COS 426
Computer Graphics
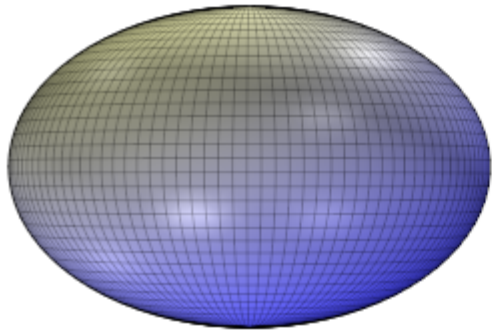# Princeton University

Tianqiang Liu

April 4, 2012

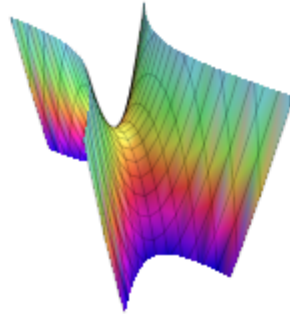*Thanks to Vova for providing slides!*
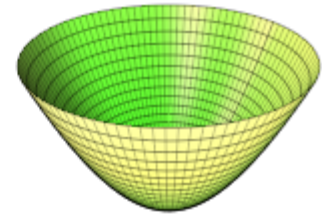
Ray Tracing

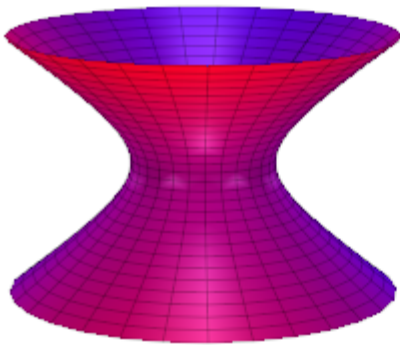- **Ray/primitive intersection**

- Acceleration
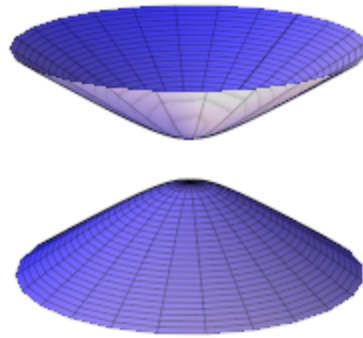
Quadrics
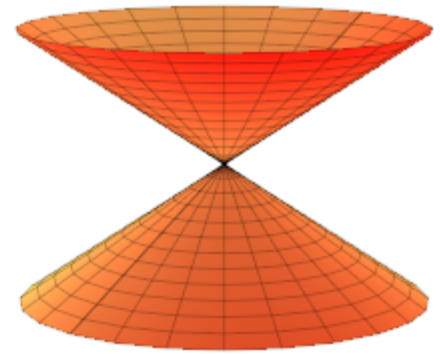

Ellipsoid

Hyperbolic paraboloid

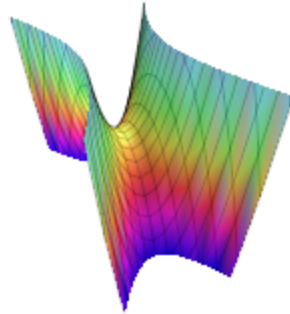Elliptic paraboloid

Hyperboloid of one sheet

Hyperboloid of two sheets

Cone

Quadrics
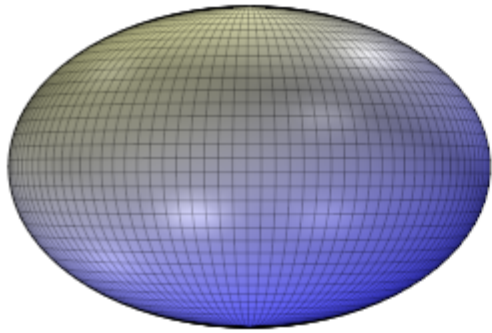
Ellipsoid

Hyperbolic paraboloid
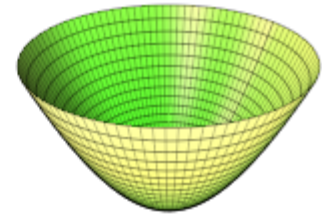
Elliptic paraboloid
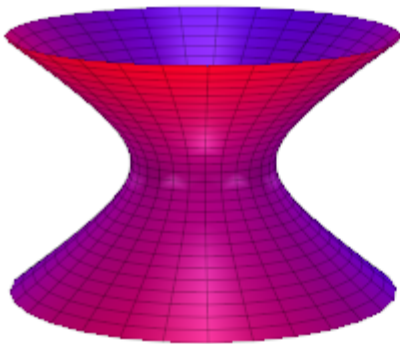
Hyperboloid of one sheet
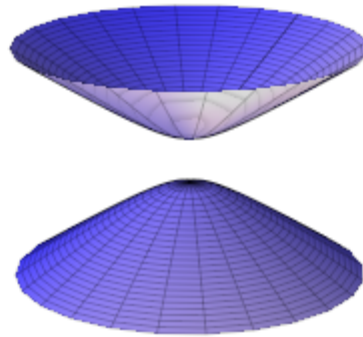
Hyperboloid of two sheets
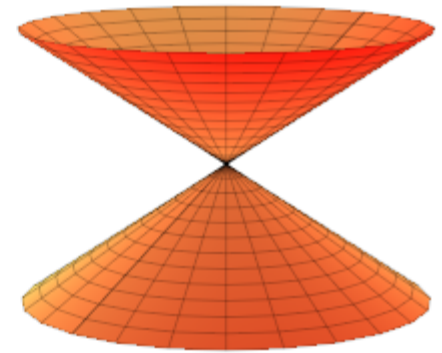
Cone

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

Quadrics

- Ray/primitive intersection:
  - Write down all equations
  - Solve for intersection

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- Ray:
  - ???

Quadrics

- Ray/primitive intersection:
  - Write down all equations
  - Solve for intersection
- Quadric:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

- Ray:

$$p = p_0 + t \cdot v$$

Quadrics

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v \quad \begin{cases} x = x_0 + t \cdot v_x \\ y = y_0 + t \cdot v_y \\ z = z_0 + t \cdot v_z \end{cases}$$

Quadrics

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

$$x = x_0 + t \cdot v_x$$
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$$K \cdot t^2 + L \cdot t + M = 0$$

Quadrics

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v \quad \begin{cases} x = x_0 + t \cdot v_x \\ y = y_0 + t \cdot v_y \\ z = z_0 + t \cdot v_z \end{cases}$$

$$K \cdot t^2 + L \cdot t + M = 0$$

A positive real solution exists

Two complex solutions

Two real negative solution

Quadrics

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v \begin{cases} x = x_0 + t \cdot v_x \\ y = y_0 + t \cdot v_y \\ z = z_0 + t \cdot v_z \end{cases}$$

$$K \cdot t^2 + L \cdot t + M = 0$$

A positive real solution exists

Two complex solutions

Two real negative solution

Pick smallest positive value to find intersection

Does not intersect

Does not intersect

Quadrics
Simpler Derivation

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

Quadrics
Simpler Derivation

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

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Quadrics
Simpler Derivation

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

$$\underset{3 \times 3}{pQp^T} + \underset{1 \times 3}{Pp^T} + \underset{1 \times 1}{R} = 0$$

Quadrics
Simpler Derivation

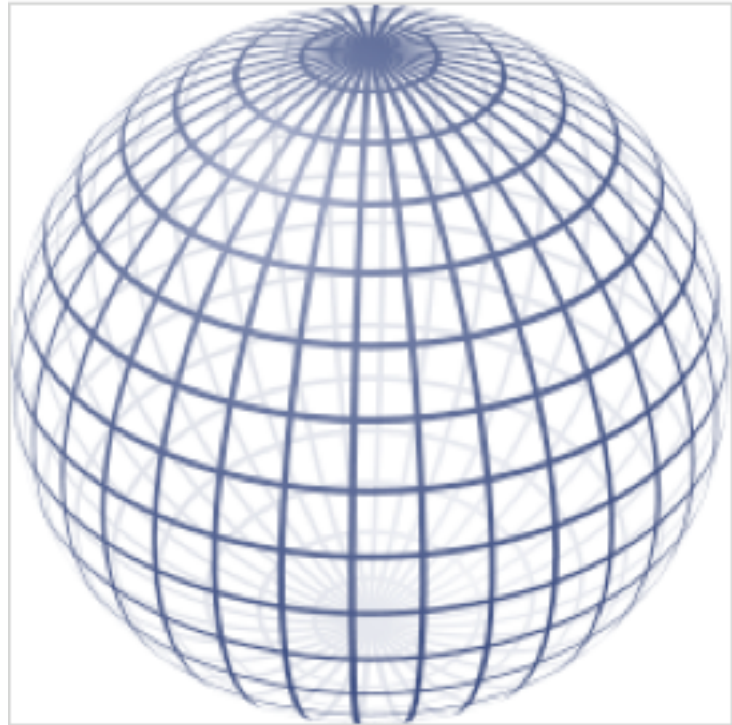$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

$$\underset{3 \times 3}{pQp^T} + \underset{1 \times 3}{Pp^T} + \underset{1 \times 1}{R} = 0$$

$$(p_0 + tv)Q(p_0 + tv)^T + P(p_0 + tv)^T + R = 0$$

Quadrics

- If you use general quadric for sphere

$$pQp^T + Pp^T + R = 0$$

Quadrics

- If you use general quadric for sphere

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  – What do you need to define?

Quadrics

- If you use general quadric for sphere

$$pQp^T + Pp^T + R = 0$$

  – What do you need to define?
  – Q, P, R

Quadrics

- If you use general quadric for sphere
$$pQp^T + Pp^T + R = 0$$
  - What do you need to define?
  - Q, P, R

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Quadrics

- If you use general quadric for sphere
$$pQp^T + Pp^T + R = 0$$
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$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Quadrics

- If you use general quadric for sphere

$$pQp^T + Pp^T + R = 0$$

  – What do you need to define?

  – Q, P, R

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = -r^2$$

$$P = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Ray Tracing

- Ray/primitive intersection

- **Acceleration**

Acceleration

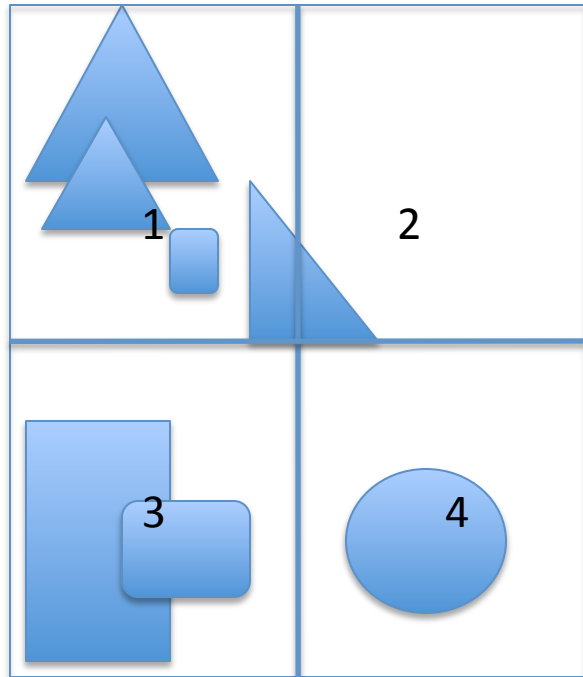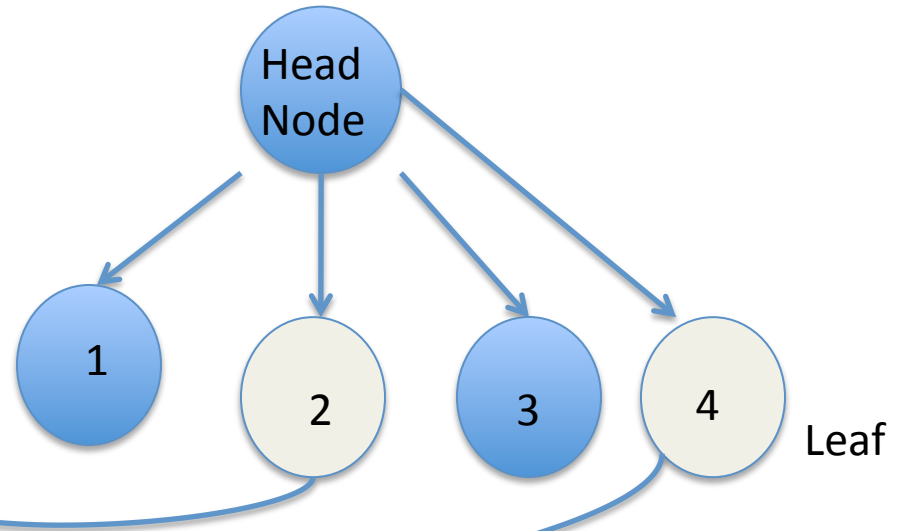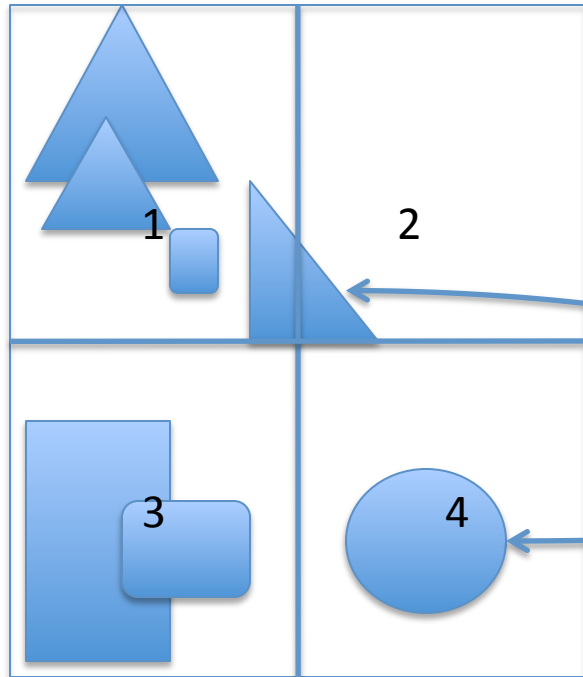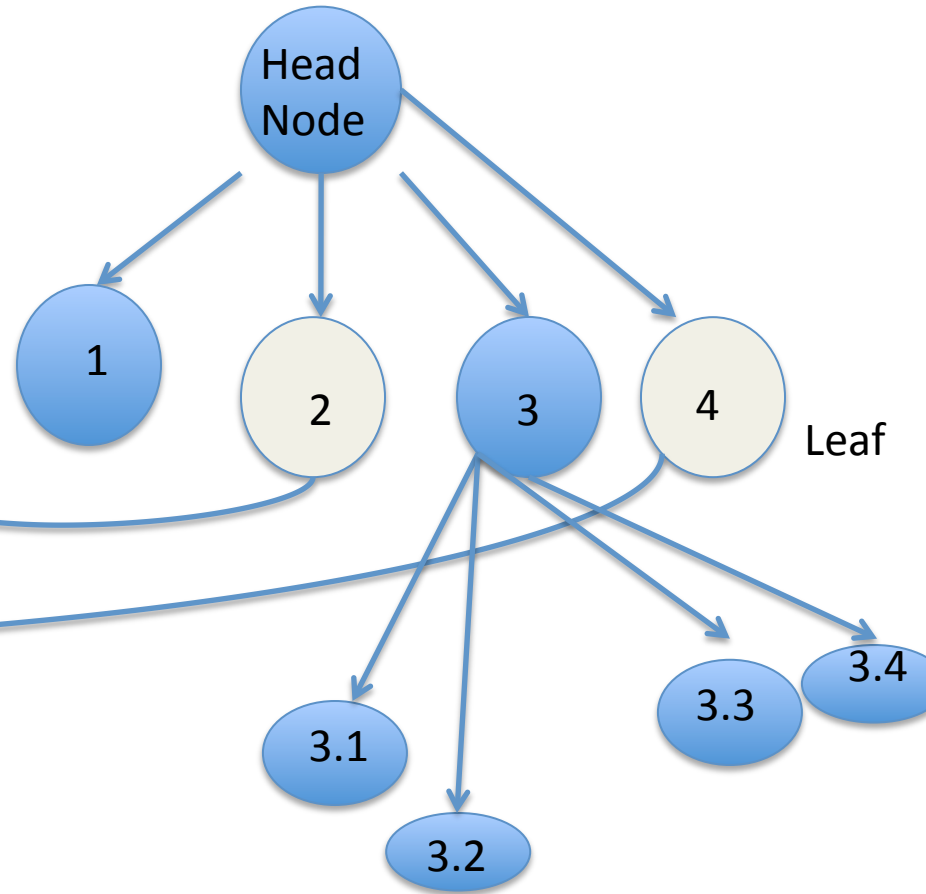- **Generate Structure (e.g. octree)**

- Traverse Strucuture

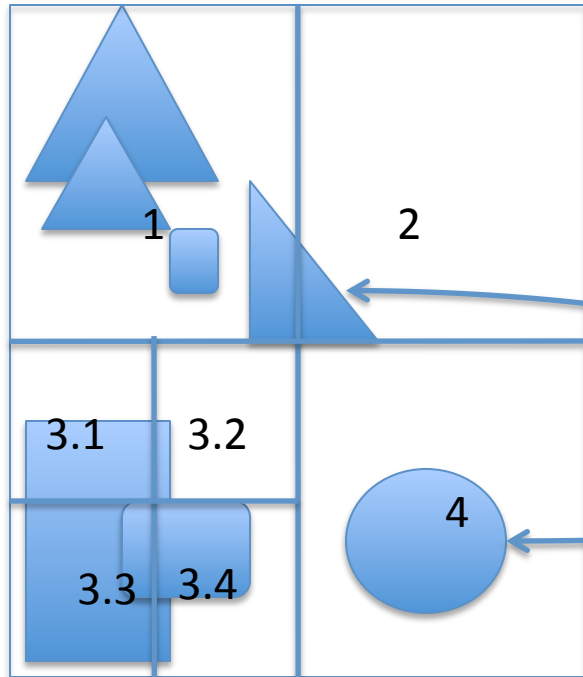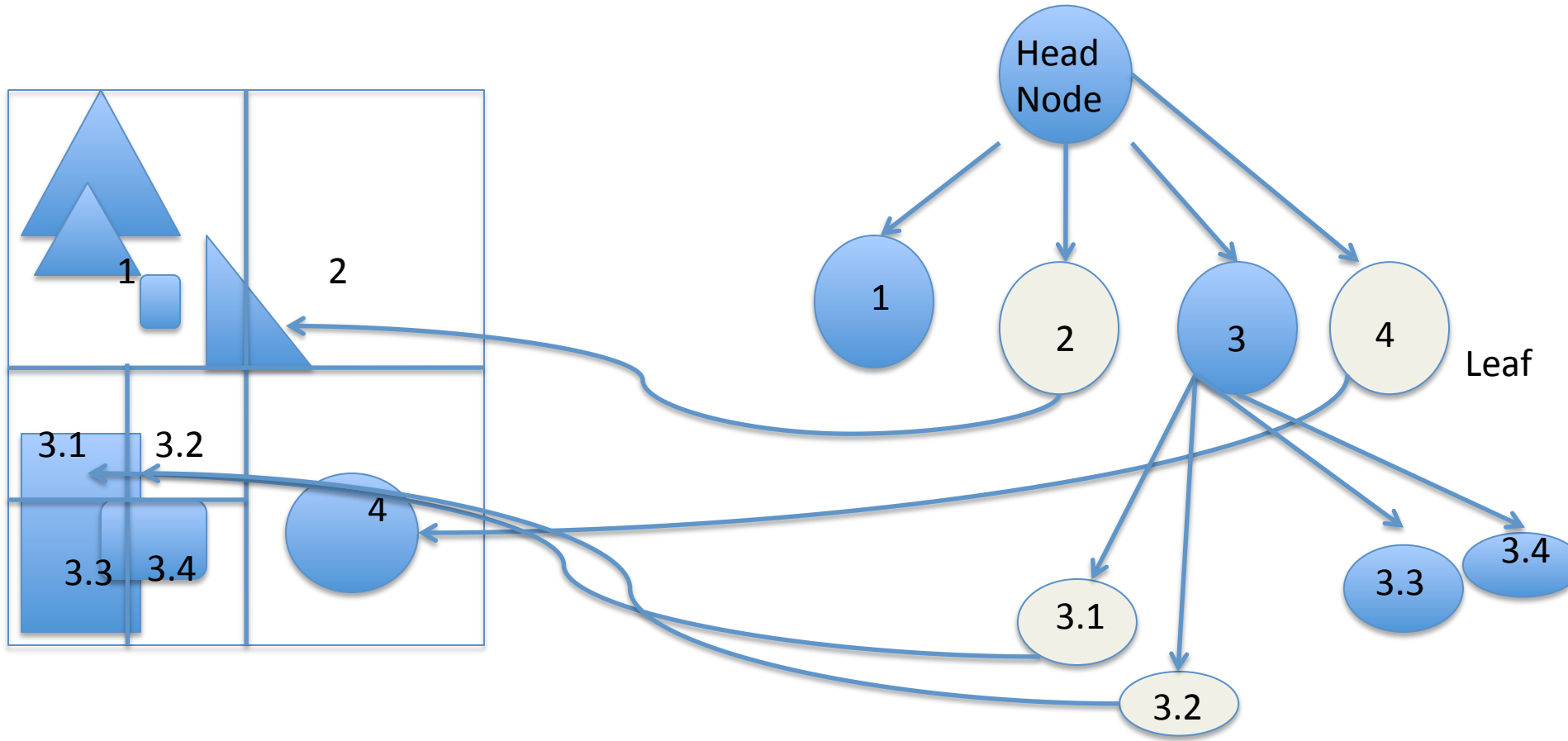# Acceleration

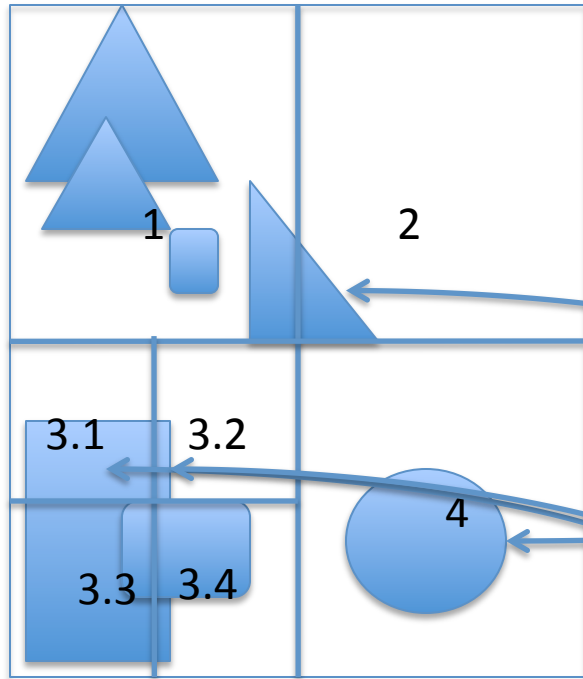# Acceleration

Acceleration

Acceleration

Acceleration

Acceleration

Acceleration

1

2

3.1   3.2

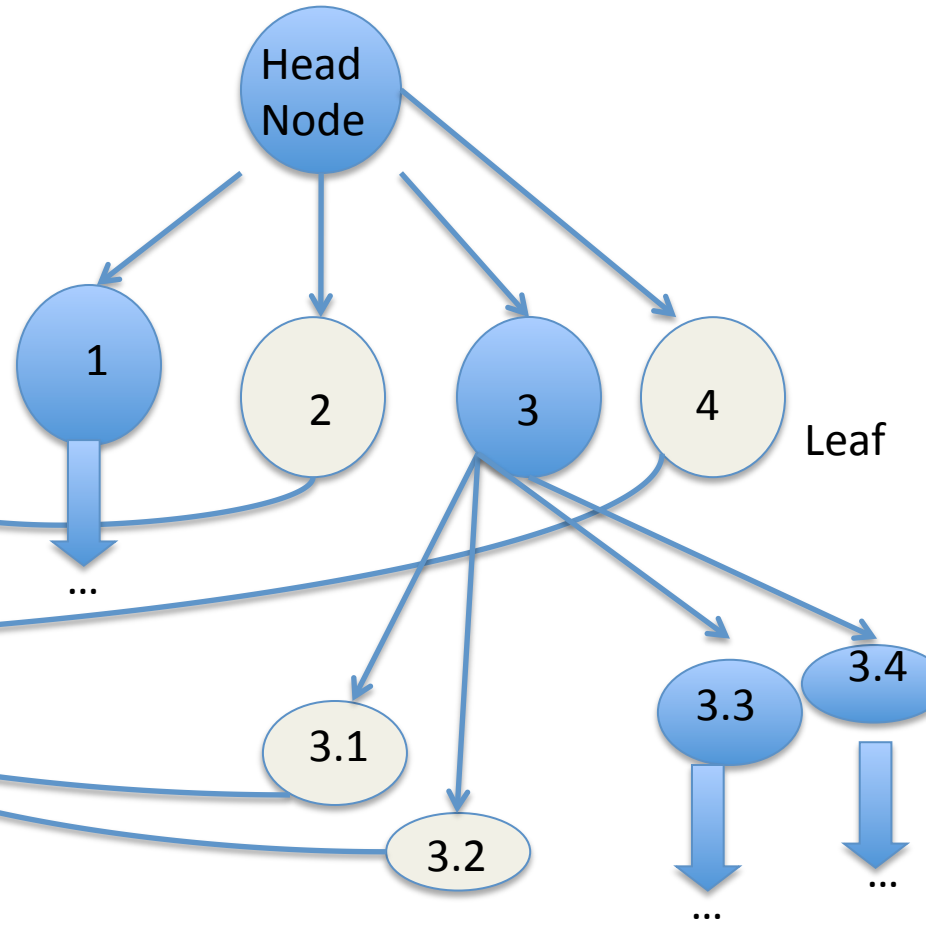3.3   3.4

4

Head Node

1

2

3

4

Leaf

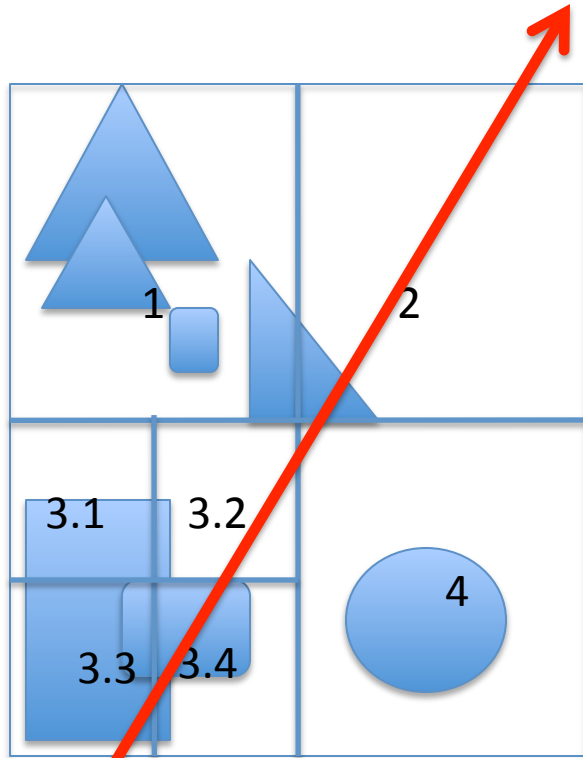...

3.1

3.2
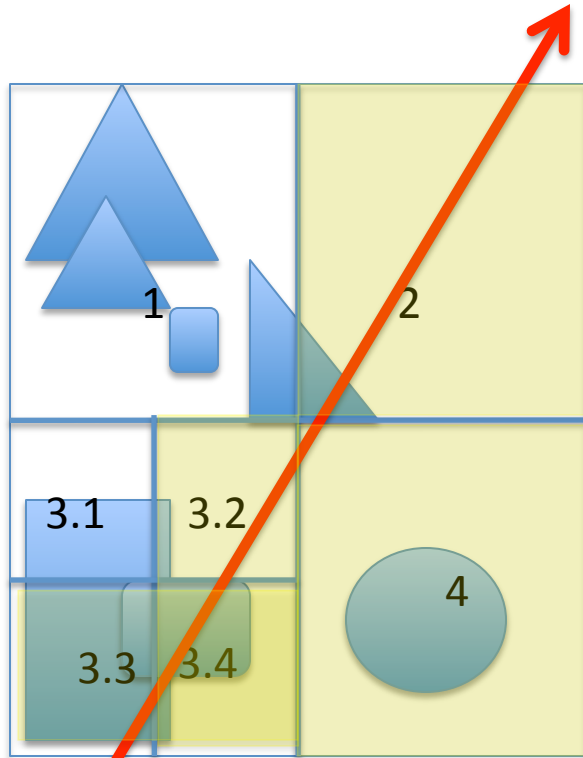
3.3

3.4

...

...

When do we stop?

Acceleration

- Generate Structure (e.g. octree)

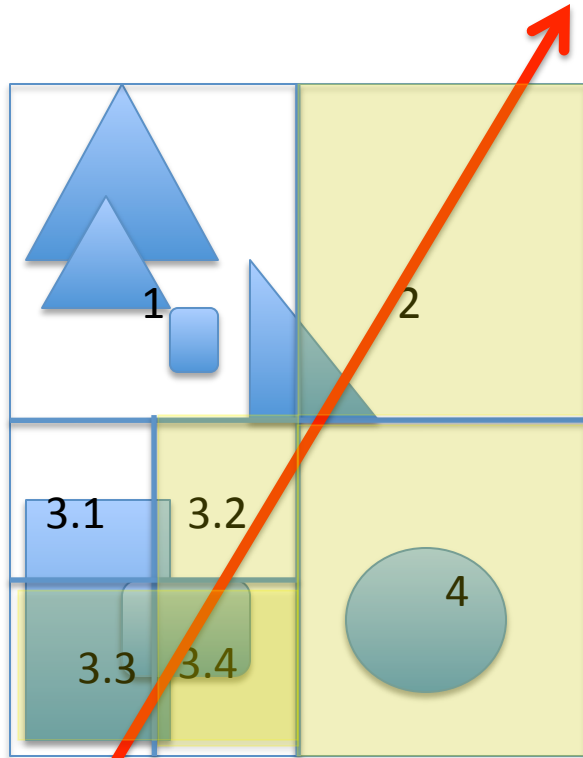- **Traverse Structure**

Acceleration

Acceleration

ORDER: 3.3, 3.4, 3.2, 2

If a ray intersected something in 3.3,
can it intersect something (with a smaller t)
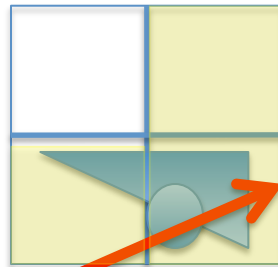in a later node?

Acceleration

ORDER: 3.3, 3.4, 3.2, 2

If a ray intersected something in 3.3,
can it intersect something (with a smaller t)
in a later node?

Think about objects on boundaries.

Example: should intersect the circle,
but the triangle is visited first

The End

Questions?