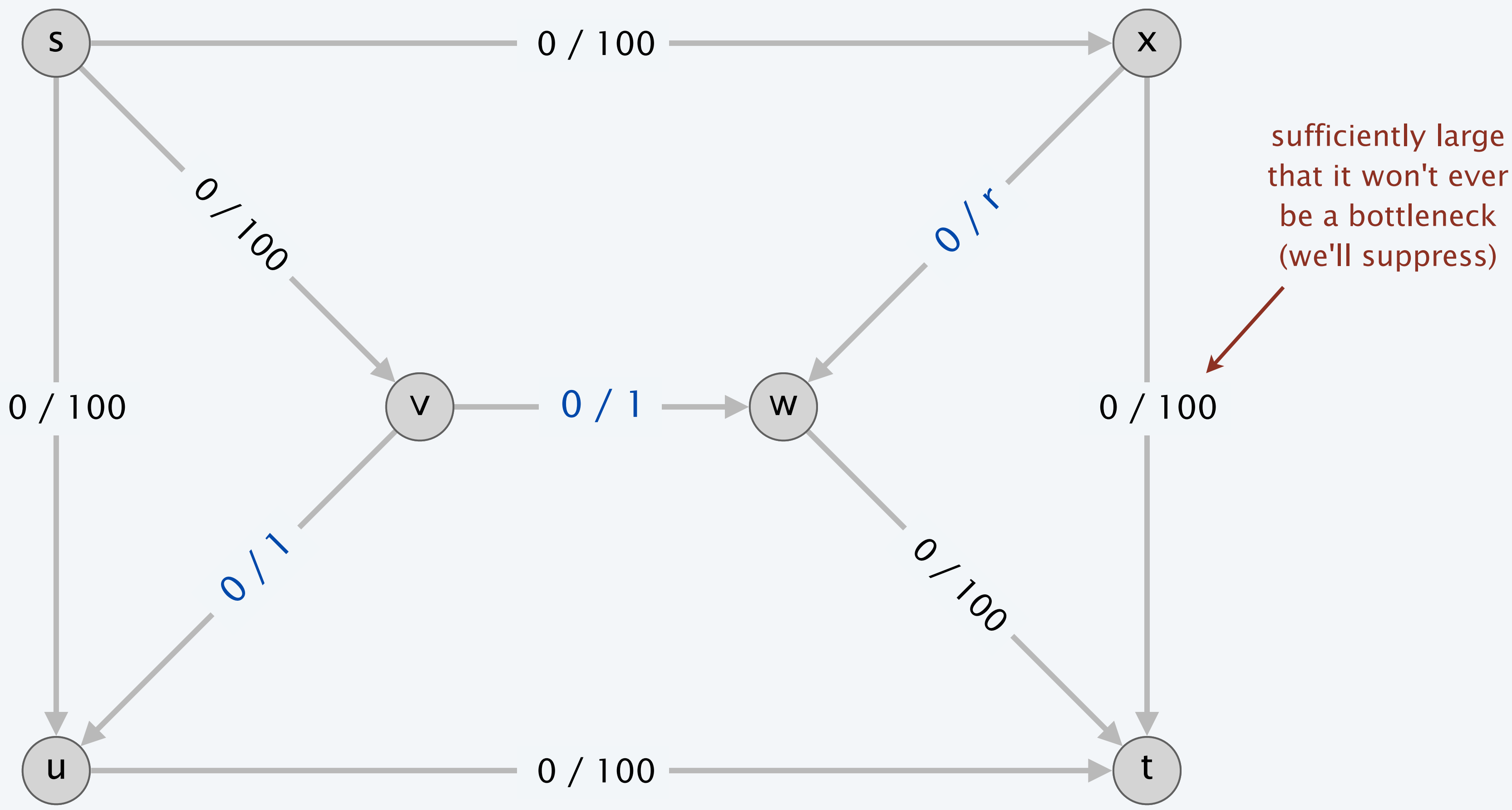
Ford-Fulkerson pathological example

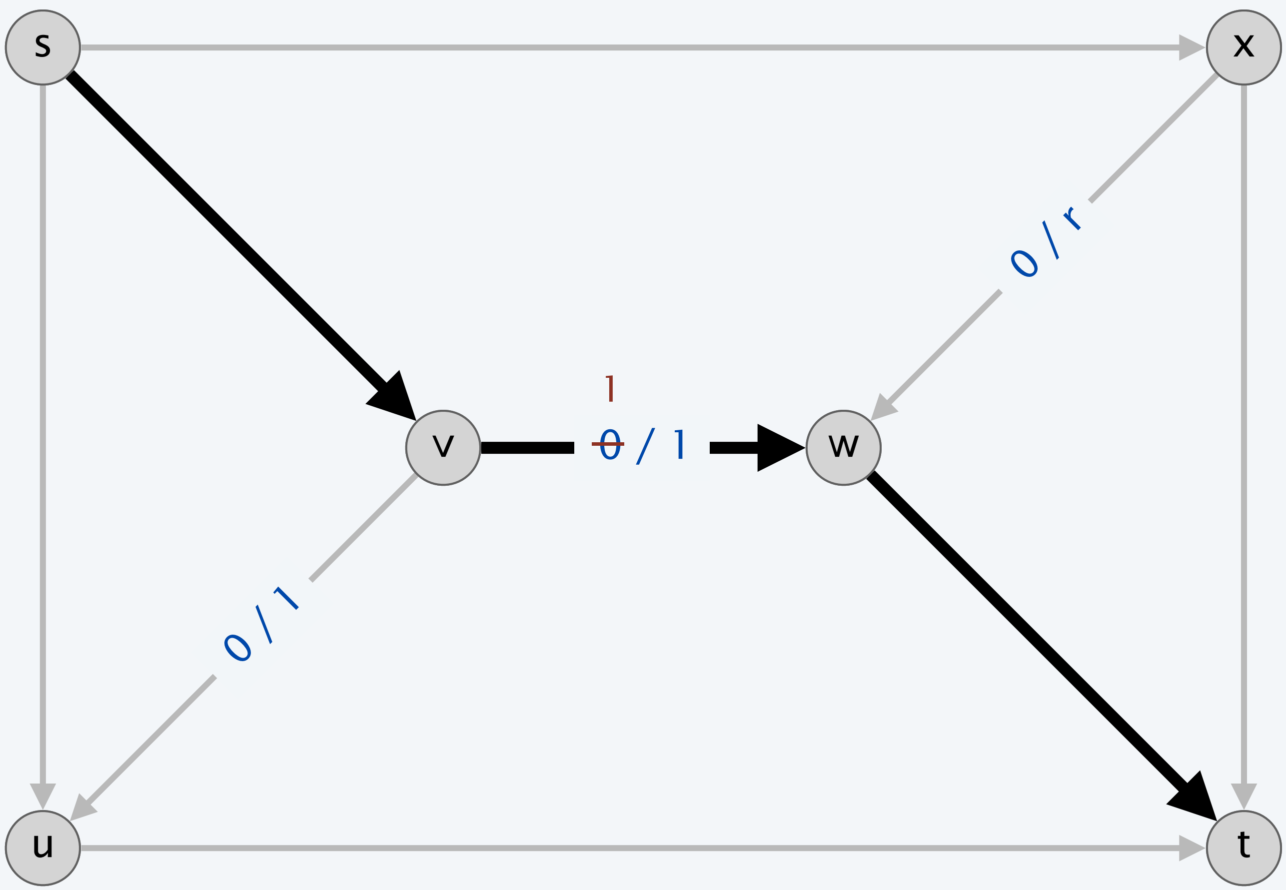
$$r = \frac{\sqrt{5} - 1}{2} \implies r^2 = 1 - r$$

network G



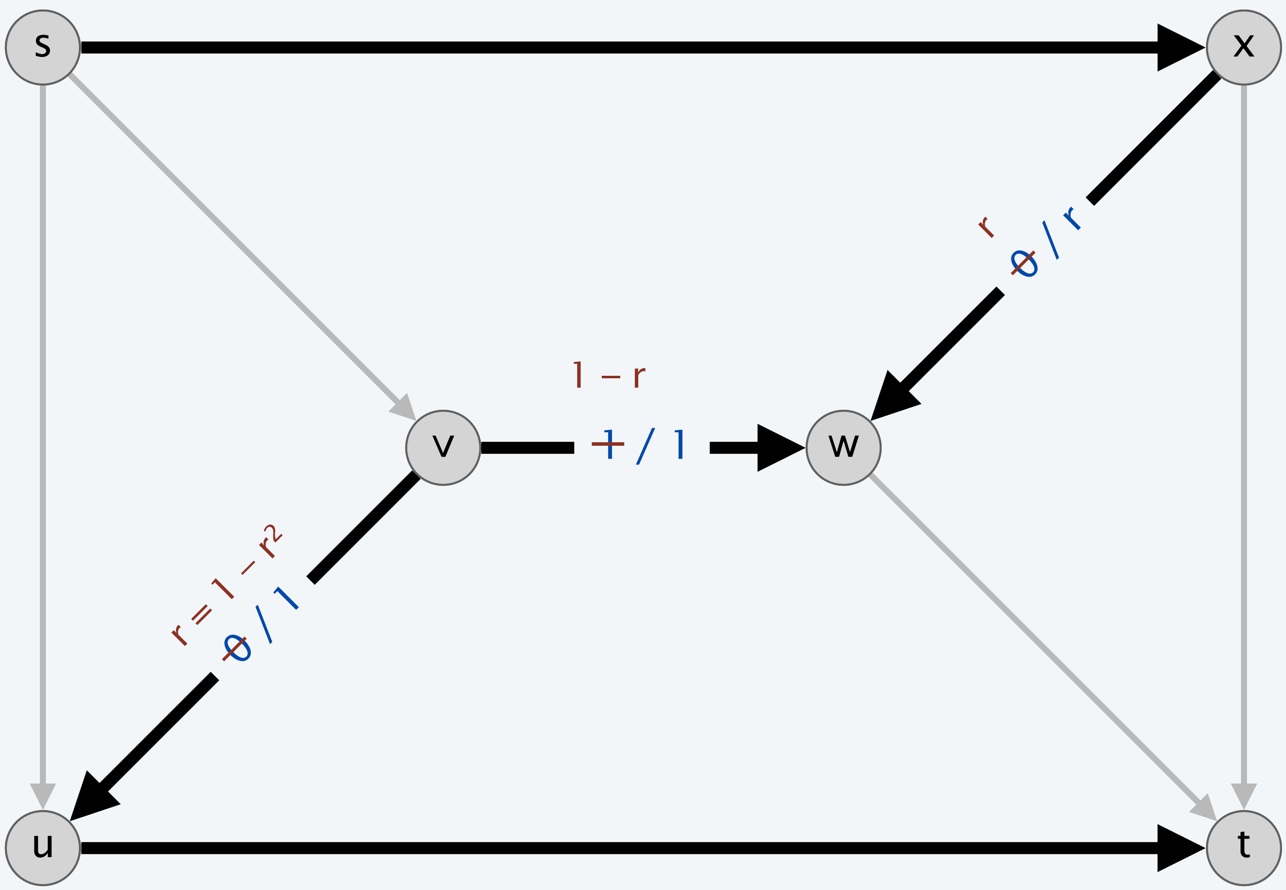
Ford-Fulkerson pathological example

augmenting path 1: s→v→w→t (bottleneck capacity = 1)



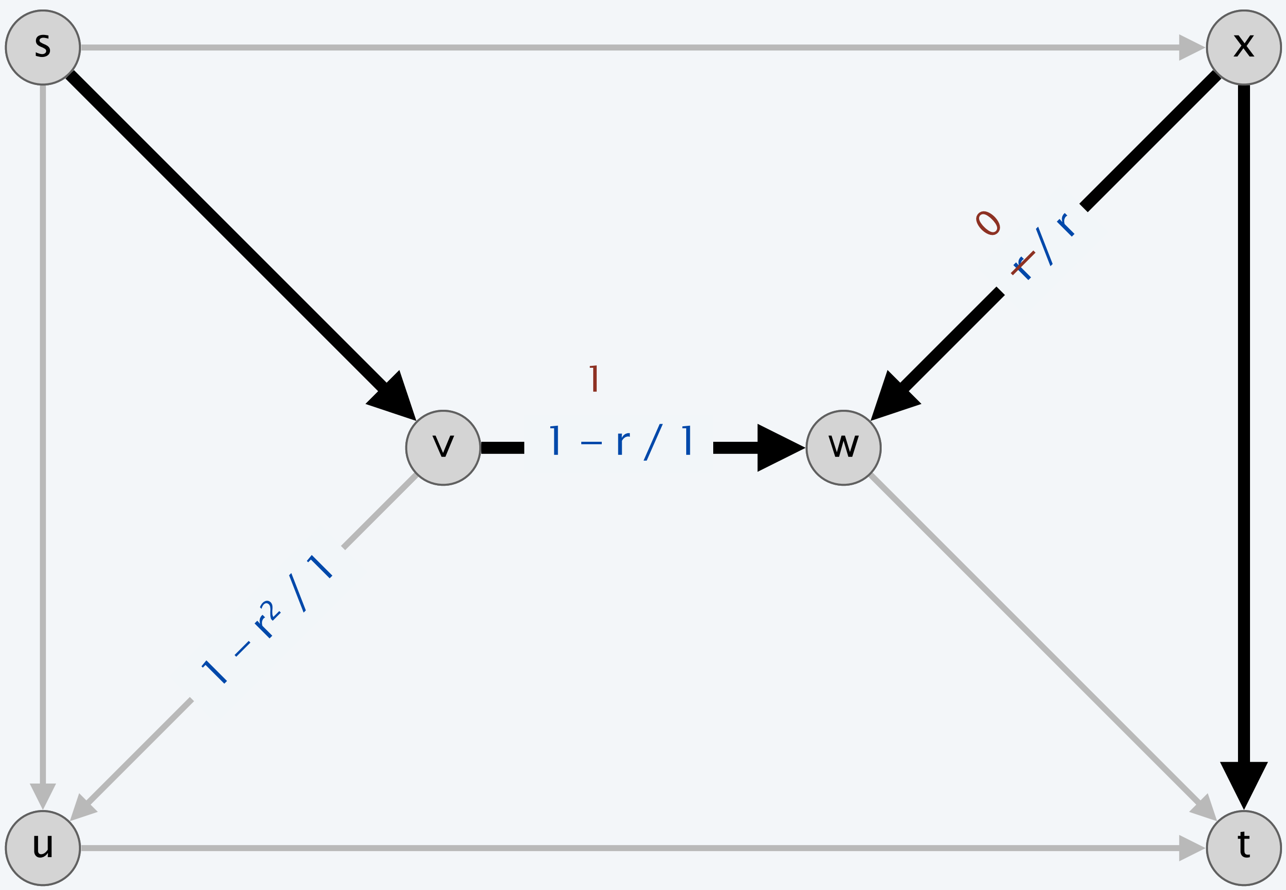
Ford-Fulkerson pathological example

augmenting path 2: s→x→w→v→u→t (bottleneck capacity = r)



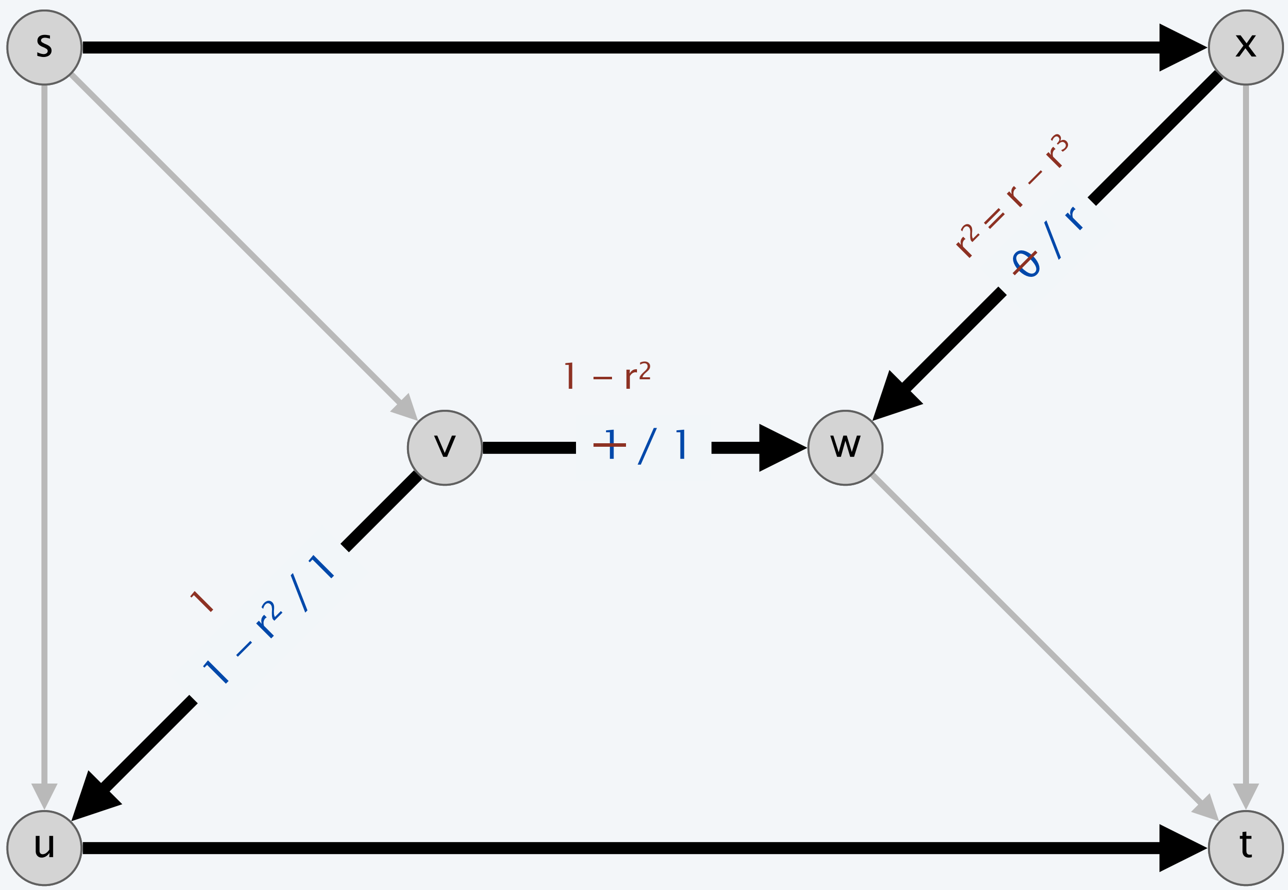
Ford-Fulkerson pathological example

augmenting path 3: s→v→w→x→t (bottleneck capacity = r)



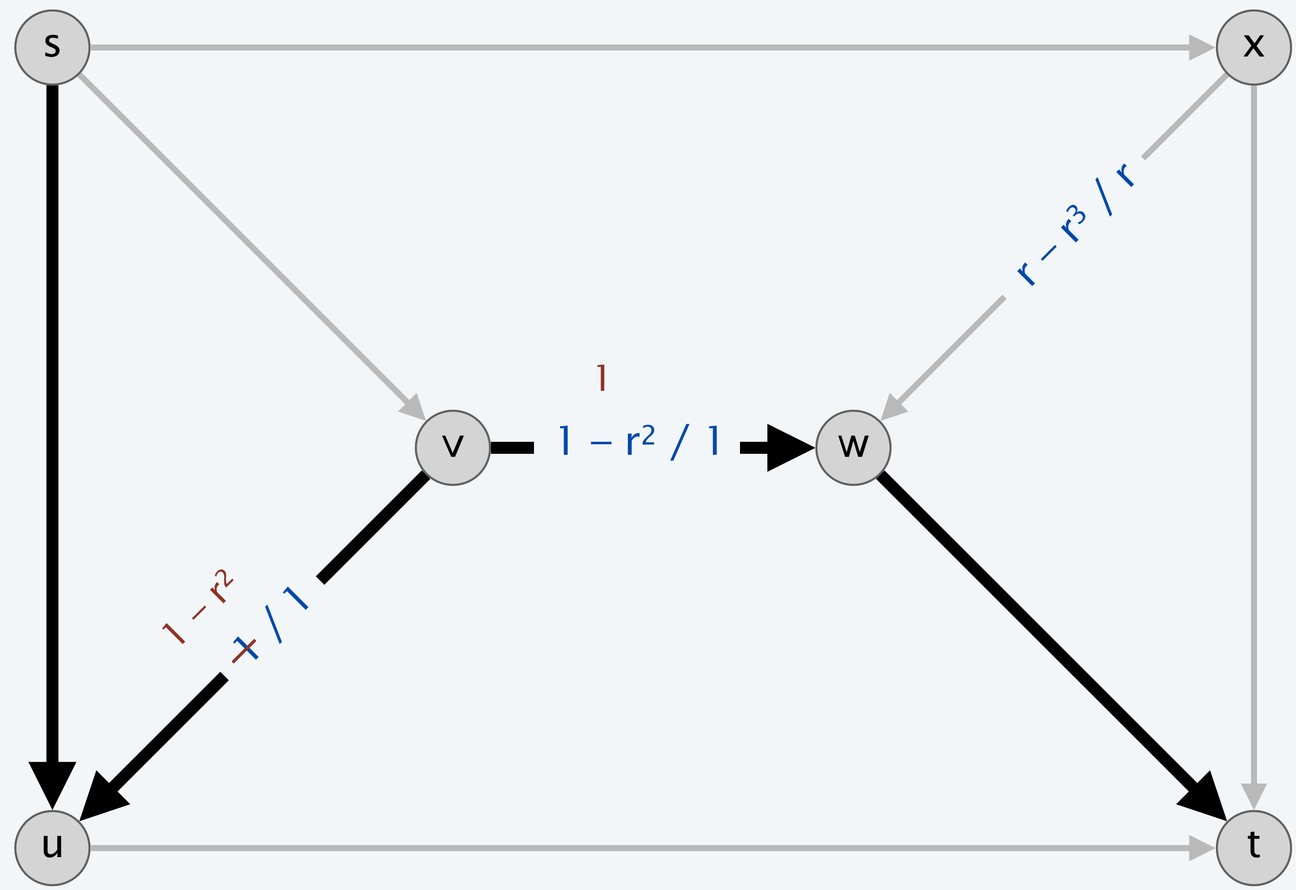
Ford-Fulkerson pathological example

augmenting path 4: s→x→w→v→u→t (bottleneck capacity = r^2)



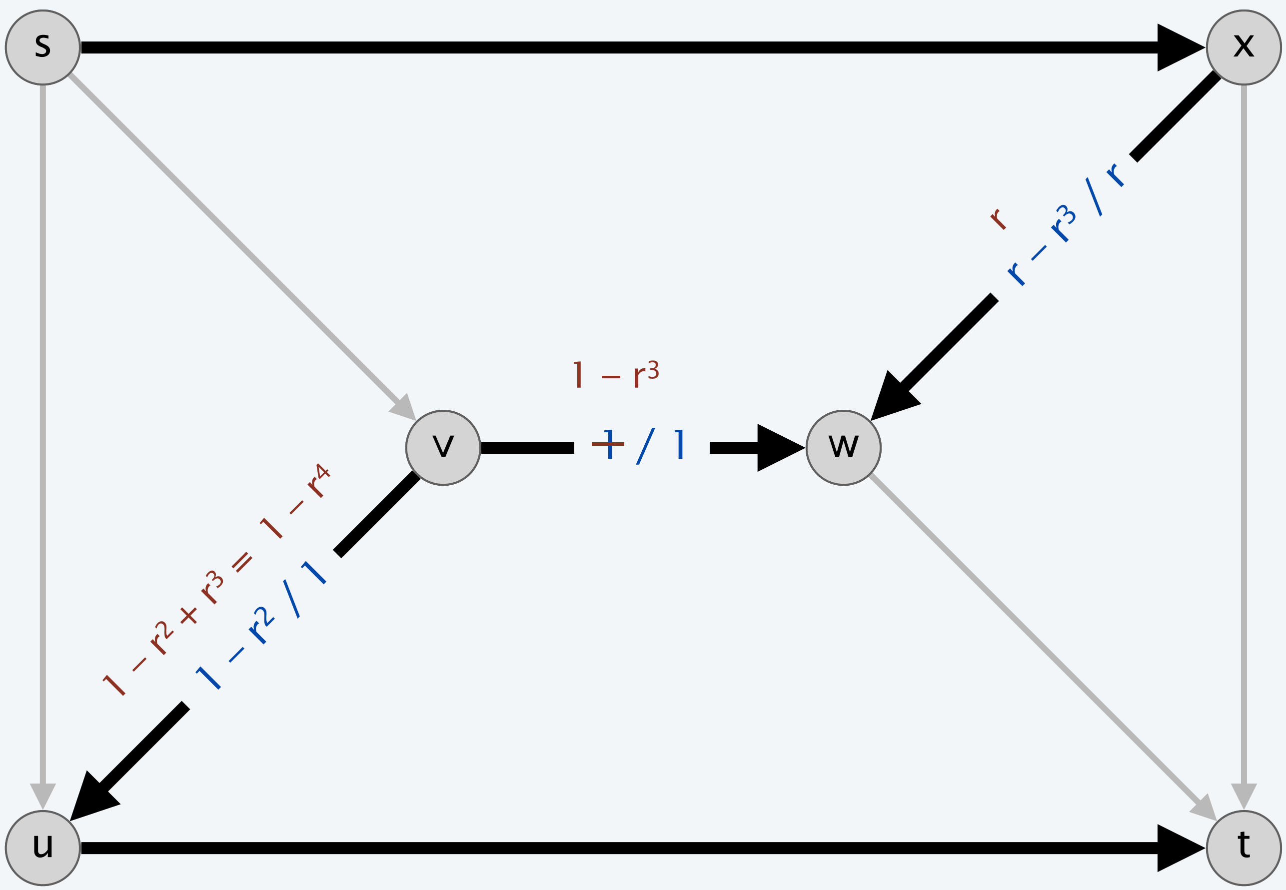
Ford-Fulkerson pathological example

augmenting path 5: s→u→v→w→t (bottleneck capacity = r^2)



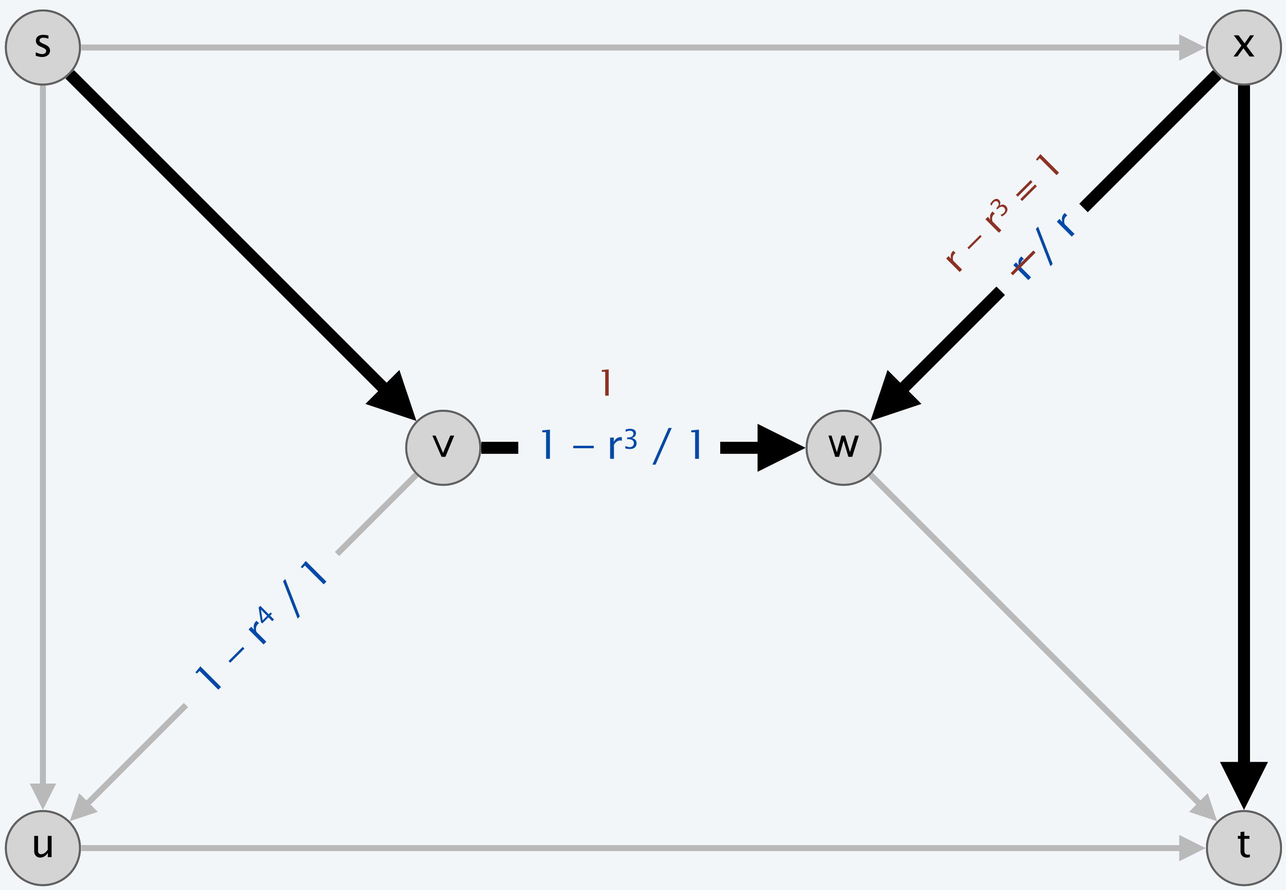
Ford-Fulkerson pathological example

augmenting path 6: s→x→w→v→u→t (bottleneck capacity = r^3)



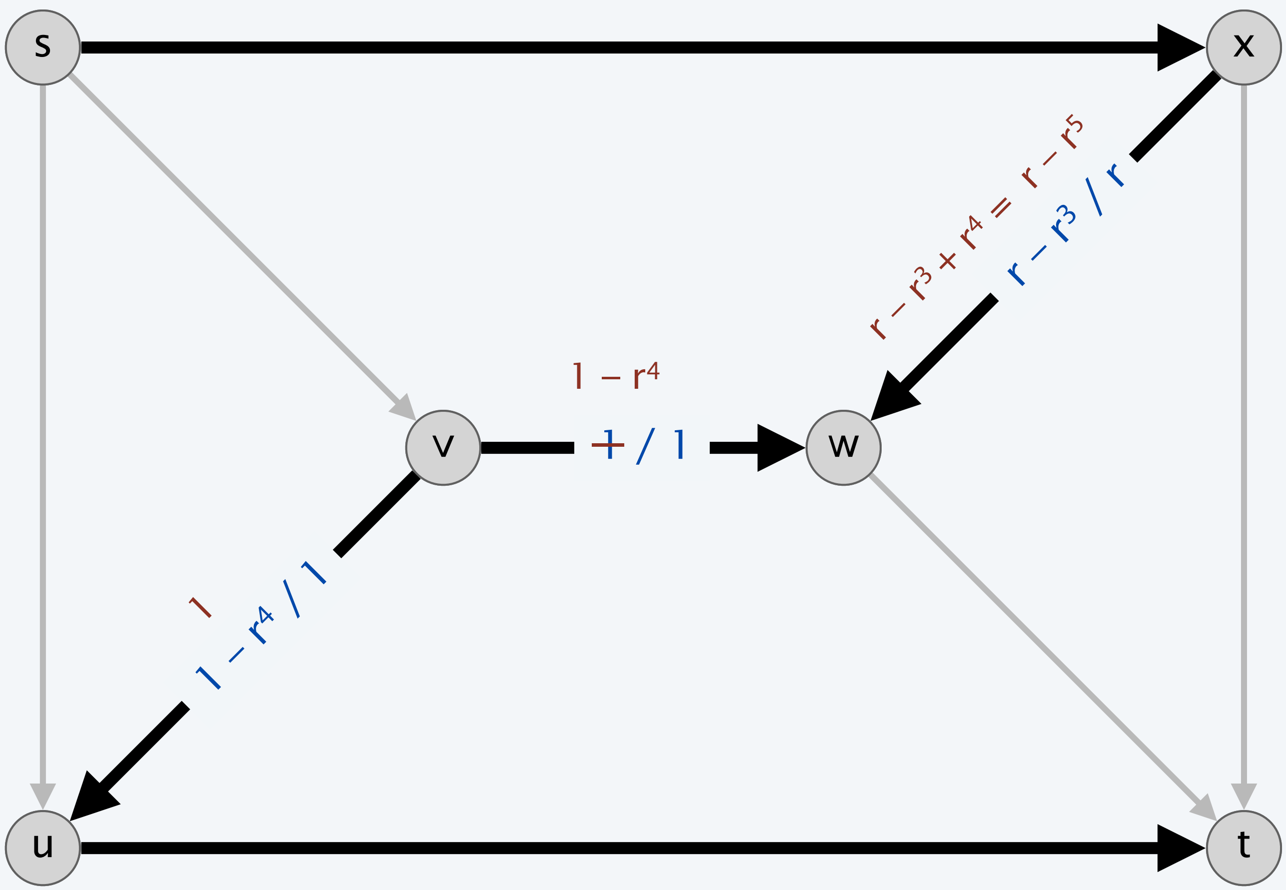
Ford-Fulkerson pathological example

augmenting path 7: s→v→w→x→t (bottleneck capacity = r^3)



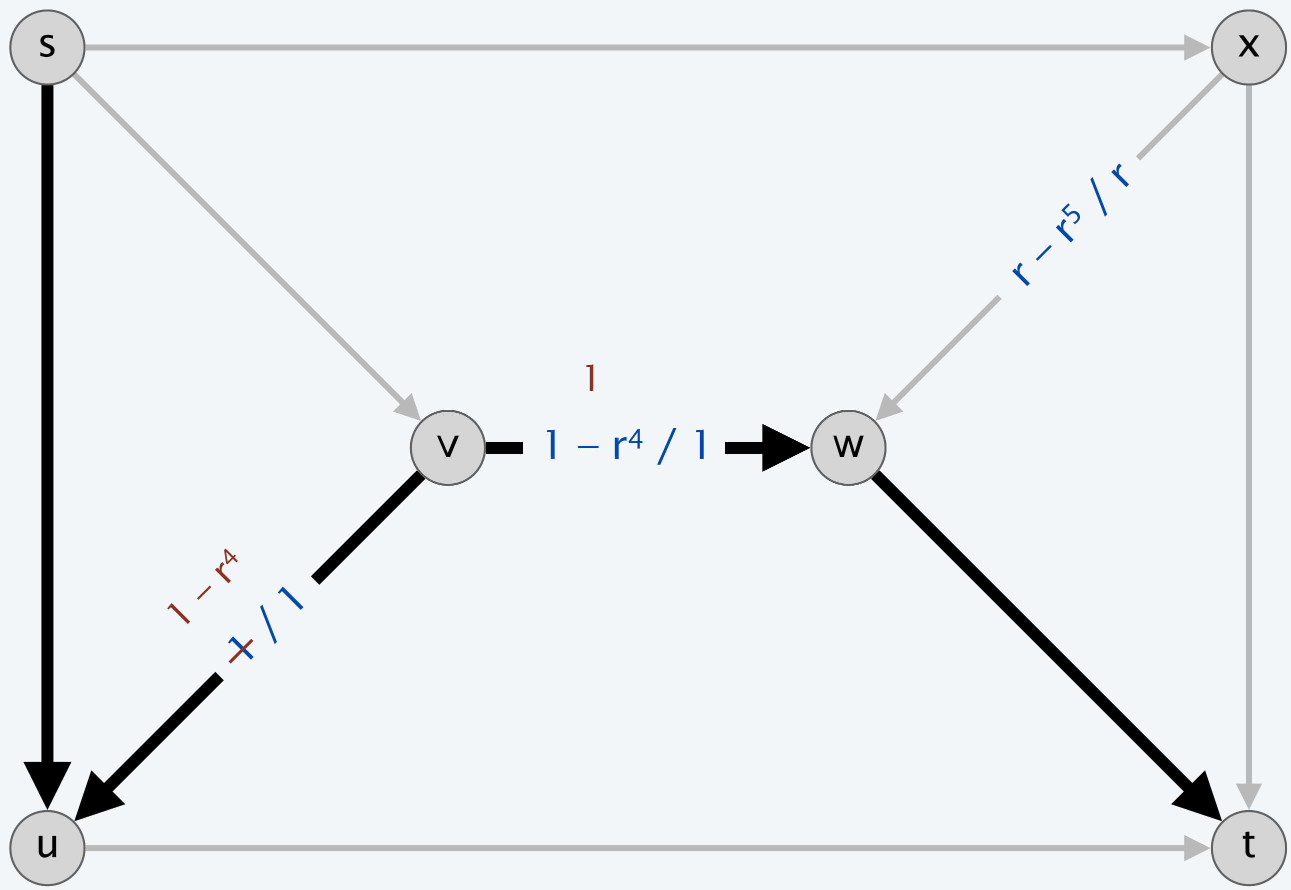
Ford-Fulkerson pathological example

augmenting path 8: s→x→w→v→u→t (bottleneck capacity = r^4)



Ford-Fulkerson pathological example

augmenting path 9: s→u→v→w→t (bottleneck capacity = r^4)

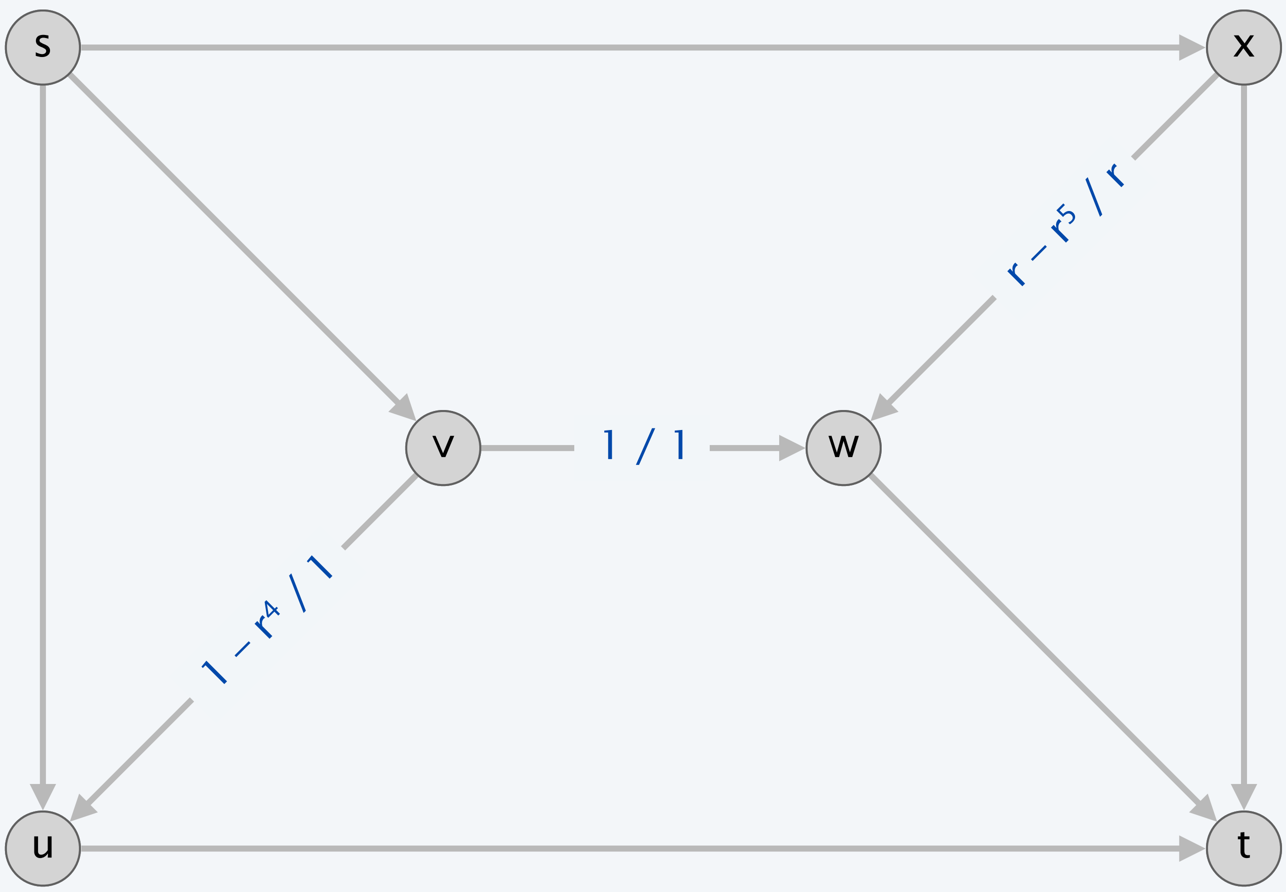


Ford-Fulkerson pathological example

after augmenting path 1: $1 - r^0, 1, r - r^1$ (flow = 1)

after augmenting path 5: $1 - r^2, 1, r - r^3$ (flow = $1 + 2r + 2r^2$)

after augmenting path 9: $1 - r^4, 1, r - r^5$ (flow = $1 + 2r + 2r^2 + 2r^3 + 2r^4$)



Ford-Fulkerson pathological example

Theorem. The Ford-Fulkerson algorithm may not terminate; moreover, it may converge a value not equal to the value of the maximum flow.

Pf.

- Using the given sequence of augmenting paths, after $(1 + 4k)^{th}$ such path, the value of the flow

$$\begin{aligned} &= 1 + 2\sum_{i=1}^{k} r^i \\ &\leq 1 + 2\sum_{i=1}^{\infty} r^i \\ &= 3 + 2r \\ &< 5 \end{aligned}$$

$$r = \frac{\sqrt{5} - 1}{2}$$

- Value of maximum flow = $200 + 1$. ▪