Name: ....

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Computer Science 426, 1st Exam

March 12, 2009

This test contains 5 questions, of equal weight, on 11 pages. Do all of your work on these pages (use the back for scratch space), giving the answer in the space provided. This is a closed-book exam – you may use one-page of notes with writing on one side during the exam. **Put your name on every page, and write out and sign the Honor Code pledge on the front page before turning in the test.**

"I pledge my honor that I have not violated the Honor Code during this examination."

Question	Score
1	
2	
3	
4	
5	
Total	

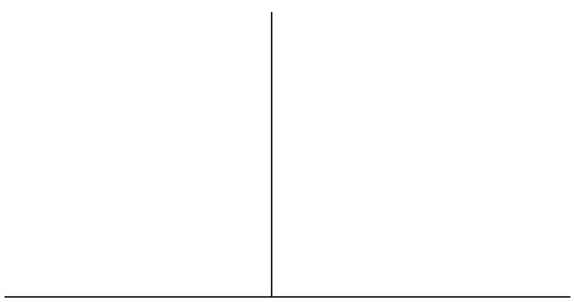
Q1. Image Sampling and Reconstruction

(a) What does it mean to bandlimit a function? (one sentence)

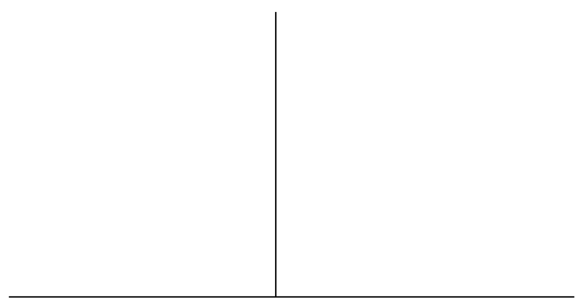
(b) Why is it important to bandlimit a function before representing it as an image? Please provide an example to support your answer. (a few sentences and an example)

(c) What filter is able to bandlimit a function perfectly? Why is that we do not use that filter in image processing applications? (two phrases or sentences)

(d) Please show the effect of bandlimiting a 1D function in the *spatial* domain. (draw a curve on each of the axes below)

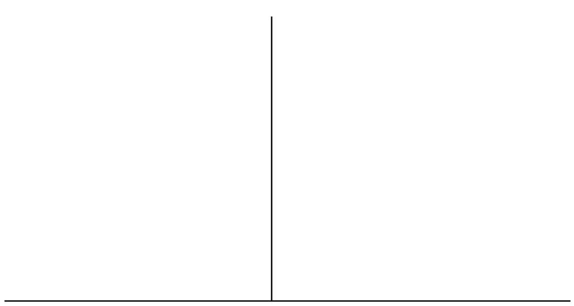


Input Function

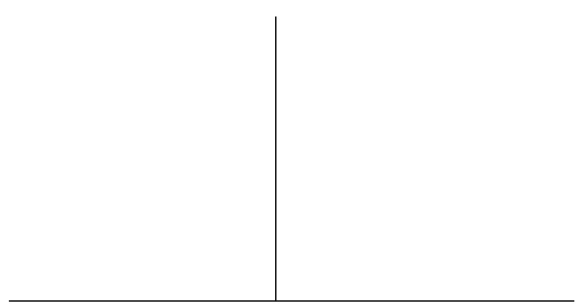


Bandlimited Function

(e) Please show the effect of bandlimiting a 1D function in the *frequency* domain. (draw a curve on each of the axes below)



Input Function



Bandlimited Function

(f) Under the interpretation that a pixel "is a little square," what filter is used to reconstruct a 1D function from a regular array of samples? (one phrase, and possibly a figure)

(g) What are the key properties of a Gaussian filter that make it more desirable for image resampling and reconstruction than other filters studied in this class (a few phrases, and possibly a labeled figure)

(h) What is the difference between "pre-aliasing" and "post-aliasing," as defined by Pat Hanrahan in his article titled "Baisc Signal Processing?" (a couple of phrases or sentences)

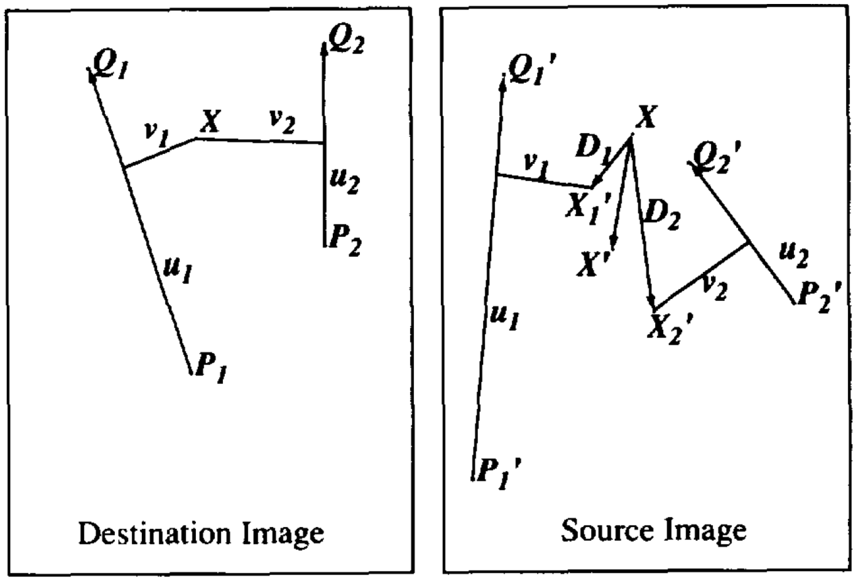
Q2. Image Processing

(a) Alvy Ray Smith wrote an article titled "Image Compositing Fundamentals" in which he argues that image composition with colors *not* premultiplied by alpha is a flawed formulation. Please explain why in a few sentences.

(b) You are the manager of a computer graphics company, and one of your employees has built a display device that has very high spatial resolution – 1M x 1M pixels on a 20 inch screen. However, the display technology allows only two intensities for each pixel (black and white). Can you suggest an effective method to display gray-scale digital photos that can have many different intensities at each pixel? Please explain why your method works. (a few sentences and possibly a figure)

(c) Assume you work for Boeing and your job is to analyze the output of your colleagues' wind tunnel simulation in which air velocity V(x,y,z) has been measured at every vertex of a uniform, rectilinear 3D grid. Write the equation for how you would use trilinear interpolation to estimate the air velocity V at an arbitrary point (x,y,z) within the grid cell defined by corners (x1,y1,z1) and (x2,y2,z2). Support your answer with a labeled drawing.

(d) Please explain the figure below, which was taken from the paper by Beier and Neely. Be sure to highlight the main purpose of the figure, define each variable (e.g., Q_1, Q_1', X, etc.), and explain what the lines in the image show. (several phrases and a few sentences)



Q3. Geometric Representations

It is possible to represent a 2D function with: 1) a mesh (where each point sample is a vertex, vertices are connected into faces that cumulatively cover the 2D domain, and vertex values are interpolated across each face) or 2) an IMAGE (a rectilinear array of point samples where values are interpolated between samples, as in assignment 1). Please explain which of these two representations is better *and why* for each of the following inputs and operations. Unless otherwise specified, we are interested in computing results at a level of accuracy that is visible on a computer screen.

For each of the following inputs and operations, please write "MESH" or "IMAGE", followed by one short phrase or sentence explaining why it is the better representation. Assume that TRI is a function with a single black triangle on a white background, FACE is a portrait of a human face, and FOREST is a photorealistic picture of an outdoor scene.

(a) Represent TRI with minimal storage.

(b) Represent FACE with minimal storage.

(c) Evaluate the color of TRI at a given position (x,y)

(d) Evaluate the color of FOREST at a given position (x,y)

(e) Detect edges in FOREST with maximal accuracy.

(f) Rotate FACE by theta degrees in the least possible time.

(g) Blur TRI with a small sigma with maximal accuracy.

(h) Blur FOREST with a small sigma in the least possible time.

(i) Warp FACE with a "swirl" function in the least possible time, where "swirl" rotates each position by an amount proportional to its distance from a given center.

(j) Which of your answers would be different if the image were represented by a quad-tree instead of a uniform grid of samples?

Q4. Mesh Data Structures

Imagine that you are given a mesh data structure that stores only the positions of vertices and vertex-vertex adjacencies. Specifically, the mesh stores a doubly linked list of vertices, and each vertex stores a 3D position and a list of pointers to adjacent vertices sorted counter-clockwise (with respect to the exterior of the surface). No explicit representation is provided for edges or faces, which are defined only by the connectivity of their vertices (assume a face can have any number of vertices). We will use the convention of specifying an edge or a face by a pair of adjacent vertices, where the face defined by adjacent vertices (v1,v2) is the one that contains vertices v1 and v2 in counter-clockwise order. For each of the following operations, please provide a brief description (or pseudo-code) of its fastest possible implementation and describe how long it will take on average as a function of V vertices, E edges, F faces, and K adjacencies per vertex.

(a)
```
// Smooth the mesh by moving the position of every vertex to an
// average of its neighbors
void Smooth(Mesh *m)
```

(b)
```
// Compute the surface normals for every vertex
// by taking an unweighted average of its adjacent face normals
R3Vector ComputeVertexNormal(Mesh *m)
```

(c)
```
// Insert a vertex at the midpoint of the edge between v1 and v2
void SubdivideEdgeAtMidpoint(Mesh *m, Vertex *v1, Vertex *v2)
```

(d)
```
// Print the coordinates of the vertices on the boundary of a face
void PrintVertexCoordinatesOfFace(Mesh *m, Vertex *v1, Vertex *v2)
```

(e)
```
// Perform one level of Loop Subdivision, assuming the input mesh
// is a manifold with no boundaries and has only triangular faces
void LoopSubdivision(Mesh *m)
```

Q5. Parametric Curves

The blending functions for a uniform, cubic Bezier curve are provided below. Please use them to mathematically prove or disprove the following claims.

$$
\begin{aligned}
Q_i(u) &= B_0(u)V_0 + B_1(u)V_1 + B_2(u)V_2 + B_3(u)V_3 \\
B_0(u) &= -u^3 + 3u^2 - 3u + 1 \\
B_1(u) &= 3u^3 - 6u^2 + 3u \\
B_2(u) &= -3u^3 + 3u^2 \\
B_3(u) &= u^3
\end{aligned}
$$

(a) A Bezier curve interpolates V_0.

(b) A Bezier curve always lies within the convex hull of its control vertices.

(c) The direction vector (first derivative) of a Bezier curve at $u = 0$ is in the same direction as the vector from V_0 to V_1.

(d) Adjacent Bezier curves defined by vertices (V_0, V_1, V_2, V_3) and (V_3, V_4, V_5, V_6) are C1 continuous if $V_2 - V_3 = V_3 - V_4$.

Please provide an argument to support or refute the following claims:

(e) The boundary curve of a cubic tensor product Bezier surface patch is equivalent to the Bezier curve defined by its four boundary control points.

(f) The boundary curve of a cubic tensor product **B-Spline** surface patch is equivalent to the **B-Spline** curve defined by its four boundary control points.

(g) It is possible to define a spline composed of uniform cubic parametric curves that interpolates all of its control vertices and has C2 continuity at every joint between curve segments.

(h) It is possible to define a spline composed of uniform cubic parametric curves that interpolates all of its control vertices and has C2 continuity at every joint between curve segments **and has local control**.