Computer Science 320: Spring 2011 Midterm Examination

You have 1:20 to answer the following questions. This midterm is closed book/closed notes. For partial credit, show all work. Put your name on every page. Write out and sign the Honor Code pledge before turning in the test.

"I pledge my honor that I have not violated the Honor Code during this examination."

Problem 1: (15%)
Provide a *Deterministic Finite Automaton* that matchs all strings of "a"s and "b"s containing an odd number of "a"s.

Problem 2: (15%)
Provide a context free grammar for the language described in Problem 1.

Problem 3: (20%)
Consider the expression language from the typing lectures, without functions, products, or subtypes, as summarized below. Define the typing context $\Gamma = [y : \mathbf{ref\ int}, b : \mathbf{bool}]$ and the expression e by

$$\mathbf{let}\ x = 3\ \mathbf{in\ if}\ (x < !y) \vee b\ \mathbf{then\ alloc}\ (x+1)\ \mathbf{else\ let}\ z = y{:=}8\ \mathbf{in}\ 4\ \mathbf{end\ end}.$$

Is there some type τ such that $\Gamma \vdash e : \tau$ is derivable using the rules? If no, say why not, i.e. show where an attempt to construct a typing derivation fails. If yes, give a suitable typing derivation.

$$\begin{array}{rcl} e & ::= & \ldots \mid -1 \mid 0 \mid 1 \mid \ldots \mid \mathbf{tt} \mid \mathbf{ff} \mid e \oplus e \mid \mathbf{if}\ e\ \mathbf{then}\ e\ \mathbf{else}\ e \mid x \\ & & \mid \mathbf{let}\ x = e\ \mathbf{in}\ e\ \mathbf{end} \mid \mathbf{alloc}\ e \mid !e \mid e{:=}e \\ \oplus & ::= & + \mid - \mid \times \mid \wedge \mid \vee \mid < \mid = \\ \tau & ::= & \mathbf{bool} \mid \mathbf{int} \mid \mathbf{ref}\ \tau \mid \mathbf{unit} \end{array}$$

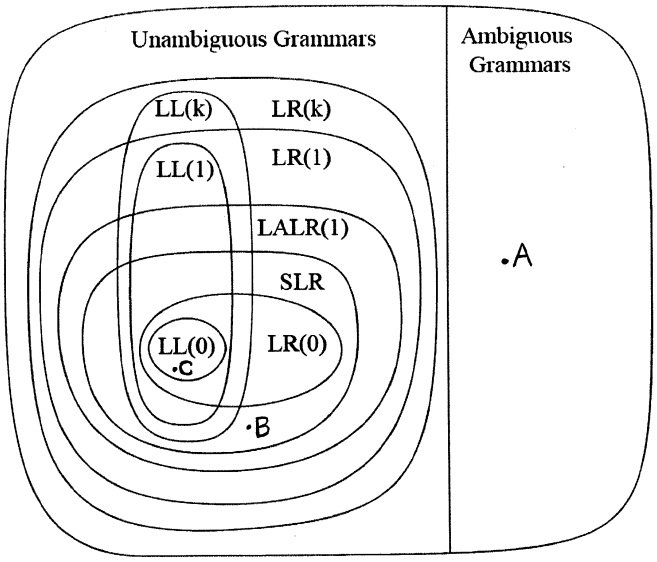
$$\textsc{Bool}\ \frac{e \in \{\mathbf{tt}, \mathbf{ff}\}}{\Gamma \vdash e : \mathbf{bool}} \qquad \textsc{Num}\ \frac{n \in \{\ldots, -1, 0, 1, \ldots\}}{\Gamma \vdash n : \mathbf{int}} \qquad \textsc{Var}\ \frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau}$$

$$\textsc{IOp}\ \frac{\Gamma \vdash e_1 : \mathbf{int} \quad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 \oplus e_2 : \mathbf{int}}\ \oplus \in \{+, -, \times\} \qquad \textsc{BOp}\ \frac{\Gamma \vdash e_1 : \mathbf{bool} \quad \Gamma \vdash e_2 : \mathbf{bool}}{\Gamma \vdash e_1 \oplus e_2 : \mathbf{bool}}\ \oplus \in \{\wedge, \vee\}$$

$$\textsc{COp}\ \frac{\Gamma \vdash e_1 : \mathbf{int} \quad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 \oplus e_2 : \mathbf{bool}}\ \oplus \in \{<, =\} \qquad \textsc{ITE}\ \frac{\Gamma \vdash e_1 : \mathbf{bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \mathbf{if}\ e_1\ \mathbf{then}\ e_2\ \mathbf{else}\ e_2 : \tau}$$

$$\textsc{Let}\ \frac{\Gamma \vdash e_1 : \sigma \quad \Gamma[x : \sigma] \vdash e_2 : \tau}{\Gamma \vdash \mathbf{let}\ x = e_1\ \mathbf{in}\ e_2\ \mathbf{end} : \tau} \qquad \textsc{Alloc}\ \frac{\Gamma \vdash e : \tau}{\Gamma \vdash \mathbf{alloc}\ e : \mathbf{ref}\ \tau}$$

$$\textsc{Read}\ \frac{\Gamma \vdash e : \mathbf{ref}\ \tau}{\Gamma \vdash !e : \tau} \qquad \textsc{Write}\ \frac{\Gamma \vdash e_1 : \mathbf{ref}\ \tau \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1{:=}e_2 : \mathbf{unit}}$$



Problem 4: (20%)

Provide a simple grammar that resides in position A in the above figure. Prove that your grammar resides in position A in an organized manner.

Problem 5: (20%)

Provide a simple grammar that resides in position B in the above figure. Prove that your grammar resides in position B in an organized manner.

Problem 6: (10%)

Provide a simple grammar that resides in position C in the above figure. Prove that your grammar resides in position C in an organized manner.