COS 320 Midterm Exam, Princeton University

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This exam is **closed book**. You have **50 minutes**.

1 Regular Expressions

Write regular expressions for the following (or explain why it can't be done).

A. Decimal numbers that are multiples of 25.

B. 8-bit signed 2's-complement numbers less than +8.

C. 8-bit signed 2's-complement numbers greater than 127.

D. Nonempty strings of (at most) the letters a,b,c, but not more than one *c*.

E. Nonempty strings of (at most) the letters a,b,c, but not more *a*'s than *c*'s.

F. Unsigned binary numbers n such that there exist natural numbers a, b, c such that $a^n + b^n = c^n$.

2 Parsing

$$\begin{array}{lcllcl}
S & \rightarrow & A\$ & & & \\
A & \rightarrow & aB & B & \rightarrow & bBC \\
A & \rightarrow & aC & B & \rightarrow & f \\
A & \rightarrow & Ad & C & \rightarrow & c \\
A & \rightarrow & Ae & & &
\end{array}$$

A. In 10 words or less, state *just one of the most obvious reasons* why this grammar cannot be in LL(1).

B. Modify this grammar so that it is in LL(1), without changing the language. *If you have extra time when you finish the rest of the exam, prove that the result is in LL(1).*

C. Is the original grammar in LR(1)? Justify your answer with a rigorous demonstration. If it is not in LR(1), suggest a grammar transformation that will likely put it into LR(1) (but you don't have to justify this part with a rigorous demonstration).

3 Type Checking

```
datatype exp = . . .
             | Let of var * exp * exp
             | LetAnd of var * exp * var * exp * exp
             | LetRec of var * tp * exp * var * tp * exp * exp
             | . . .
```

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma[x_1 : \tau_1] \vdash e_2 : \tau_2}{\Gamma \vdash (\texttt{let}\ x_1 = e_1\ \texttt{in}\ e_2) : \tau_2}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2 \quad \Gamma[x_1 : \tau_1][x_2 : \tau_2] \vdash e_3 : \tau_3}{\Gamma \vdash (\texttt{let}\ x_1 = e_1\ \texttt{and}\ x_2 = e_2\ \texttt{in}\ e_3) : \tau_3}$$

$$\frac{\Gamma[x_1 : \tau_1][x_2 : \tau_2] \vdash e_1 : \tau_1 \quad \Gamma[x_1 : \tau_1][x_2 : \tau_2] \vdash e_2 : \tau_2 \quad \Gamma[x_1 : \tau_1][x_2 : \tau_2] \vdash e_3 : \tau_3}{\Gamma \vdash (\texttt{let rec}\ x_1 : \tau_1 = e_1\ \texttt{and}\ x_2 : \tau_2 = e_2\ \texttt{in}\ e_3) : \tau_3}$$

```
fun complain (msg: string) : unit = . . .
fun sub (t1: tp, t2: tp) : bool = . . .

fun typecheck (ctxt: tp Symbol.table) (e: exp) : tp =
 case e
  of   . . .
   | Let (x1,e1,e2) =>                (* A *)
   | LetAnd (x1,e1,x2,e2,e3) =>       (* B *)
   | LetRec (x1,t1,e1,x2,t2,e2,e3) => (* C *)
   |   . . .
```

A. What should go in the blank `A`?
B. What should go in the blank `B`?
C. What should go in the blank `C`?