Subdivision Surfaces

COS 426

3D Object Representations

- Raw data
  - o Voxels
  - o Point cloud
  - o Range image
  - o Polygons
- Surfaces
  - o Mesh
  - o Subdivision
  - o Parametric
  - o Implicit
- Solids
  - o Octree
  - o BSP tree
  - o CSG
  - o Sweep
- High-level structures
  - o Scene graph
  - o Application specific

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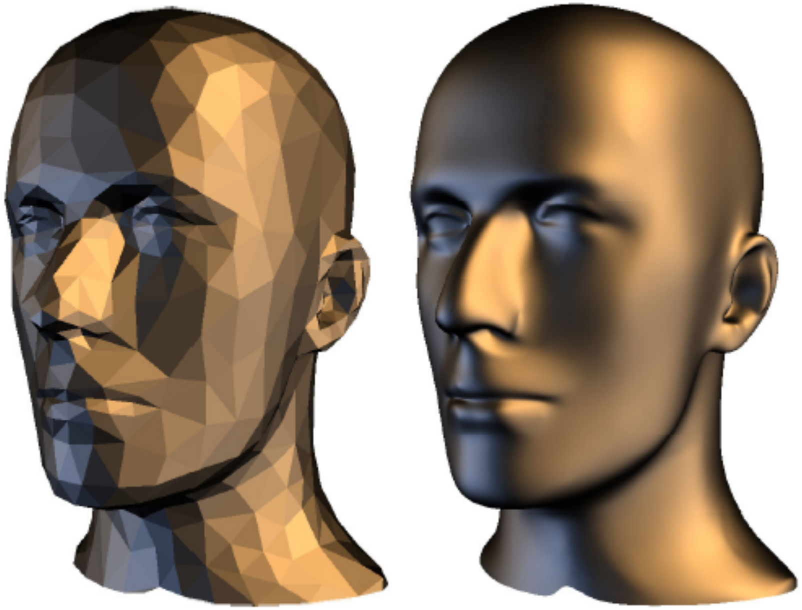
Subdivision Surfaces

- What makes a good surface representation?
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections



Reif & Schroeder 2000

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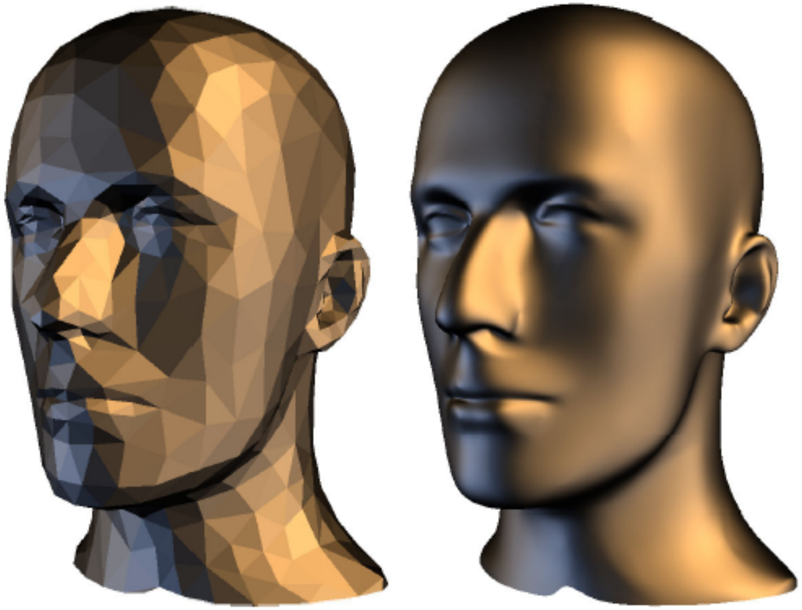
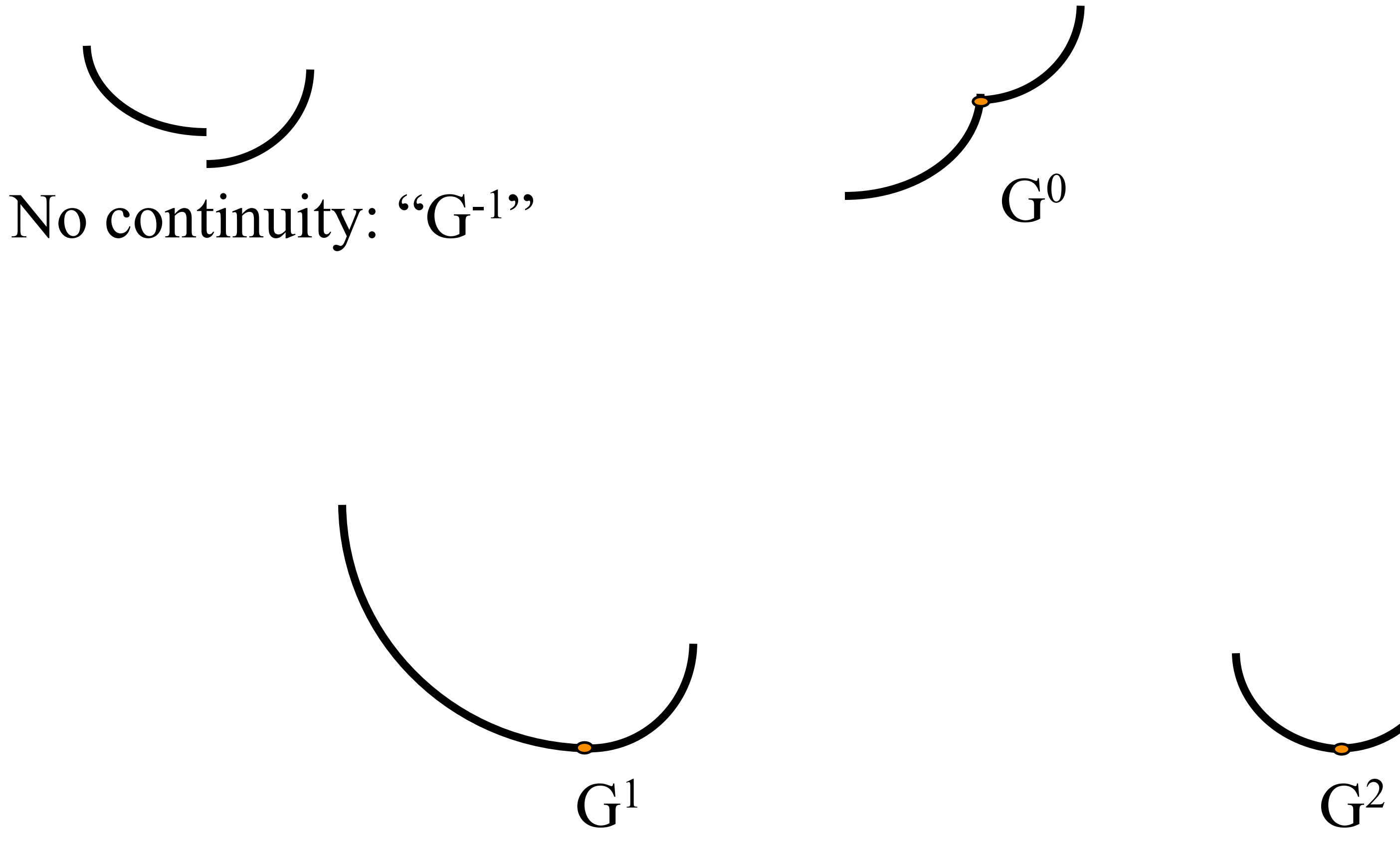


Reif & Schroeder 2000

Continuity

- A curve / surface with G^k continuity has a continuous k-th derivative



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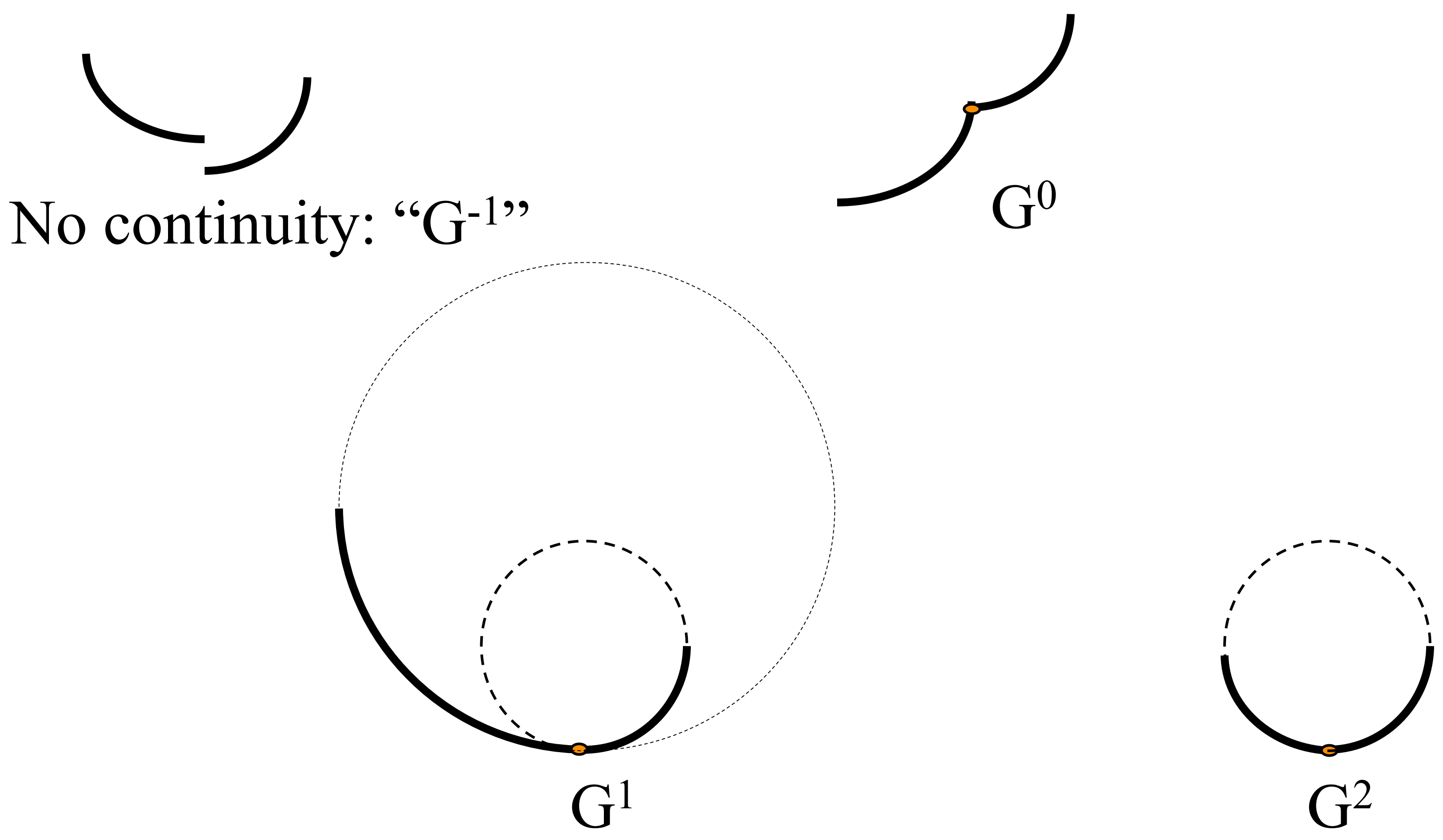


Continuity

Evaluated per point, but often speak of the minimum degree of continuity over entire surface

- e.g., a "G^k surface" is at least G^k at all points, but with at least one point that's not G^{k+1}

Parametric Continuity

- Curve x = f(t), y = g(t) is C^k continuous iff

  $$\frac{d^k f}{dt^k} \text{ and } \frac{d^k g}{dt^k} \text{ are continuous}$$

- For surfaces: x = f(s,t), y = g(s,t), z = h(s,t)

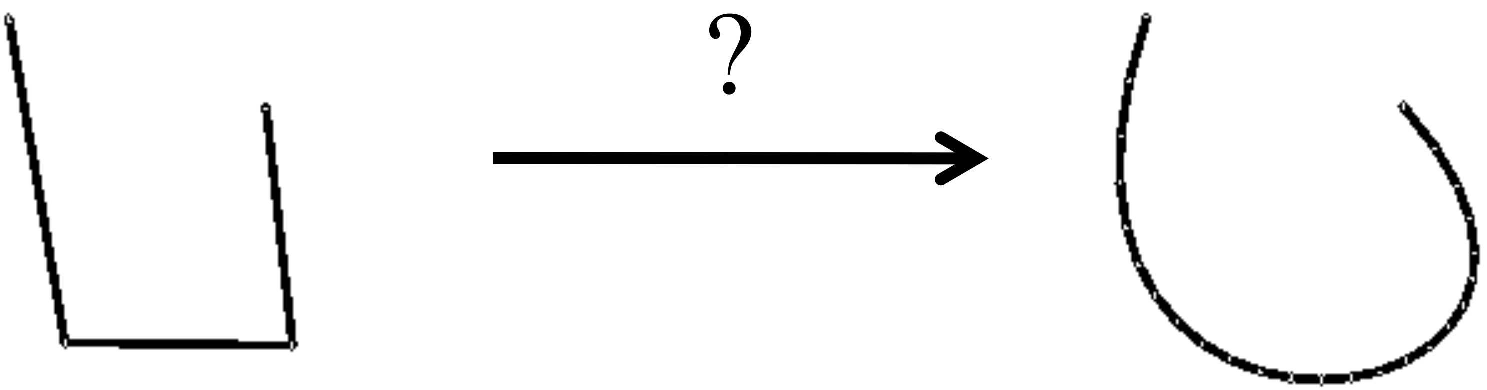
  C^k requires continuous $\frac{\partial^k f}{\partial s^k}, \frac{\partial^k f}{\partial t^k}, \frac{\partial^k g}{\partial s^k}, \frac{\partial^k g}{\partial t^k}, \frac{\partial^k h}{\partial s^k}, \frac{\partial^k h}{\partial t^k}$

  (and all mixed partial derivatives of order k)

- Often easier to check / prove things about than G^k continuity

Subdivision

- How do you make a curve with guaranteed continuity?



Subdivision

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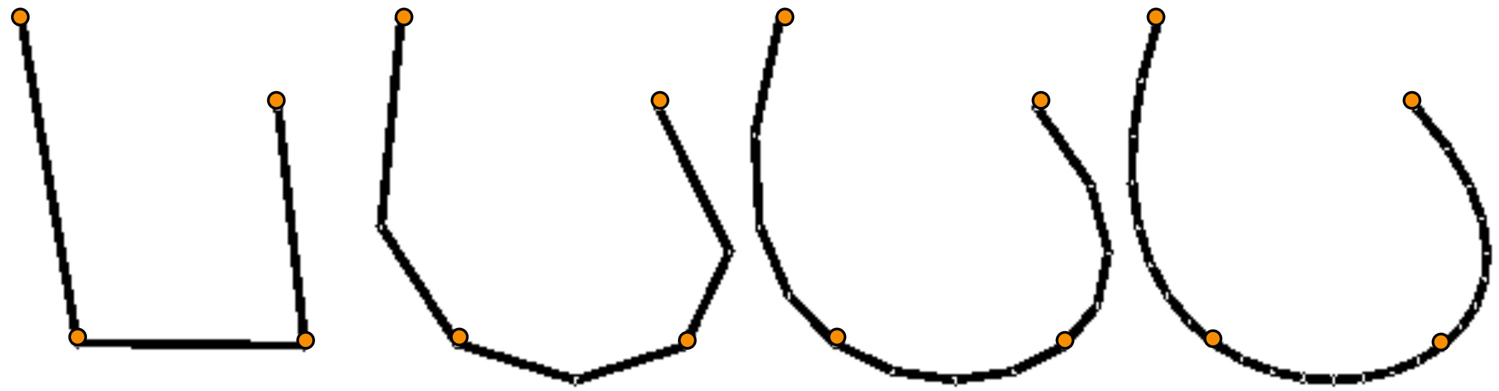

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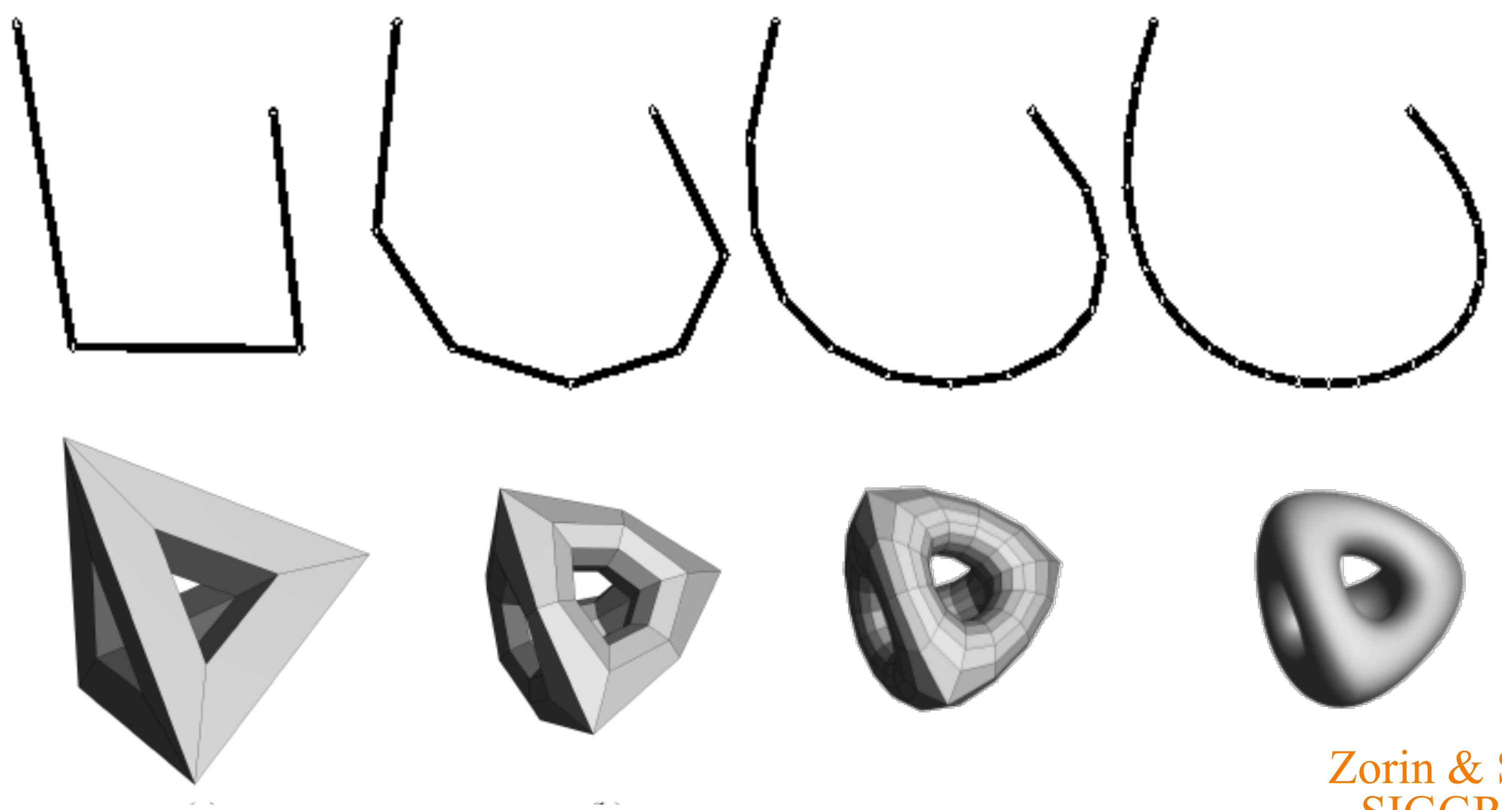
Course Notes

Subdivision

- How do you make a surface with guaranteed continuity?



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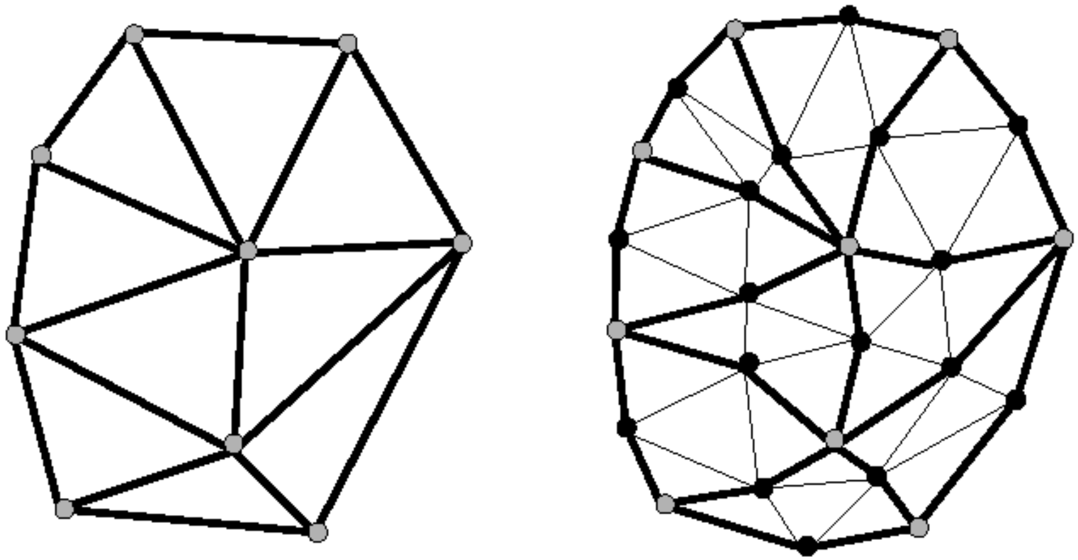
Subdivision Surfaces

- Repeated application of
  - o Topology refinement (splitting faces)
  - o Geometry refinement (weighted averaging)



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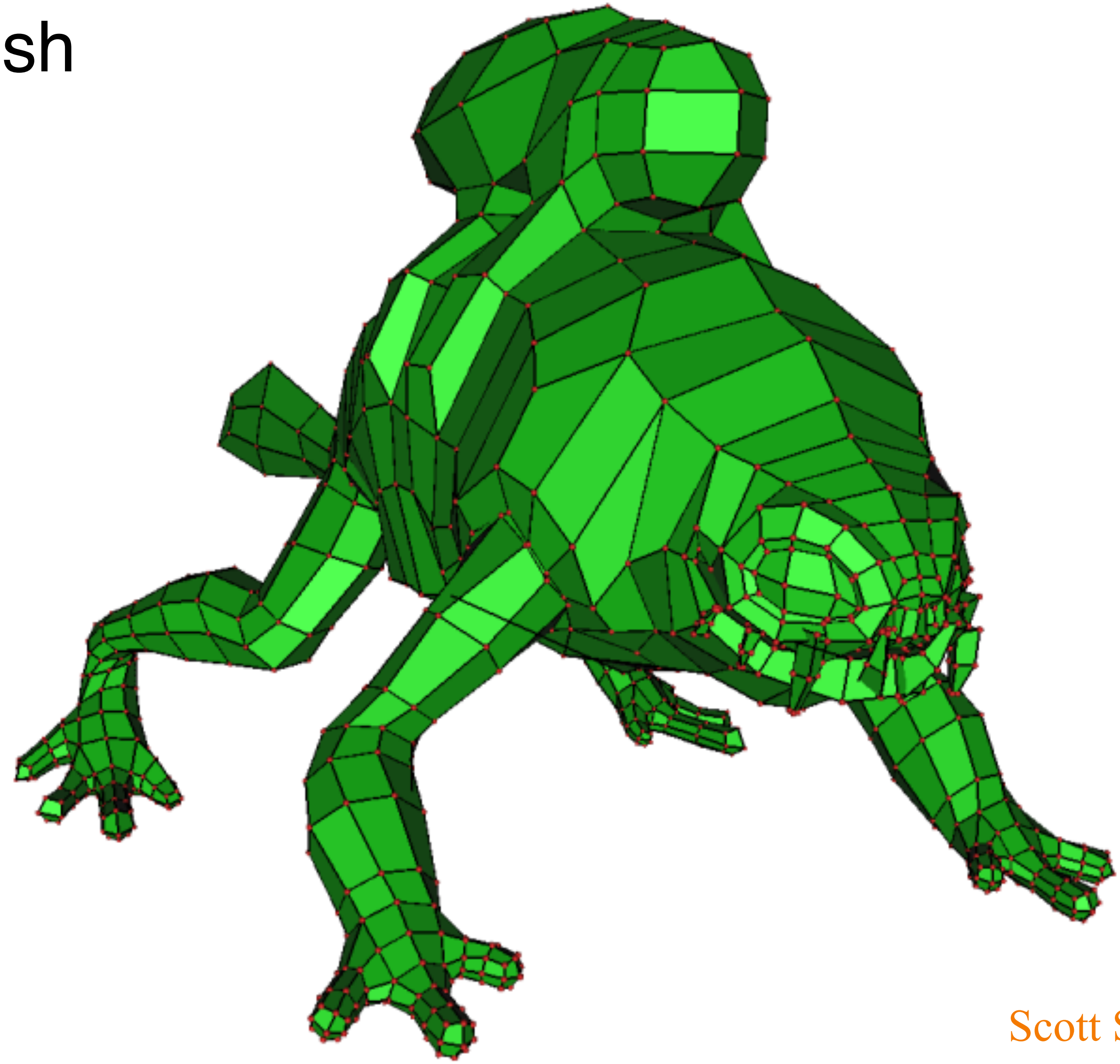
Subdivision Surfaces – Examples

- Base mesh



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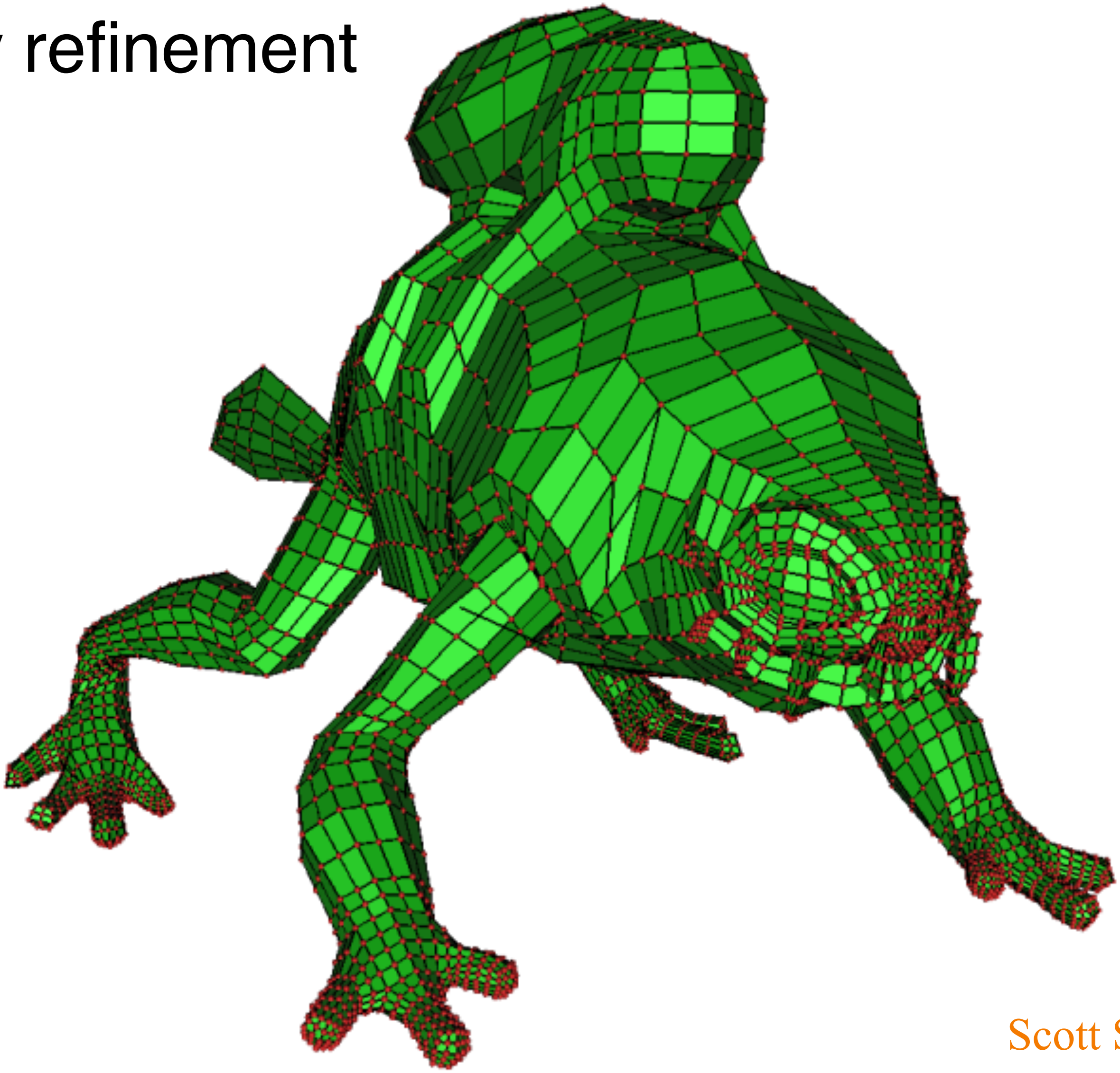
Subdivision Surfaces – Examples

- Topology refinement



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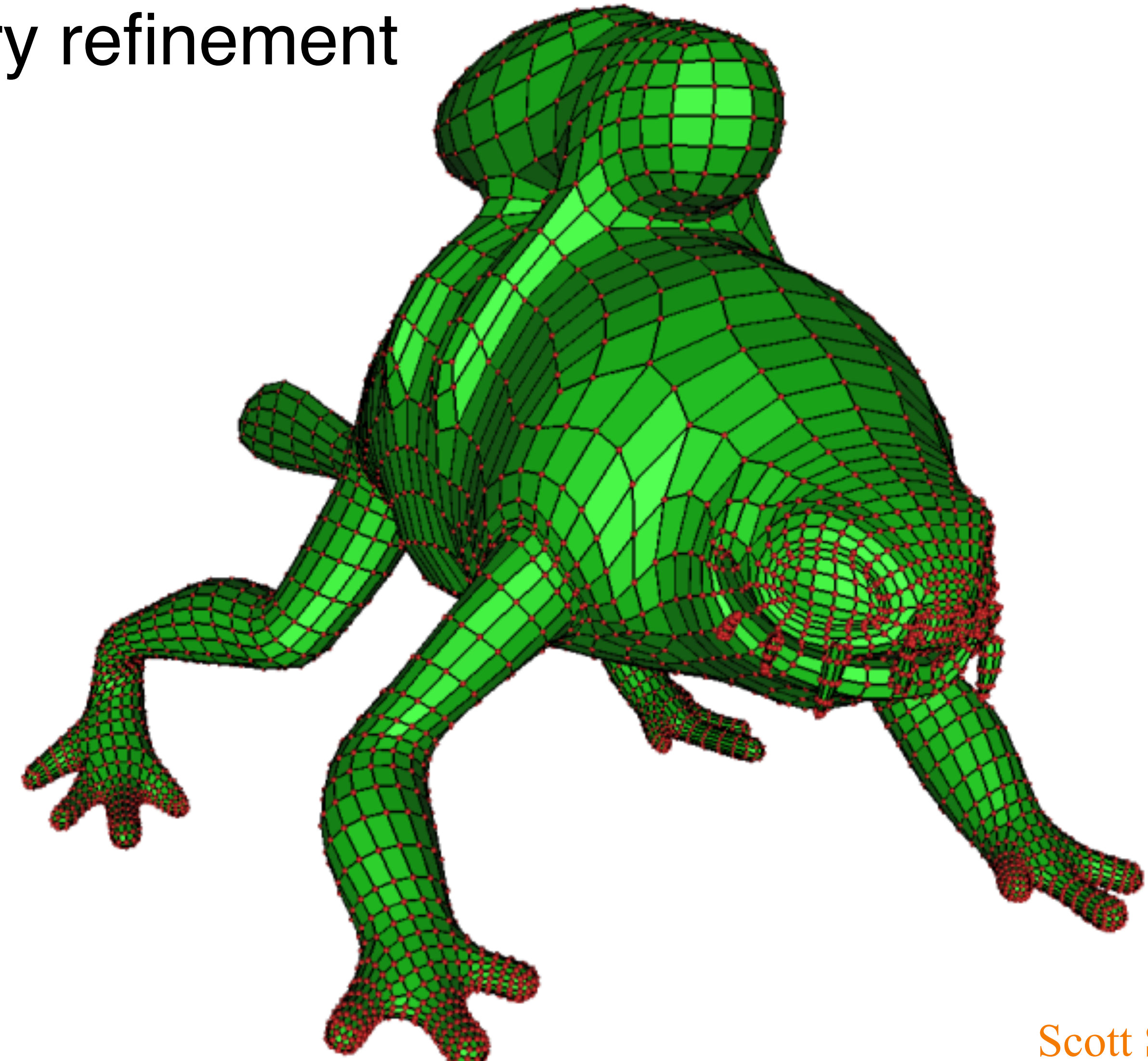
Subdivision Surfaces – Examples

- Geometry refinement



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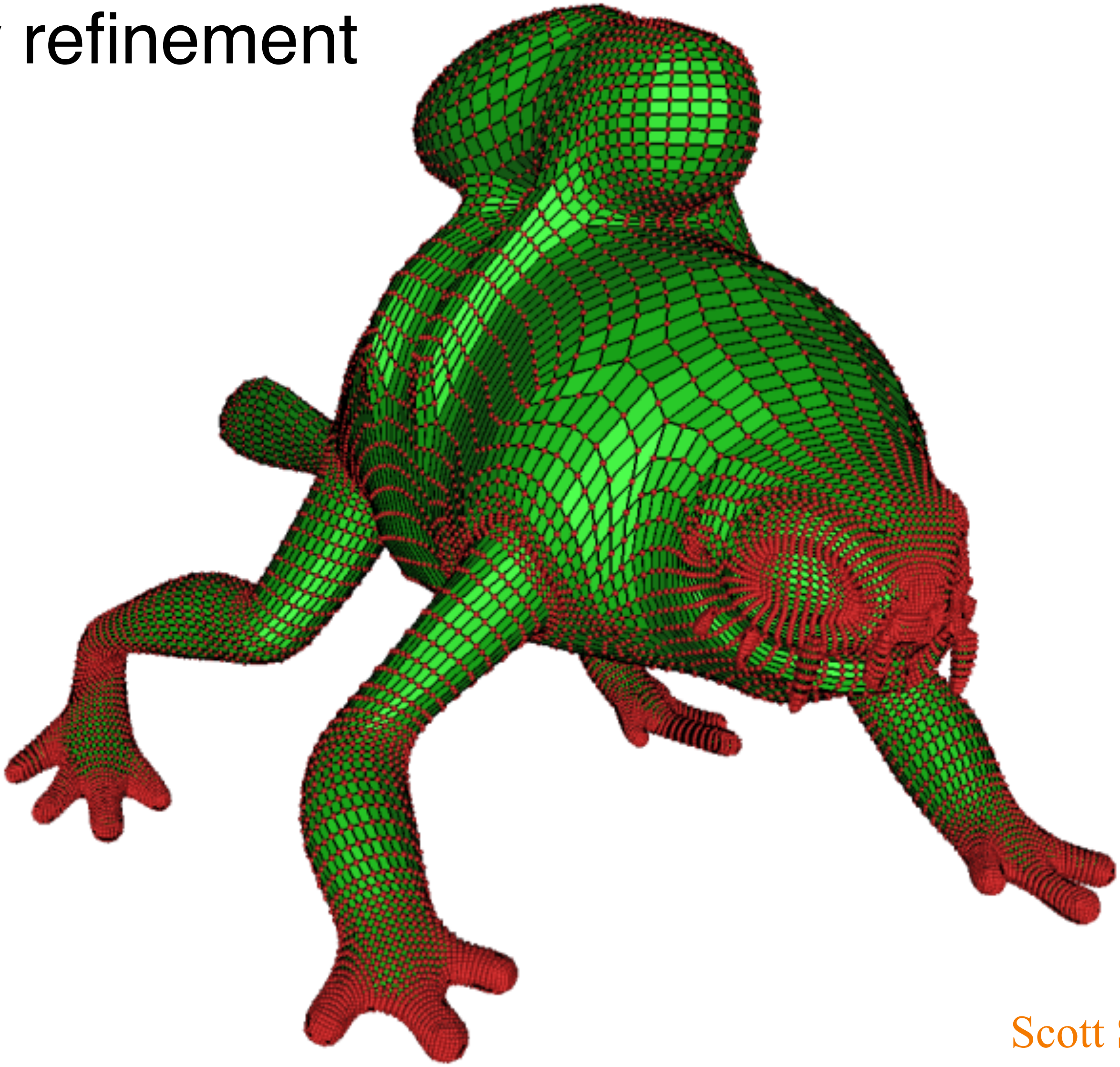
Subdivision Surfaces – Examples

- Topology refinement



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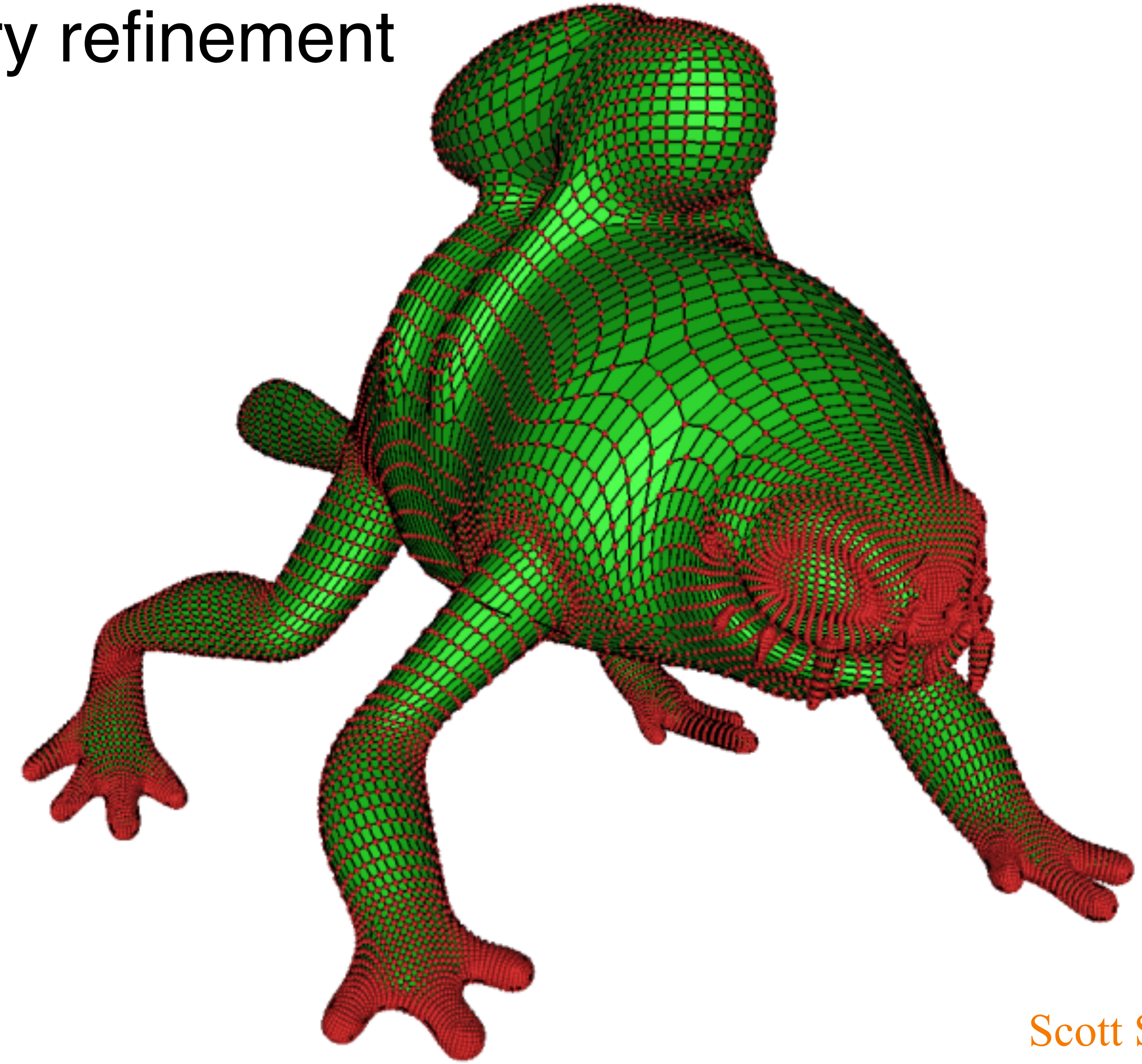
Subdivision Surfaces – Examples

- Geometry refinement



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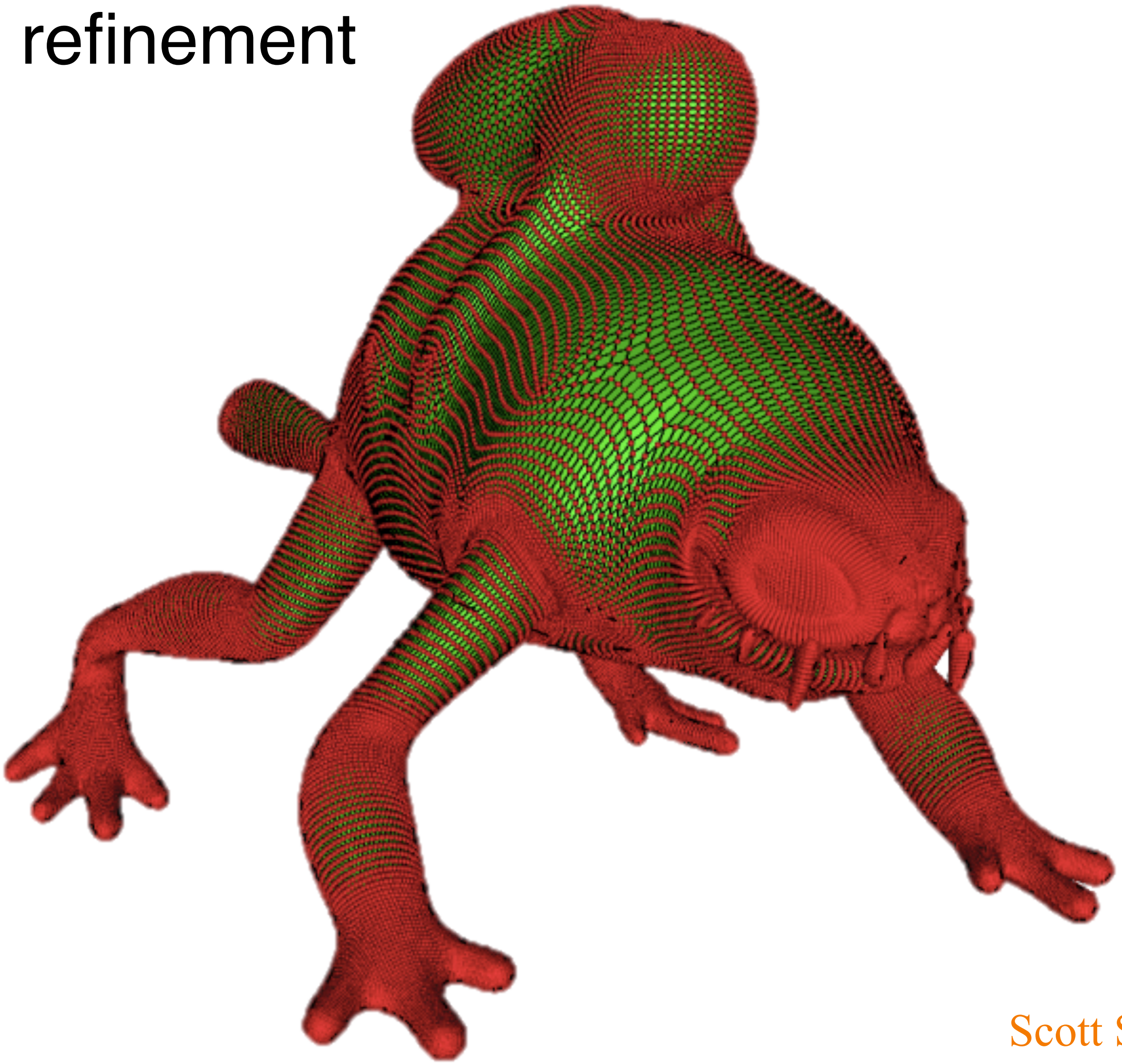
Subdivision Surfaces – Examples

- Topology refinement



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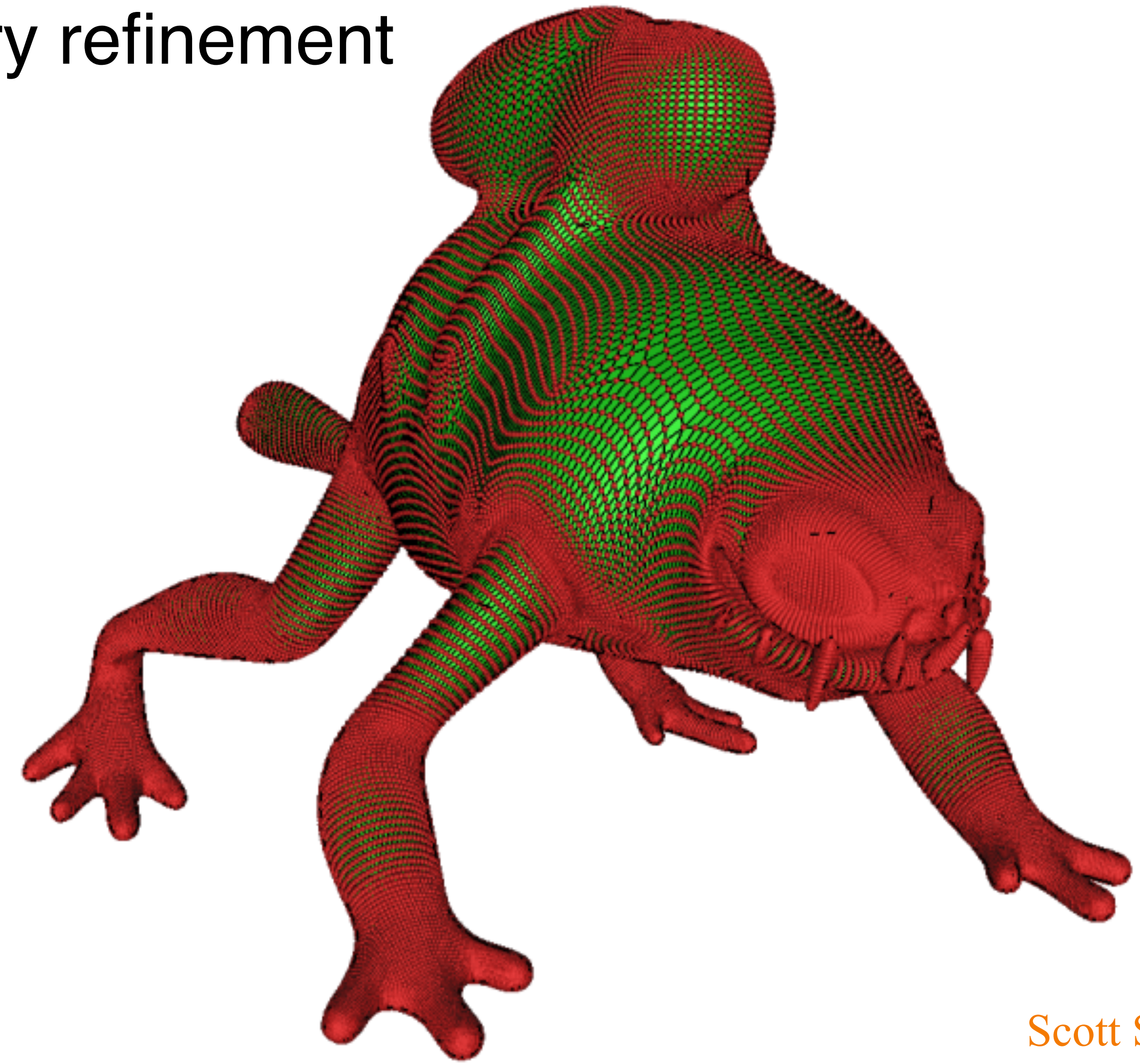
Subdivision Surfaces – Examples

- Geometry refinement



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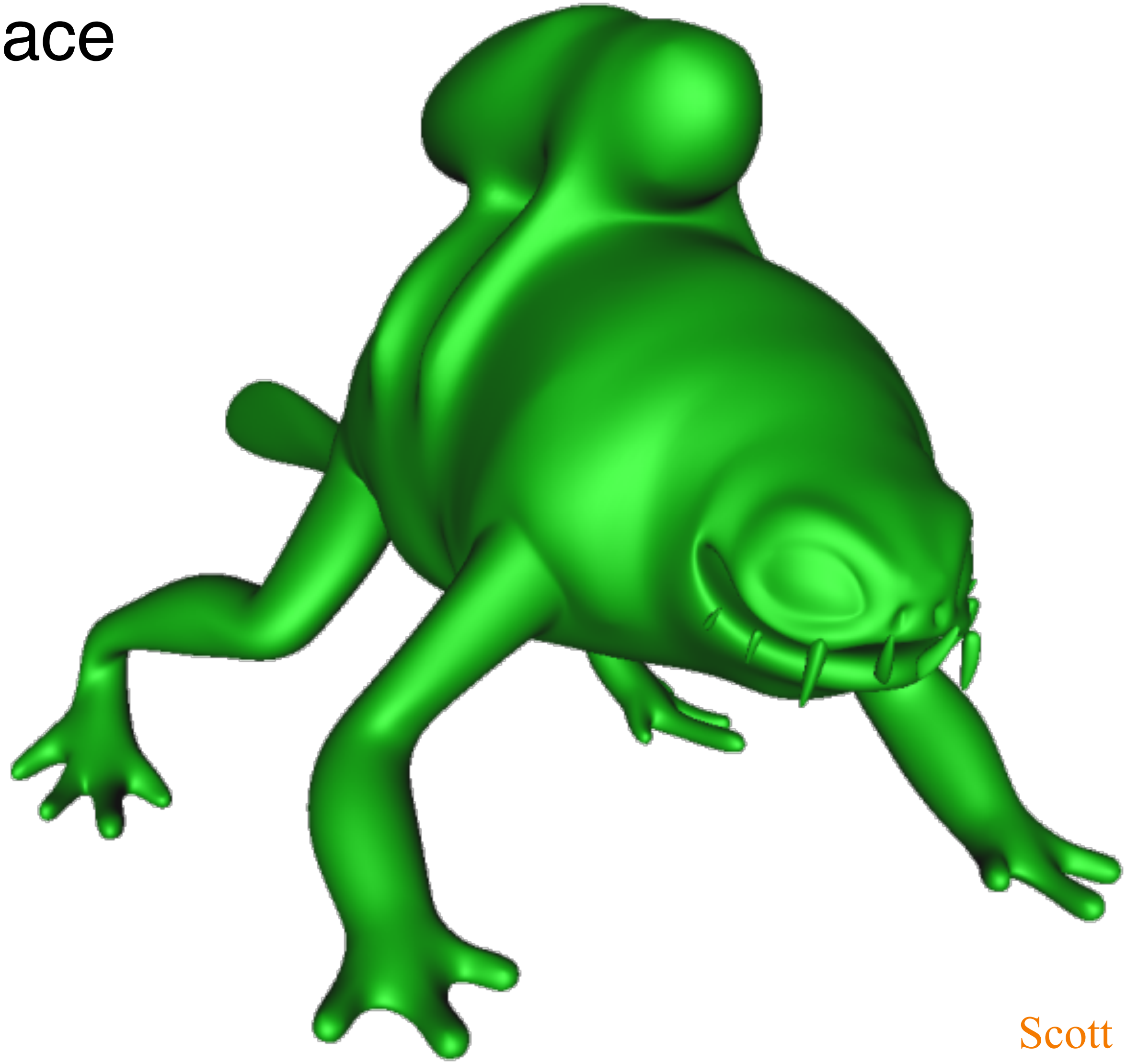
Subdivision Surfaces – Examples

- Limit surface



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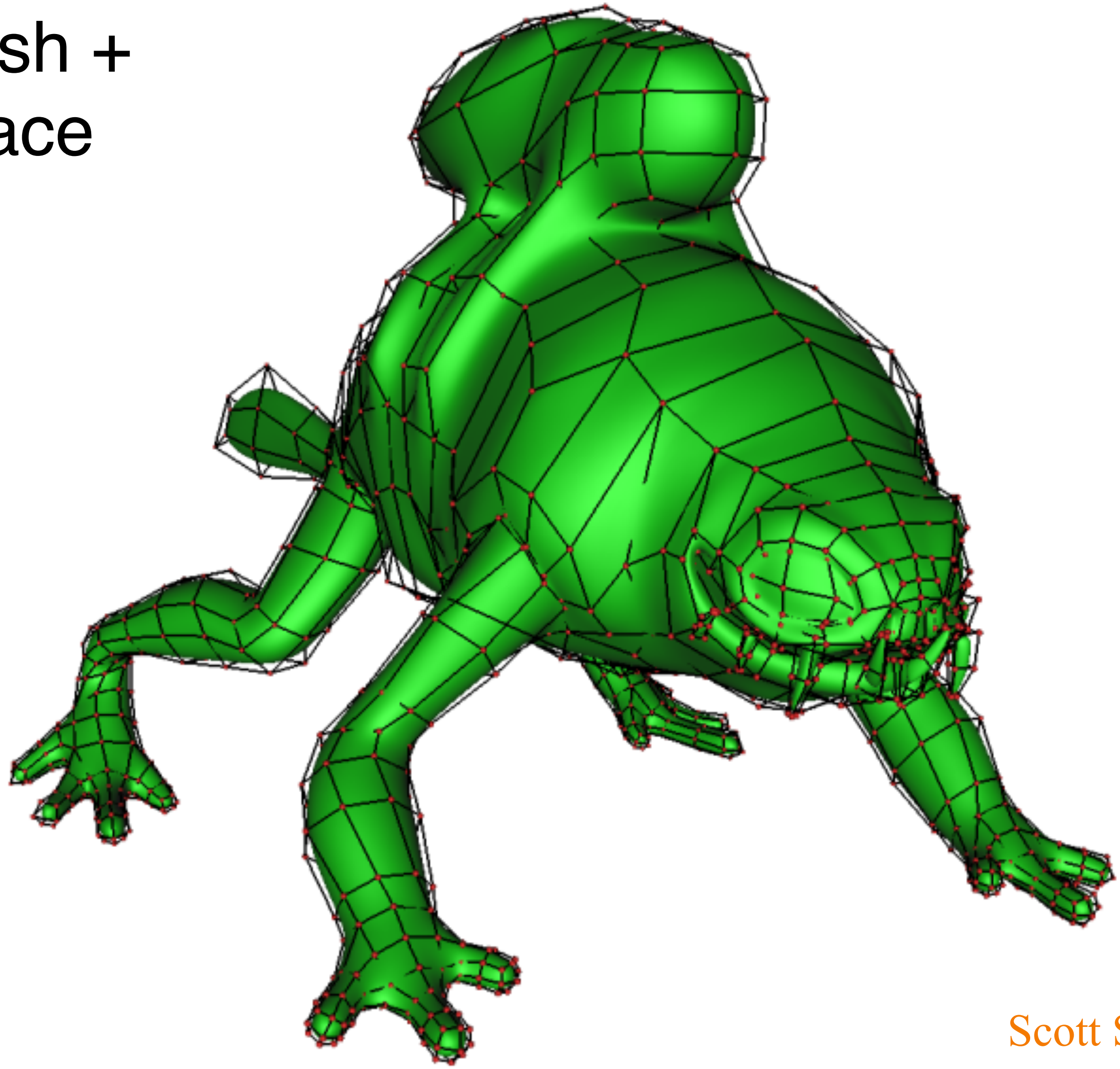
Subdivision Surfaces – Examples

- Base mesh + limit surface



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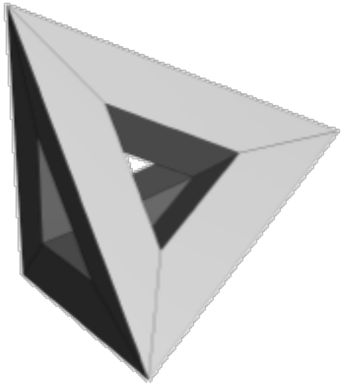
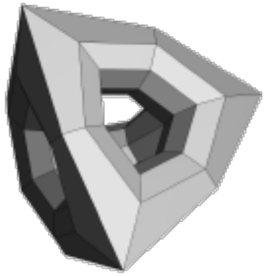
Design of Subdivision Rules

- What types of input?
  - o Quad meshes, triangle meshes, etc.
- How to refine topology?
  - o Simple implementations
- How to refine geometry?
  - o Smoothness guarantees in limit surface
    - » Continuity (C^0, C^1, C^2, …?)
  - o Provable relationships between limit surface and original control mesh
    - » Interpolation of vertices?



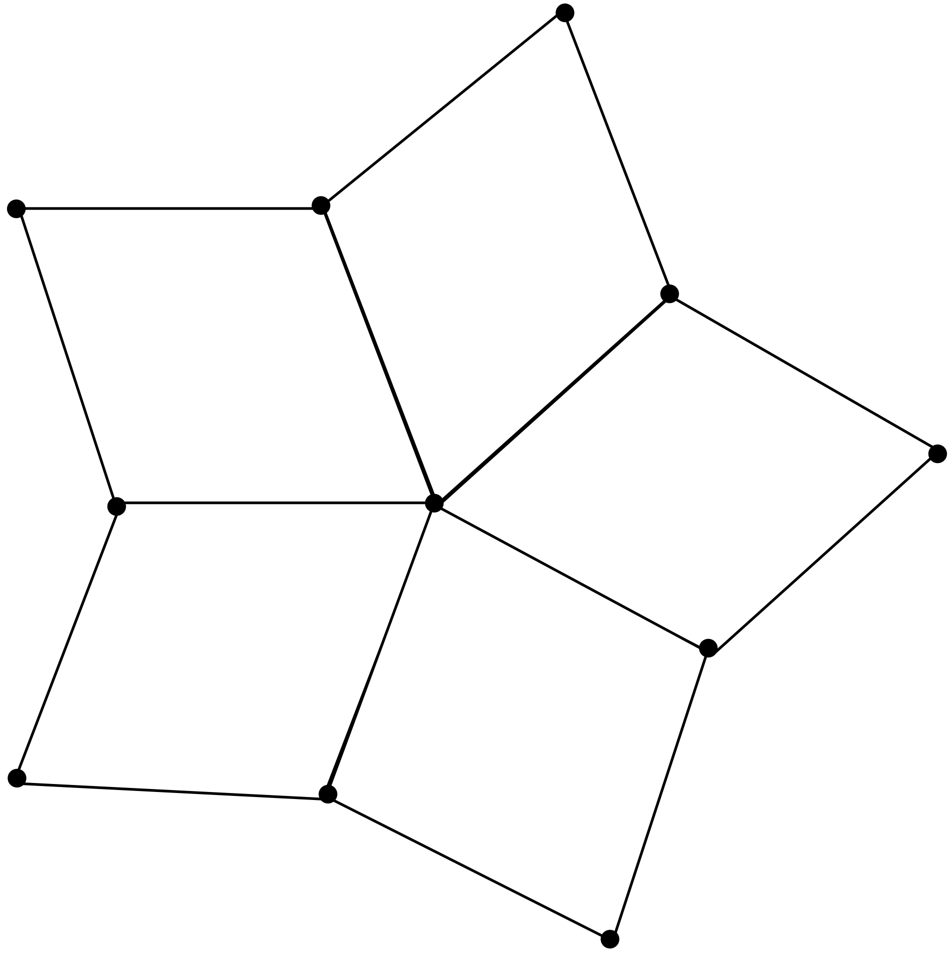


Linear Subdivision

- Type of input
  - o Quad mesh -- four-sided polygons (*quads*)
  - o Any number of quads may touch each vertex
- Topology refinement rule
  - o Split every quad into four at midpoints
- Geometry refinement rule
  - o Average vertex positions

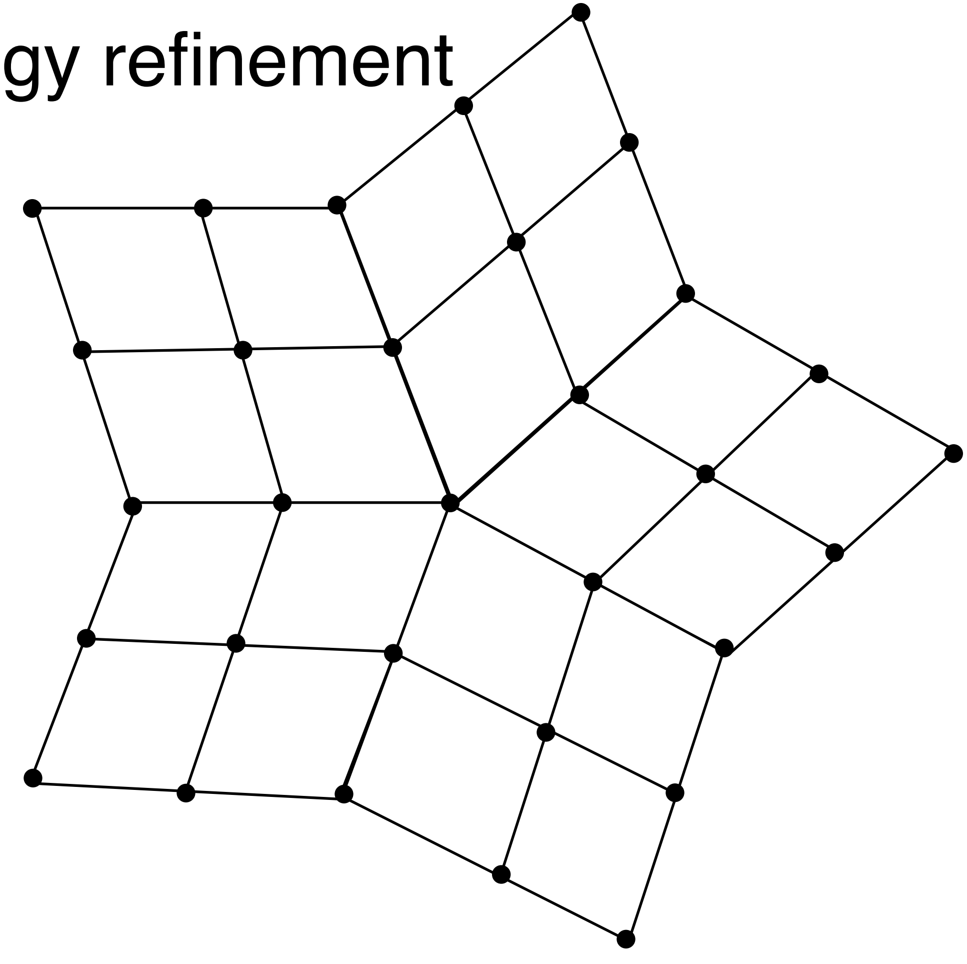
This is a simple example to demonstrate how subdivision schemes work

Linear Subdivision



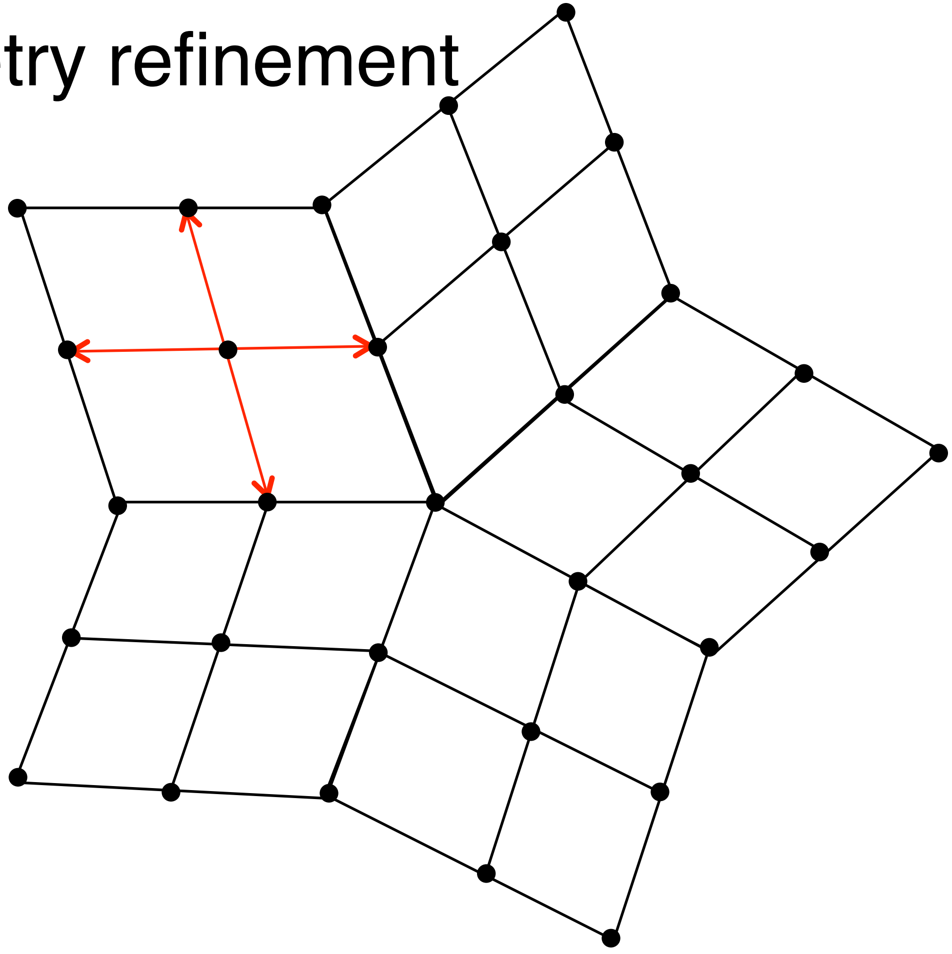
Linear Subdivision

- Topology refinement



Linear Subdivision

- Geometry refinement



Linear Subdivision

LinearSubivision (F_0, V_0, k)

for i = 1 …k levels

(F_i, V_i) = RefineTopology(F_{i-1}, V_{i-1})

RefineGeometry(F_i, V_i)

return (F_k, V_k)

Linear Subdivision

RefineTopology (F, V)

$newV = V$

$newF = \{\}$

for each face F_i

Insert new vertex c at centroid of F_i into $newV$

for j = 1 to 4

Insert/lookup in $newV$ new vertex e_j at centroid of each edge ($F_{i,j}$, $F_{i,j+1}$)

for j = 1 to 4

Insert new face ($F_{i,j}$, e_j, c, e_{j-1}) into $newF$

return ($newF$, $newV$)

Linear Subdivision

RefineGeometry(F, V)

$newV$ = 0 * V

wt = array of 0 whose size is number of vertices

$newF$ = F

for each face F_i

$cent$ = centroid for F_i

$newV[F_i]$ += $cent$ // syntax: repeat for all vtx indices in F_i

$wt[F_i]$ += 1 // syntax: repeat for all vtx indices in F_i

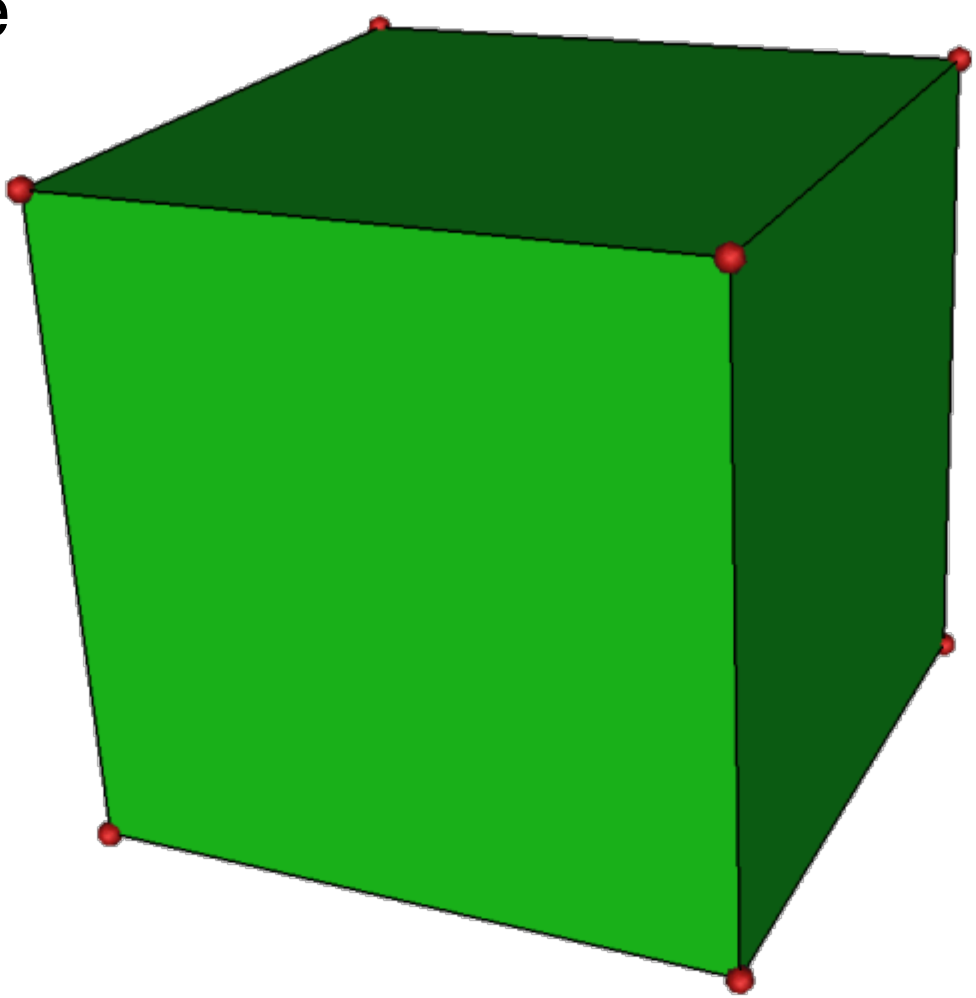
for each vertex $newV[i]$

$newV[i]$ /= $wt[i]$

return ($newF$, $newV$)

Linear Subdivision

- Example



Input mesh

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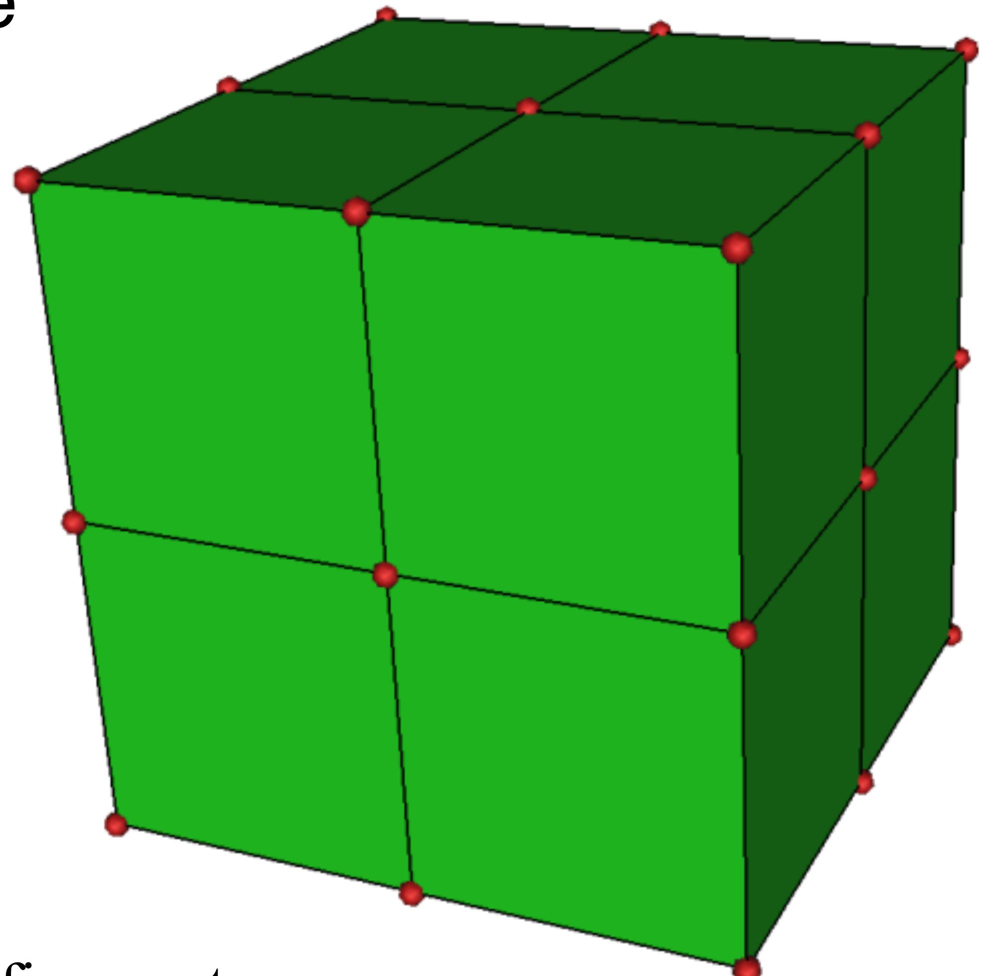
Linear Subdivision

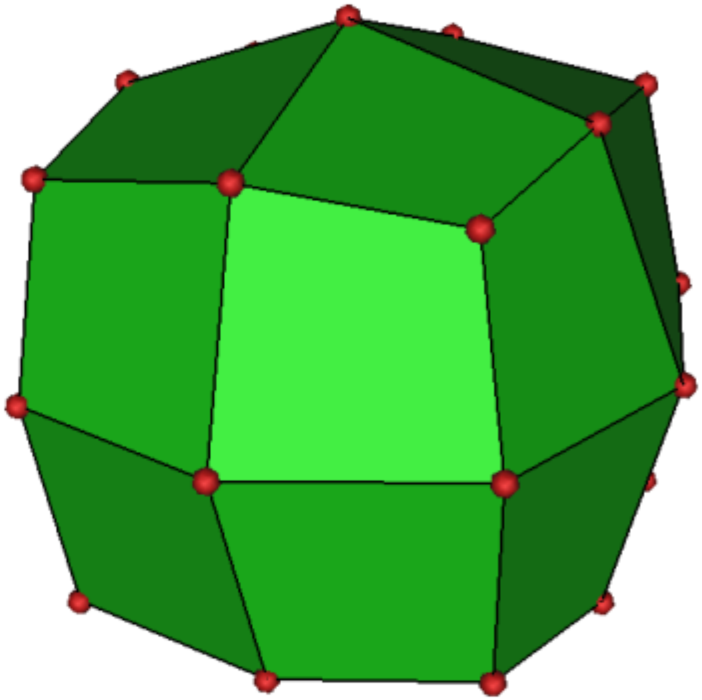
- Example



Topology refinement

Linear Subdivision

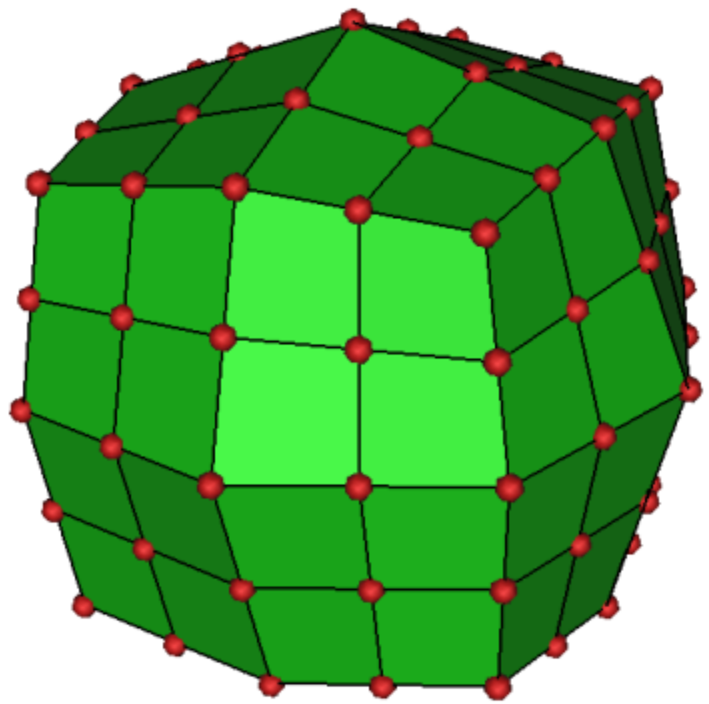
- Example



Geometry refinement

Linear Subdivision

- Example



Topology refinement

Linear Subdivision

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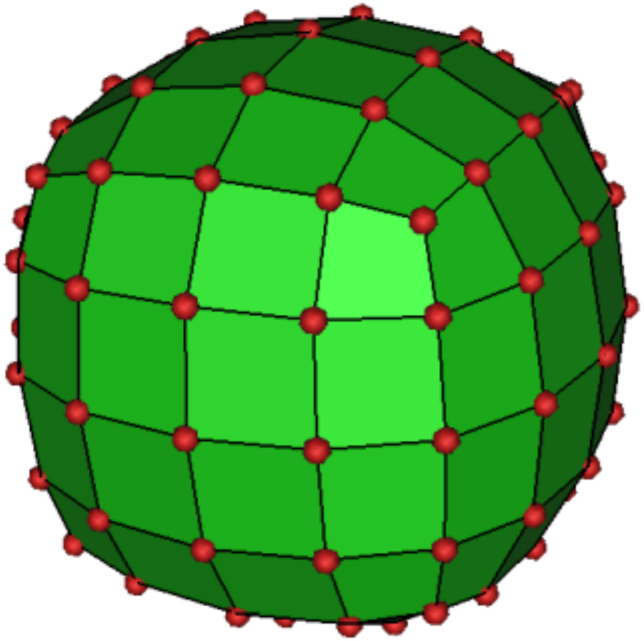

Geometry refinement

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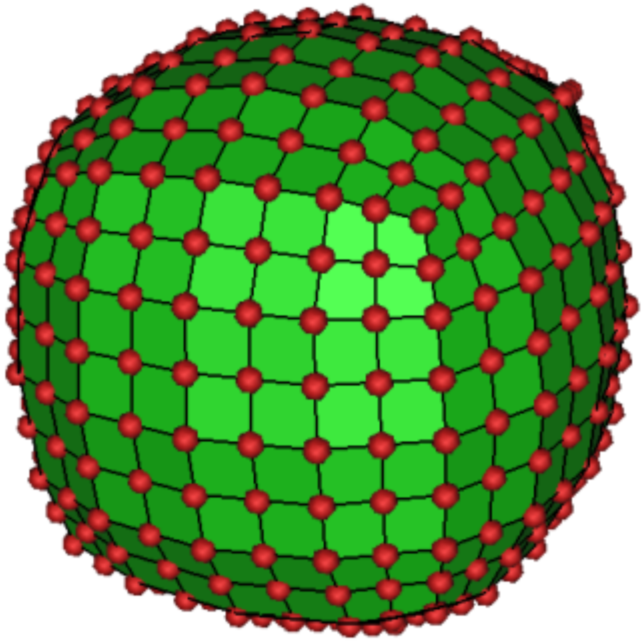
Linear Subdivision

- Example



Topology refinement

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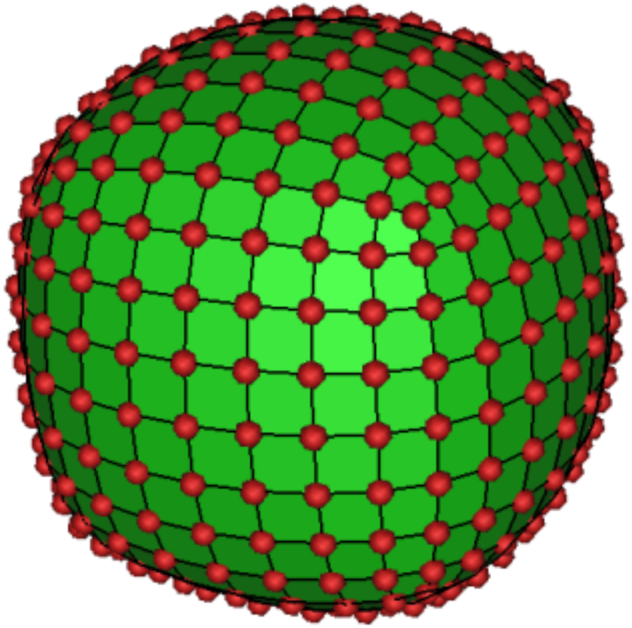
Linear Subdivision

- Example



Geometry refinement

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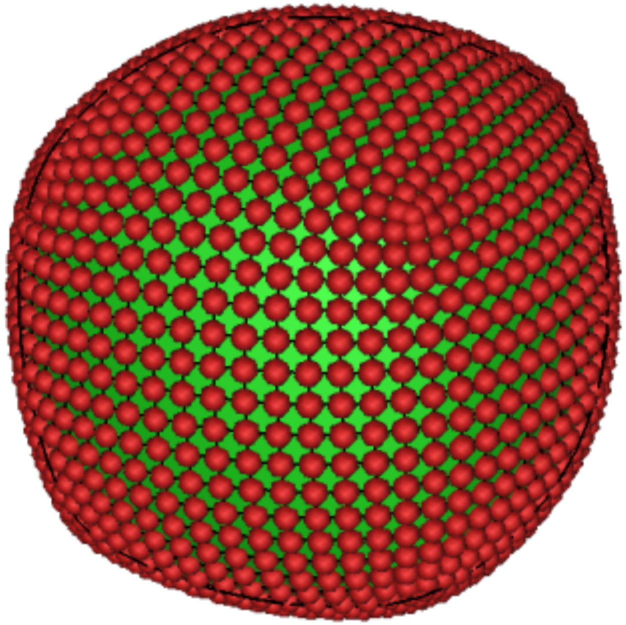
Linear Subdivision

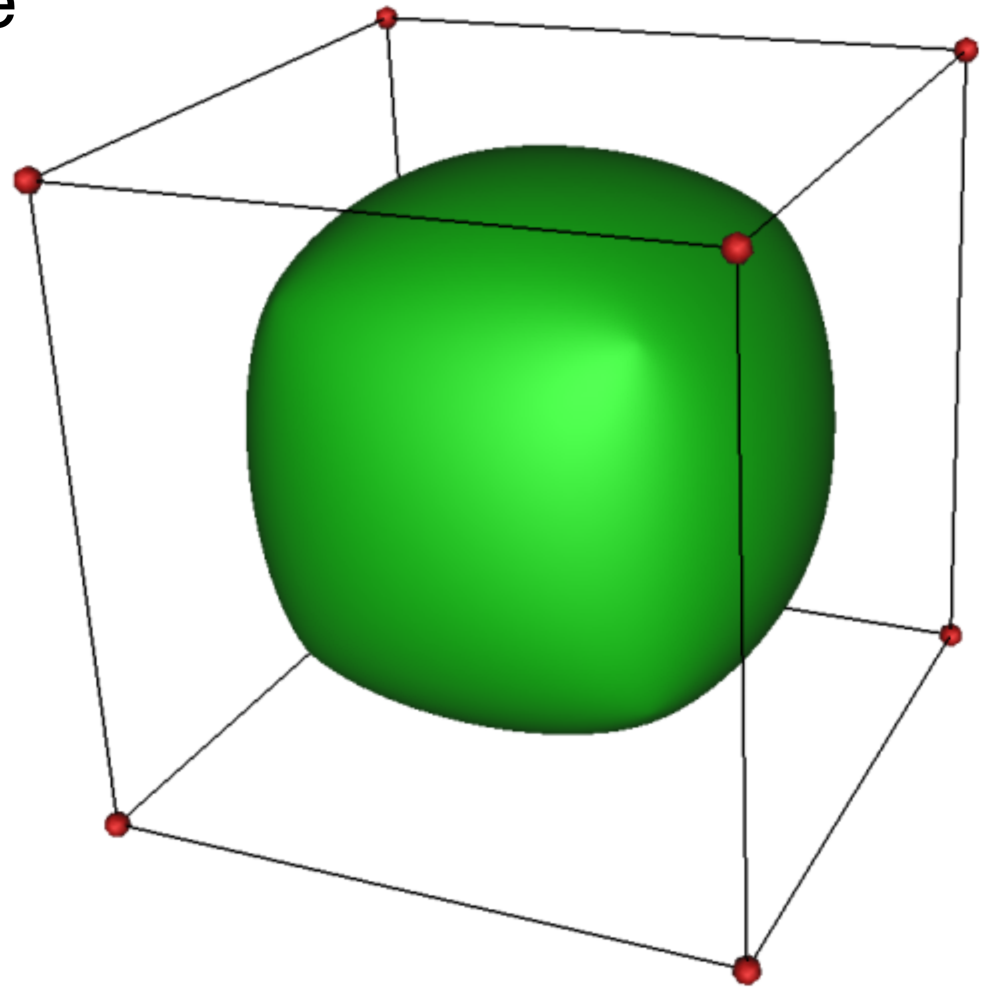
- Example



Topology refinement

Linear Subdivision

- Example



Final result

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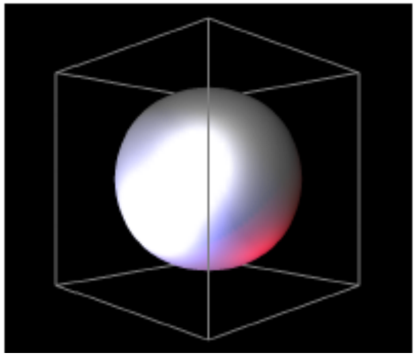
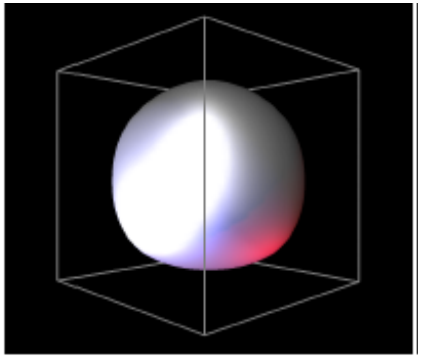
Subdivision Schemes

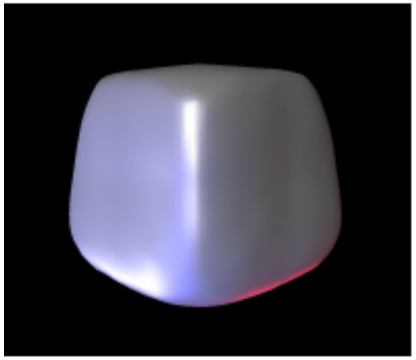
- Common subdivision schemes
  - o Catmull-Clark
  - o Loop
  - o Many others
- Differ in ...
  - o Input topology
  - o How refine topology
  - o How refine geometry

… which makes differences in …

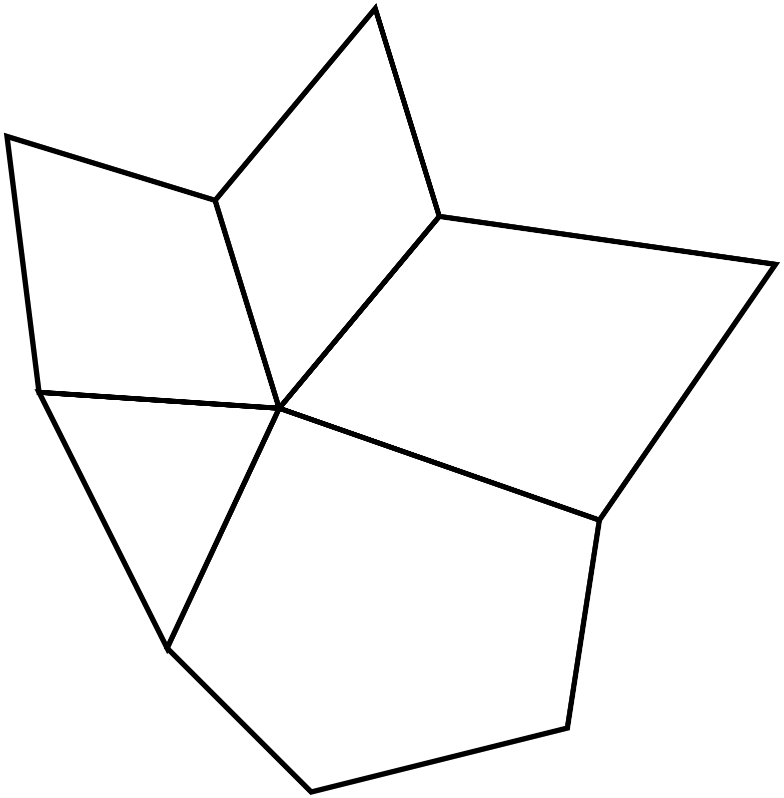
- o Provable properties







Catmull-Clark Subdivision



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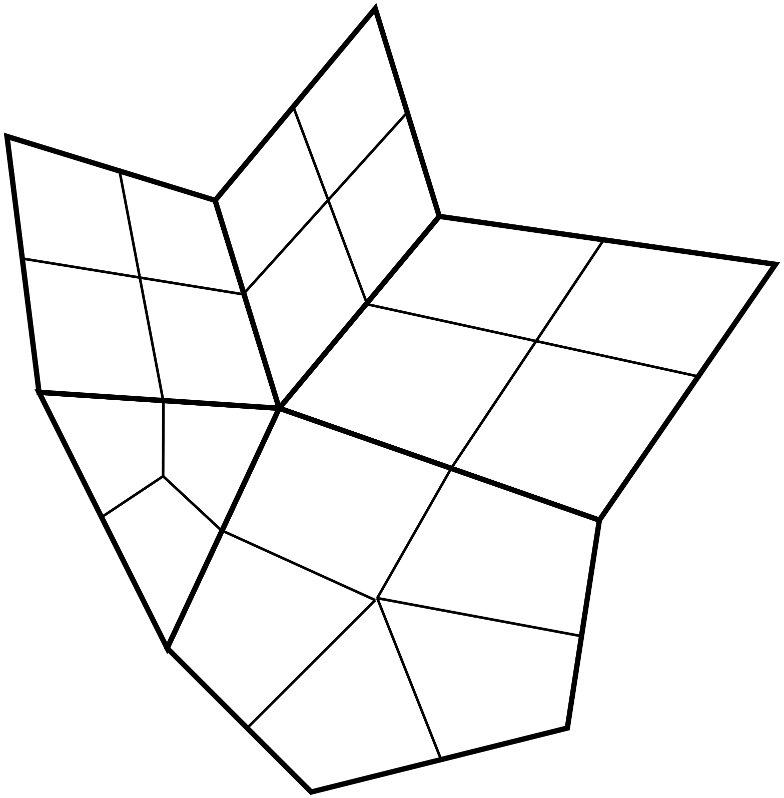
Catmull-Clark Subdivision



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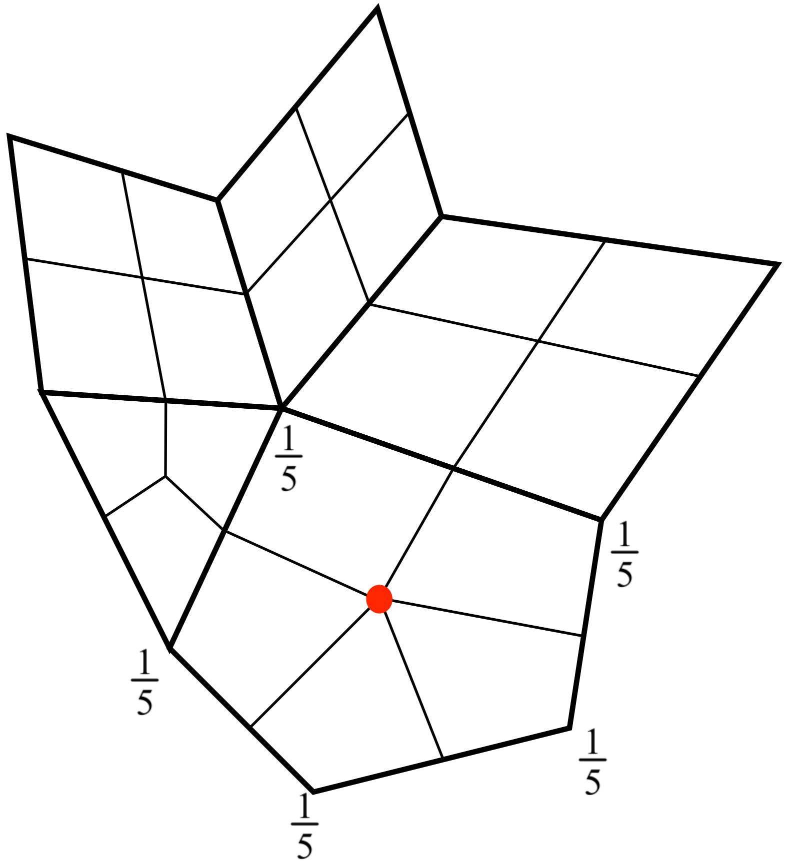
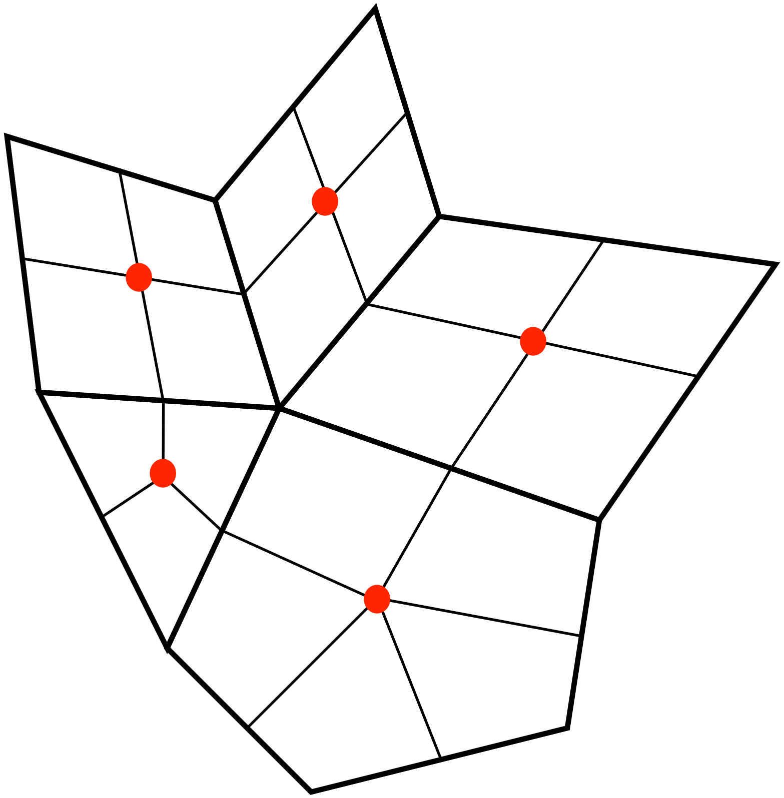
Catmull-Clark Subdivision



Catmull-Clark Subdivision



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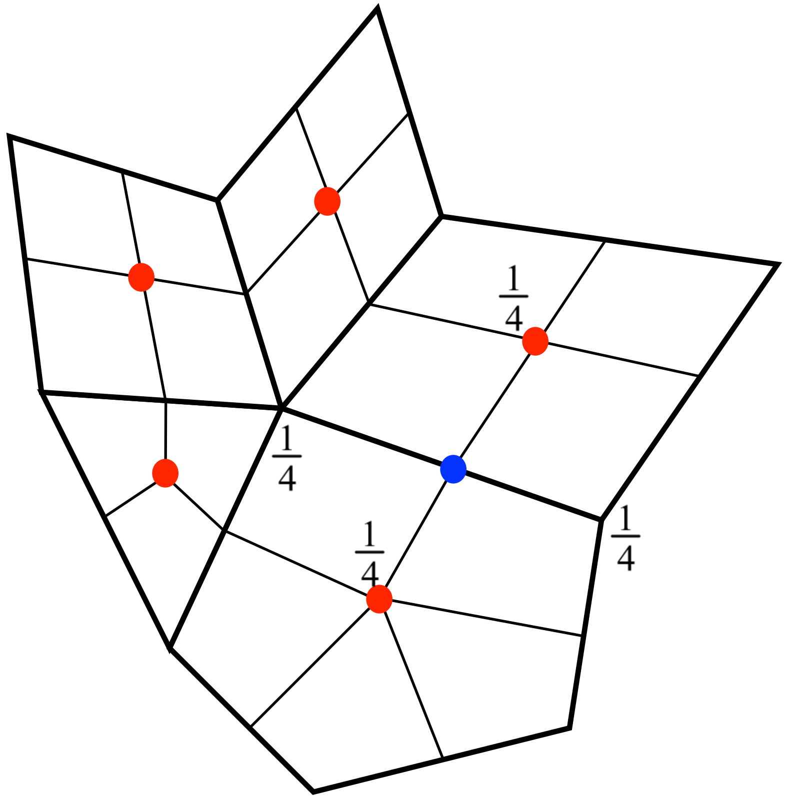
Catmull-Clark Subdivision



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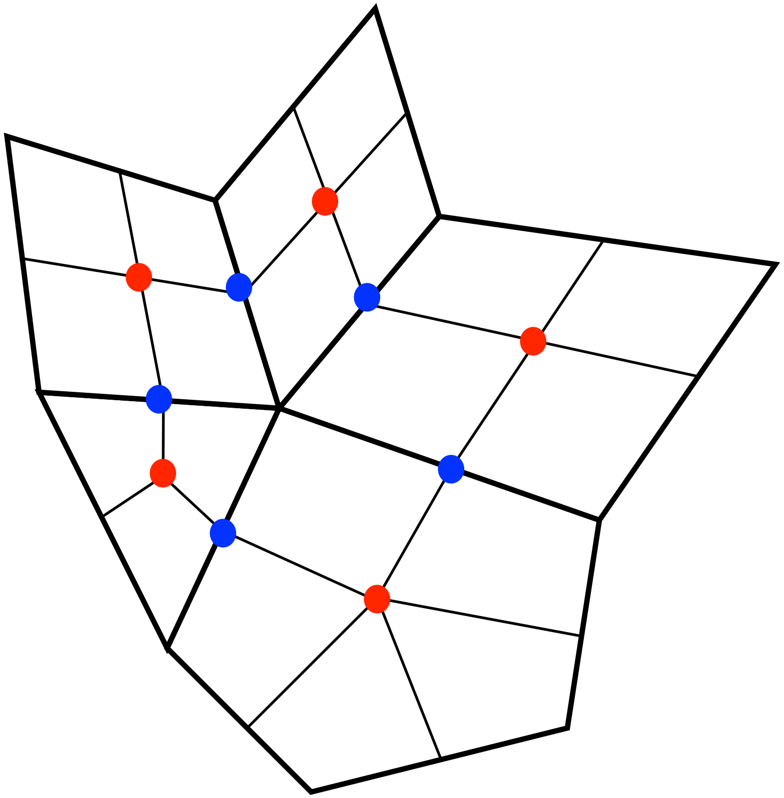
Catmull-Clark Subdivision



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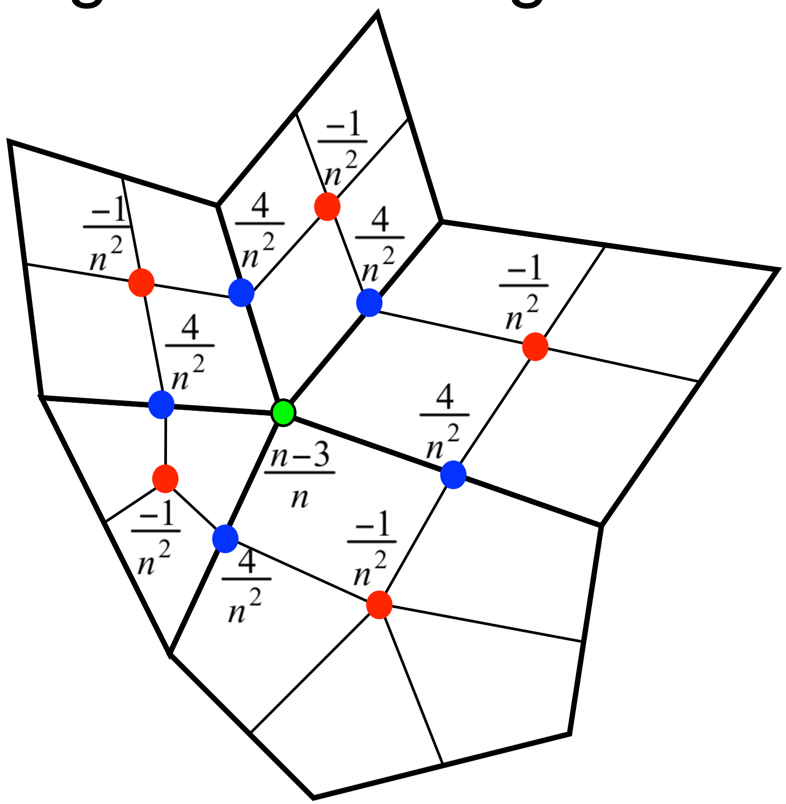
Catmull-Clark Subdivision

New ● = (4 * avg of ● –1 * avg of ● + (n-3) * ●) / n



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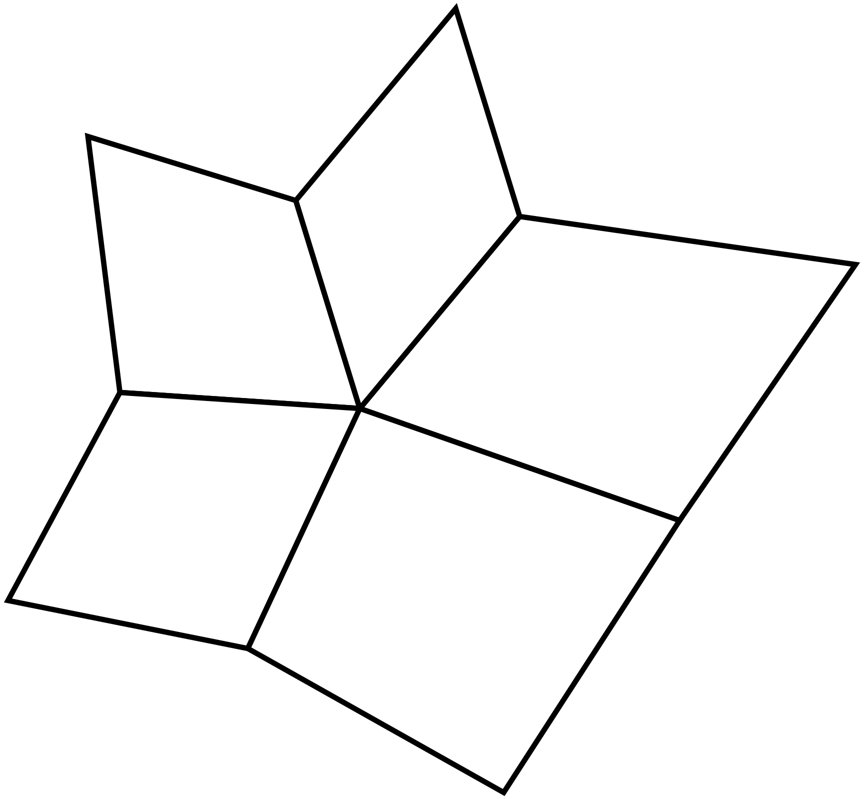
Catmull-Clark Subdivision



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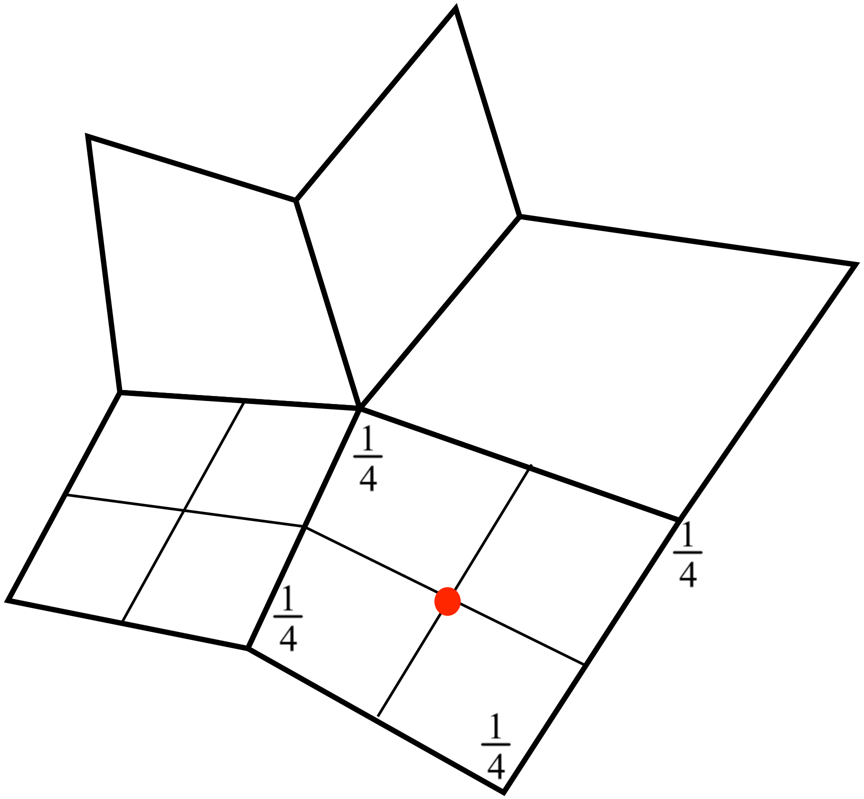

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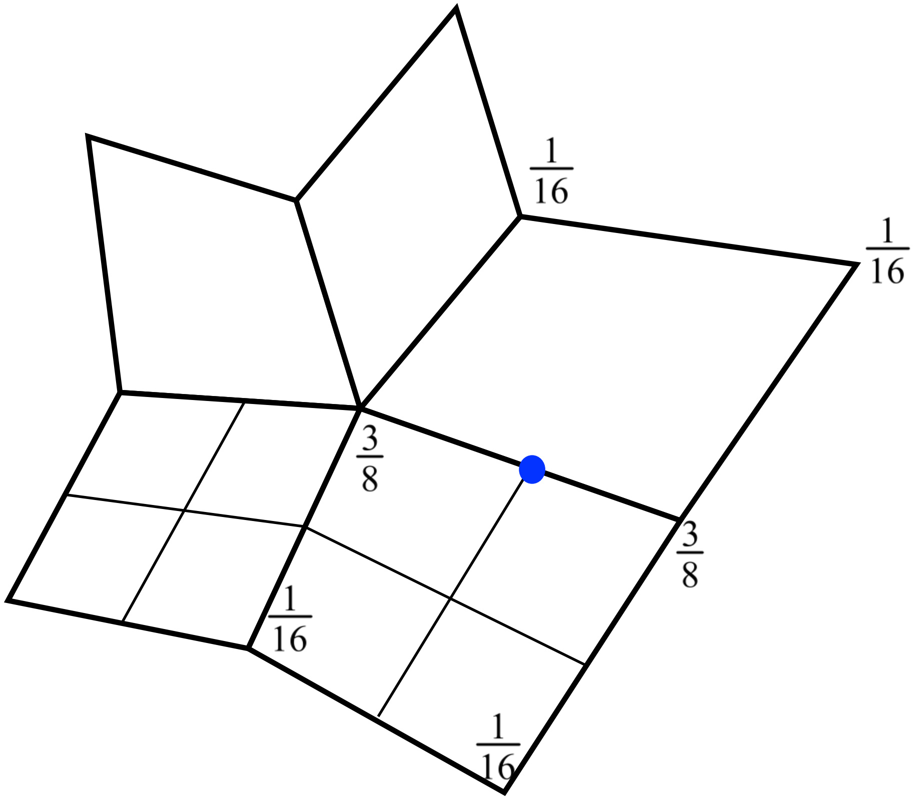

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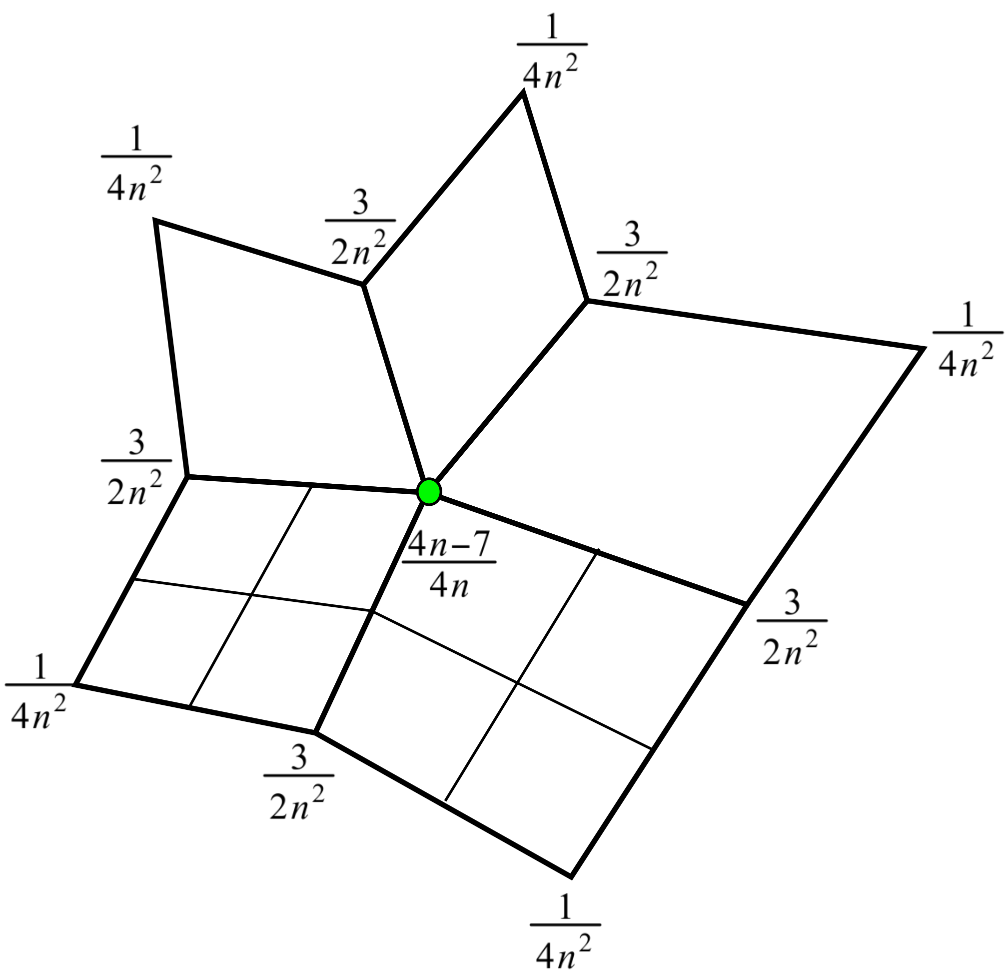
Catmull-Clark Subdivision



Catmull-Clark Subdivision

Repeated Averaging

Catmull-Clark Subdivision

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Catmull-Clark Subdivision

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Catmull-Clark Subdivision

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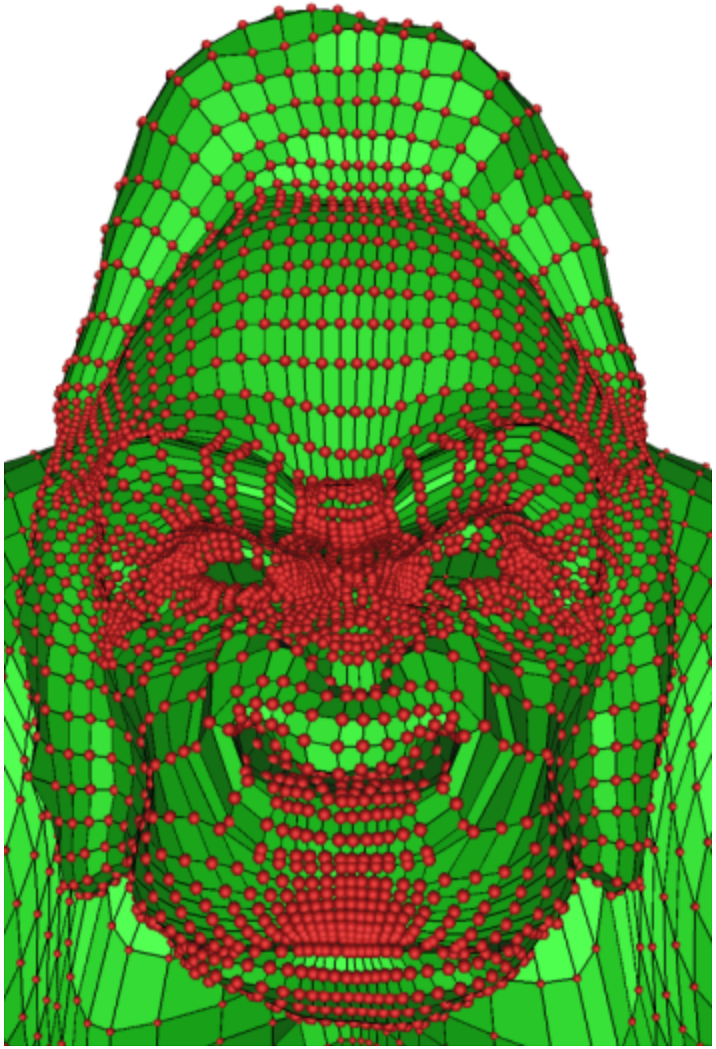
Catmull-Clark Subdivision



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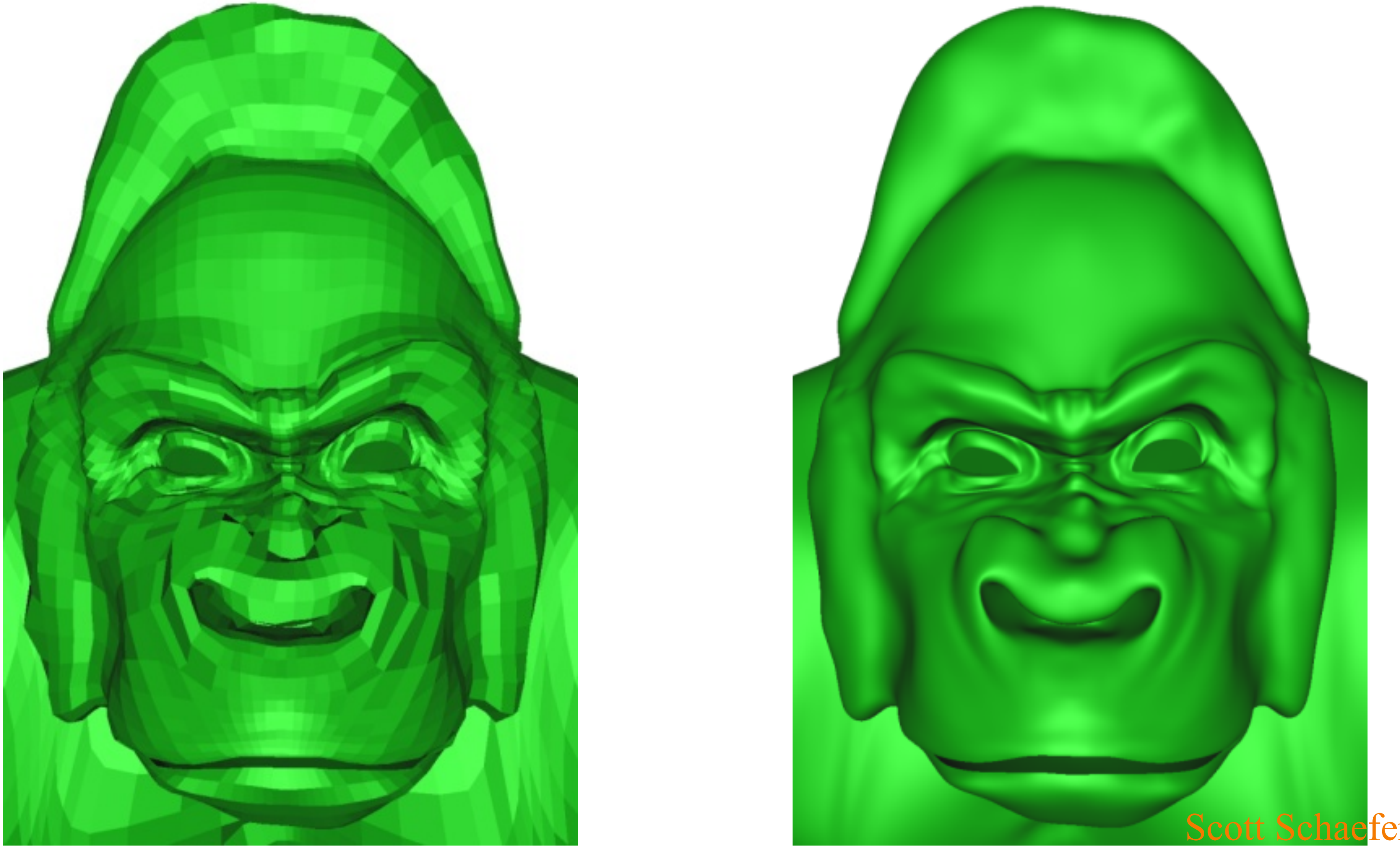
Catmull-Clark Subdivision



Catmull-Clark Subdivision

- One round of subdivision produces all quads
- Smoothness of limit surface
  - o C^2 almost everywhere
  - o C^1 at vertices with valence $\neq$ 4
- Relationship to control mesh
  - o Does not interpolate input vertices
  - o Within convex hull
- Most commonly used subdivision scheme in the movies…

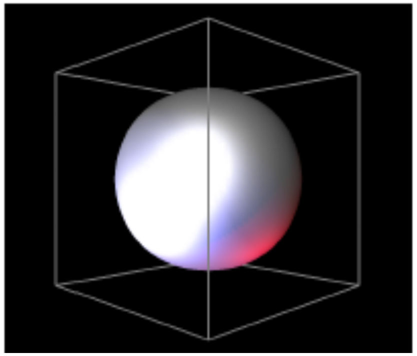
Pixar

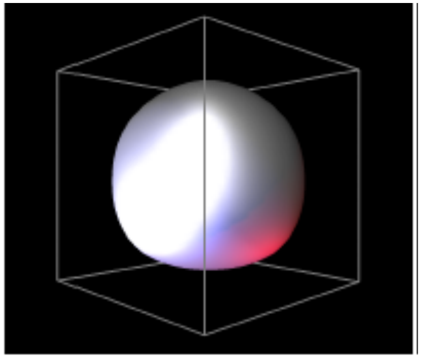
Subdivision Schemes

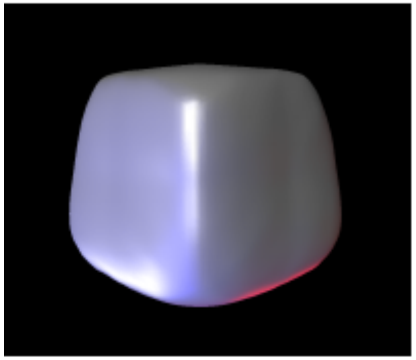
- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others
- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

… which makes differences in …

- Provable properties

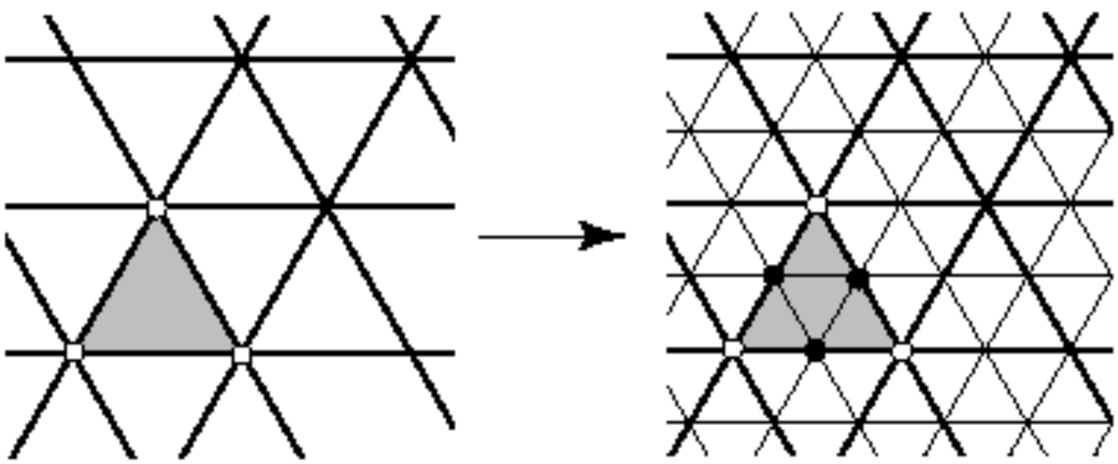






Loop Subdivision

- Operates on pure triangle meshes
- Subdivision rules
  - o Linear subdivision
  - o Averaging rules for “even / odd” (white / black) vertices



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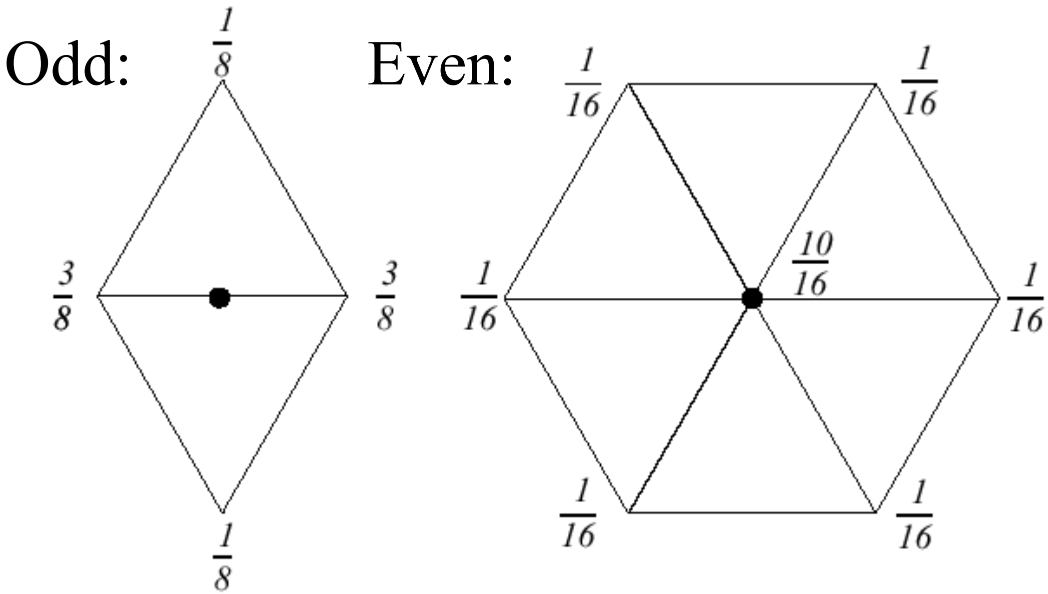
Loop Subdivision

- Averaging rules
  - o Weights for “odd” and “even” vertices



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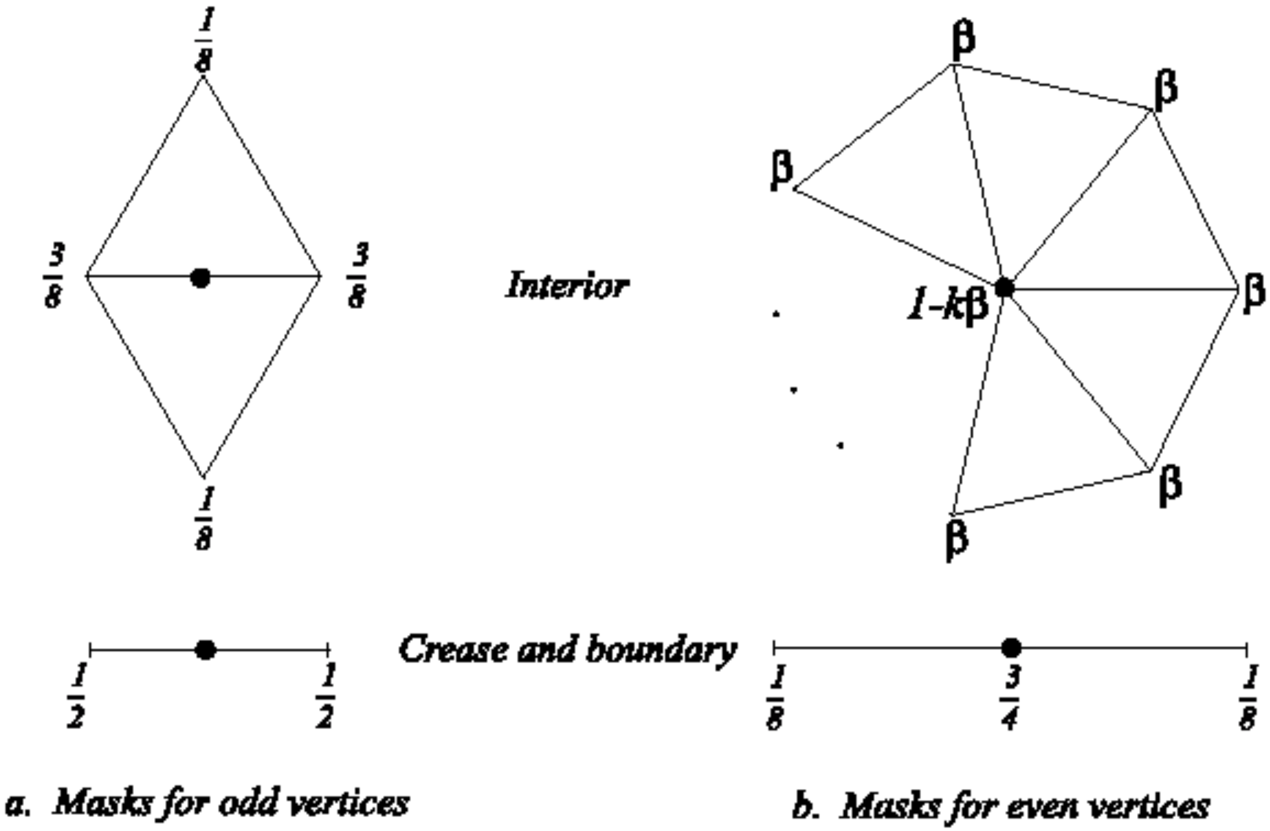
Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:



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Loop Subdivision

- How to choose β?
  - Analyze properties of limit surface
  - Interested in continuity of surface and smoothness
  - Involves calculating eigenvalues of matrices

    - » Original Loop

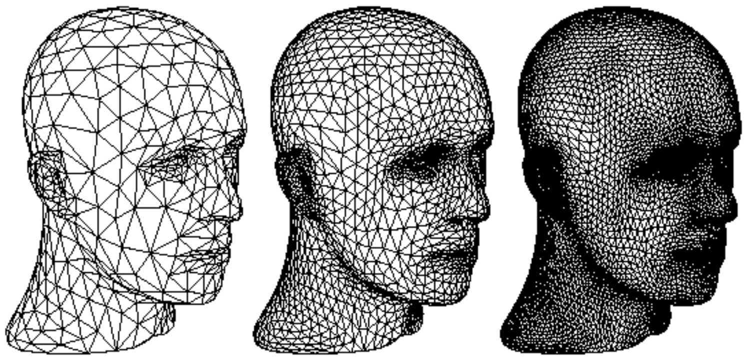
$$\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

    - » Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

Loop Subdivision

- Operates only on triangle meshes
- Smoothness of limit surface
  - o C^2 almost everywhere
  - o C^1 at vertices with valence ≠ 6
- Relationship to control mesh
  - o Does not interpolate input vertices
  - o Within convex hull



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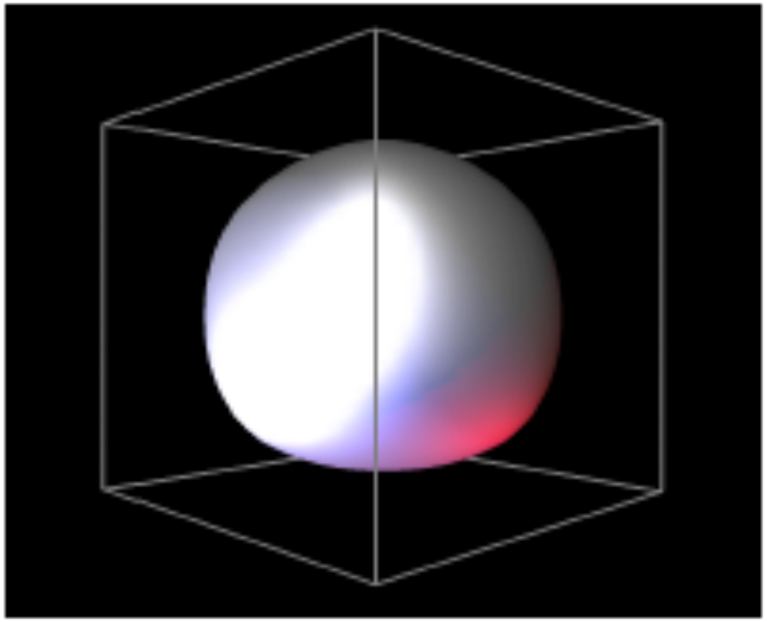
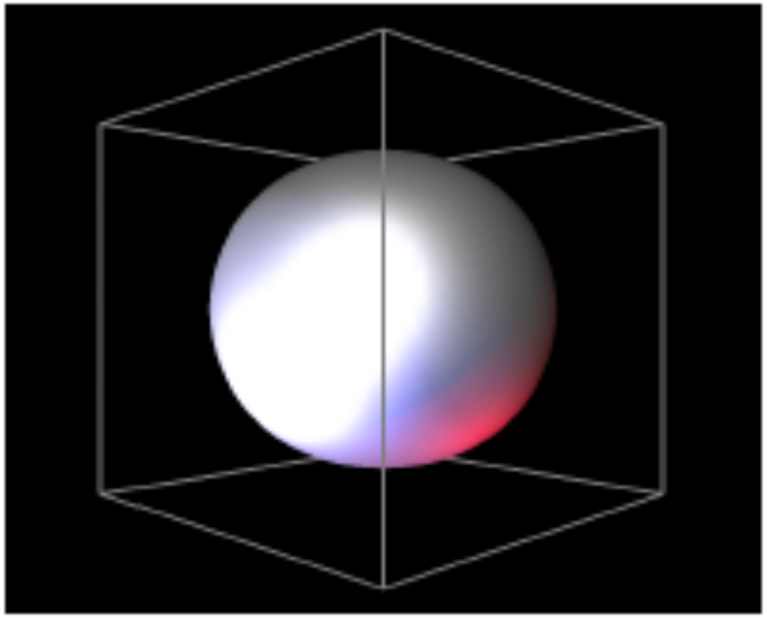
Subdivision Schemes

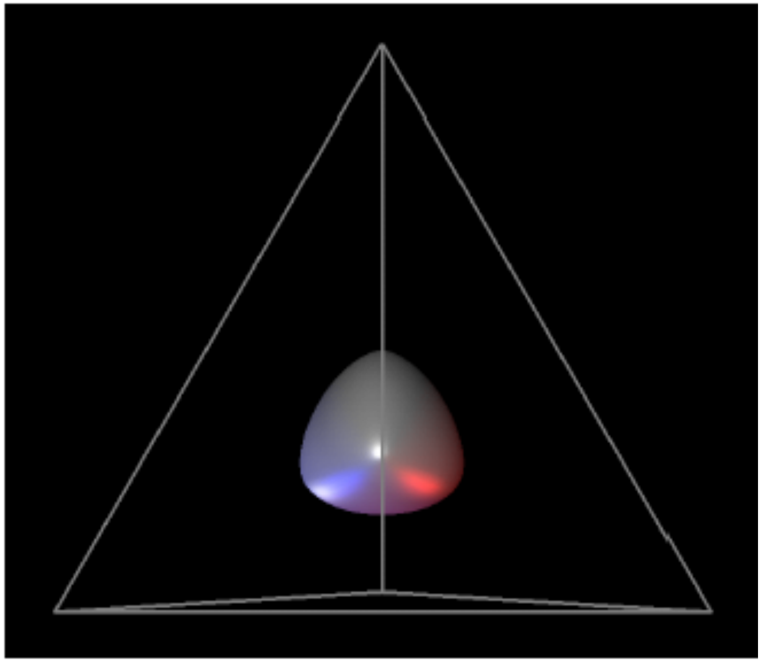


Loop

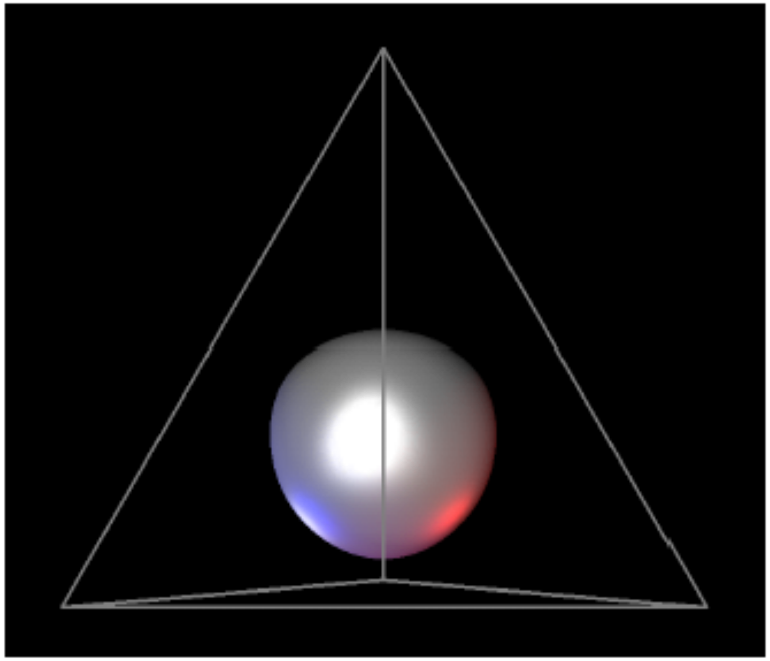


Catmull-Clark

Subdivision Schemes



Loop



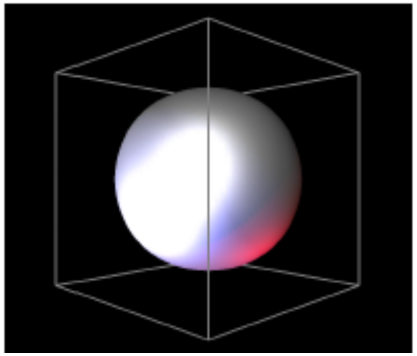
Catmull-Clark

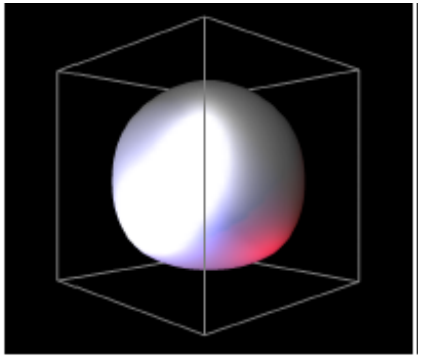
Subdivision Schemes

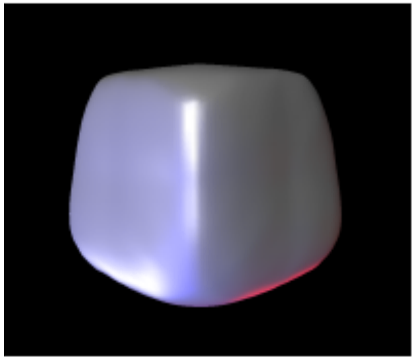
- Common subdivision schemes
  - o Catmull-Clark
  - o Loop
  - o Many others
- Differ in ...
  - o Input topology
  - o How refine topology
  - o How refine geometry

… which makes differences in …

- o Provable properties







Subdivision Schemes

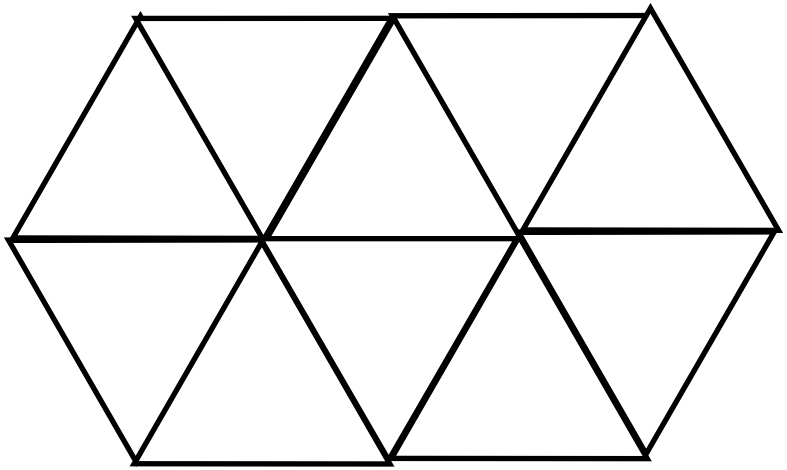
- Other subdivision schemes

Face split		
	Triangular meshes	*Quad. meshes*
Approximating	Loop (C^2)	Catmull-Clark (C^2)
Interpolating	Mod. Butterfly (C^1)	Kobbelt (C^1)

Vertex split
Doo-Sabin, Midedge (C^1)
Biquartic (C^2)

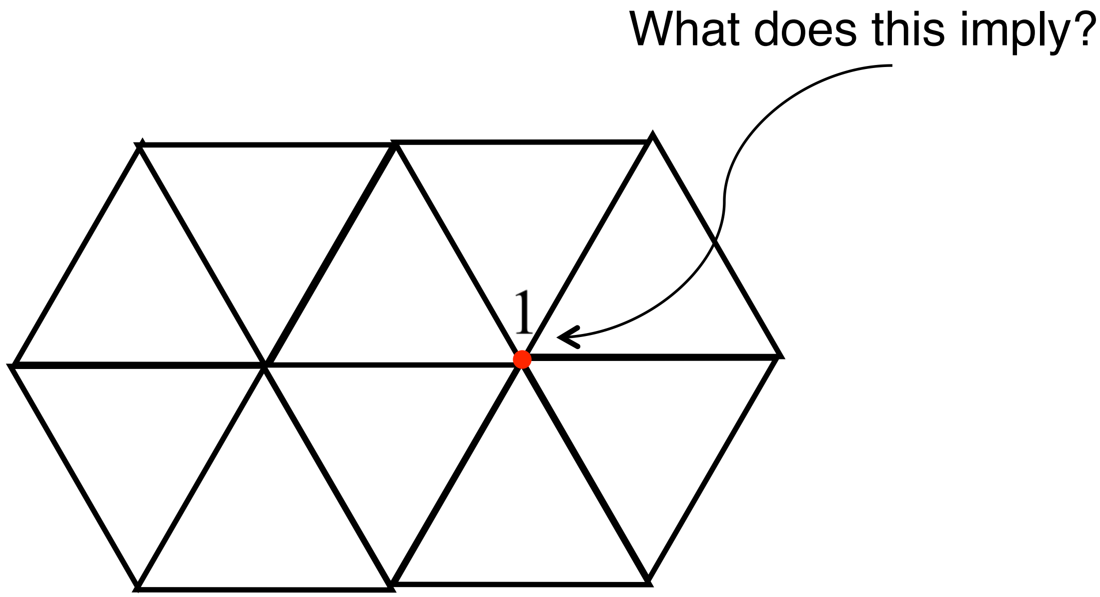
Other Subdivision Schemes

- Butterfly subdivision



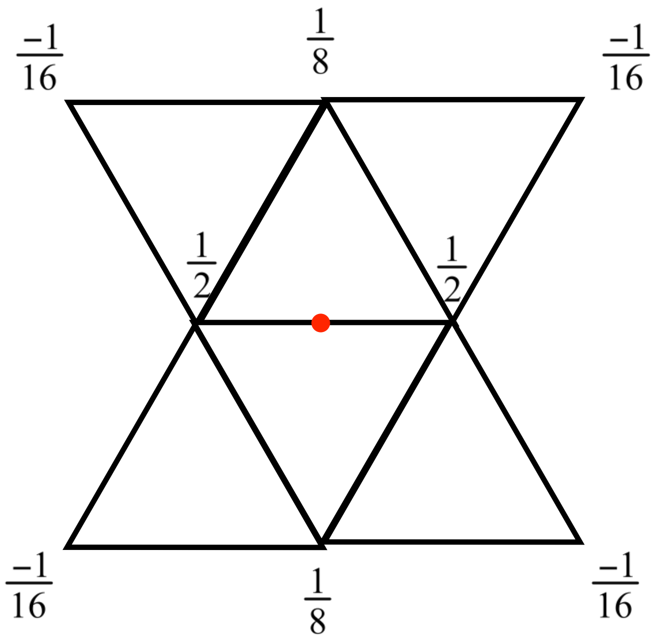
Other Subdivision Schemes

- Butterfly subdivision

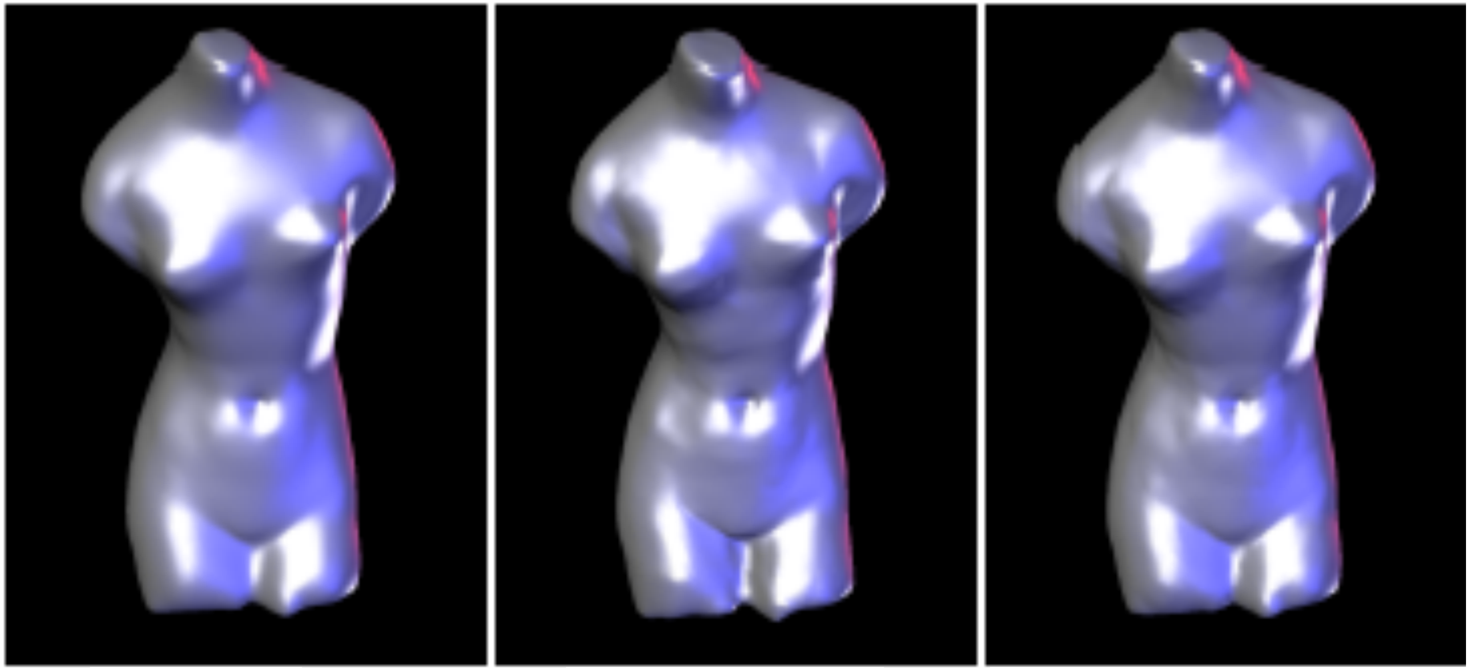


Other Subdivision Schemes

- Butterfly subdivision



Other Subdivision Schemes



Loop Butterfly Catmull-Clark

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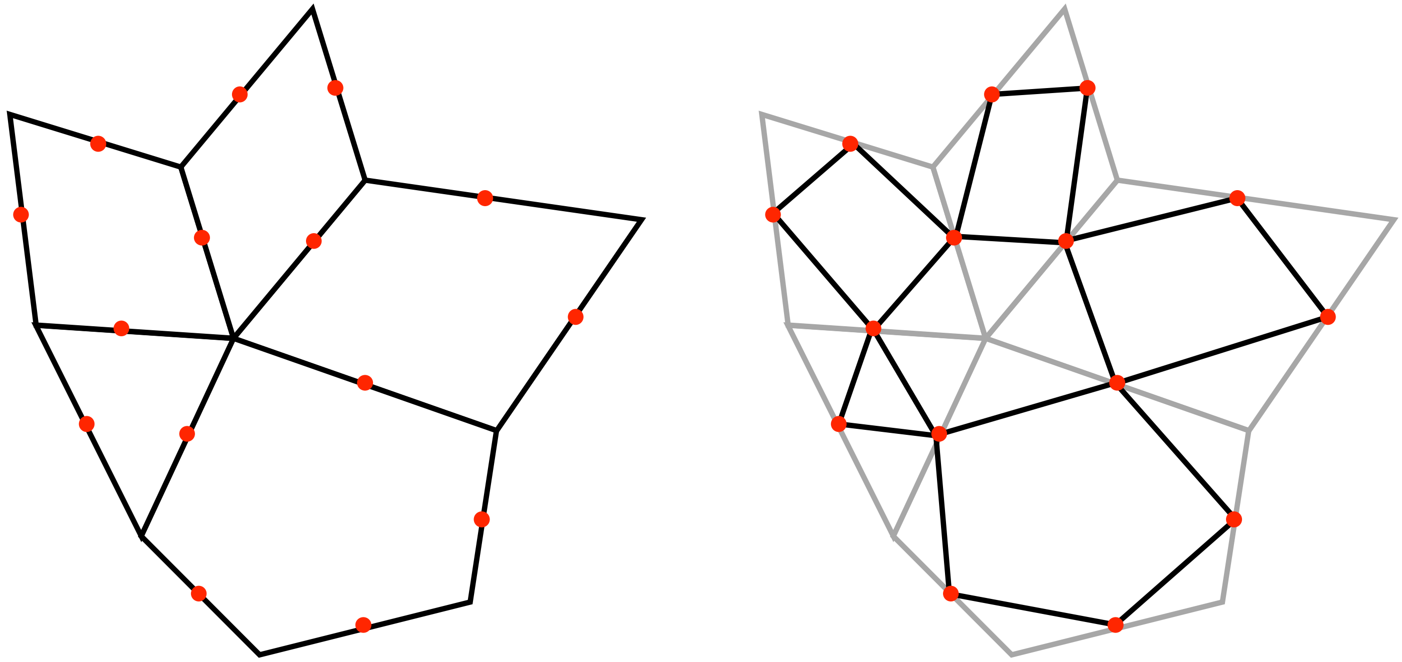
Other Subdivision Schemes

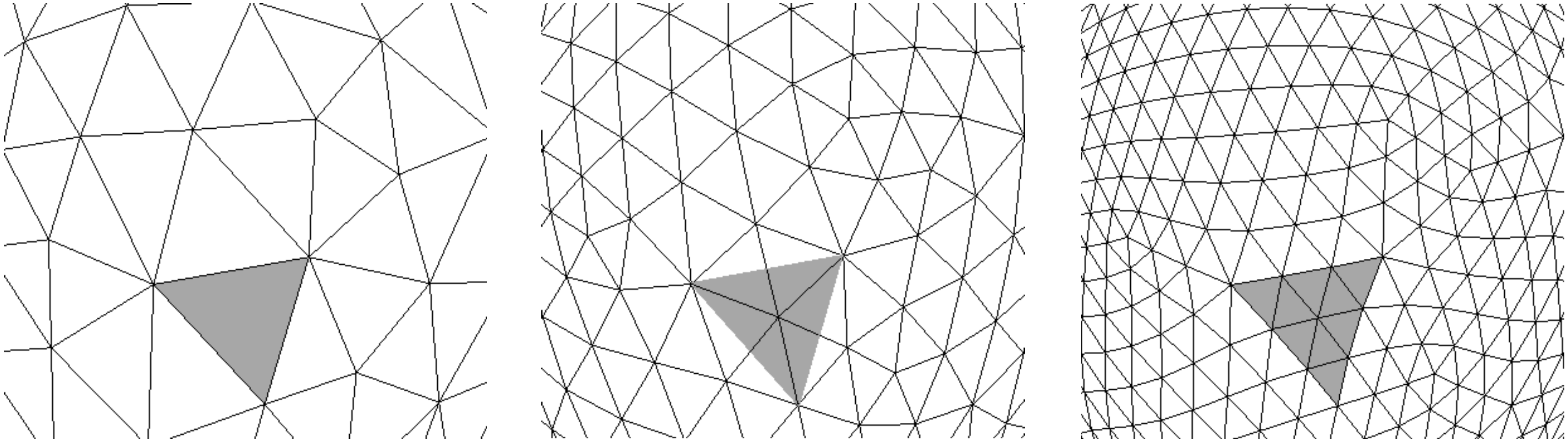
- Vertex-split subdivision
  (Doo-Sabin, Midedge, Biquartic)



One step of Midegde subdivision

Other Subdivision Schemes

- Sqrt(3) subdivision



Rotating grid of sqrt(3) subdivision

[Kobbelt 2000]

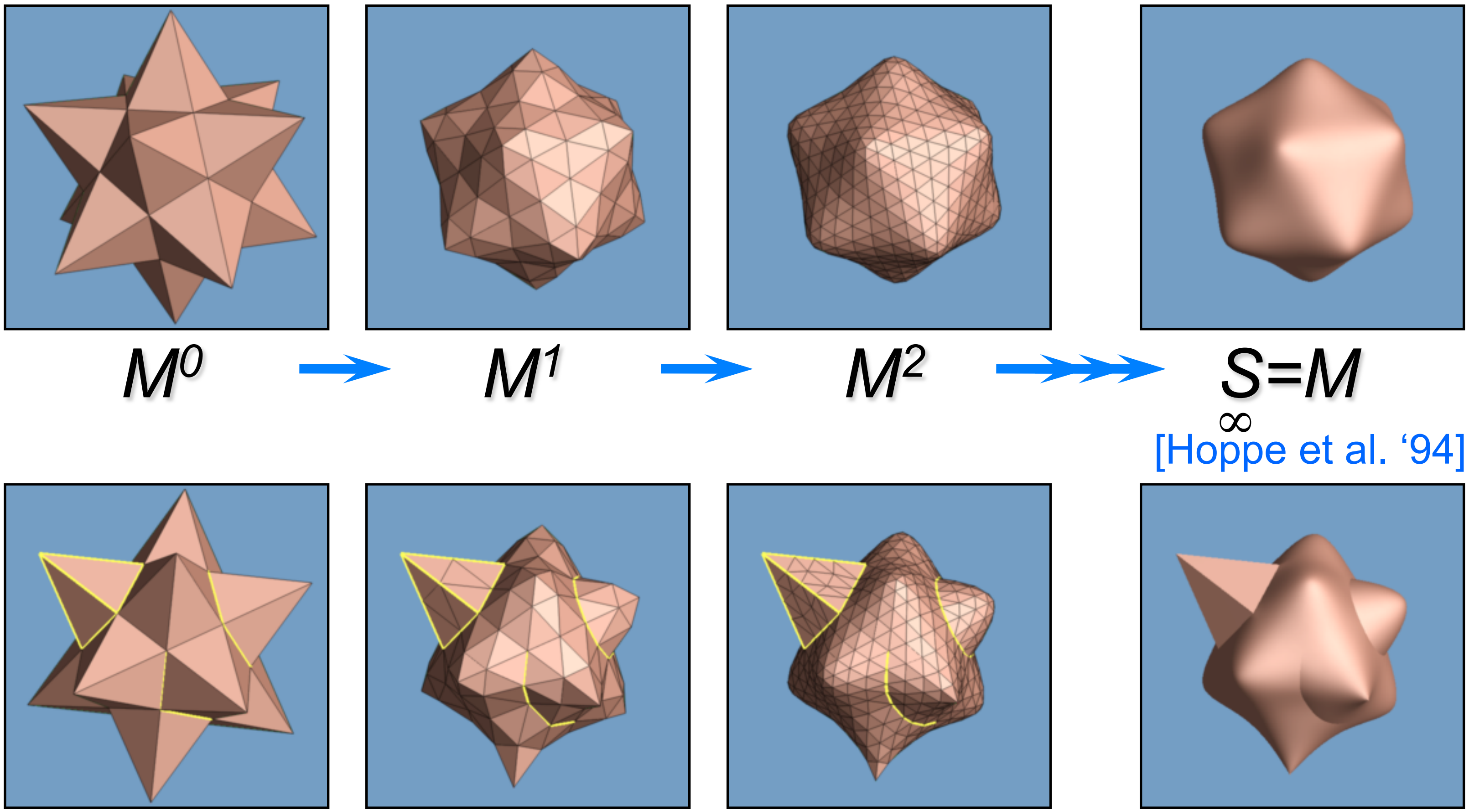
Extensions

- Common schemes assume *stationary* rules
- Locally altering subdivision rules allows for additional effects
- Examples for non-stationary rules:
  - Edge preservation
  - Adaptive subdivision

Edge Preservation

- Treat selected control mesh edges as boundary:

[Loop ‘87]



tagged mesh

Adaptive Subdivision

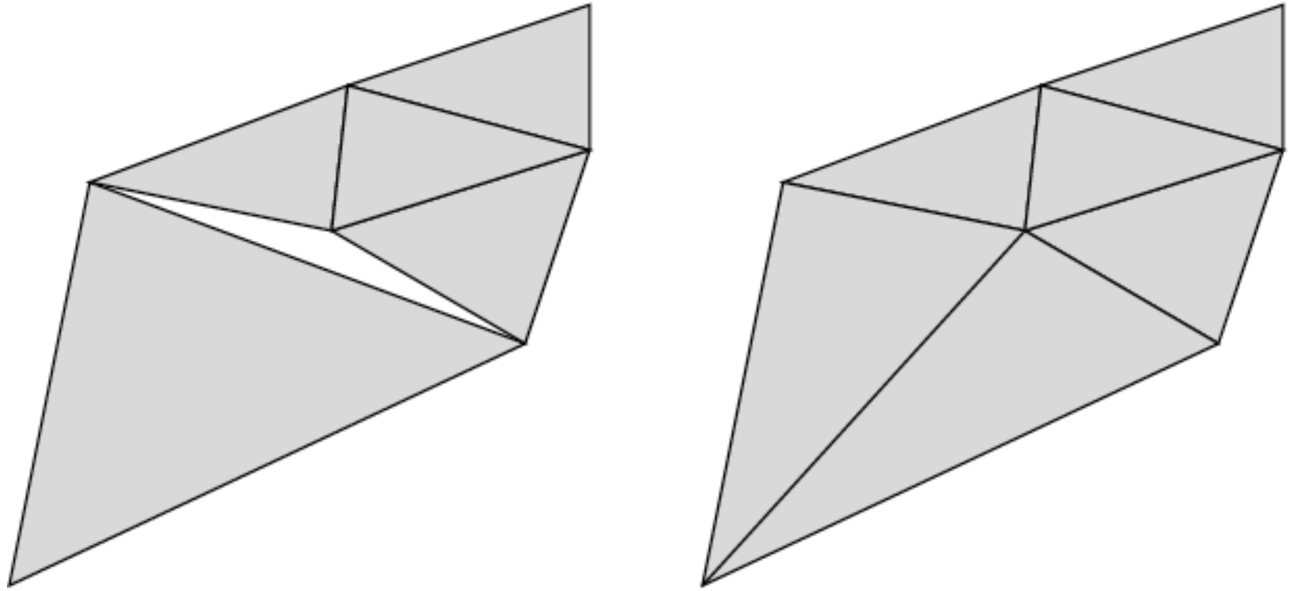
- Goal:
  - o Best possible approximation of smooth limit surface
  - o With limited triangle budget
- Quality of approximation can be defined by
  - o Projected (screen) area of final triangles
  - o Local surface curvature

Adaptive Subdivision

- Goal:
  - o Best possible approximation of smooth limit surface
  - o With limited triangle budget
- Quality of approximation can be defined by
  - o Projected (screen) area of final triangles
  - o Local surface curvature
- Solution:
  - o Stop subdivision at different levels across the surface
  - o Stop-criterion depending on quality measure
  - o Project each vertex onto limit surface

Adaptive Subdivision

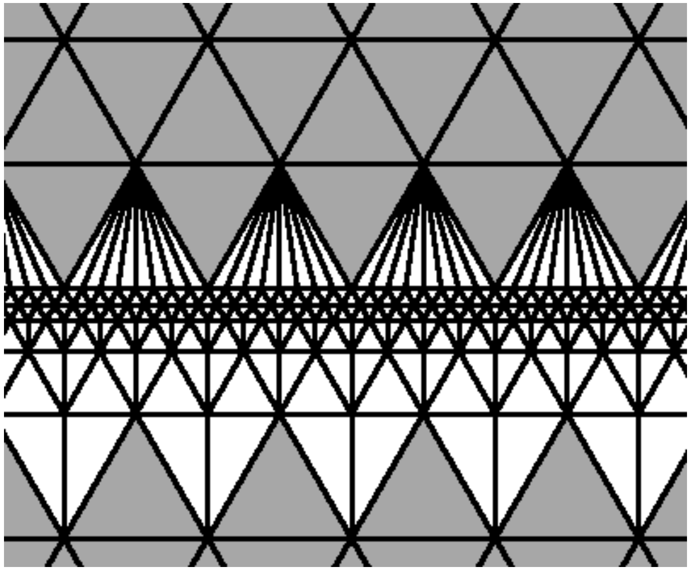
- Problem:
  - o Different levels of subdivision may lead to gaps in the surface



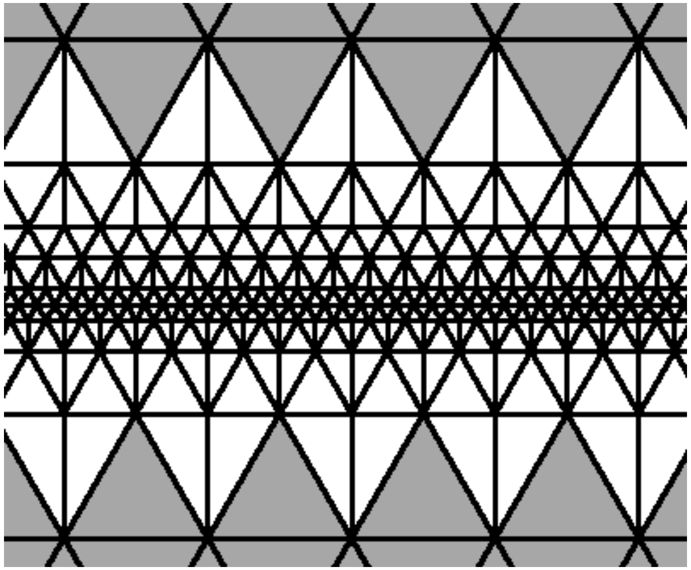
[Kobbelt 2000]

Adaptive Subdivision

- Solution:
  - o Replacing incompatible coarse triangles by *triangle fan*
  - o Balanced subdivision: neighboring subdivision levels must not differ by more than one



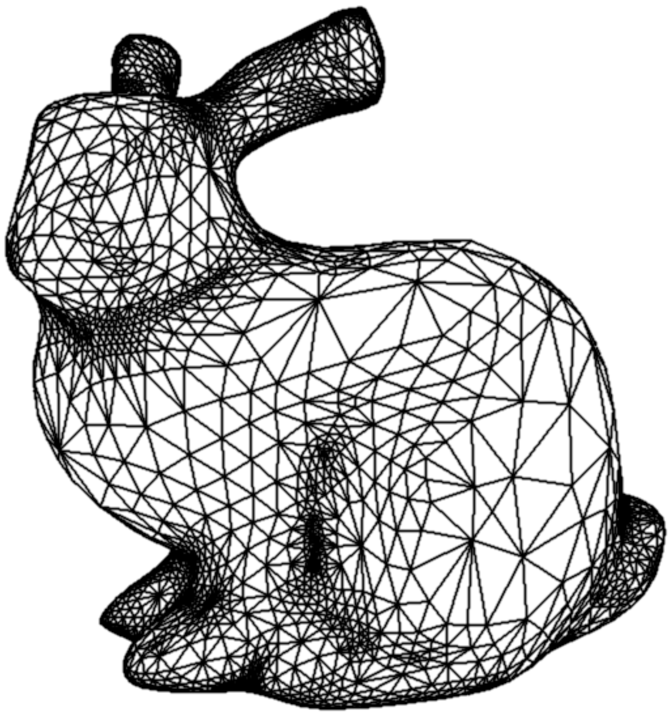
Unbalanced



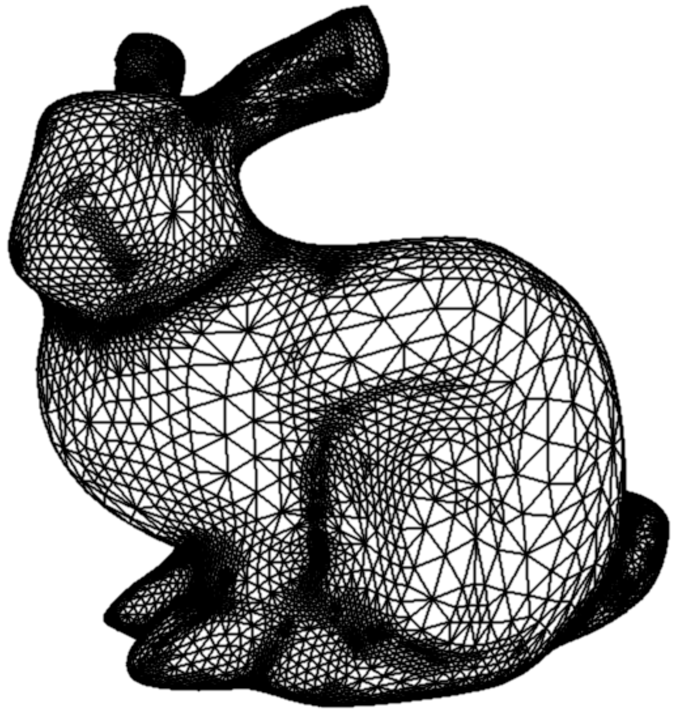
Balanced

[Kobbelt 2000]

Adaptive Subdivision



10072 Triangles



228654 Triangles

[Kobbelt 2000]

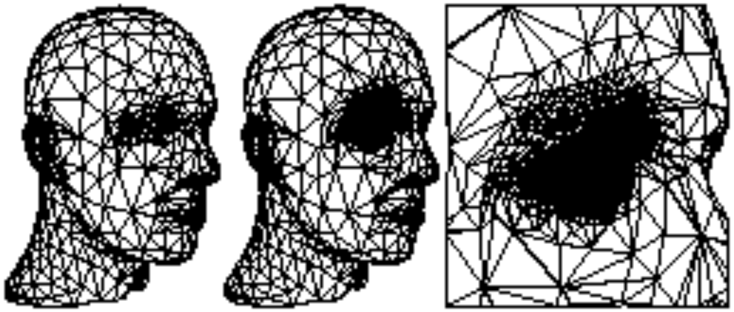
Subdivision Surfaces

- Properties:
  - o Accurate
  - o Concise
  - o Intuitive specification
  - o Local support
  - o Affine invariant
  - o Arbitrary topology
  - o Guaranteed continuity
  - o Natural parameterization
  - o Efficient display
  - o Efficient intersections

Pixar

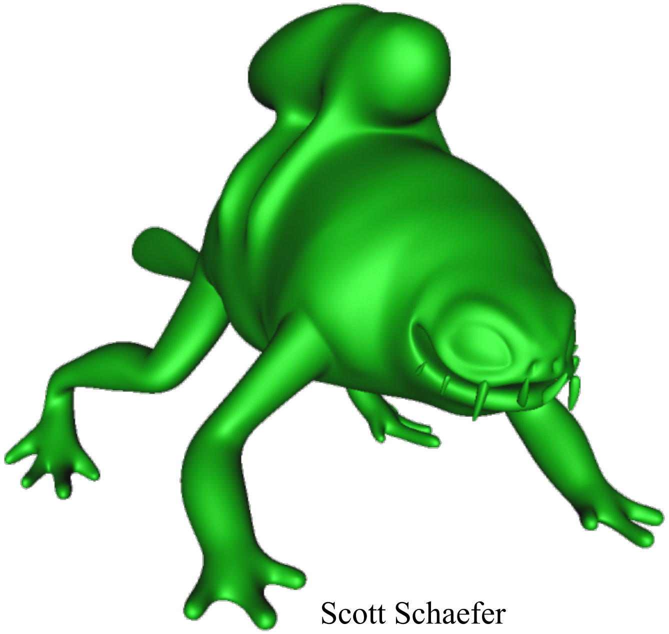
Subdivision Surfaces

- Advantages:
  - o Simple method for describing complex surfaces
  - o Relatively easy to implement
  - o Arbitrary topology
  - o Local support
  - o Guaranteed continuity
  - o Multiresolution
- Difficulties:
  - o Intuitive specification
  - o Parameterization
  - o Intersections

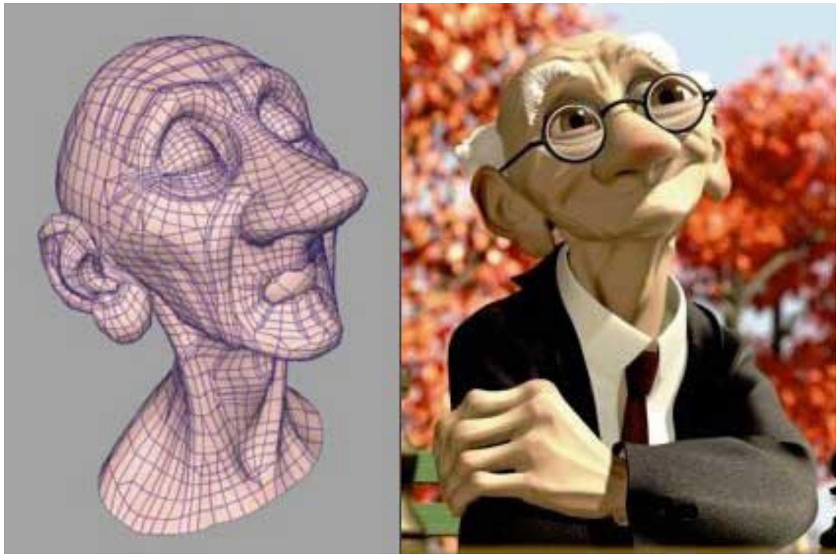


Subdivision Surfaces

- Used in movie and game industries
- Supported by most 3D modeling software



Scott Schaefer



Geri's Game © Pixar Animation Studios

Summary

Feature	Polygonal Mesh	Subdivision Surface
Accurate	No	Yes
Concise	No	Yes
Intuitive specification	No	No
Local support	Yes	Yes
Affine invariant	Yes	Yes
Arbitrary topology	Yes	Yes
Guaranteed continuity	No	Yes
Natural parameterization	No	No
Efficient display	Yes	Yes
Efficient intersections	No	No