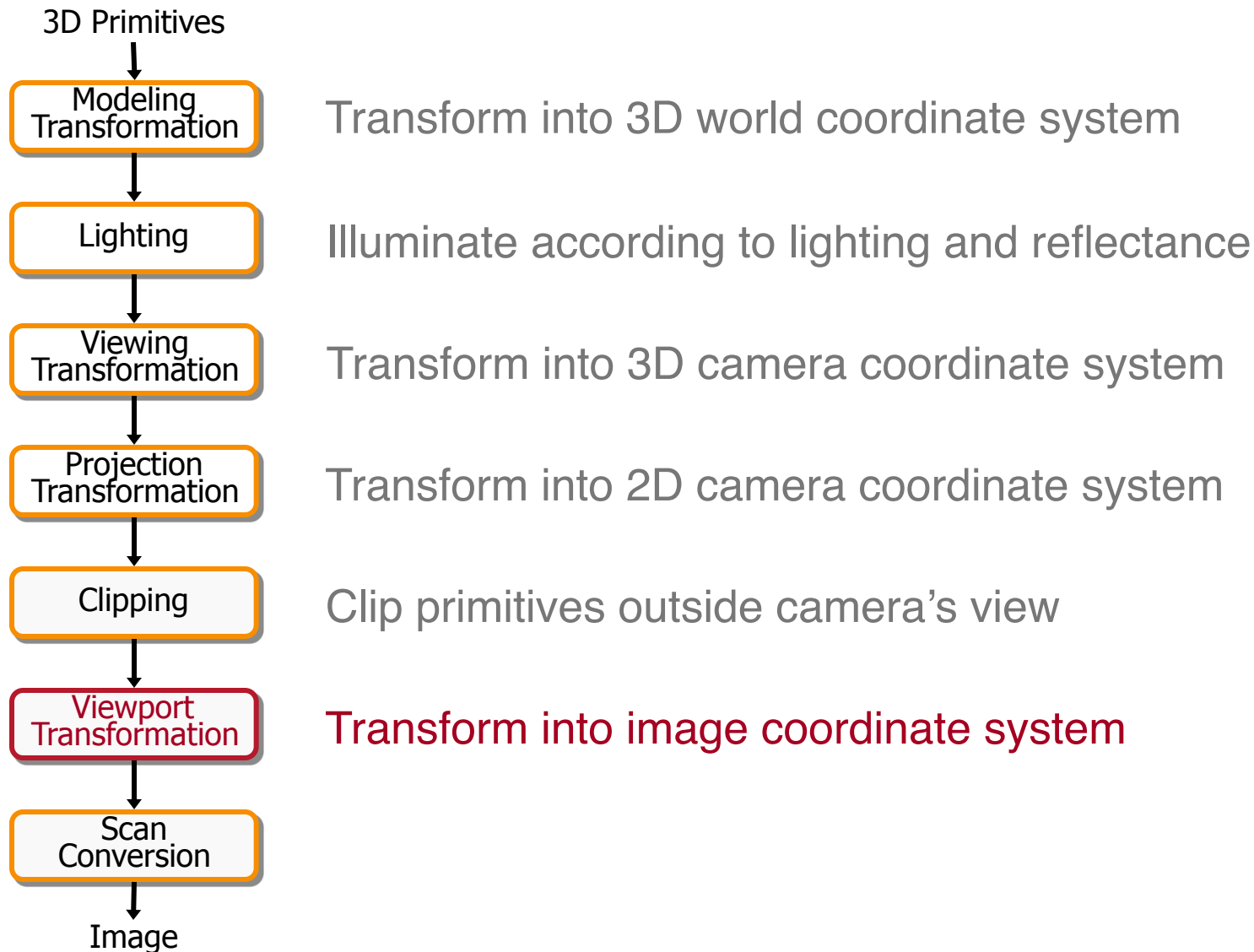
3D Rendering Pipeline (for direct illumination)



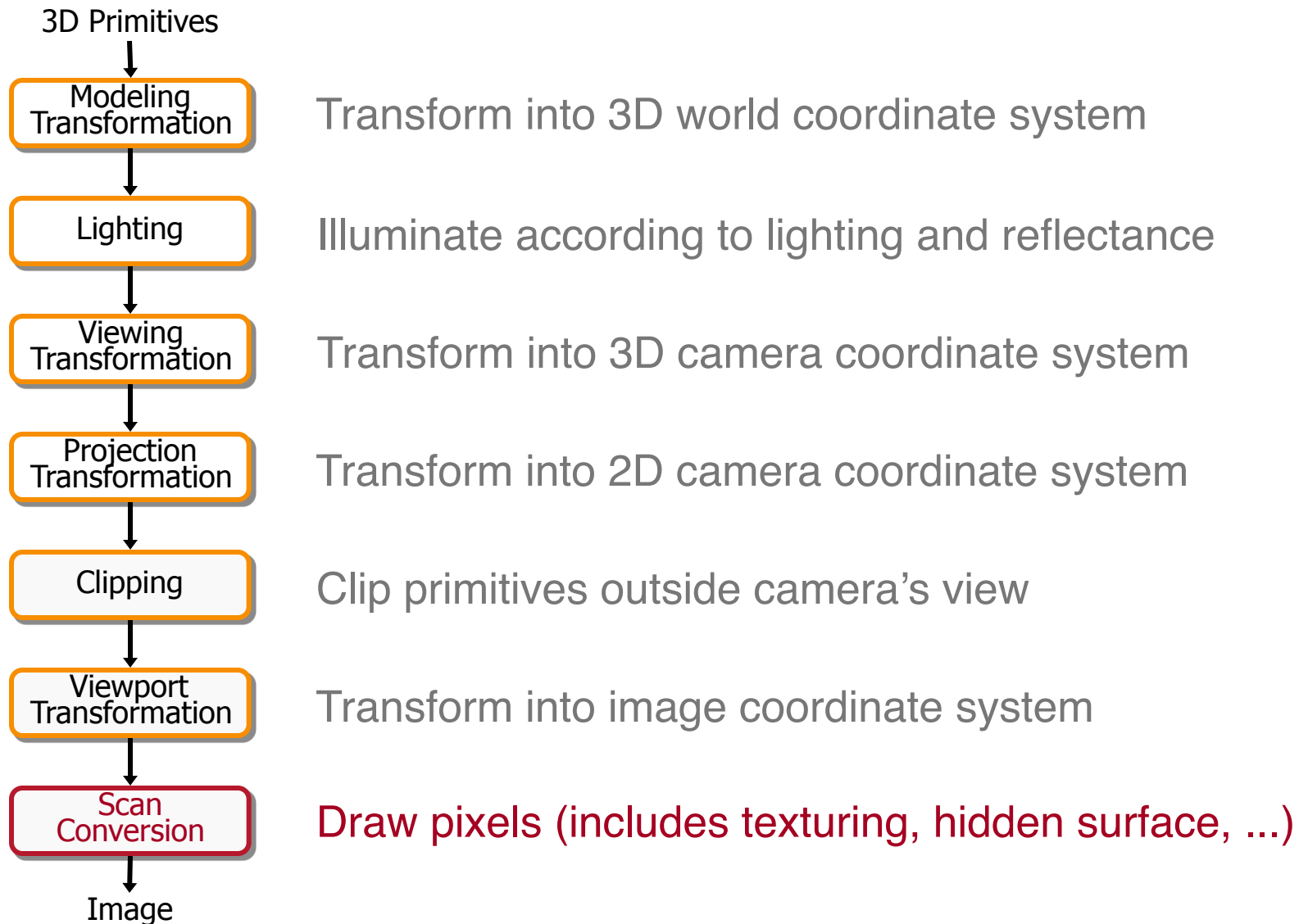
3D Rendering Pipeline (for direct illumination)



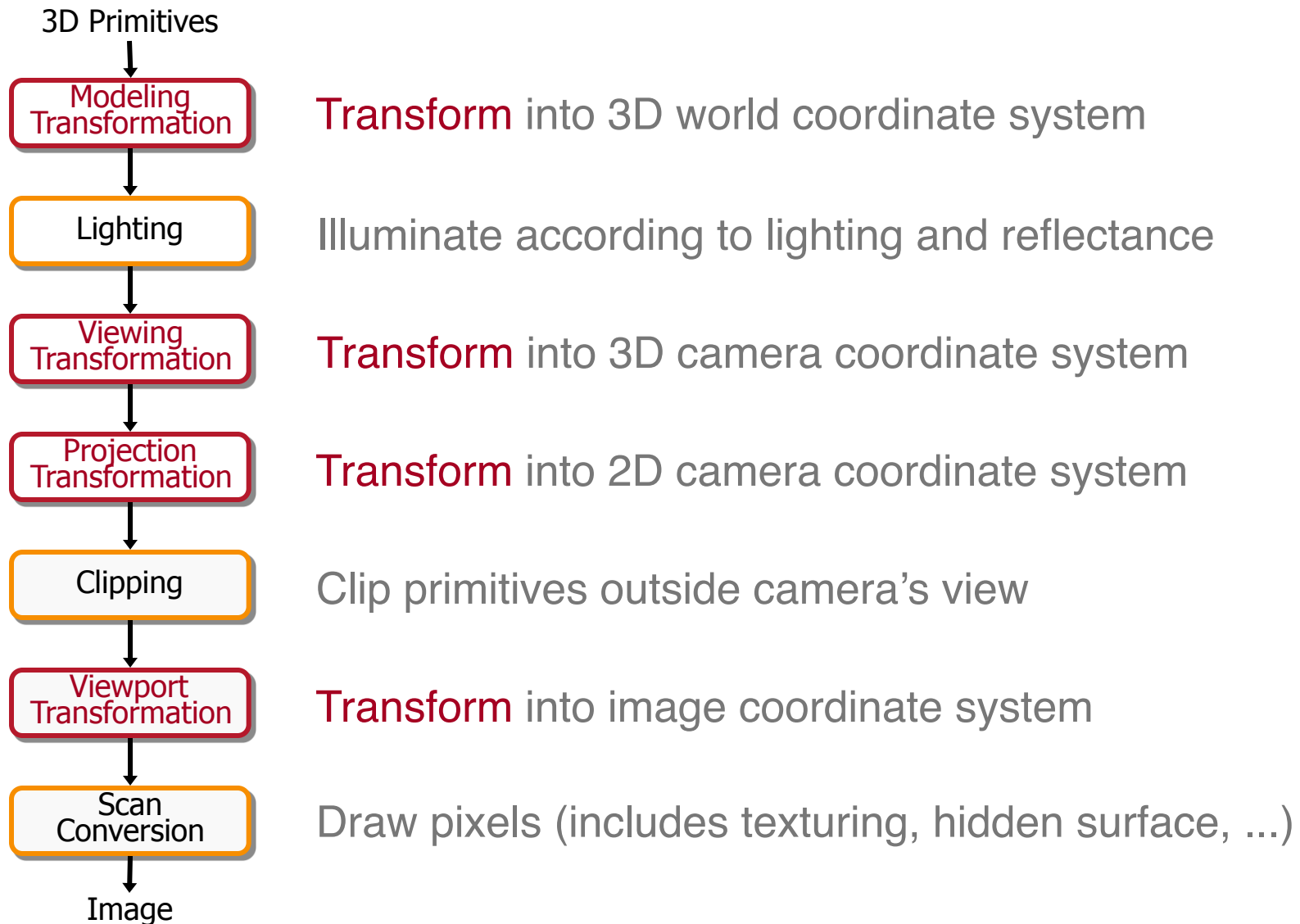
3D Rendering Pipeline (for direct illumination)



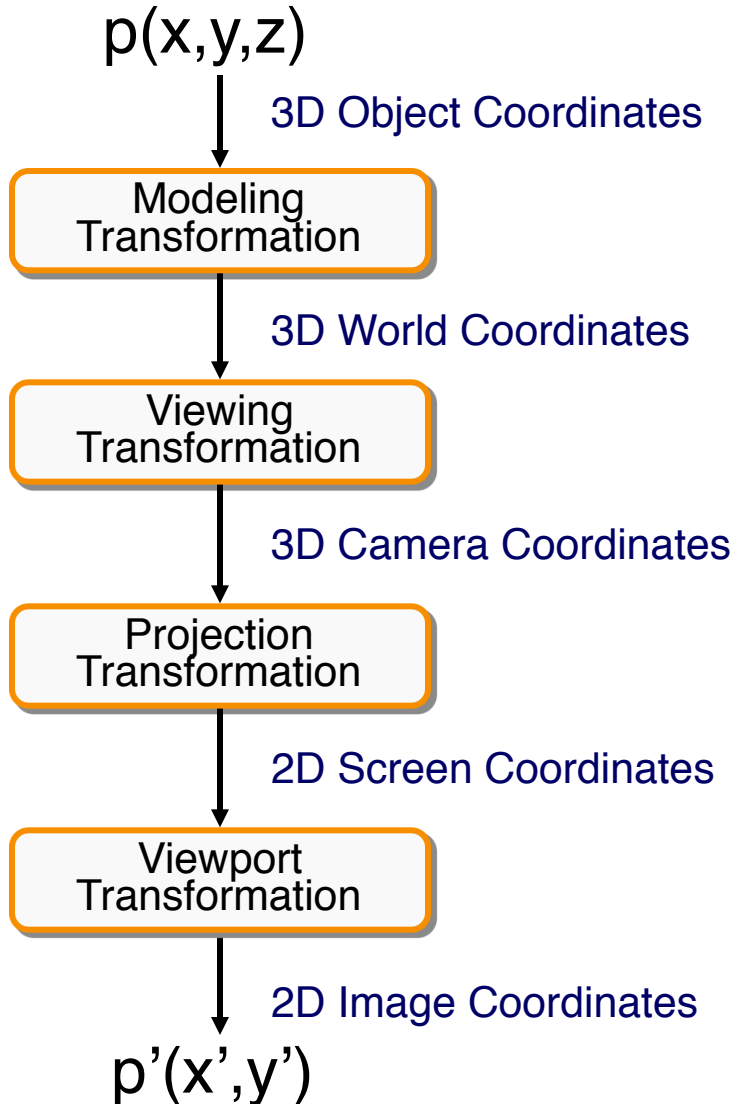
3D Rendering Pipeline (for direct illumination)



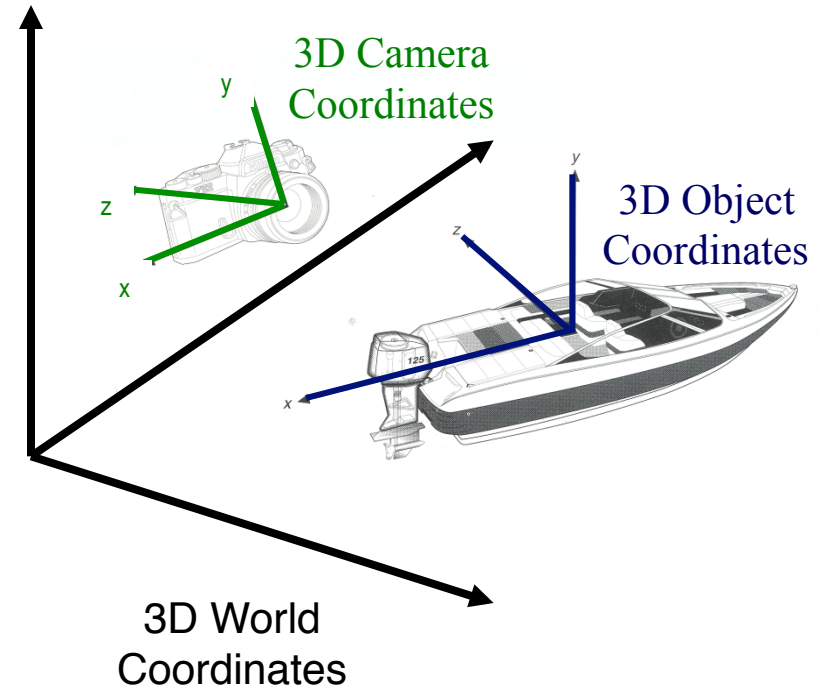
3D Rendering Pipeline (for direct illumination)



Transformations

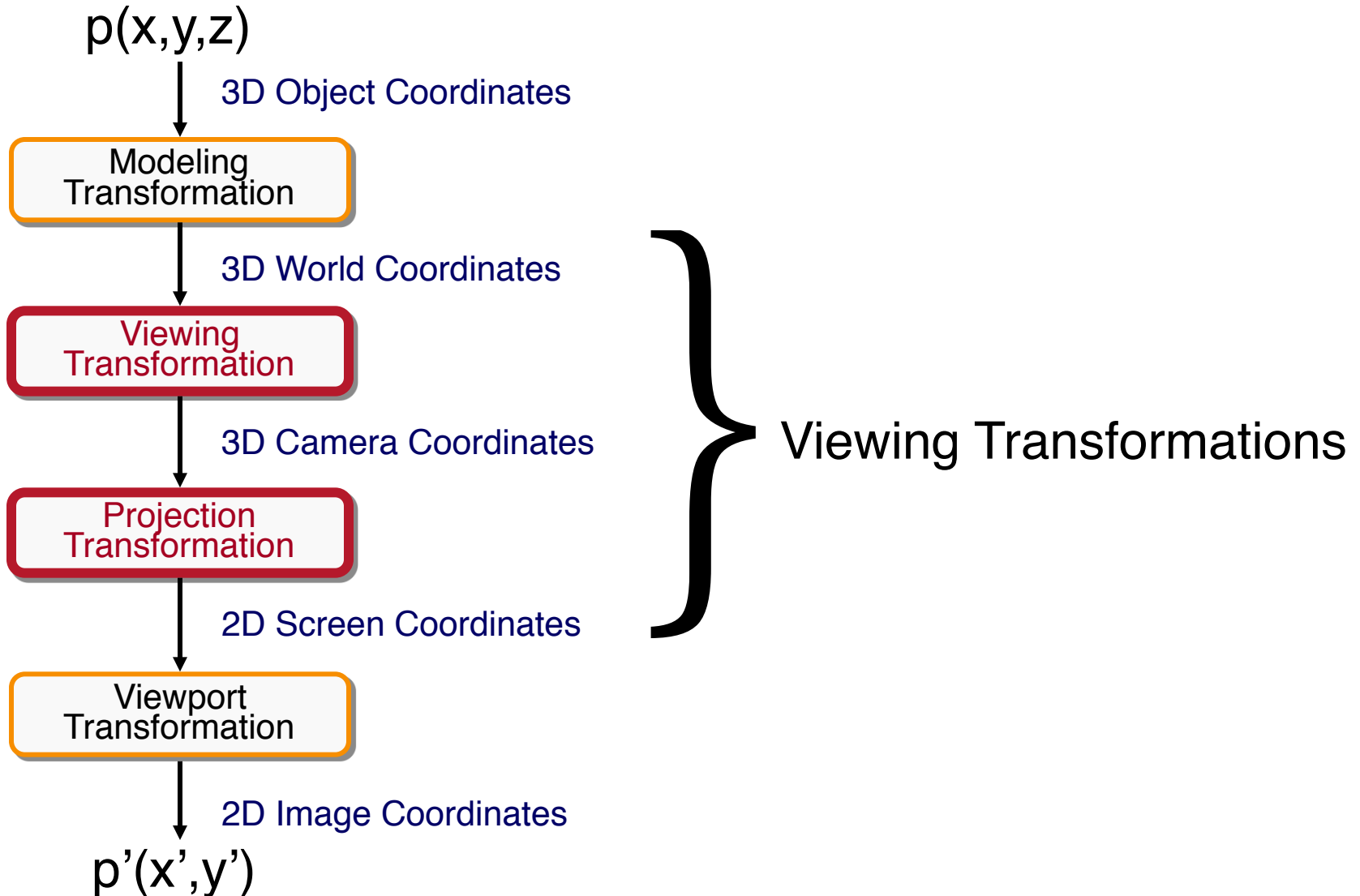


Transformations map points from one coordinate system to another





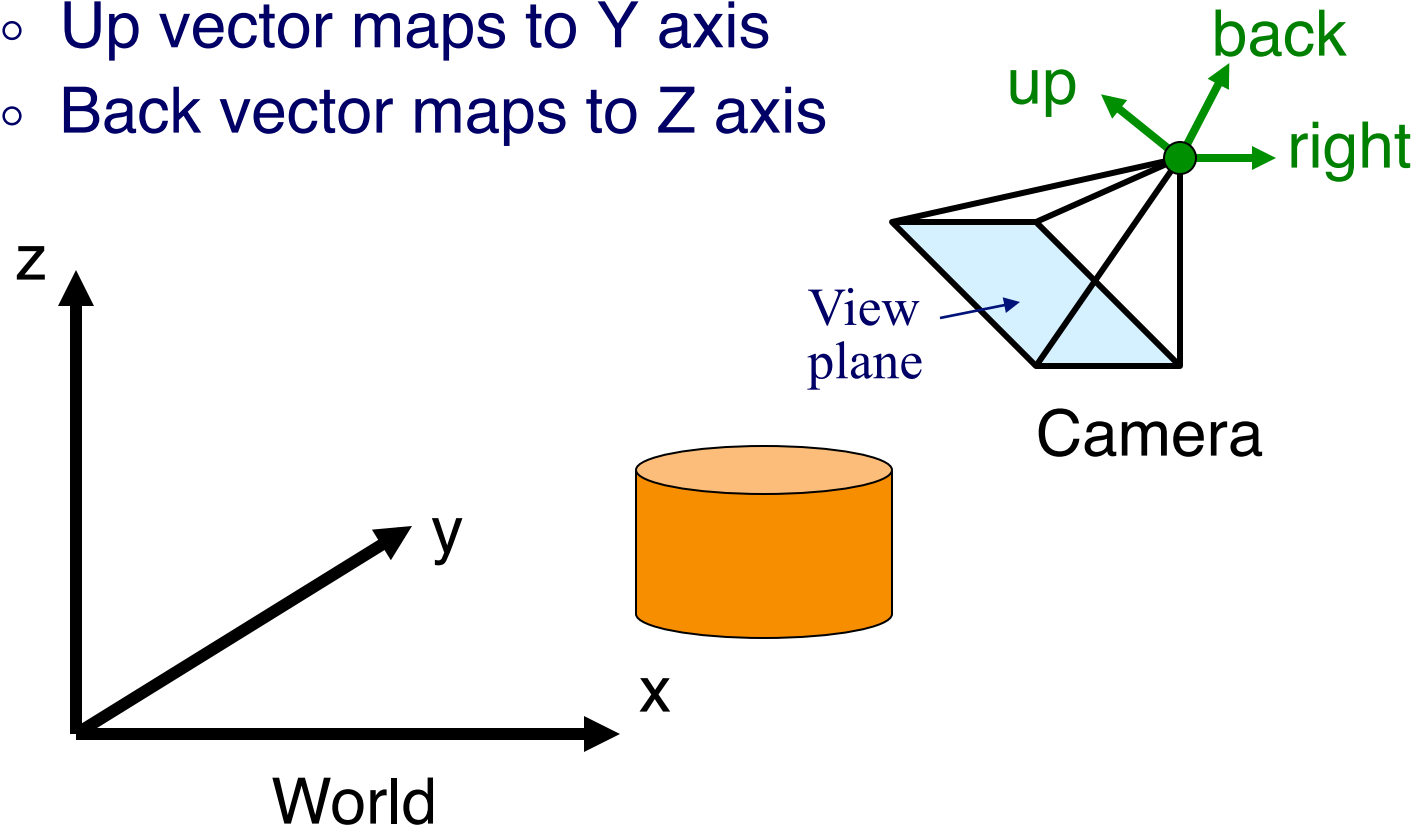
Viewing Transformations





Review: Viewing Transformation

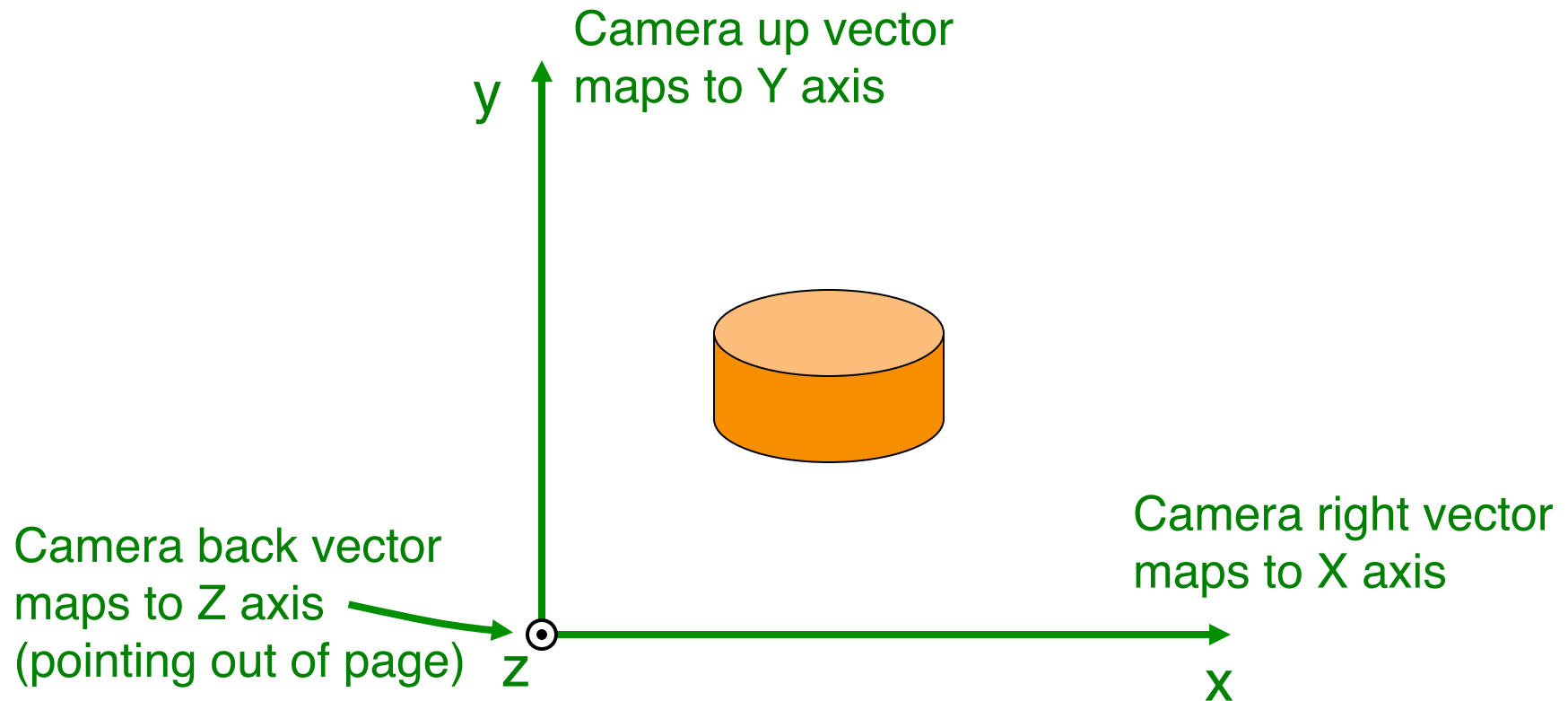
- Mapping from world to camera coordinates
 - Eye position maps to origin
 - Right vector maps to X axis
 - Up vector maps to Y axis
 - Back vector maps to Z axis





Review: Camera Coordinates

- Canonical coordinate system
 - Convention is right-handed (looking down -z axis)
 - Convenient for projection, clipping, etc.



Finding the viewing transformation



- We have the camera (in world coordinates)
- We want T taking objects from world to camera

$$p^c = T p^w$$

- Trick: find T^{-1} taking objects in camera to world

$$p^w = T^{-1} p^c$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$



Finding the Viewing Transformation



- Trick: map from camera coordinates to world
 - Origin maps to eye position
 - Z axis maps to Back vector
 - Y axis maps to Up vector
 - X axis maps to Right vector

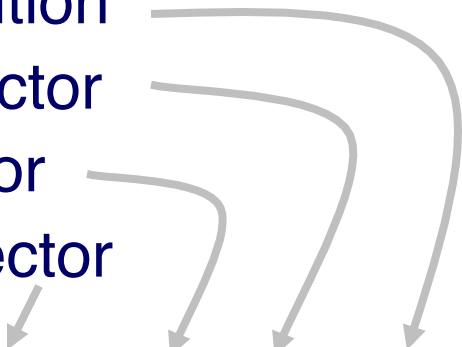


Diagram showing arrows from the list items to the matrix columns:

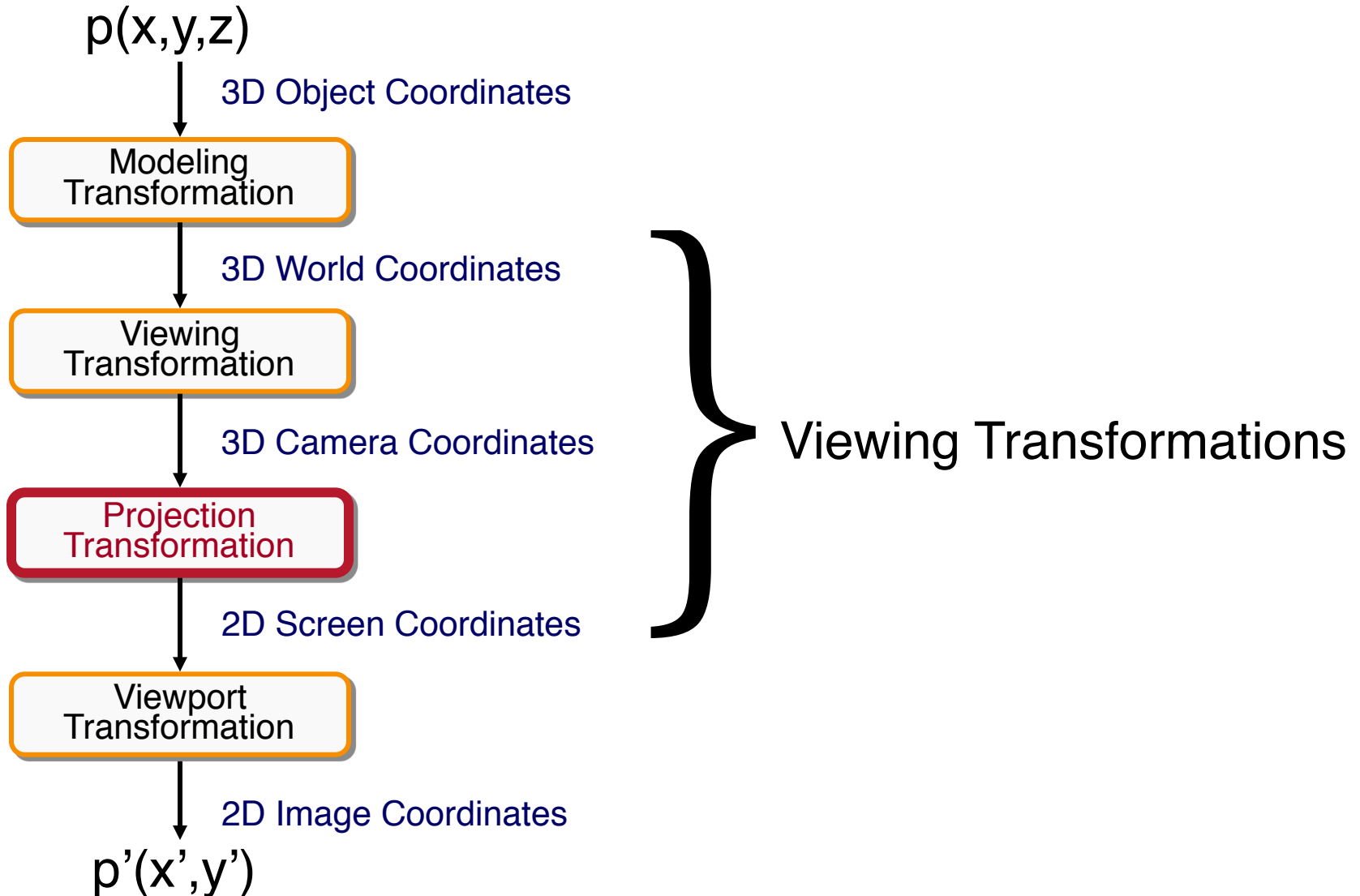
- From 'Origin maps to eye position' to the E_x column.
- From 'Z axis maps to Back vector' to the B_x column.
- From 'Y axis maps to Up vector' to the U_x column.
- From 'X axis maps to Right vector' to the R_x column.

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_y & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

- This matrix is T^{-1} so we invert it to get T ... easy!

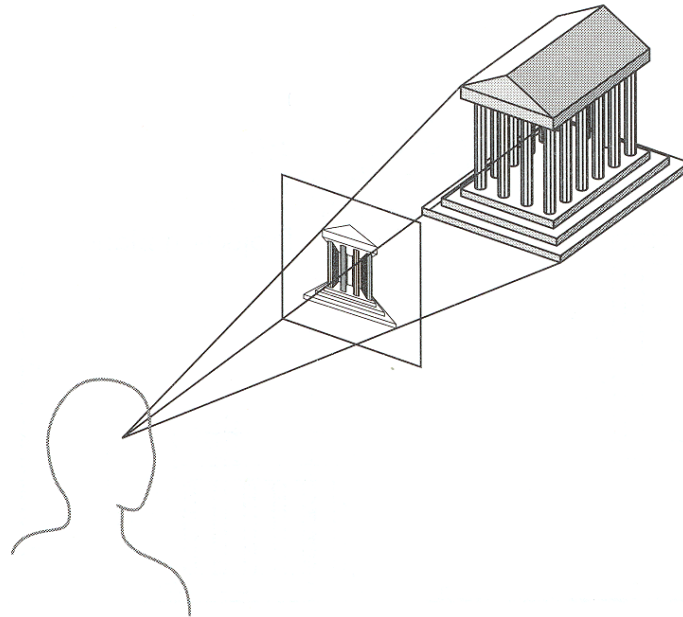


Viewing Transformations

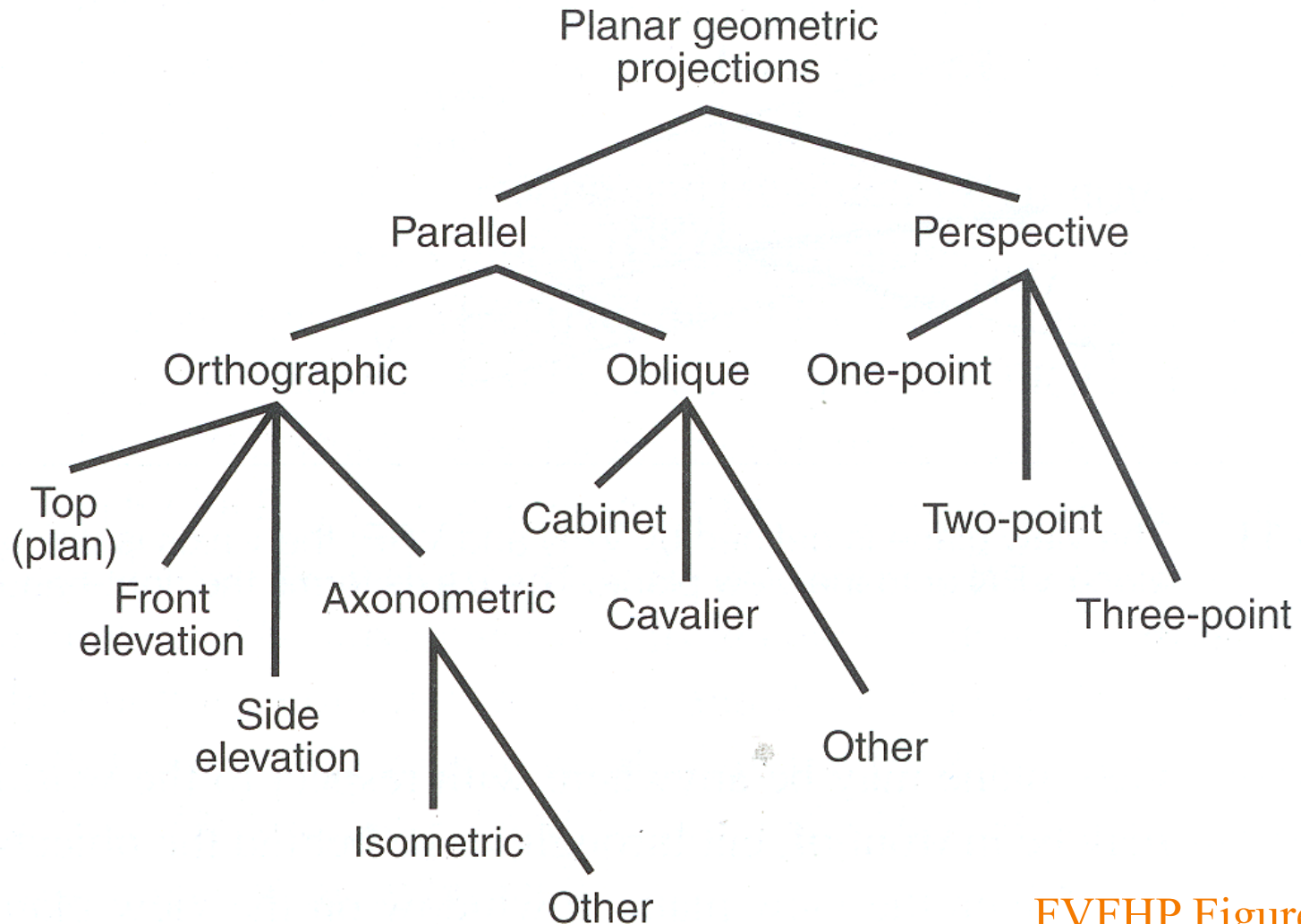


Projection

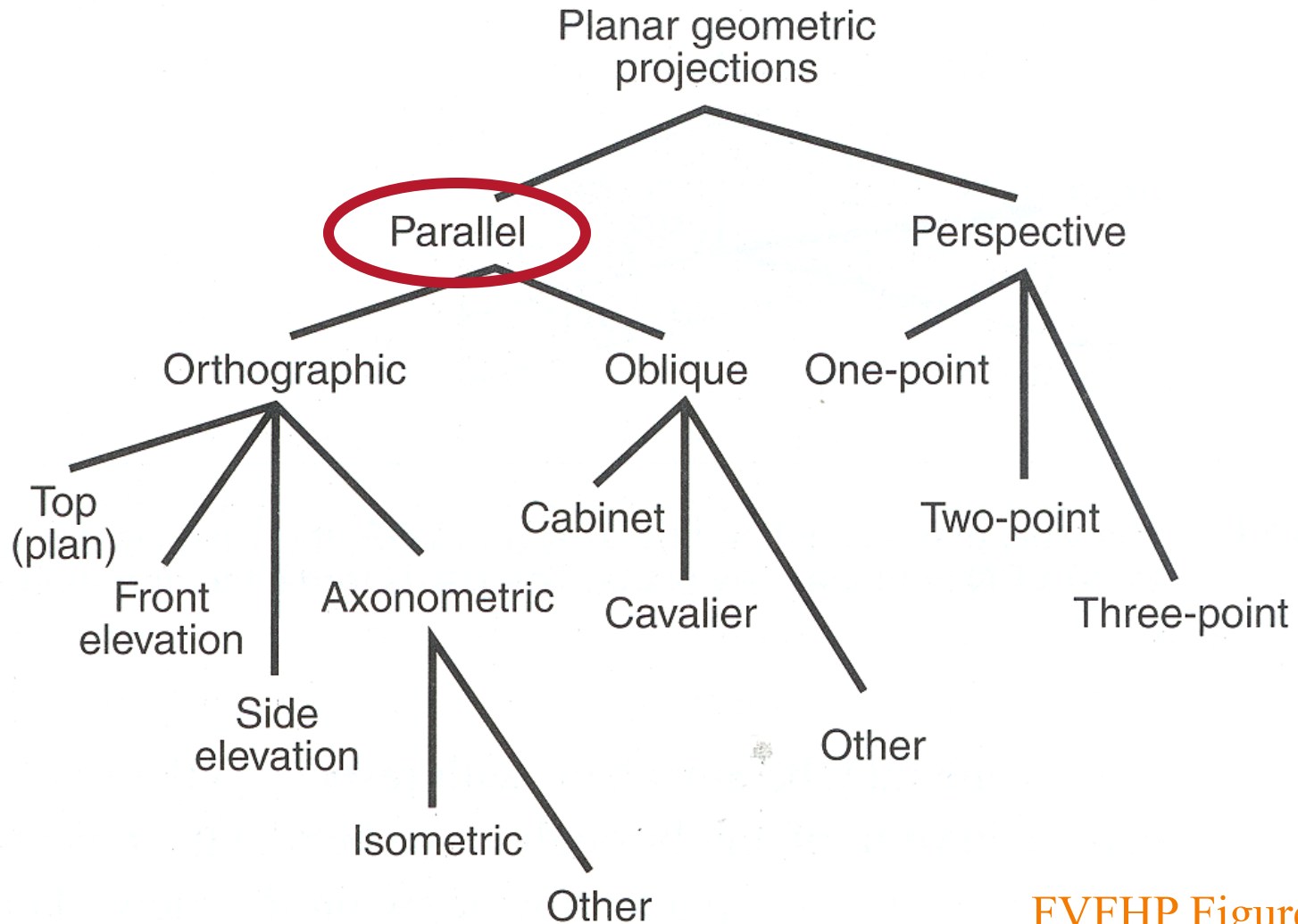
- General definition:
 - Transform points in n -space to m -space ($m < n$)
- In computer graphics:
 - Map 3D camera coordinates to 2D screen coordinates



Taxonomy of Projections

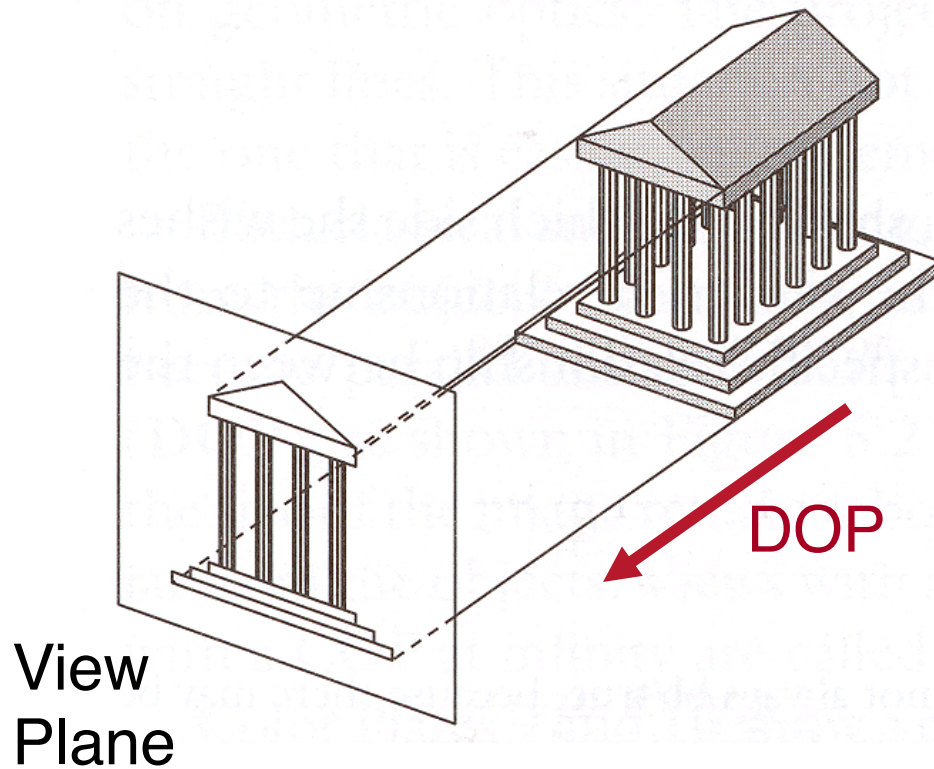


Taxonomy of Projections



Parallel Projection

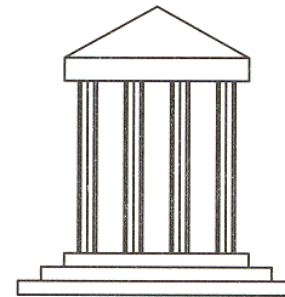
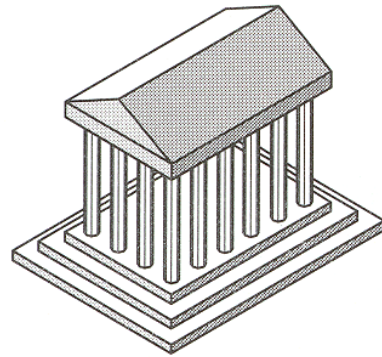
- Center of projection is at infinity
 - Direction of projection (DOP) same for all points



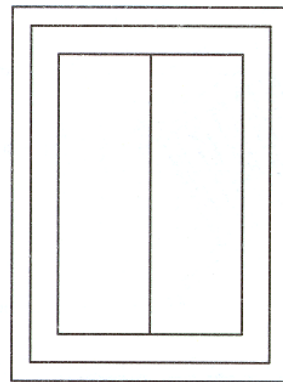
Orthographic Projections



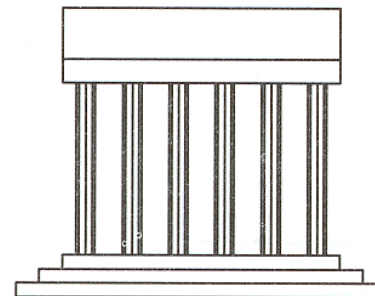
- DOP perpendicular to view plane



Front



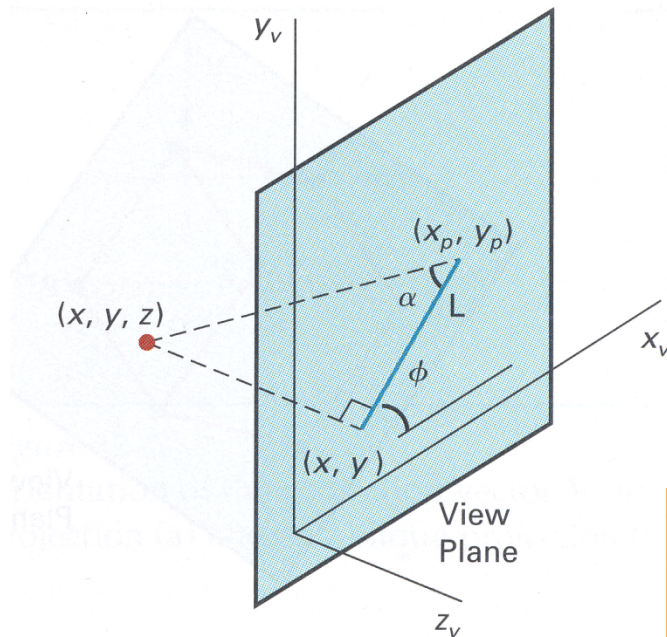
Top



Side

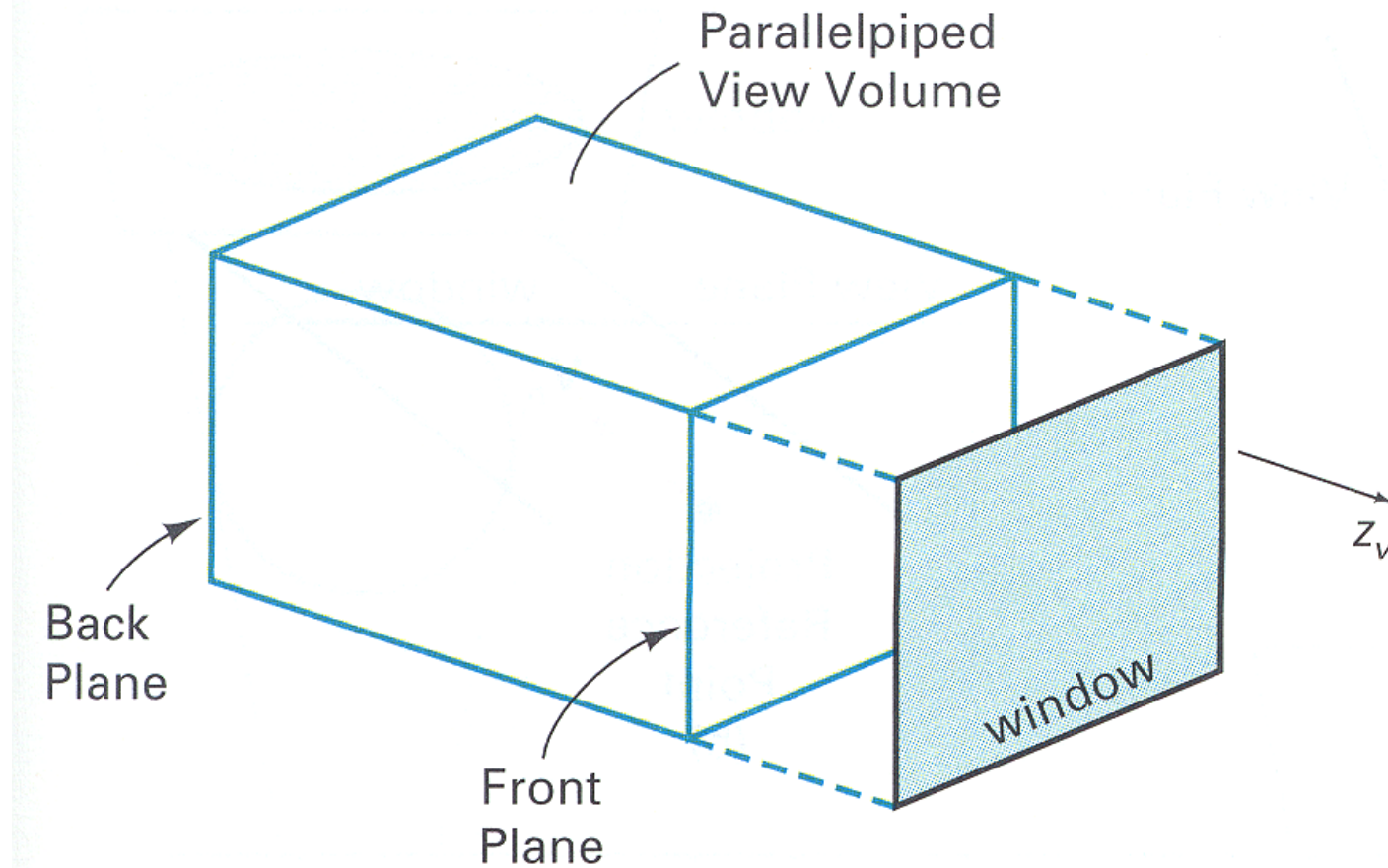
Parallel Projection Matrix

- General parallel projection transformation:



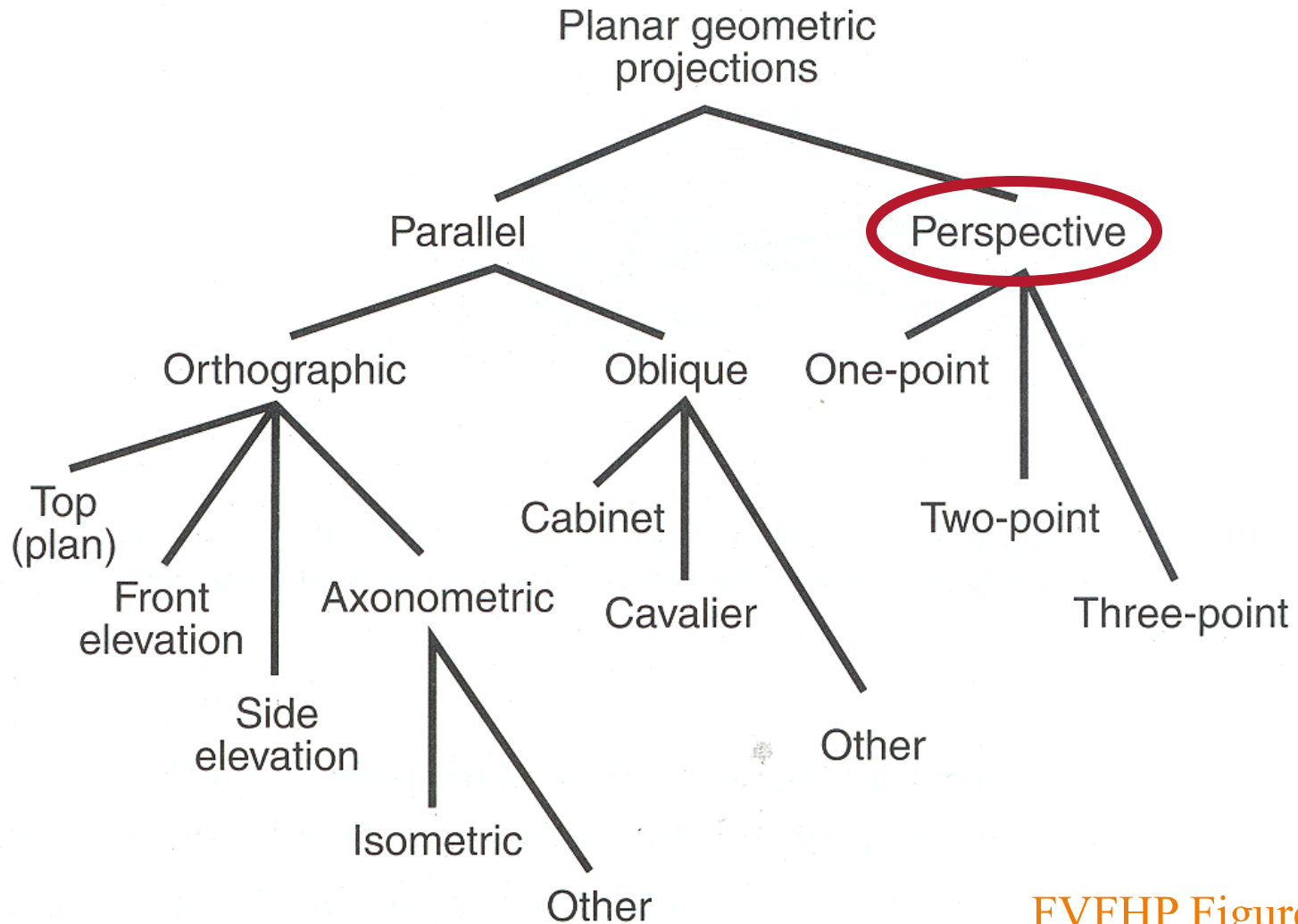
$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & L \cos \phi & 0 \\ 0 & 1 & L \sin \phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Parallel Projection View Volume



H&B Figure 12.30

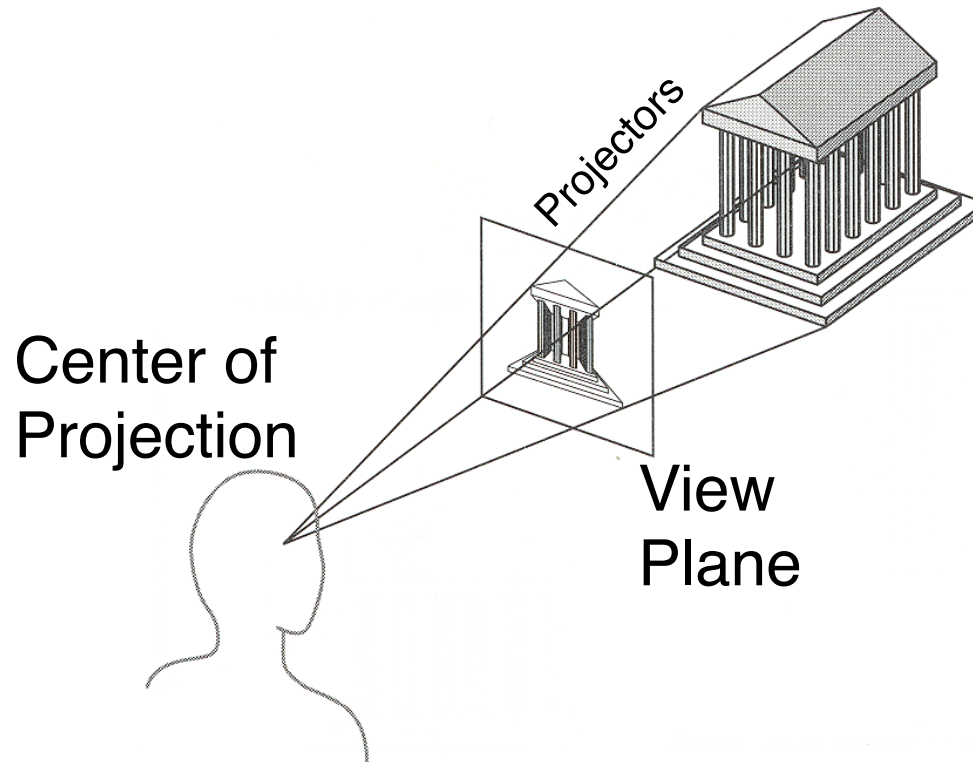
Taxonomy of Projections





Perspective Projection

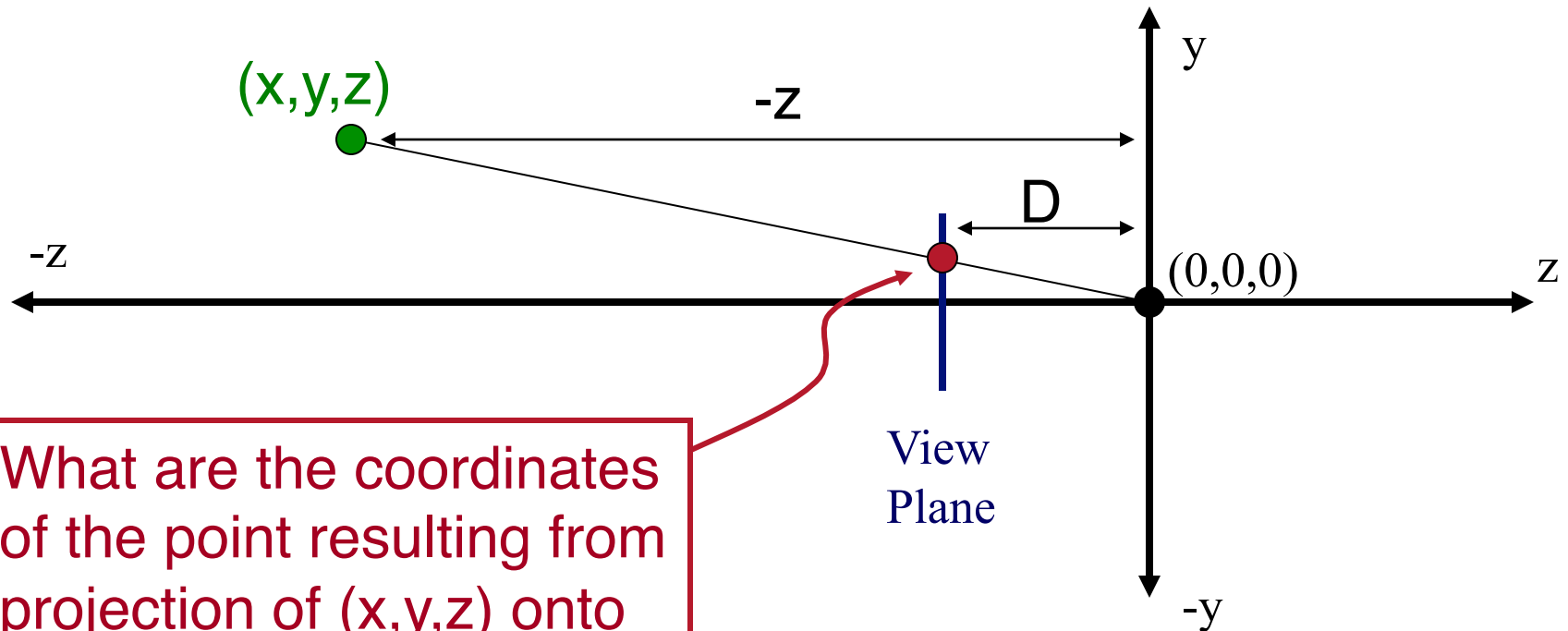
- Map points onto “view plane” along “projectors” emanating from “center of projection” (COP)





Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles

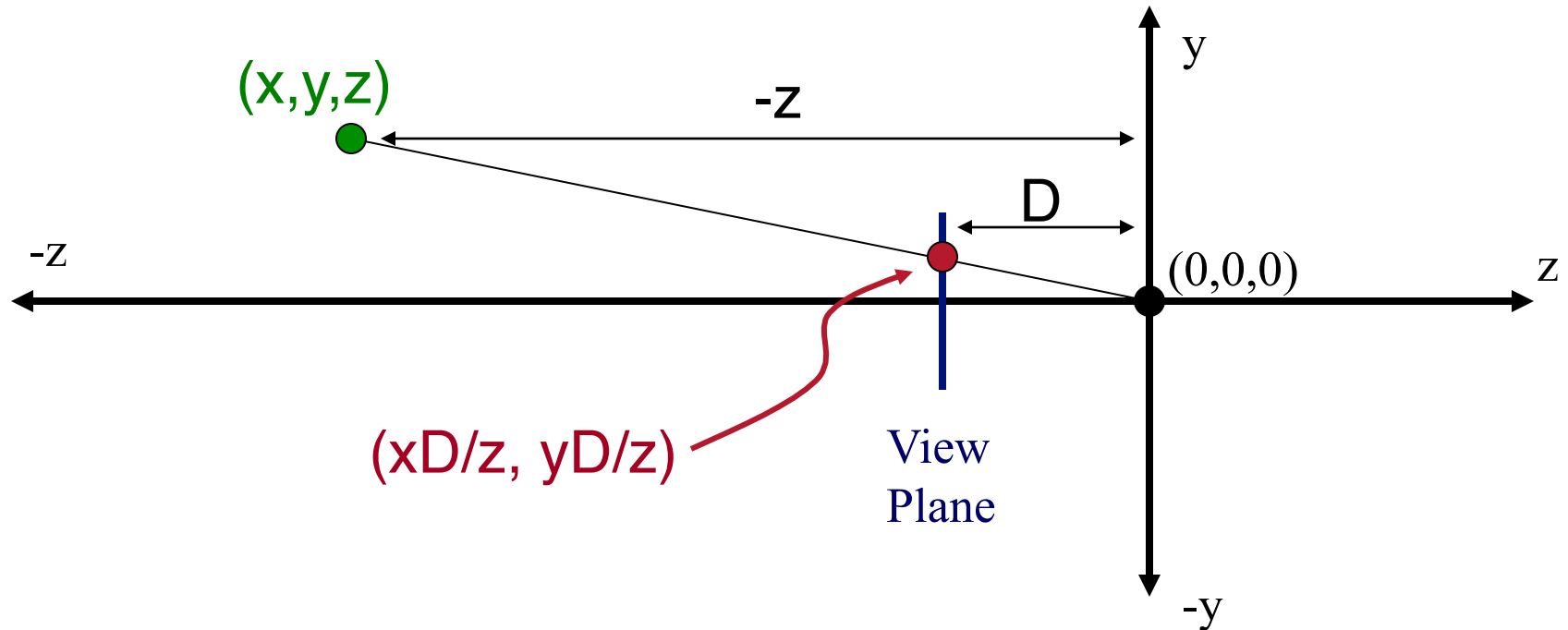


What are the coordinates of the point resulting from projection of (x,y,z) onto the view plane?



Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles





Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$

$$y_s = y_c D / z_c$$

$$z_s = D$$

$$w_s = 1$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$



Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$

$$y_s = y_c D / z_c$$

$$z_s = D$$

$$w_s = 1$$

$$x_s = x' / w'$$

$$y_s = y' / w'$$

$$z_s = z' / w'$$

$$x' = x_c$$

$$y' = y_c$$

$$z' = z_c$$

$$w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$



Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$

$$y_s = y_c D / z_c$$

$$z_s = D$$

$$w_s = 1$$

$$x_s = x' / w'$$

$$y_s = y' / w'$$

$$z_s = z' / w'$$

$$x' = x_c$$

$$y' = y_c$$

$$z' = z_c$$

$$w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$



Perspective Projection Matrix

- In practice, want to compute a value related to depth to include in z-buffer

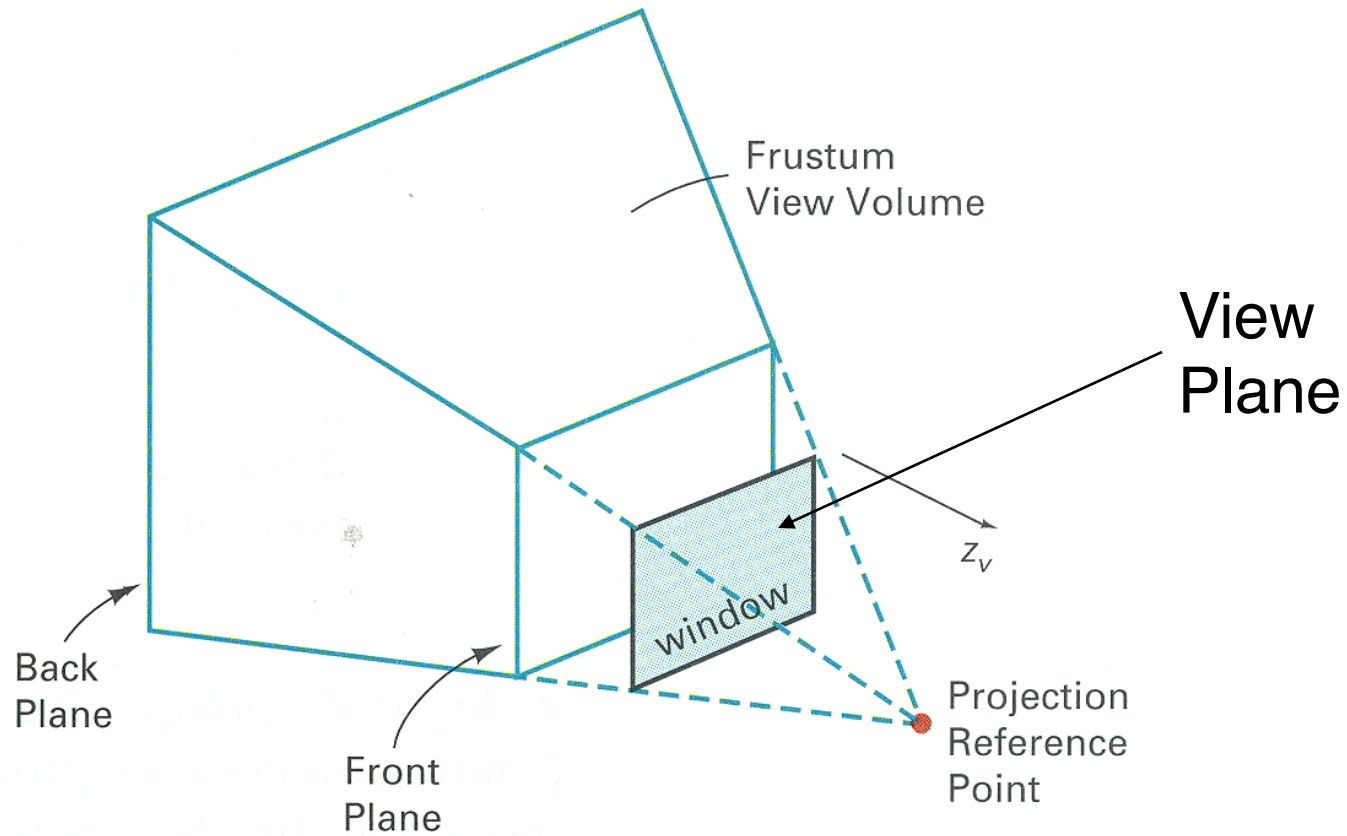
$$\begin{aligned}x_s &= x_c D / z_c \\y_s &= y_c D / z_c \\z_s &= -D / z_c \\w_s &= 1\end{aligned}$$

$$\begin{aligned}x_s &= x' / w' \\y_s &= y' / w' \\z_s &= z' / w'\end{aligned}$$

$$\begin{aligned}x' &= x_c \\y' &= y_c \\z' &= -1 \\w' &= z_c / D\end{aligned}$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection View Volume

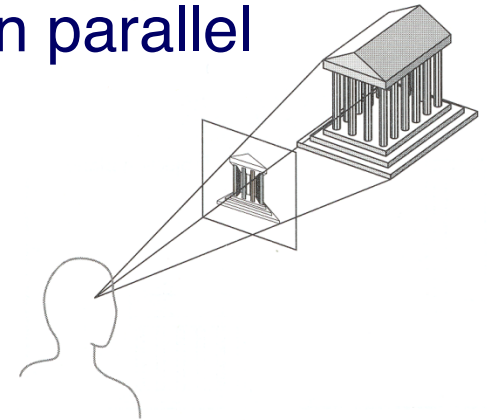


H&B Figure 12.30

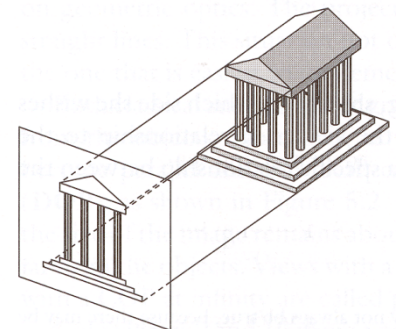
Perspective vs. Parallel



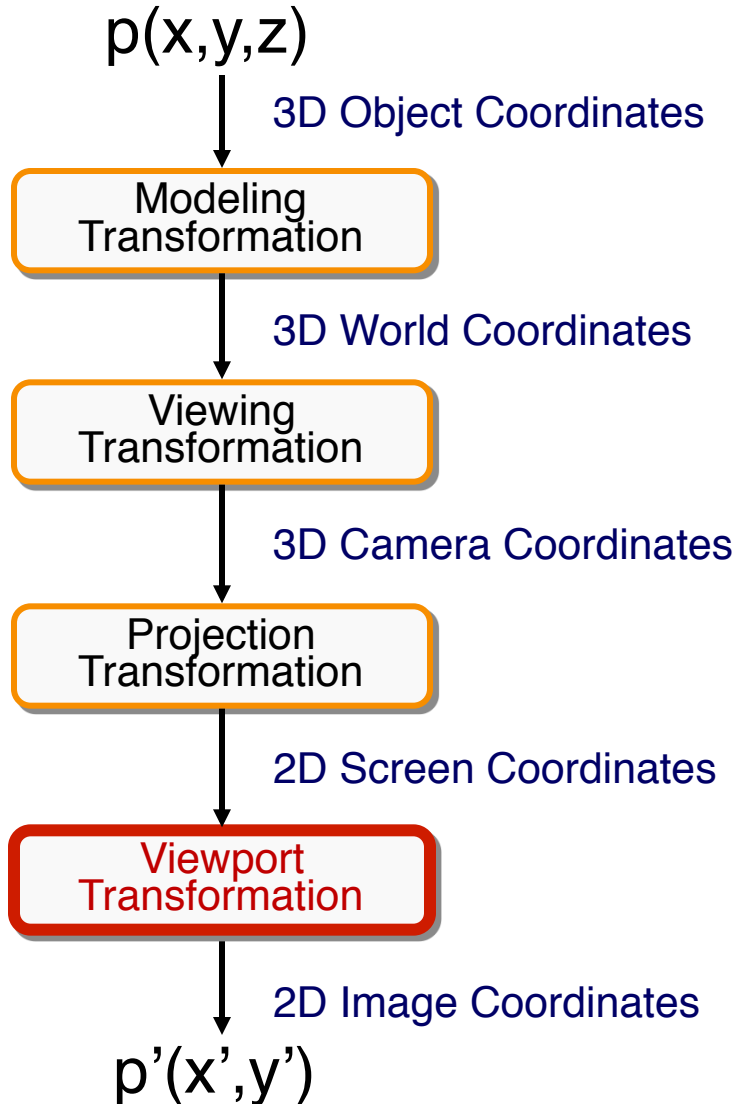
- Perspective projection
 - + Size varies inversely with distance - looks realistic
 - Distance and angles are not (in general) preserved
 - Parallel lines do not (in general) remain parallel



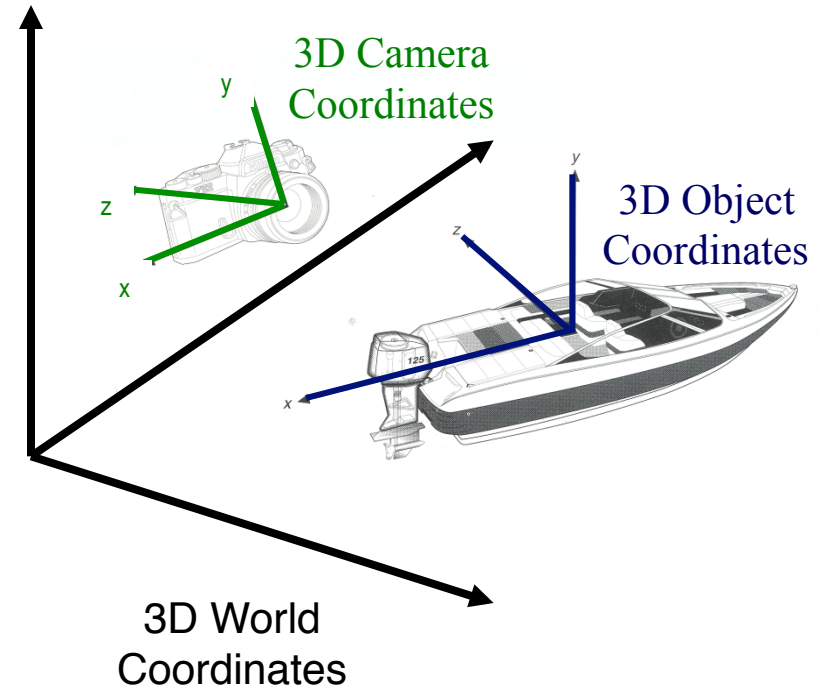
- Parallel projection
 - + Good for exact measurements
 - + Parallel lines remain parallel
 - Angles are not (in general) preserved
 - Less realistic looking



Transformations

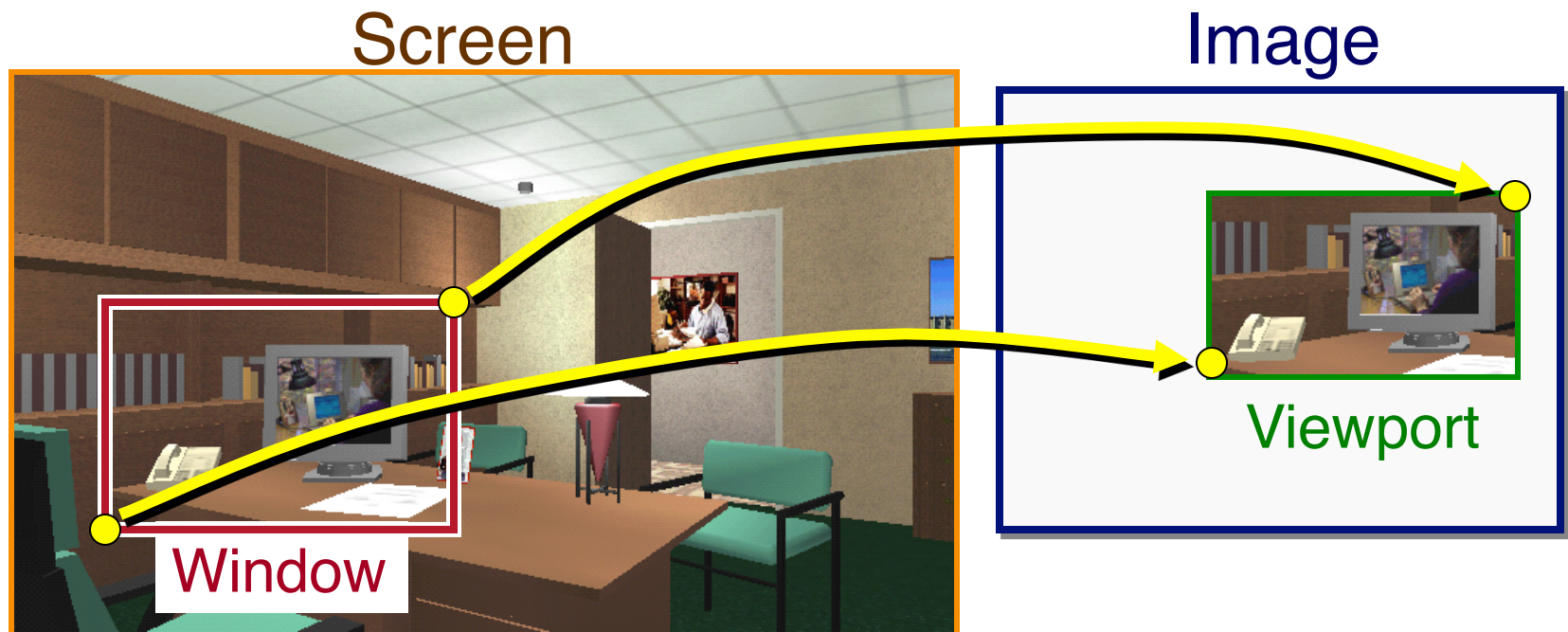


Transformations map points from one coordinate system to another



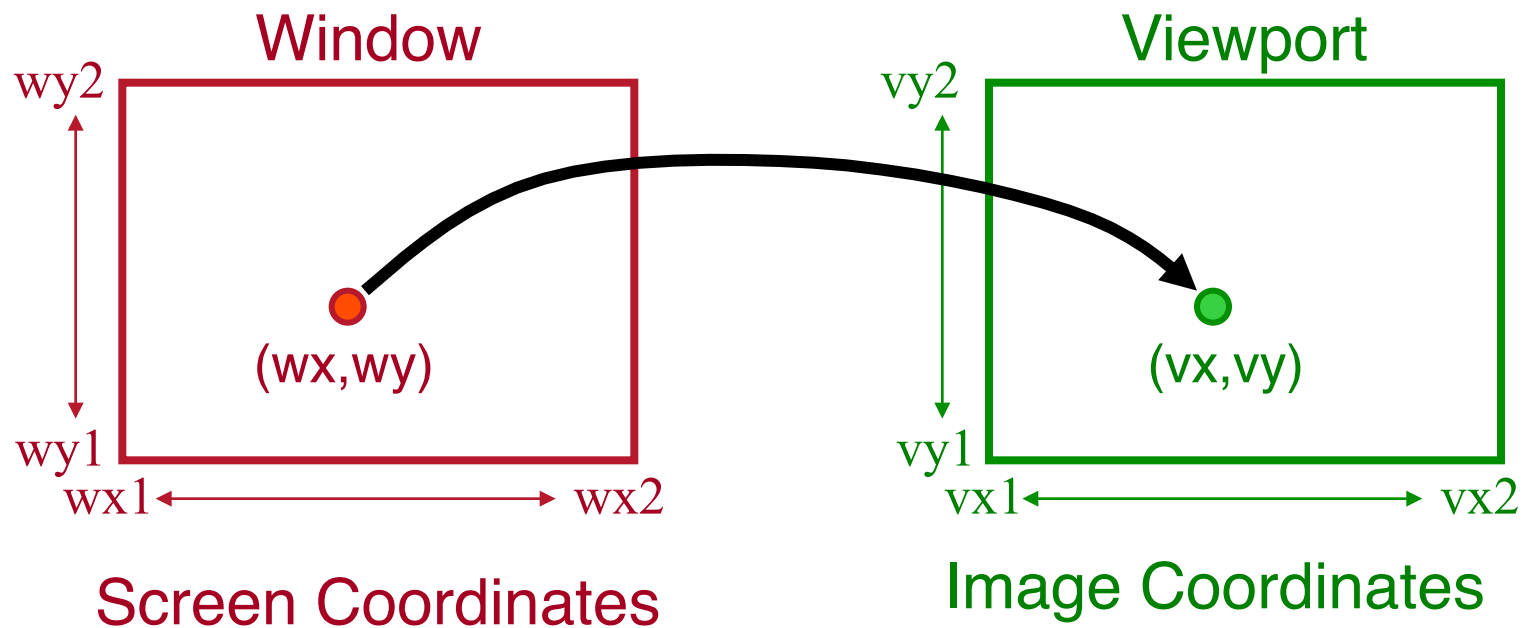
Viewport Transformation

- Transform 2D geometric primitives from screen coordinate system (normalized device coordinates) to image coordinate system (pixels)



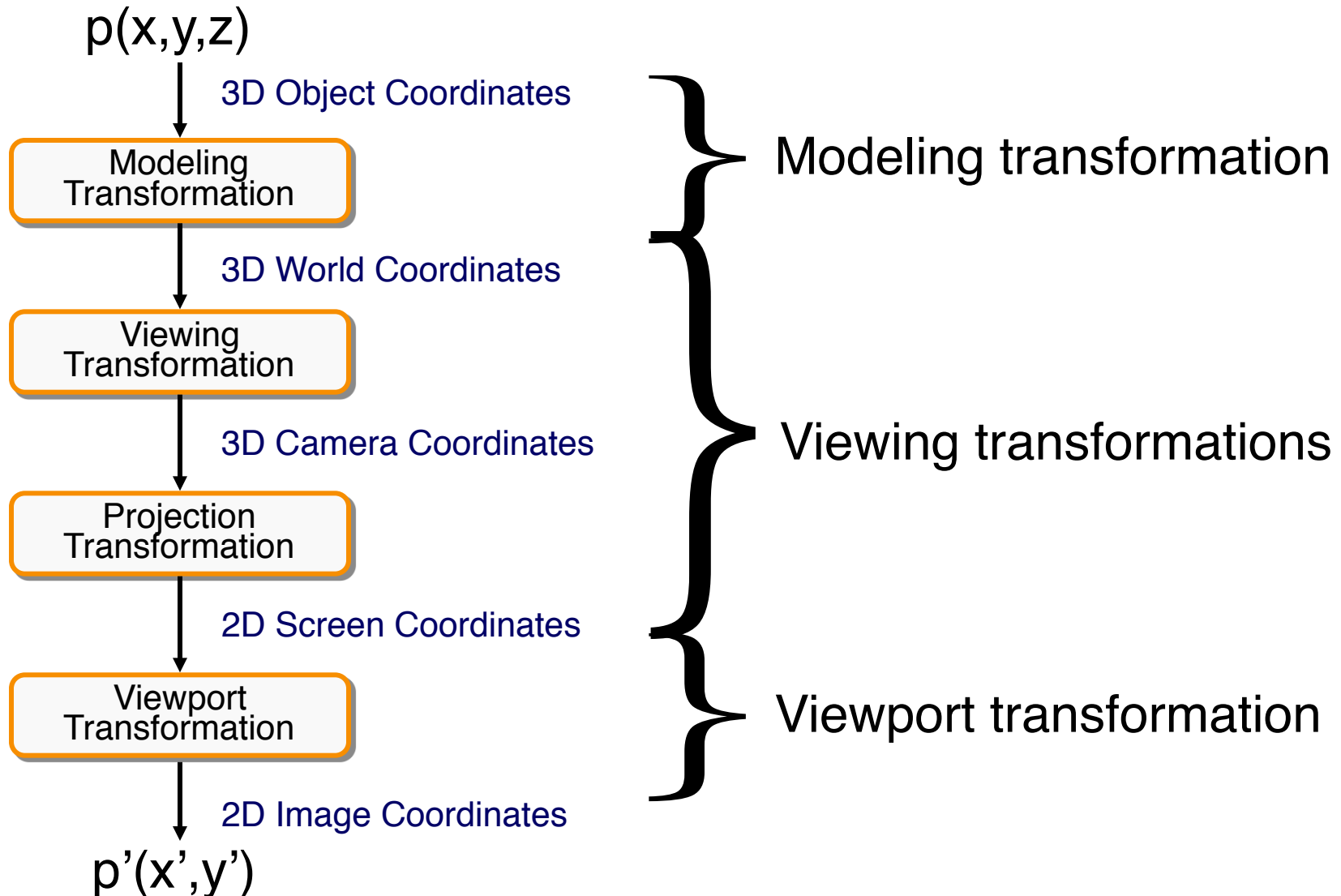
Viewport Transformation

- Window-to-viewport mapping

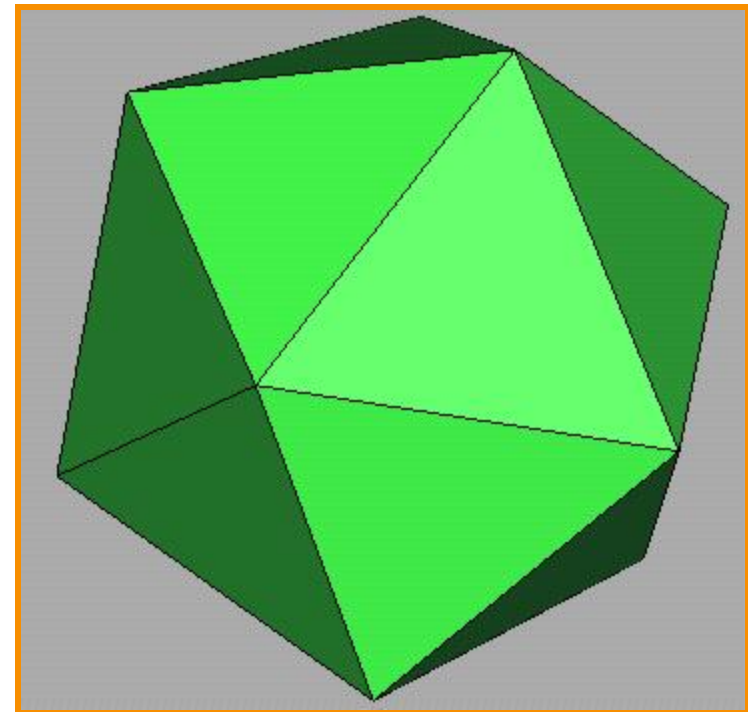
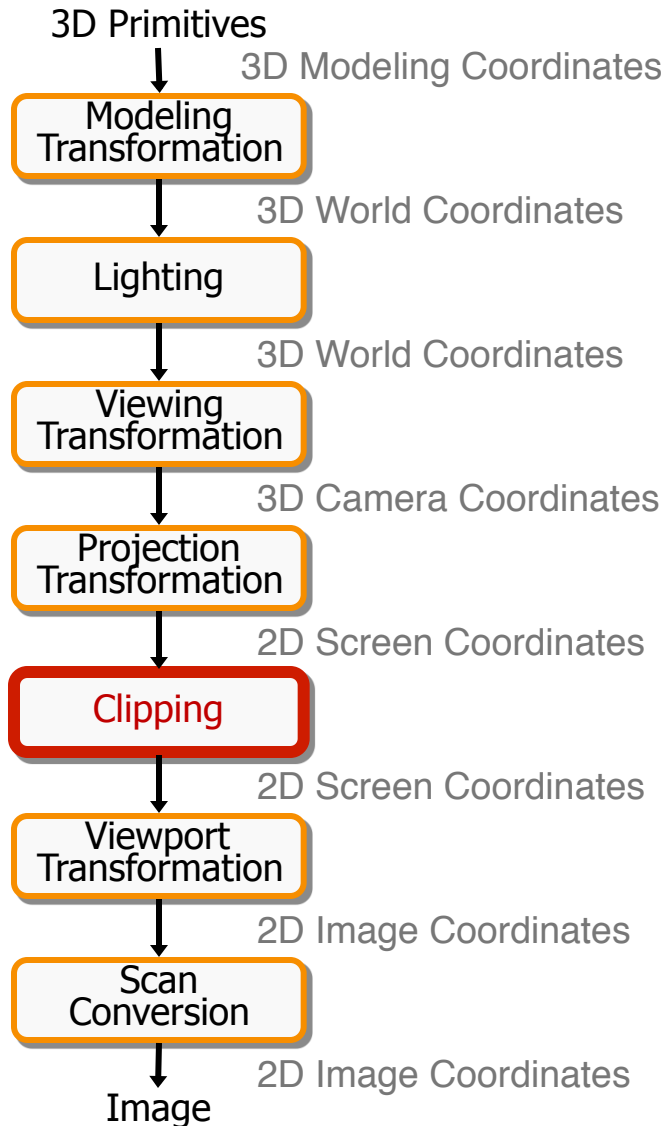


$$\begin{aligned} vx &= vx1 + (wx - wx1) * (vx2 - vx1) / (wx2 - wx1); \\ vy &= vy1 + (wy - wy1) * (vy2 - vy1) / (wy2 - wy1); \end{aligned}$$

Summary of Transformations

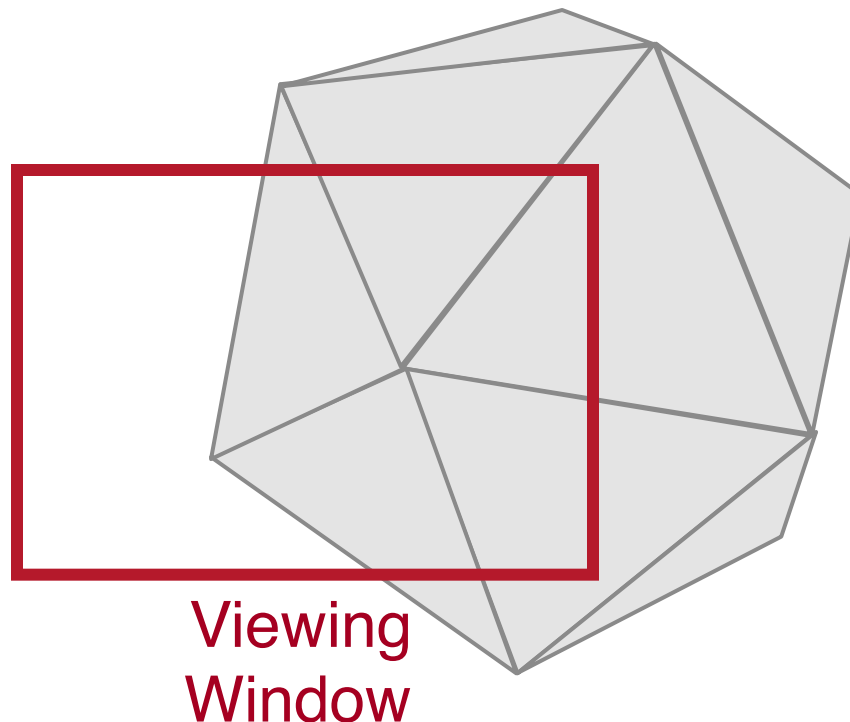


3D Rendering Pipeline (for direct illumination)



Clipping

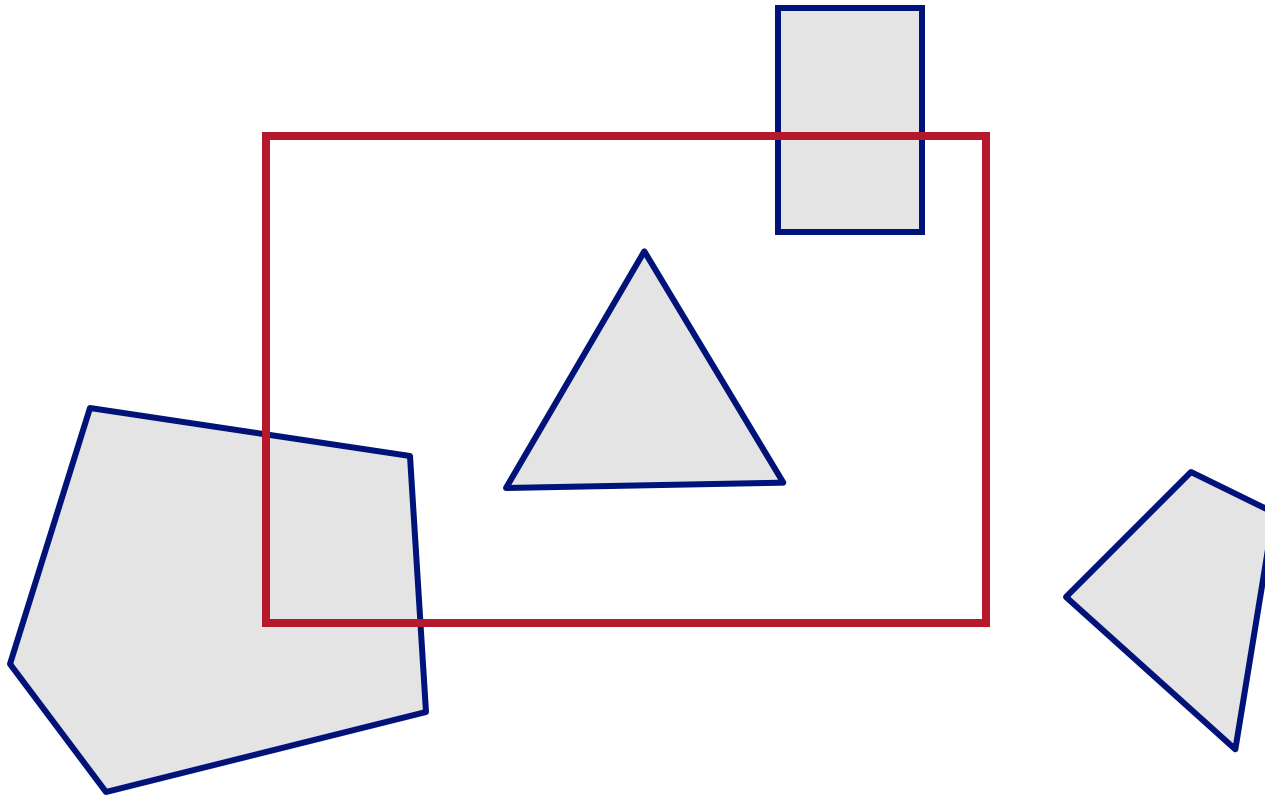
- Avoid drawing parts of primitives outside window
 - Window defines part of scene being viewed
 - Must draw geometric primitives only inside window





Polygon Clipping

- Find the part of a polygon inside the clip window?

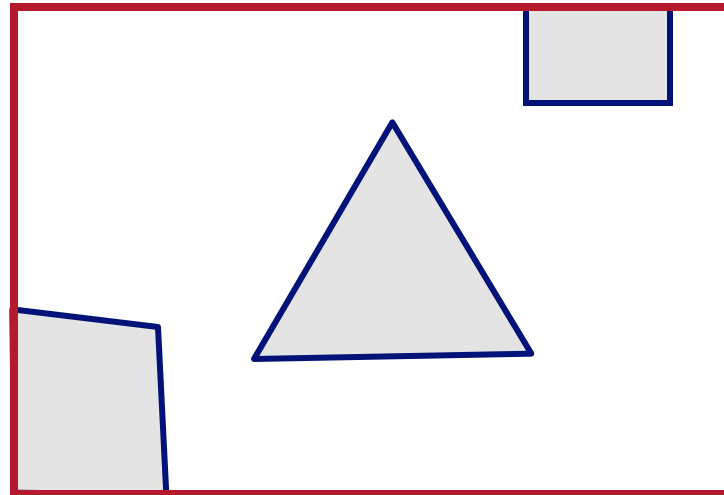


Before Clipping



Polygon Clipping

- Find the part of a polygon inside the clip window?

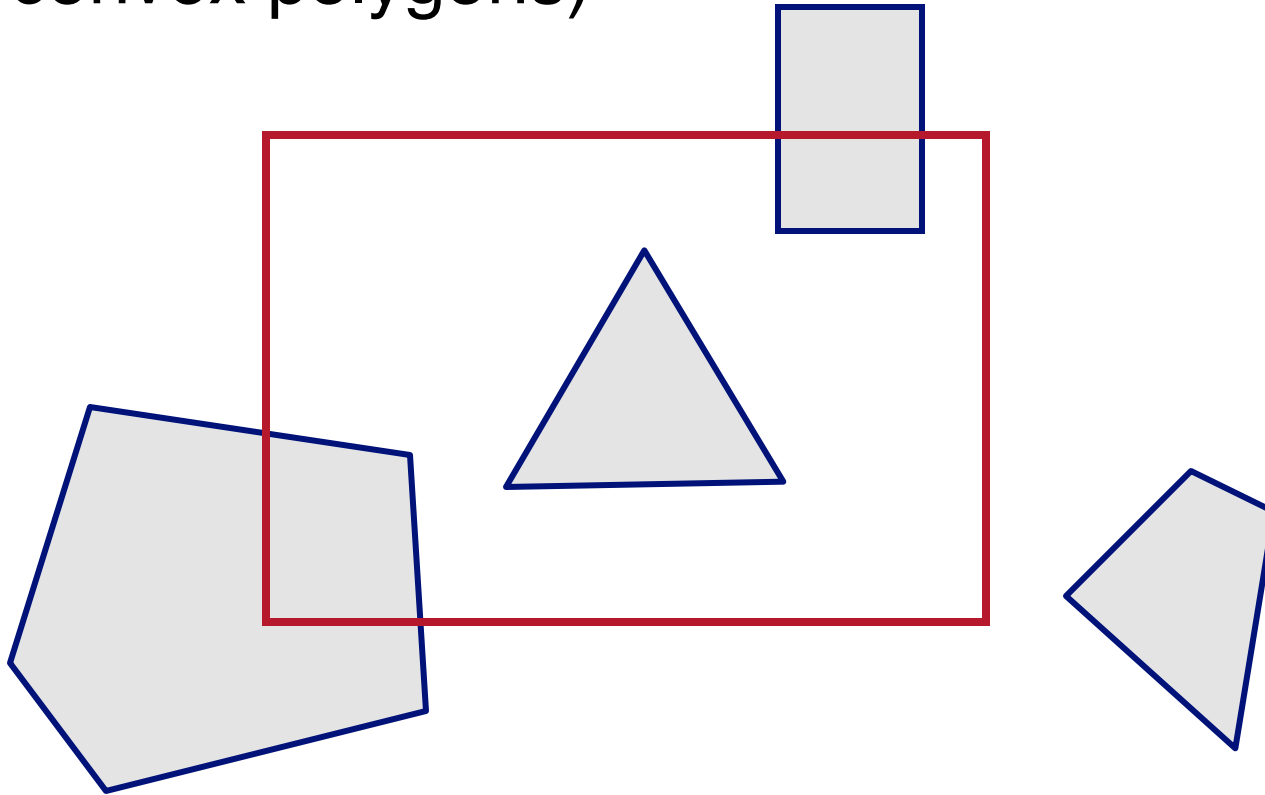


After Clipping

Sutherland Hodgeman Clipping



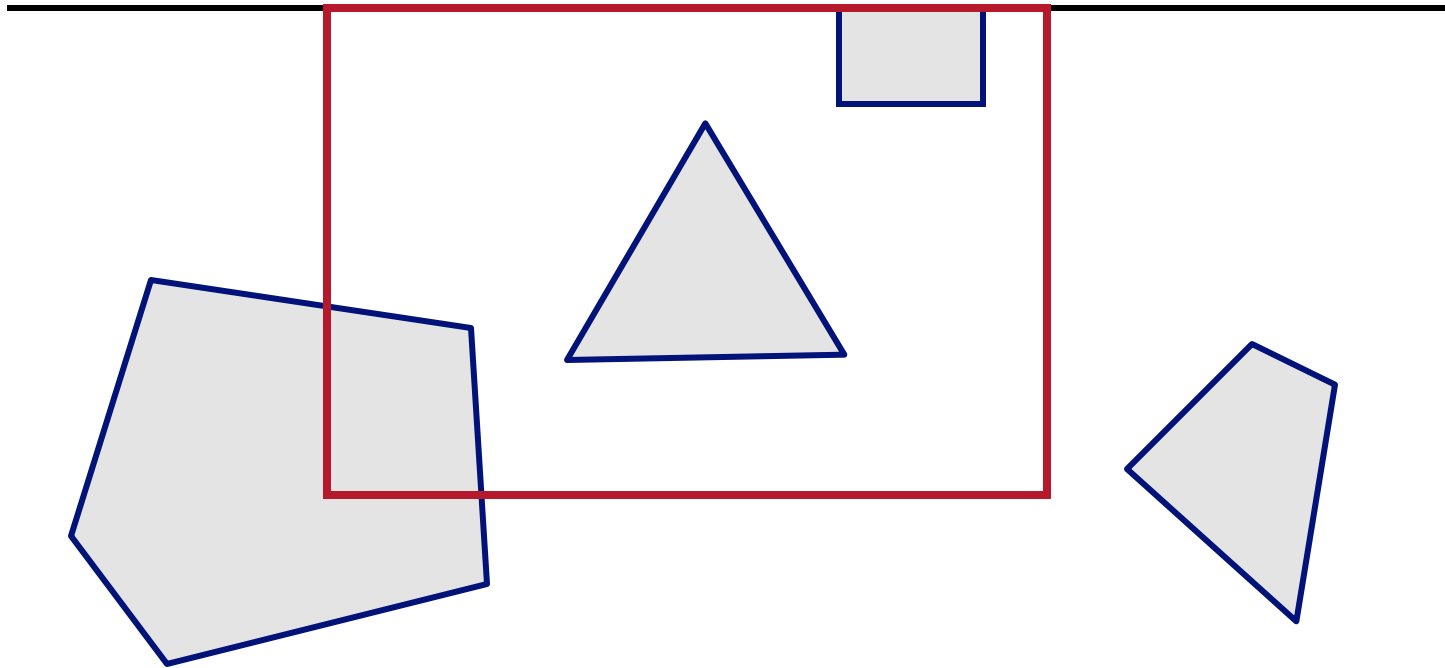
- Clip to each window boundary one at a time (for convex polygons)



Sutherland Hodgeman Clipping



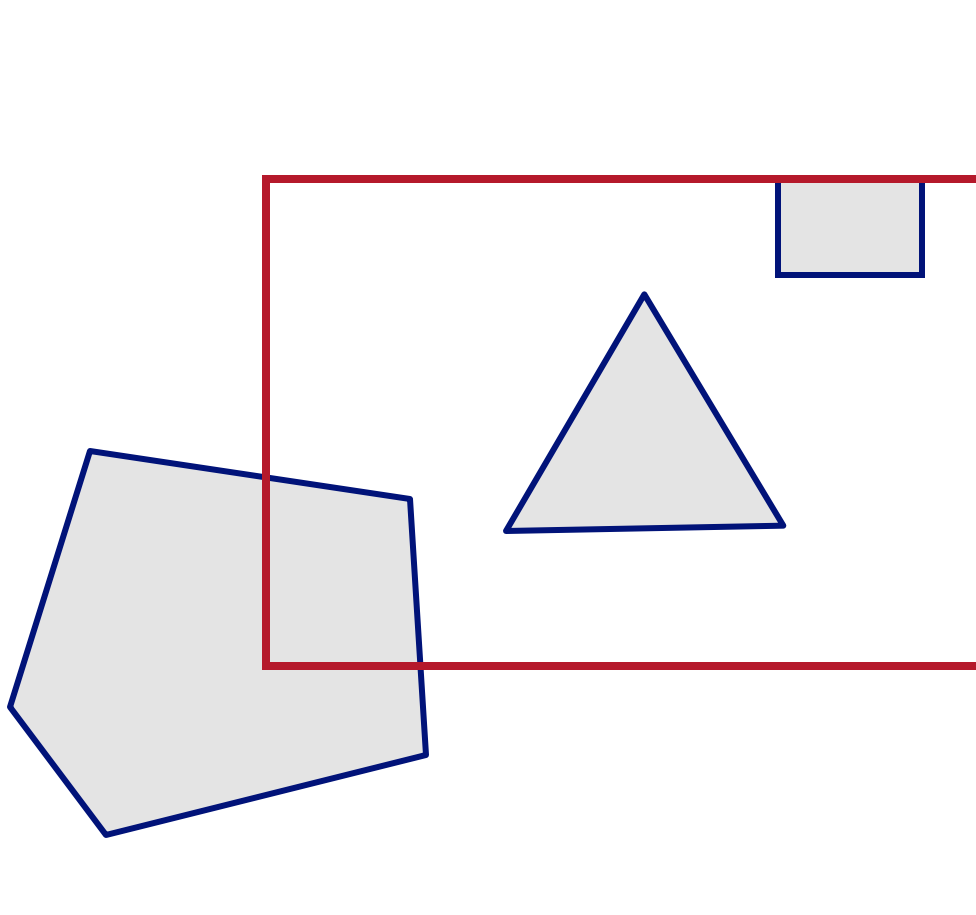
- Clip to each window boundary one at a time



Sutherland Hodgeman Clipping



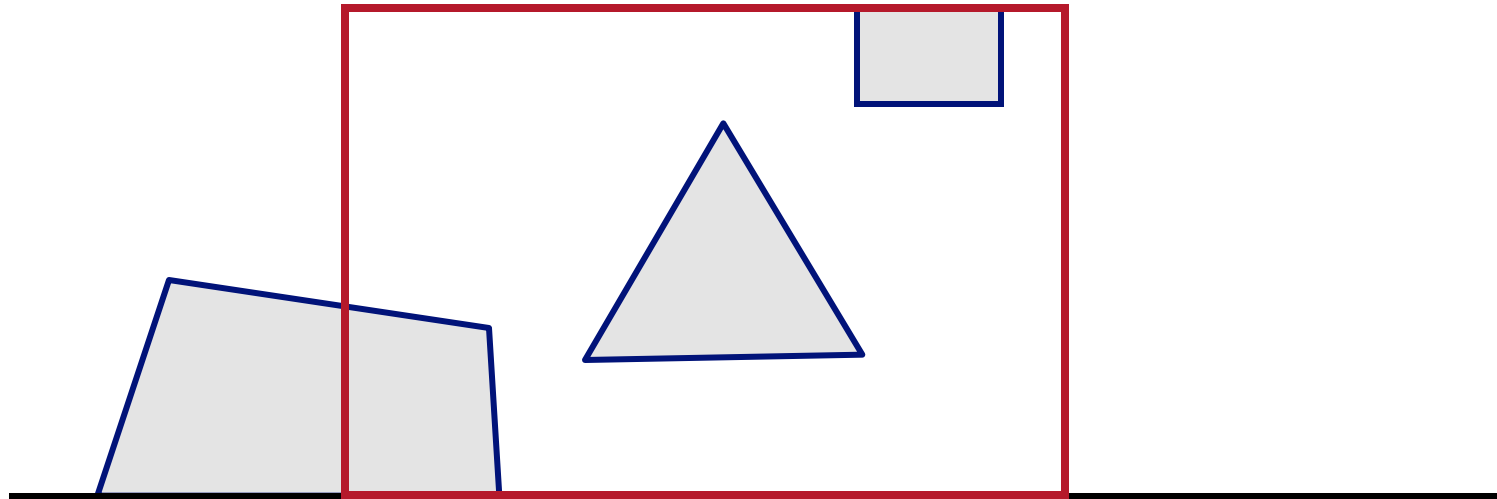
- Clip to each window boundary one at a time



Sutherland Hodgeman Clipping



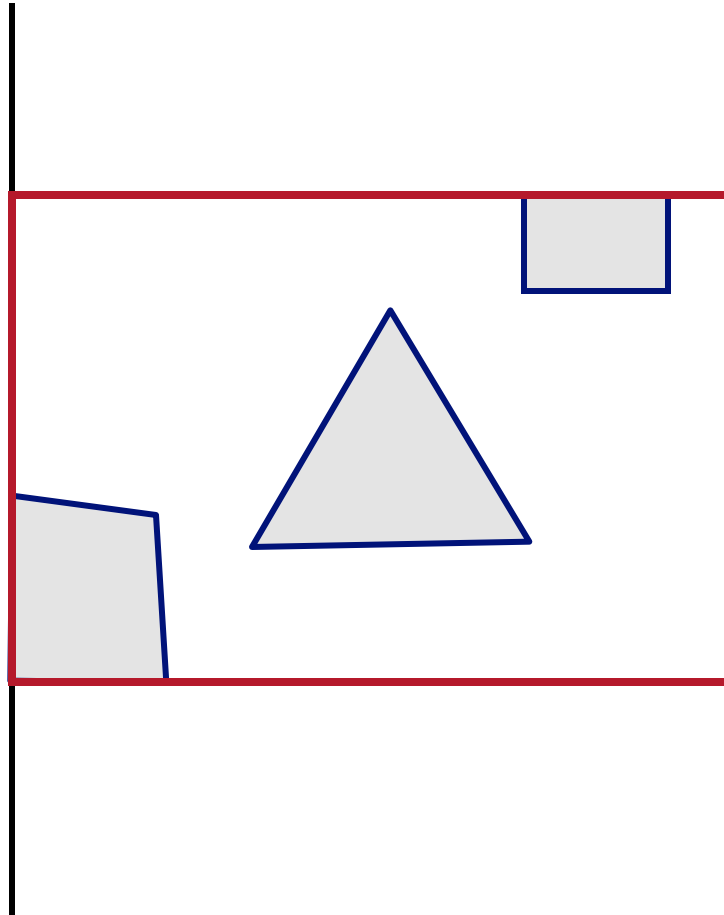
- Clip to each window boundary one at a time



Sutherland Hodgeman Clipping



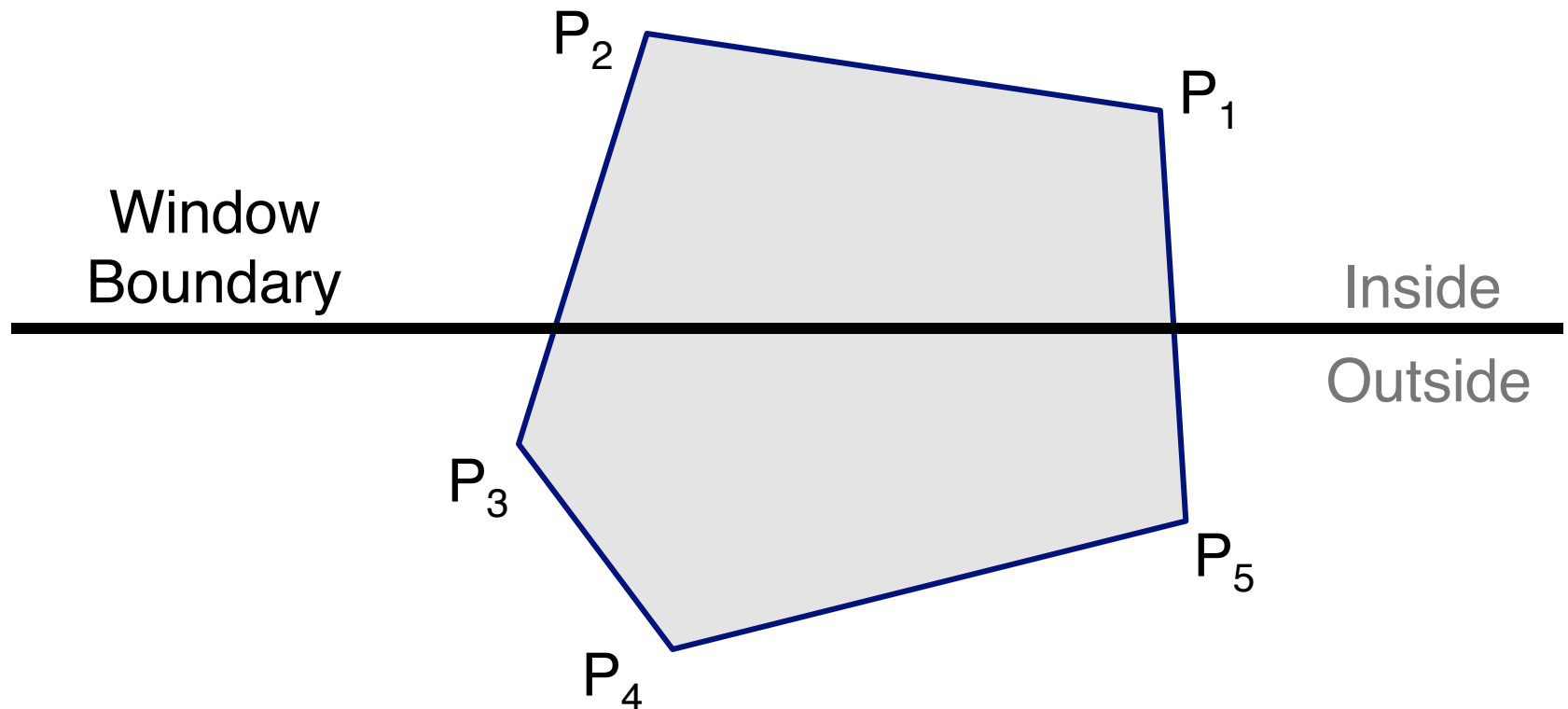
- Clip to each window boundary one at a time





Clipping to a Boundary

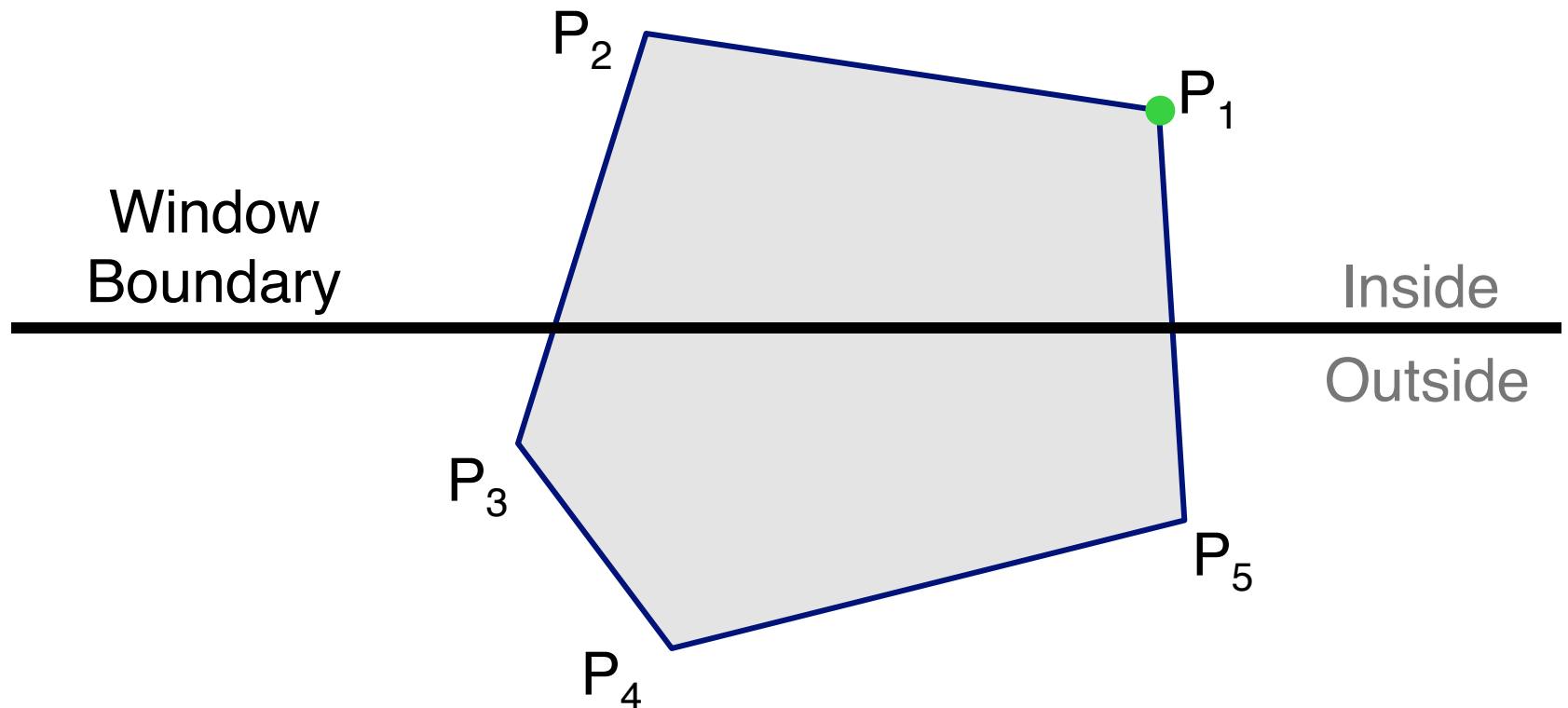
- Do **inside** test for each point in sequence,
Insert new points when cross window boundary,
Remove points outside window boundary





Clipping to a Boundary

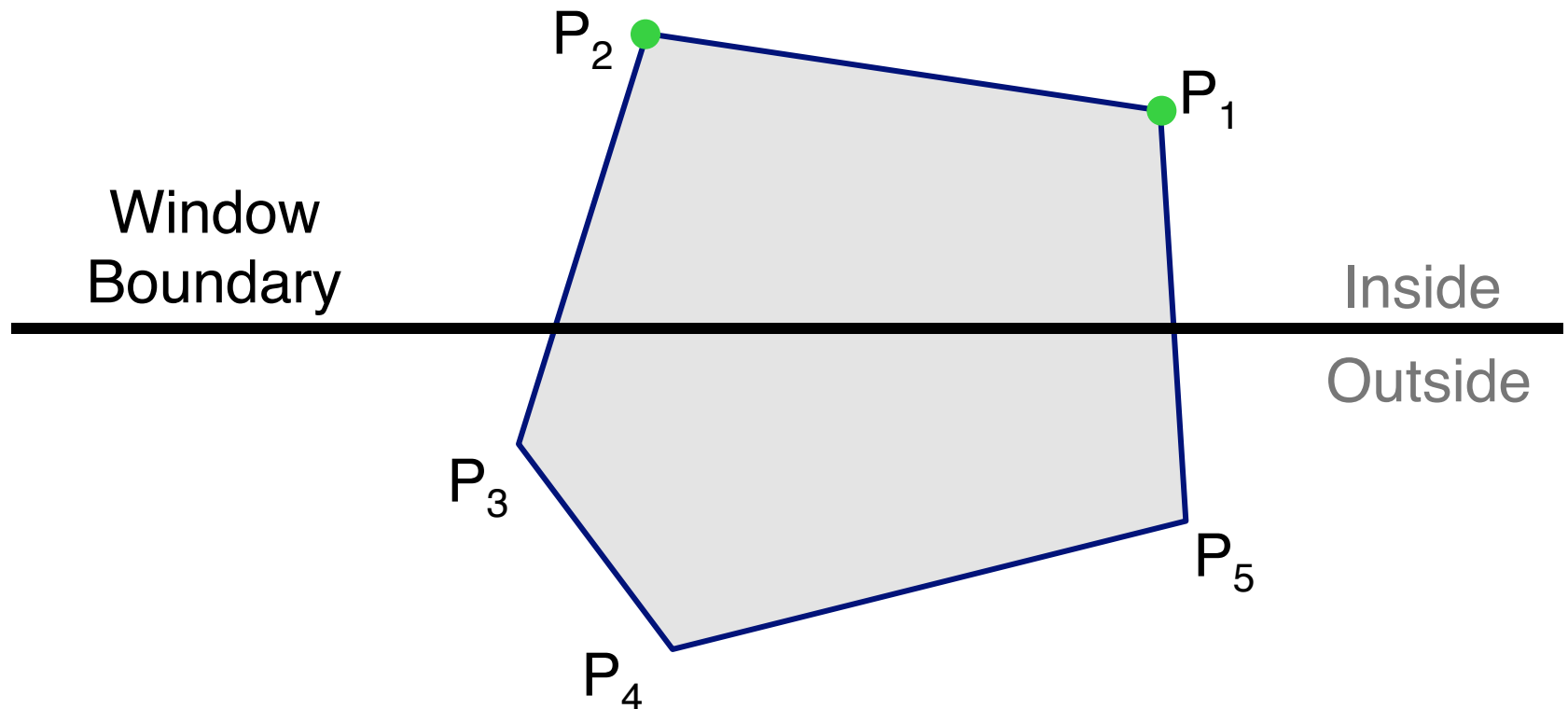
- Do **inside** test for each point in sequence,
Insert new points when cross window boundary,
Remove points outside window boundary





Clipping to a Boundary

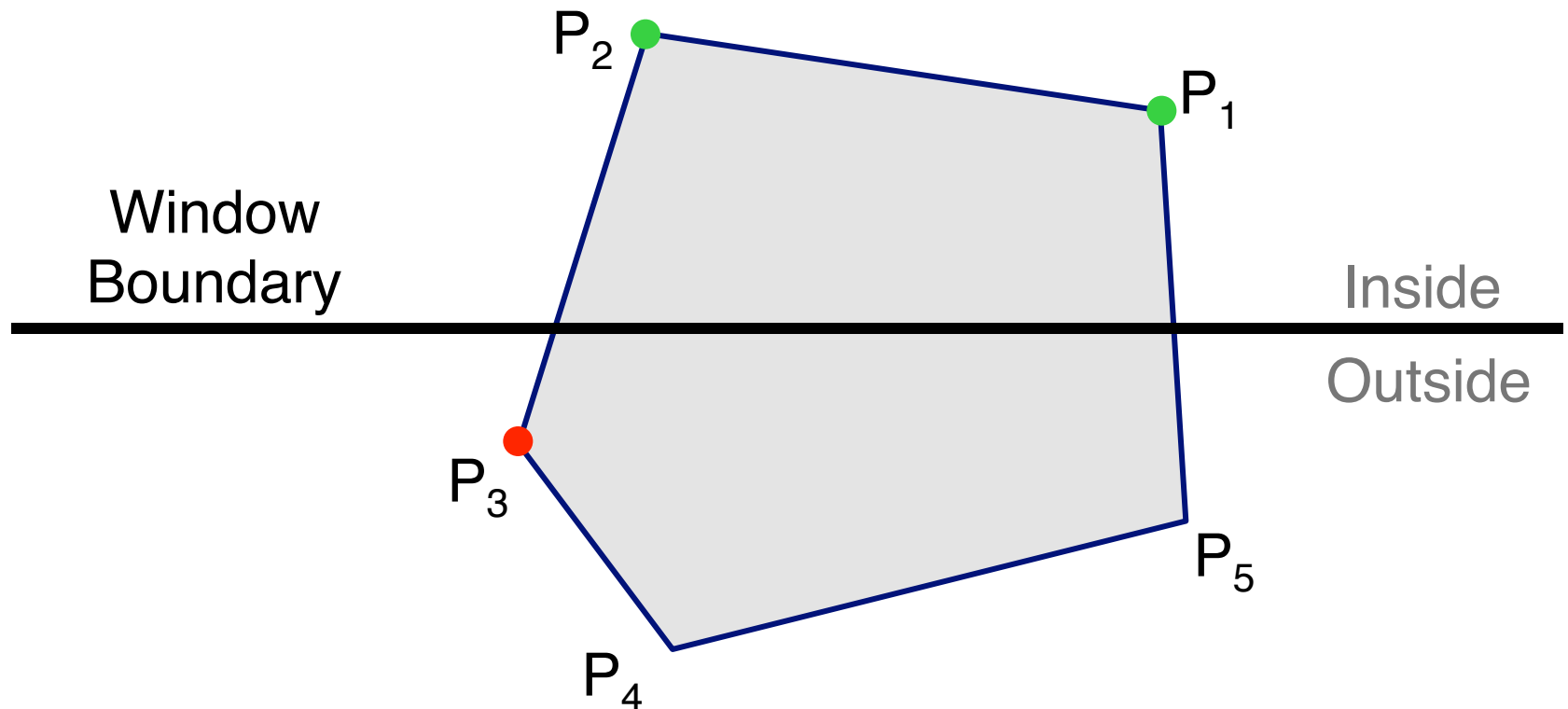
- Do **inside** test for each point in sequence,
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Remove points outside window boundary





Clipping to a Boundary

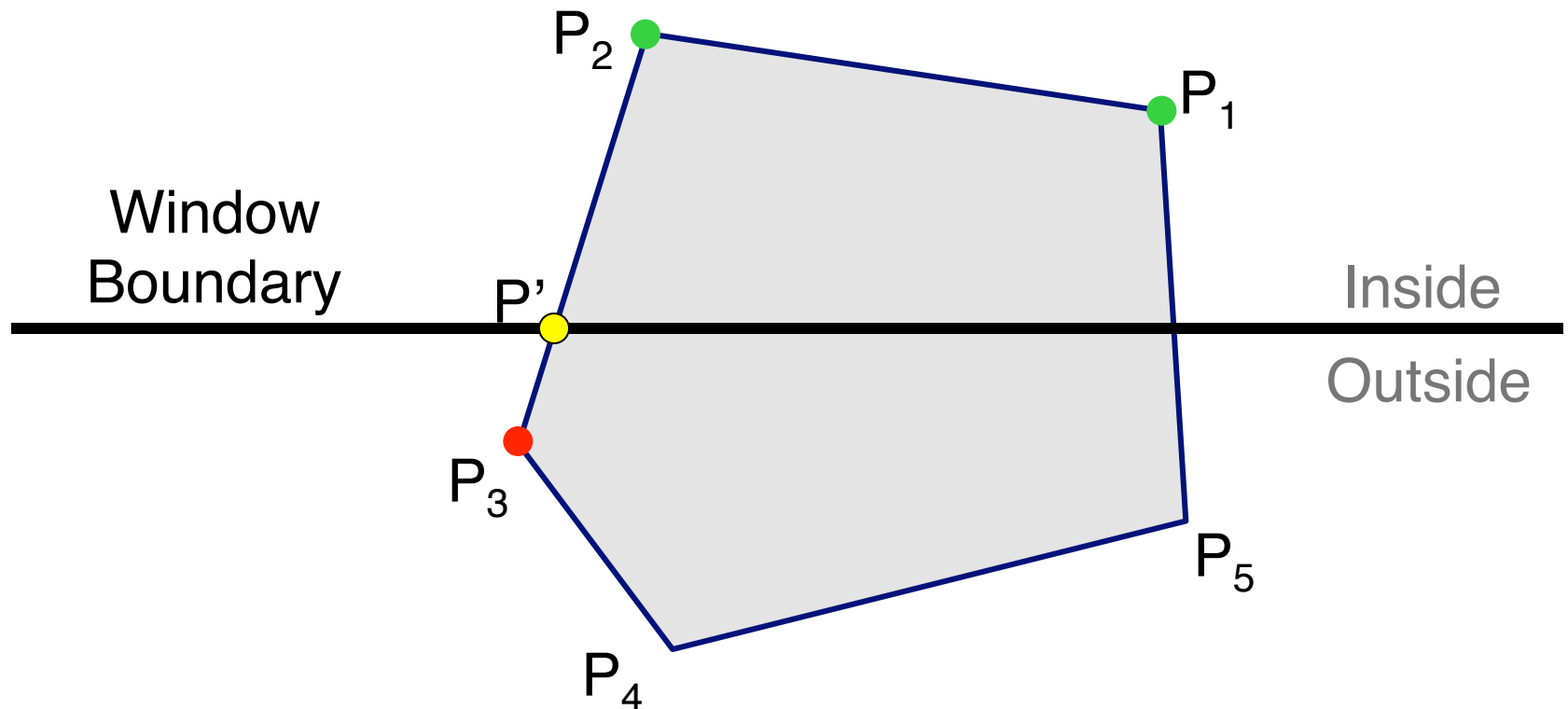
- Do **inside** test for each point in sequence,
Insert new points when cross window boundary,
Remove points outside window boundary





Clipping to a Boundary

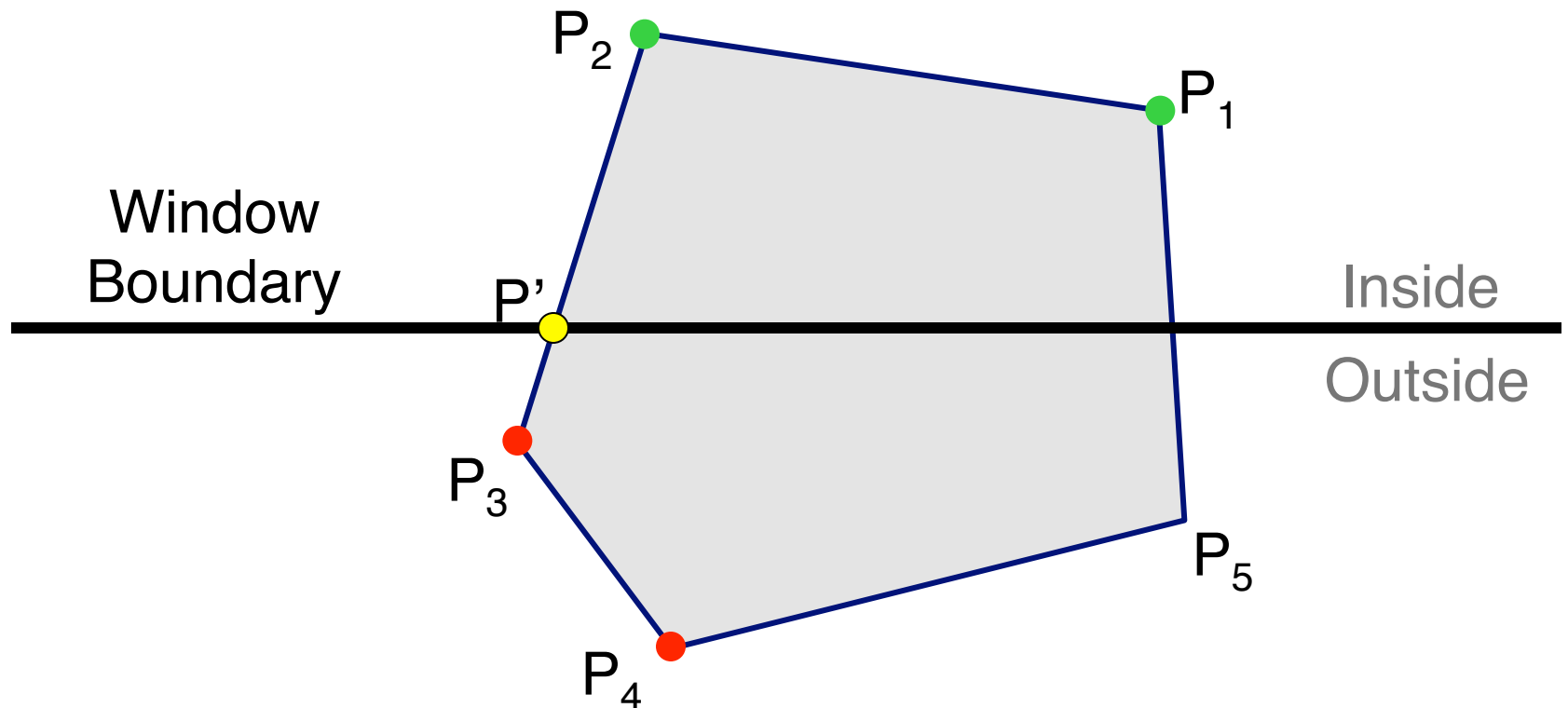
- Do **inside** test for each point in sequence,
Insert new points when cross window boundary,
Remove points outside window boundary





Clipping to a Boundary

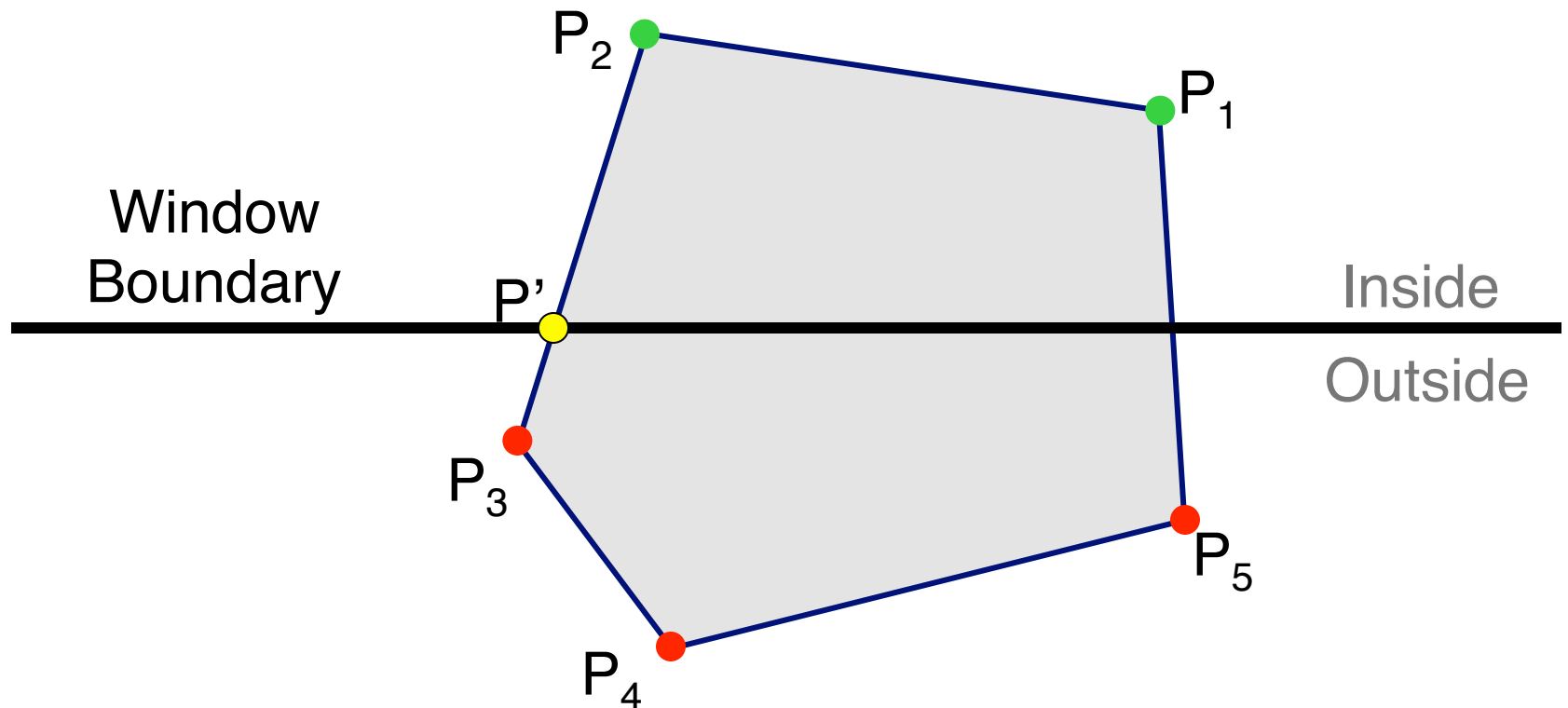
- Do **inside** test for each point in sequence,
Insert new points when cross window boundary,
Remove points outside window boundary





Clipping to a Boundary

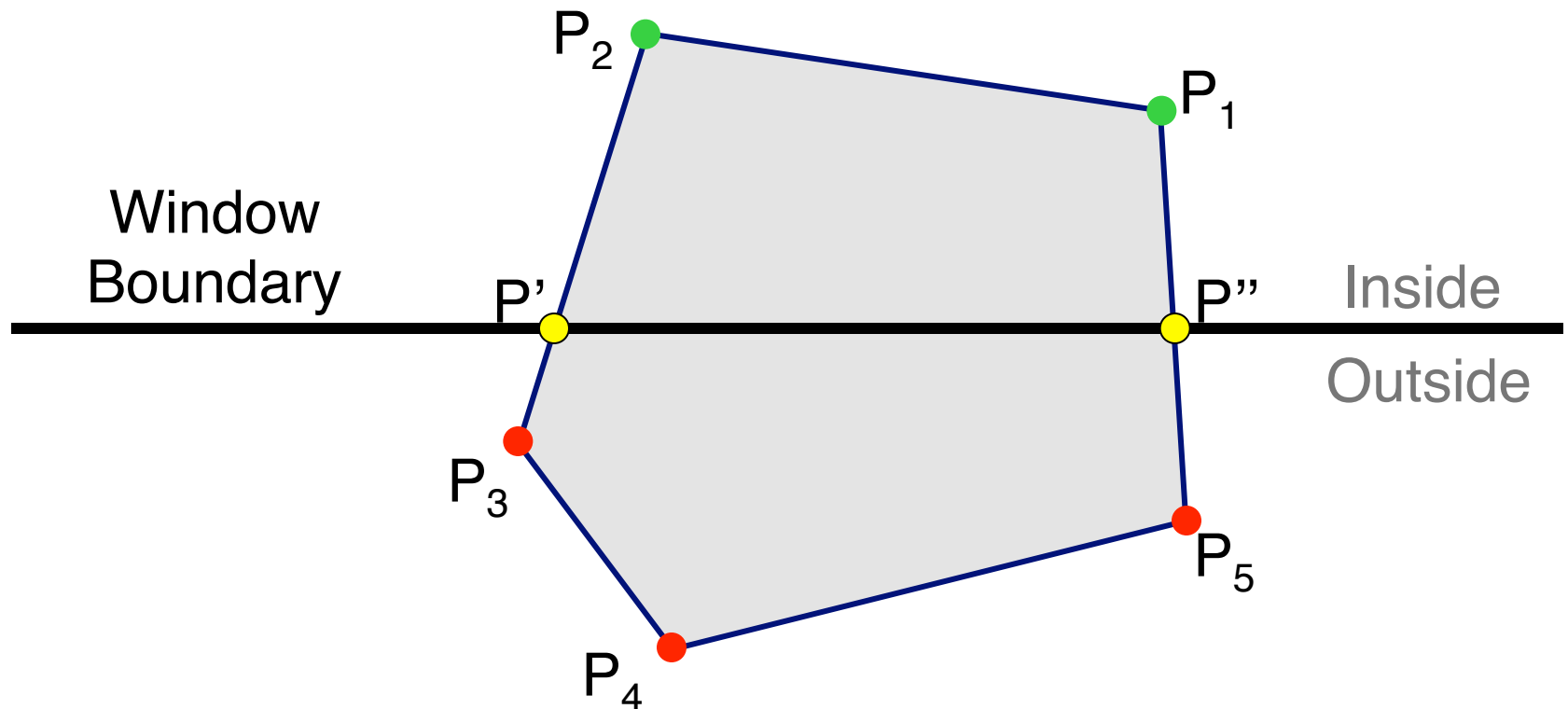
- Do **inside** test for each point in sequence,
Insert new points when cross window boundary,
Remove points outside window boundary





Clipping to a Boundary

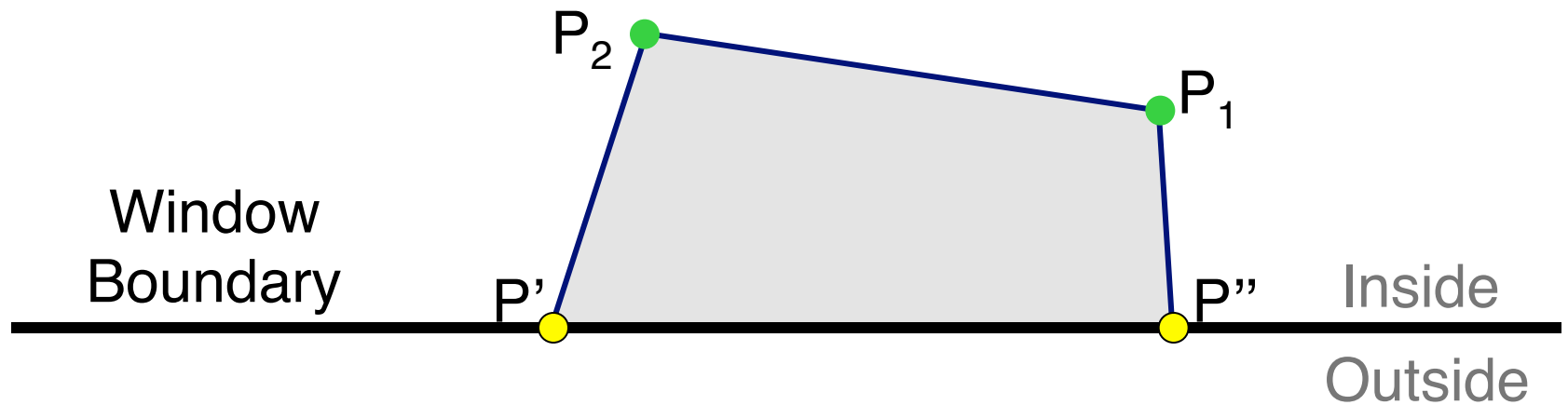
- Do **inside** test for each point in sequence,
Insert new points when cross window boundary,
Remove points outside window boundary



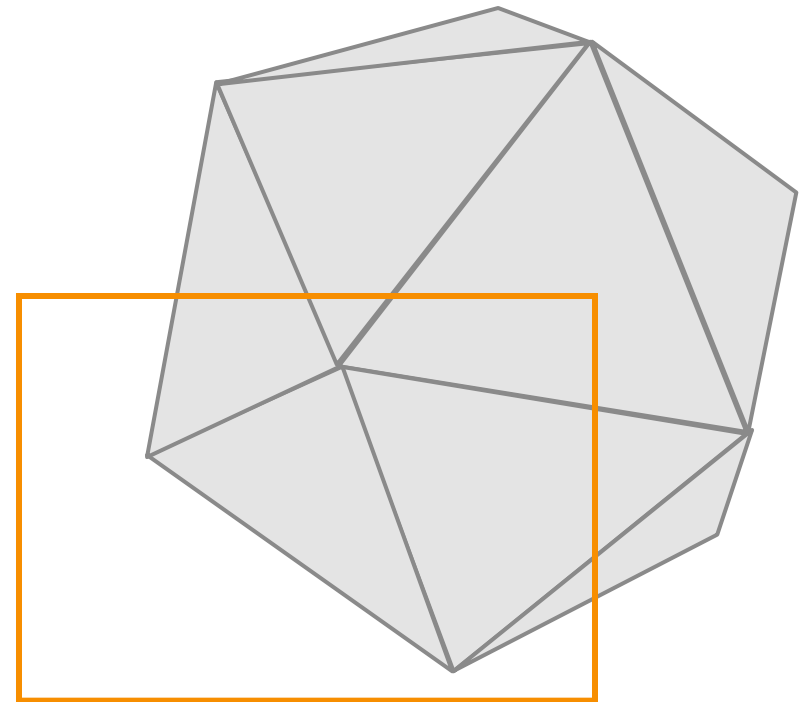
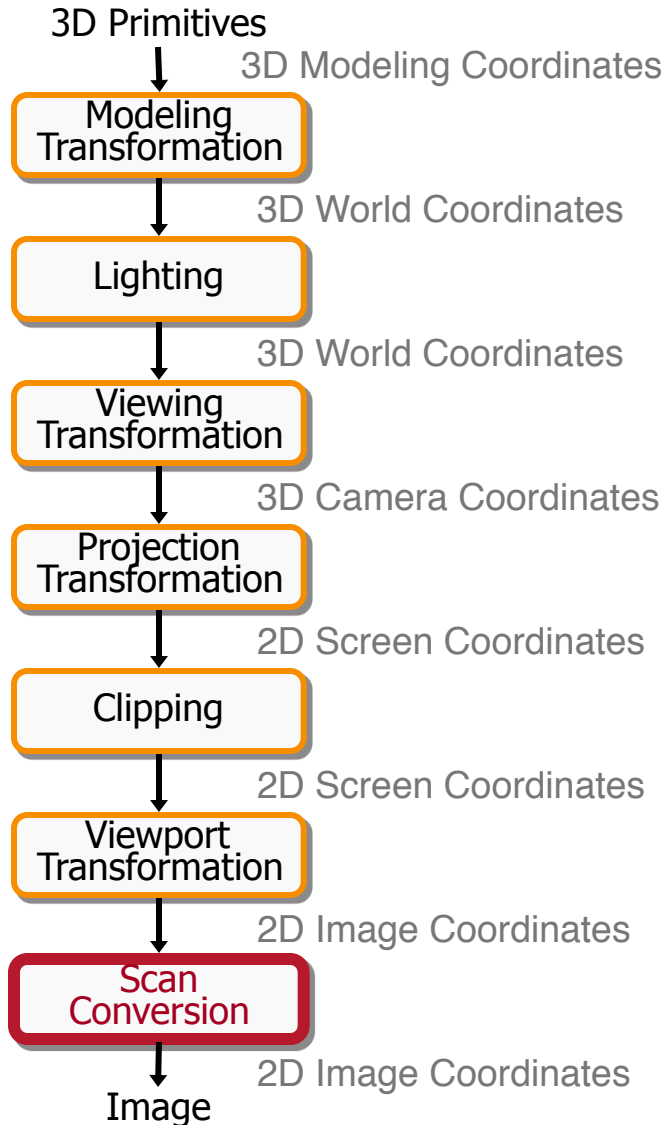


Clipping to a Boundary

- Do **inside** test for each point in sequence,
Insert new points when cross window boundary,
Remove points outside window boundary

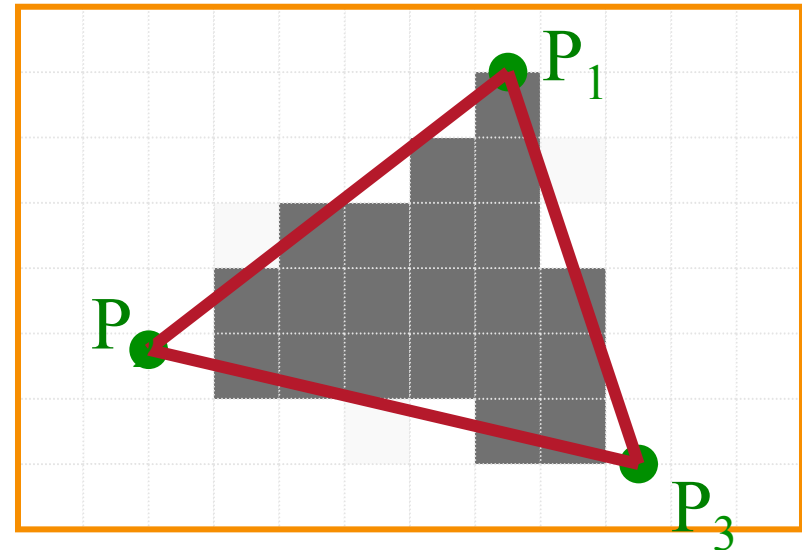
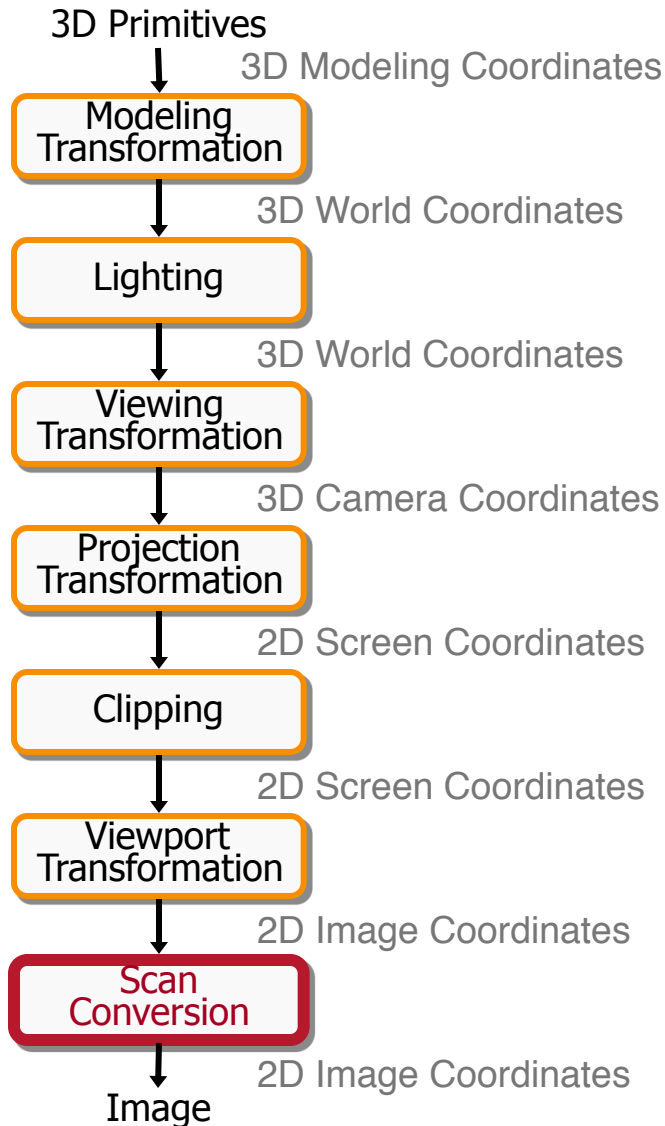


3D Rendering Pipeline (for direct illumination)



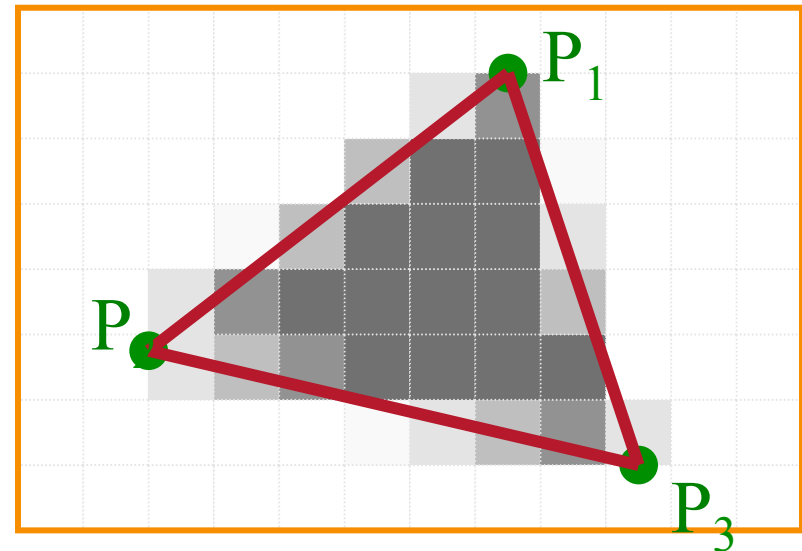
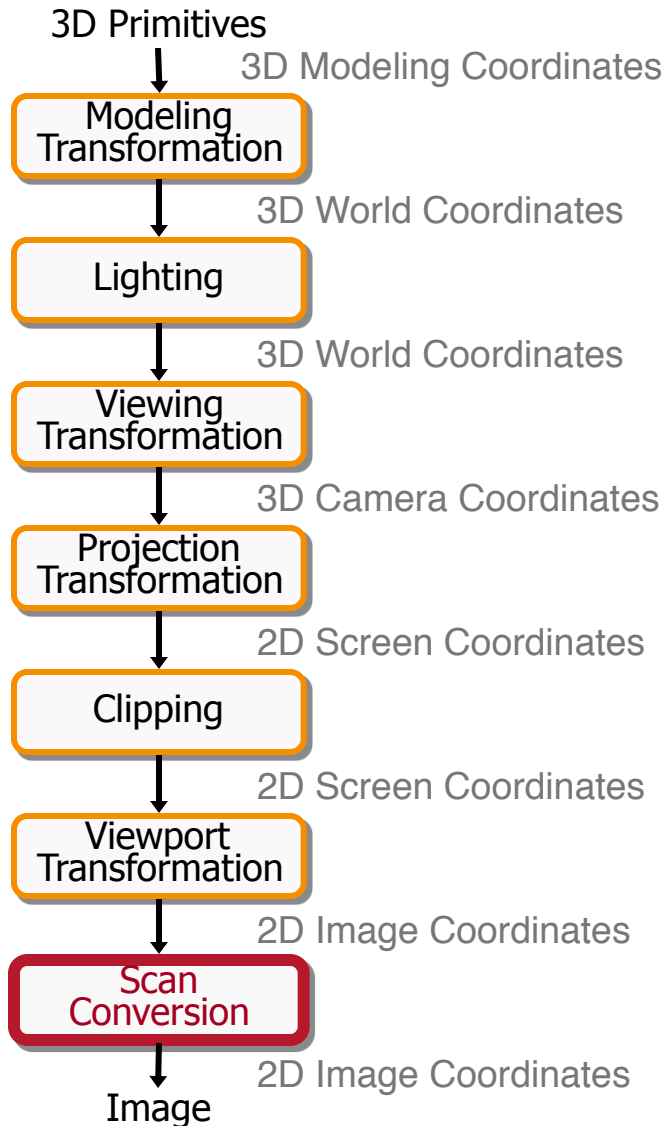
Viewing Window

3D Rendering Pipeline (for direct illumination)



Standard (aliased)
Scan Conversion

3D Rendering Pipeline (for direct illumination)



Antialiased
Scan Conversion

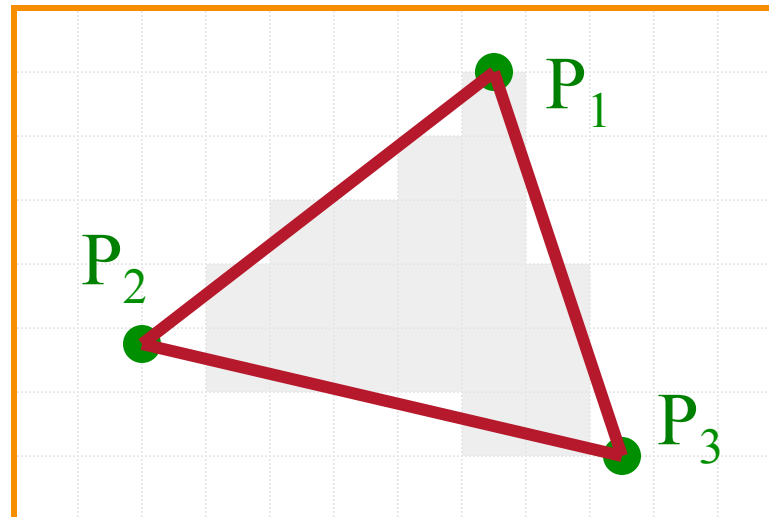


Scan Conversion

- Render an image of a geometric primitive by setting pixel colors

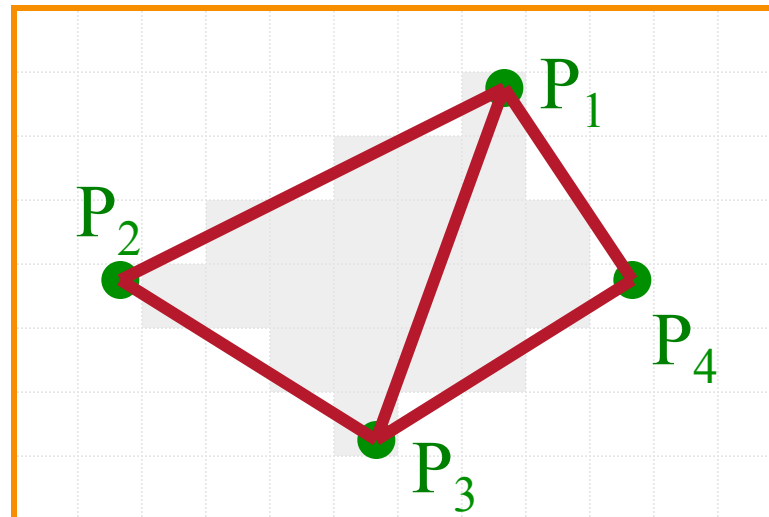
```
void SetPixel(int x, int y, Color rgba)
```

- Example: Filling the inside of a triangle



Triangle Scan Conversion

- Properties of a good algorithm
 - Symmetric
 - Straight edges
 - **No cracks between adjacent primitives**
 - (Antialiased edges)
 - **FAST!**

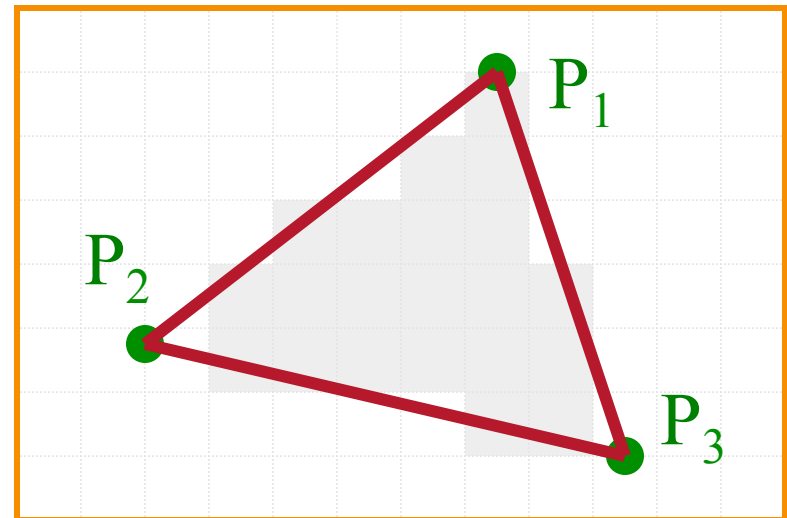




Simple Algorithm

- Color all pixels inside triangle

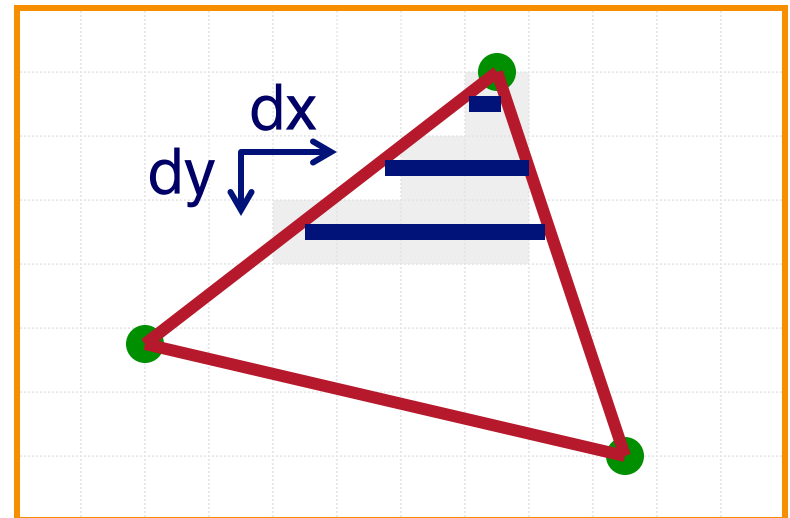
```
void ScanTriangle(Triangle T, Color rgba) {  
    for each pixel P in bbox(T) {  
        if (Inside(T, P))  
            SetPixel(P.x, P.y, rgba);  
    }  
}
```



Triangle Sweep-Line Algorithm



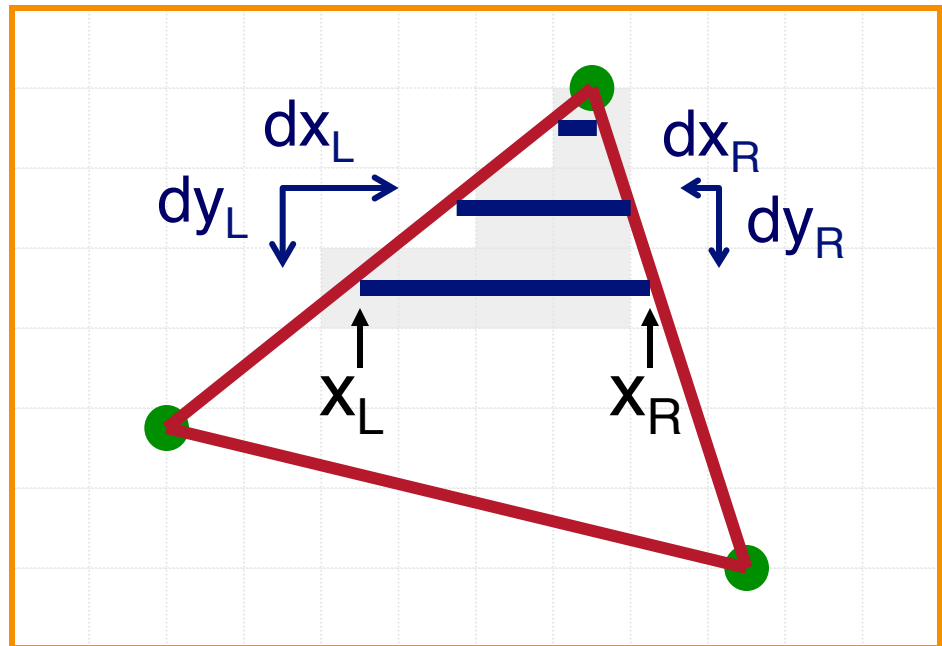
- Take advantage of spatial coherence
 - Compute which pixels are inside using horizontal spans
 - Process horizontal spans in scan-line order
- Take advantage of edge linearity
 - Use edge slopes to update coordinates incrementally





Triangle Sweep-Line Algorithm

```
void ScanTriangle(Triangle T, Color rgba) {  
    for each edge pair {  
        initialize  $x_L$ ,  $x_R$ ;  
        compute  $dx_L/dy_L$  and  $dx_R/dy_R$ ;  
        for each scanline at  $y$   
            for (int  $x = x_L$ ;  $x \leq x_R$ ;  $x++$ )  
                SetPixel( $x$ ,  $y$ , rgba);  
         $x_L += dx_L/dy_L$ ;  
         $x_R += dx_R/dy_R$ ;  
    }  
}
```

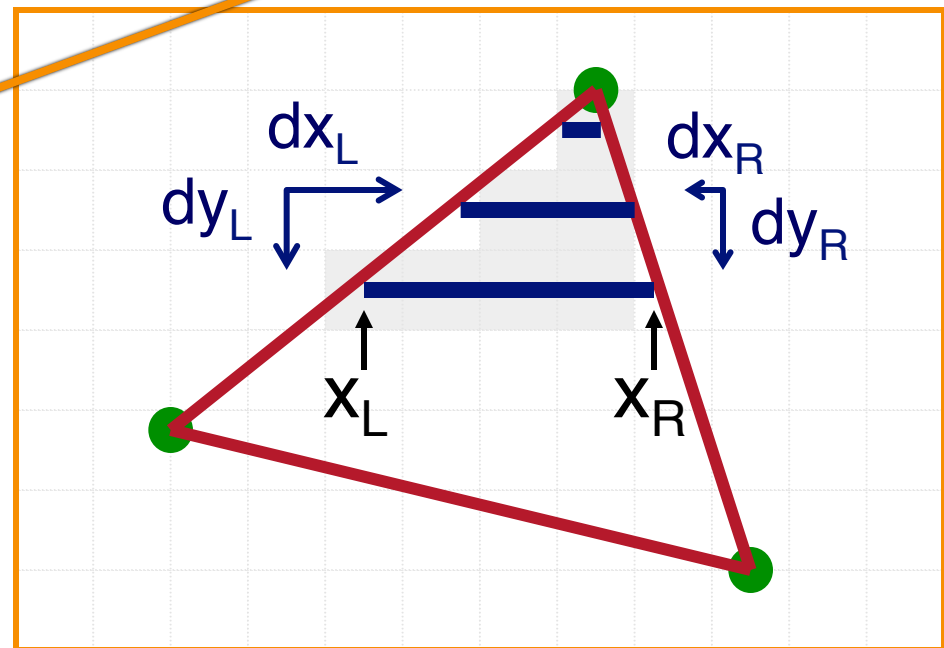




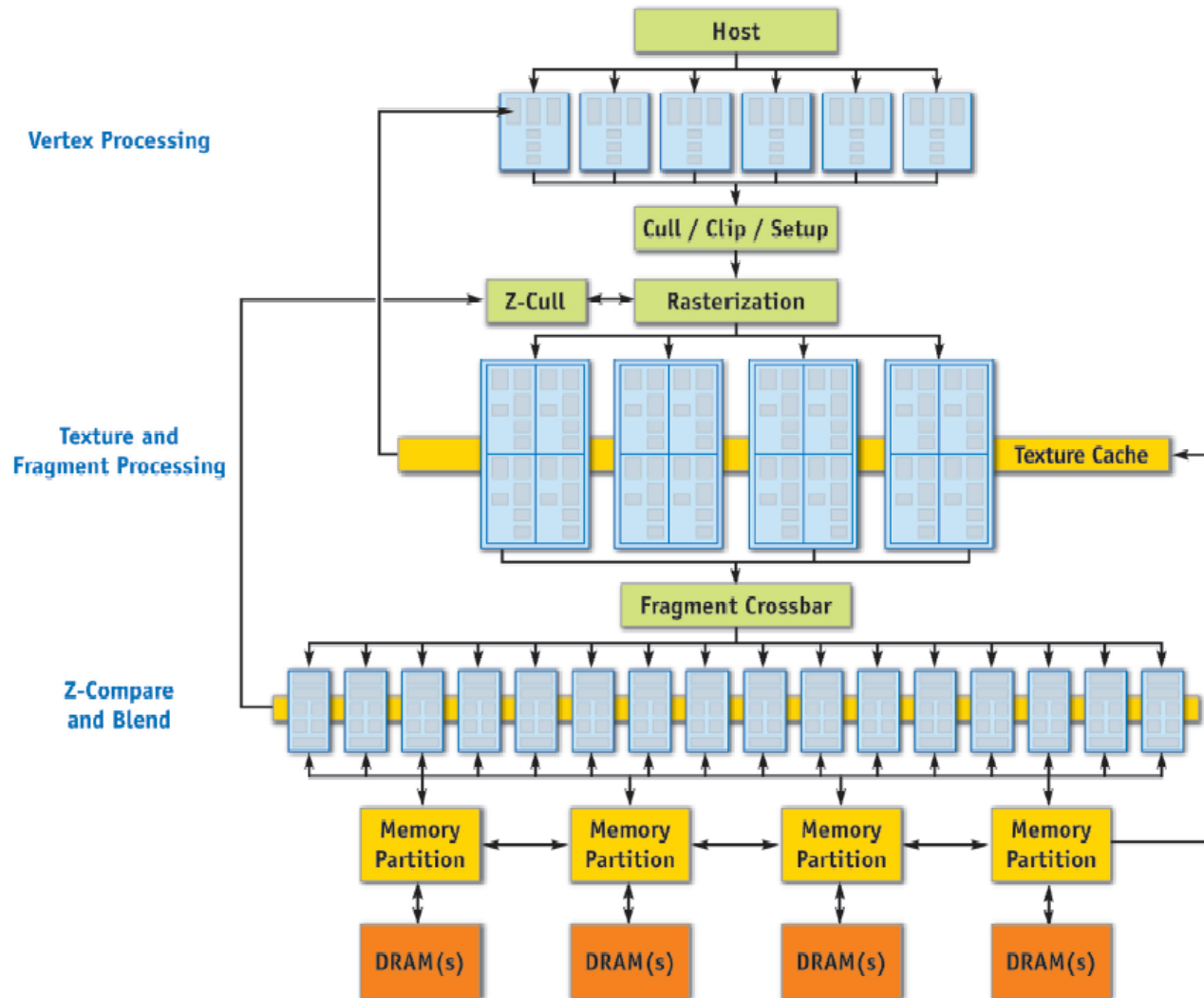
Triangle Sweep-Line Algorithm

```
void ScanTriangle(Triangle T, Color rgba) {  
  for each edge pair {  
    initialize  $x_L$ ,  $x_R$ ;  
    compute  $dx_L/dy_L$  and  $dx_R/dy_R$ ;  
    for each scanline at  $y$   
      for (int  $x = x_L$ ;  $x \leq x_R$ ;  $x++$ )  
        SetPixel( $x$ ,  $y$ , rgba);  
       $x_L += dx_L/dy_L$ ;  
       $x_R += dx_R/dy_R$ ;  
    }  
  }  
}
```

Minimize computation
in inner loops

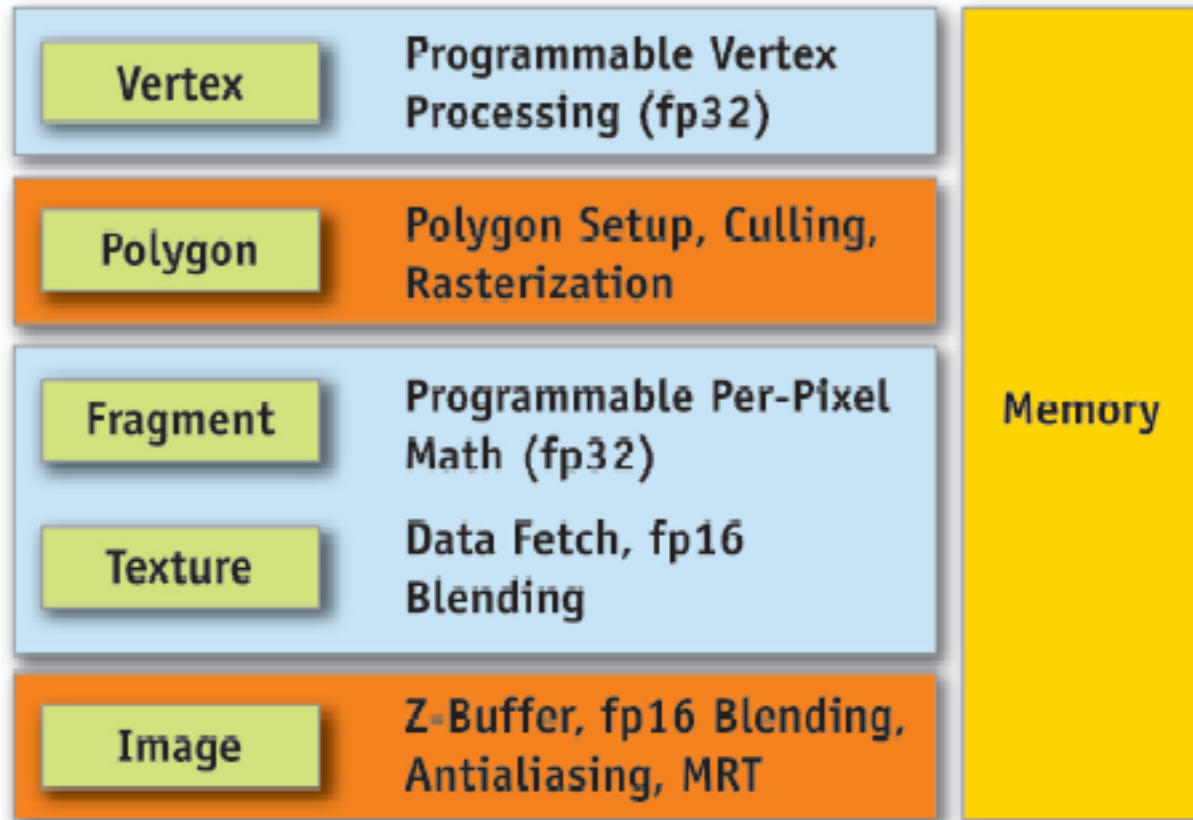


GPU Architecture



GeForce 6 Series Architecture

GPU Architecture



GeForce 6 Series Architecture