

6.5 REDUCTIONS



- ▶ designing algorithms
- ▶ establishing lower bounds
- ▶ classifying problems
- ▶ intractability

Algorithms, 4th Edition · Robert Sedgwick and Kevin Wayne · Copyright © 2002–2012 · April 30, 2012 5:16:12 AM

Bird's-eye view

Desiderata. Classify **problems** according to computational requirements.

complexity	order of growth	examples
linear	N	min, max, median, Burrows-Wheeler transform, ...
linearithmic	$N \log N$	sorting, convex hull, closest pair, farthest pair, ...
quadratic	N^2	?
⋮	⋮	⋮
exponential	c^N	?

Frustrating news. Huge number of problems have defied classification.

Bird's-eye view

Desiderata. Classify **problems** according to computational requirements.

Desiderata'.

Suppose we could (could not) solve problem X efficiently.

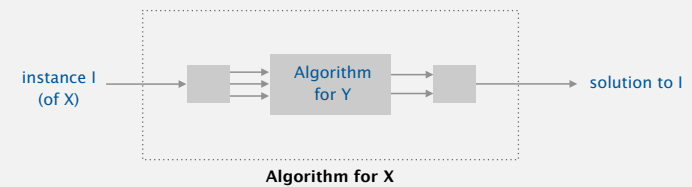
What else could (could not) we solve efficiently?



“ Give me a lever long enough and a fulcrum on which to place it, and I shall move the world. ” — Archimedes

Reduction

Def. Problem X **reduces to** problem Y if you can use an algorithm that solves Y to help solve X .

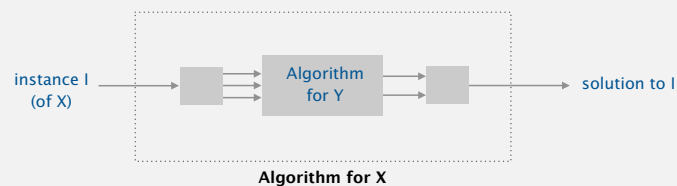


Cost of solving X = total cost of solving Y + cost of reduction.

↑ perhaps many calls to Y on problems of different sizes ↑ preprocessing and postprocessing

Reduction

Def. Problem X **reduces to** problem Y if you can use an algorithm that solves Y to help solve X .



Ex 1. [element distinctness reduces to sorting]

To solve element distinctness on N items:

- Sort N items.
- Check adjacent pairs for equality.

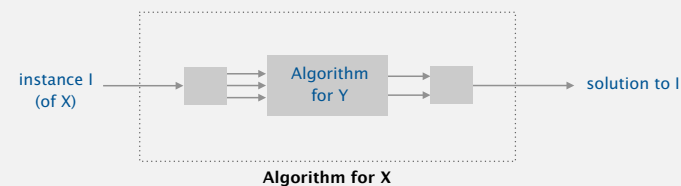
Cost of solving element distinctness. $N \log N + N$.

cost of sorting (pointing to $N \log N$)
cost of reduction (pointing to N)

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Reduction

Def. Problem X **reduces to** problem Y if you can use an algorithm that solves Y to help solve X .



Ex 2. [3-collinear reduces to sorting]

To solve 3-collinear instance on N points in the plane:

- For each point, sort other points by polar angle or slope.
 - check adjacent triples for collinearity

Cost of solving 3-collinear. $N^2 \log N + N^2$.

cost of sorting (pointing to $N^2 \log N$)
cost of reduction (pointing to N^2)

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Reduction: design algorithms

Def. Problem X **reduces to** problem Y if you can use an algorithm that solves Y to help solve X .

Design algorithm. Given algorithm for Y , can also solve X .

Ex.

- Element distinctness reduces to sorting.
- 3-collinear reduces to sorting.
- CPM reduces to topological sort. [shortest paths lecture]
- h-v line intersection reduces to 1d range searching. [geometric BST lecture]
- Baseball elimination reduces to maxflow. [assignment 7]
- Burrows-Wheeler transform reduces to suffix sort. [assignment 8]
- ...

Mentality. Since I know how to solve Y , can I use that algorithm to solve X ?

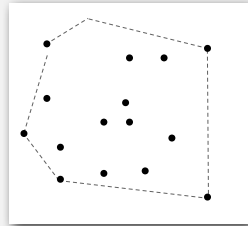
↑
programmer's version: I have code for Y . Can I use it for X ?

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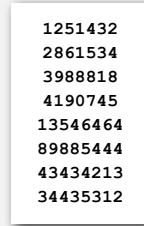
Convex hull reduces to sorting

Sorting. Given N distinct integers, rearrange them in ascending order.

Convex hull. Given N points in the plane, identify the extreme points of the convex hull (in counterclockwise order).



convex hull



sorting

Proposition. Convex hull reduces to sorting.

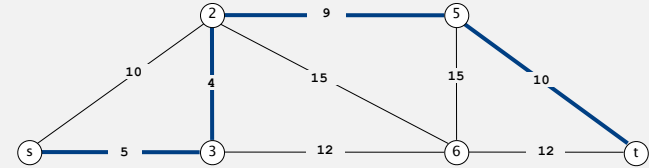
Pf. Graham scan algorithm.

cost of sorting cost of reduction

Cost of convex hull. $N \log N + N$.

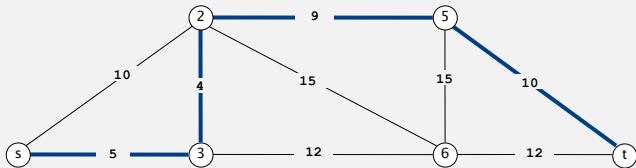
Shortest paths on edge-weighted graphs and digraphs

Proposition. Undirected shortest paths (with nonnegative weights) reduces to directed shortest path.

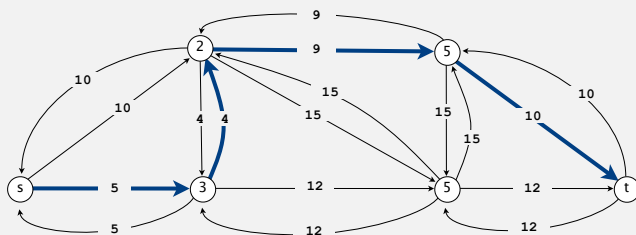


Shortest paths on edge-weighted graphs and digraphs

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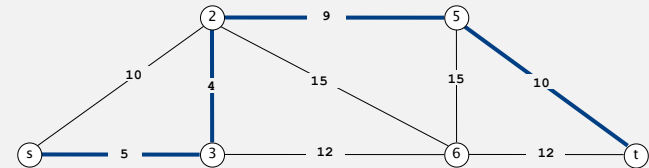


Pf. Replace each undirected edge by two directed edges.



Shortest paths on edge-weighted graphs and digraphs

Proposition. Undirected shortest paths (with nonnegative weights) reduces to directed shortest path.

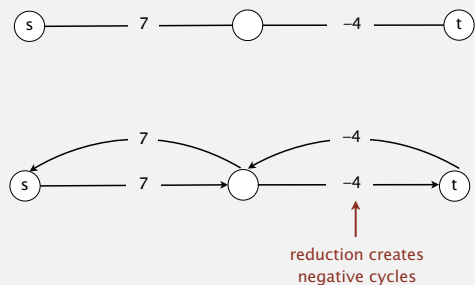


cost of shortest paths in digraph cost of reduction

Cost of undirected shortest paths. $E \log V + E$.

Shortest paths with negative weights

Caveat. Reduction is invalid for edge-weighted graphs with negative weights (even if no negative cycles).



Remark. Can still solve shortest-paths problem in undirected graphs (if no negative cycles), but need more sophisticated techniques.

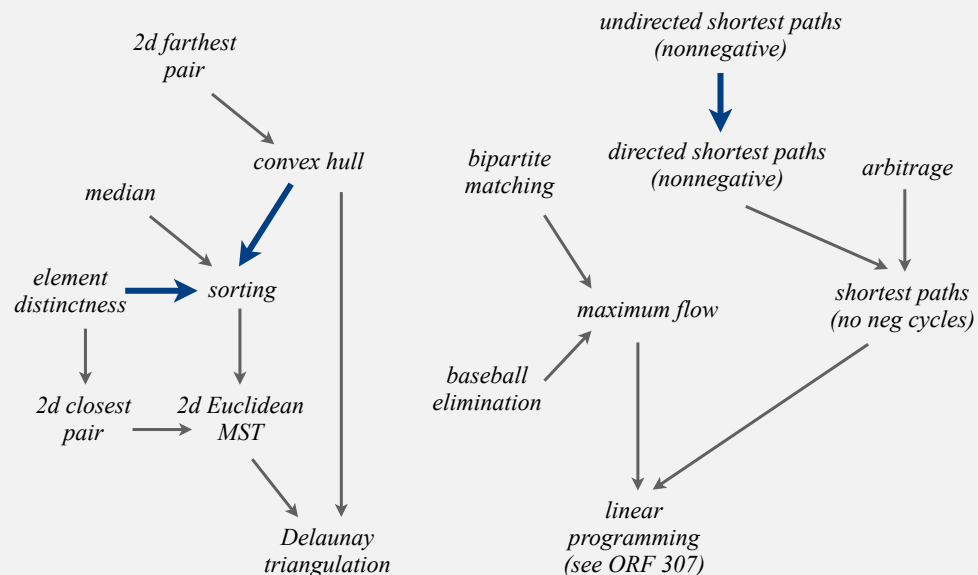
reduces to weighted non-bipartite matching (!)

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Some reductions involving familiar problems

computational geometry

combinatorial optimization



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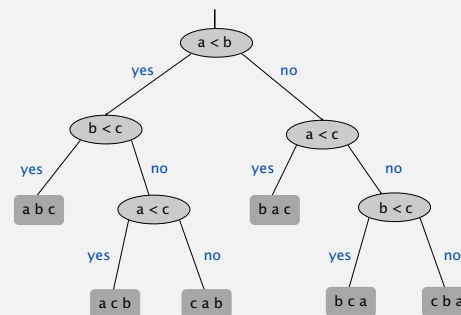
- ▶ designing algorithms
- ▶ establishing lower bounds
- ▶ classifying problems
- ▶ intractability

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Bird's-eye view

Goal. Prove that a problem requires a certain number of steps.

Ex. In decision tree model, any compare-based sorting algorithm requires $\Omega(N \log N)$ compares in the worst case.



argument must apply to all conceivable algorithms

Bad news. Very difficult to establish lower bounds from scratch.

Good news. Can spread $\Omega(N \log N)$ lower bound to Y by reducing sorting to Y .

assuming cost of reduction is not too high

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Linear-time reductions

Def. Problem X **linear-time reduces** to problem Y if X can be solved with:

- Linear number of standard computational steps.
- Constant number of calls to Y .

Ex. Almost all of the reductions we've seen so far. [Which ones weren't?]

Establish lower bound:

- If X takes $\Omega(N \log N)$ steps, then so does Y .
- If X takes $\Omega(N^2)$ steps, then so does Y .

Mentality.

- If I could easily solve Y , then I could easily solve X .
- I can't easily solve X .
- Therefore, I can't easily solve Y .

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Lower bound for convex hull

Proposition. In quadratic decision tree model, any algorithm for sorting N integers requires $\Omega(N \log N)$ steps.

allows linear or quadratic tests:
 $x_i < x_j$ or $(x_j - x_i)(x_k - x_i) - (x_j - x_i)(x_j - x_i) < 0$

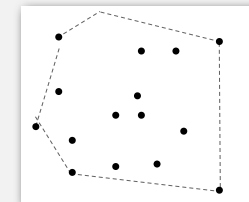
Proposition. Sorting linear-time reduces to convex hull.

Pf. [see next slide]

lower-bound mentality:
 if I can solve convex hull efficiently, I can sort efficiently

1251432
 2861534
 3988818
 4190745
 13546464
 89885444
 43434213

sorting



convex hull

linear or quadratic tests

Implication. Any ccw-based convex hull algorithm requires $\Omega(N \log N)$ ops.

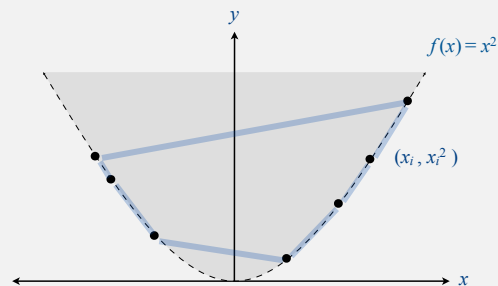
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Sorting linear-time reduces to convex hull

Proposition. Sorting linear-time reduces to convex hull.

- **Sorting instance:** x_1, x_2, \dots, x_N .
- **Convex hull instance:** $(x_1, x_1^2), (x_2, x_2^2), \dots, (x_N, x_N^2)$.

lower-bound mentality:
 if I can solve convex hull efficiently, I can sort efficiently



Pf.

- Region $\{x : x^2 \geq x\}$ is convex \Rightarrow all points are on hull.
- Starting at point with most negative x , counterclockwise order of hull points yields integers in ascending order.

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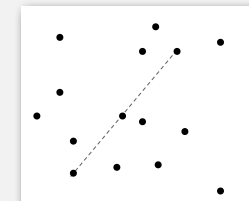
Lower bound for 3-COLLINEAR

3-SUM. Given N distinct integers, are there three that sum to 0?

3-COLLINEAR. Given N distinct points in the plane, are there 3 that all lie on the same line? ← recall Assignment 3

1251432
 -2861534
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3-sum



3-collinear

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Lower bound for 3-COLLINEAR

3-SUM. Given N distinct integers, are there three that sum to 0?

3-COLLINEAR. Given N distinct points in the plane, are there 3 that all lie on the same line?

Proposition. 3-SUM linear-time reduces to 3-COLLINEAR.

Pf. [next two slides]

Conjecture. Any algorithm for 3-SUM requires $\Omega(N^2)$ steps.

Implication. No sub-quadratic algorithm for 3-COLLINEAR likely.

your $N^2 \log N$ algorithm was pretty good

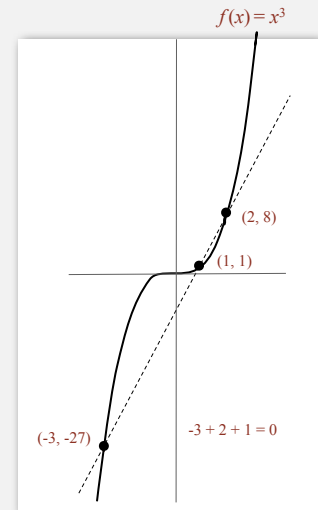
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3-SUM linear-time reduces to 3-COLLINEAR

Proposition. 3-SUM linear-time reduces to 3-COLLINEAR.

- 3-SUM instance: x_1, x_2, \dots, x_N .
- 3-COLLINEAR instance: $(x_1, x_1^3), (x_2, x_2^3), \dots, (x_N, x_N^3)$.

Lemma. If $a, b,$ and c are distinct, then $a + b + c = 0$ if and only if $(a, a^3), (b, b^3),$ and (c, c^3) are collinear.



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3-SUM linear-time reduces to 3-COLLINEAR

Proposition. 3-SUM linear-time reduces to 3-COLLINEAR.

- 3-SUM instance: x_1, x_2, \dots, x_N .
- 3-COLLINEAR instance: $(x_1, x_1^3), (x_2, x_2^3), \dots, (x_N, x_N^3)$.

Lemma. If $a, b,$ and c are distinct, then $a + b + c = 0$ if and only if $(a, a^3), (b, b^3),$ and (c, c^3) are collinear.

Pf. Three distinct points $(a, a^3), (b, b^3),$ and (c, c^3) are collinear iff:

$$\begin{aligned}
 0 &= \begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix} \\
 &= a(b^3 - c^3) - b(a^3 - c^3) + c(a^3 - b^3) \\
 &= (a - b)(b - c)(c - a)(a + b + c)
 \end{aligned}$$

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More linear-time reductions and lower bounds

sorting

element distinctness
($N \log N$ lower bound)

sorting

2d closest pair

2d convex hull

2d Euclidean MST

Delaunay triangulation

3-sum

3-sum
(conjectured N^2 lower bound)

3-collinear

dihedral rotation

3-concurrent

min area triangle

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Establishing lower bounds: summary

Establishing lower bounds through reduction is an important tool in guiding algorithm design efforts.

Q. How to convince yourself no linear-time convex hull algorithm exists?

A1. [hard way] Long futile search for a linear-time algorithm.

A2. [easy way] Linear-time reduction from sorting.

Q. How to convince yourself no sub-quadratic *3-COLLINEAR* algorithm likely.

A1. [hard way] Long futile search for a sub-quadratic algorithm.

A2. [easy way] Linear-time reduction from *3-SUM*.

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- ▶ designing algorithms
- ▶ establishing lower bounds
- ▶ **classifying problems**
- ▶ intractability

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Classifying problems: summary

Desiderata. Problem with algorithm that matches lower bound.

Ex. Sorting, convex hull, and closest pair have complexity $N \log N$.

Desiderata'. Prove that two problems X and Y have the same complexity.

- First, show that problem X linear-time reduces to Y .
- Second, show that Y linear-time reduces to X .
- Conclude that X and Y have the same complexity.

even if we don't know what it is!

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Primality testing

PRIME. Given an integer x (represented in binary), is x prime?

COMPOSITE. Given an integer x , does x have a nontrivial factor?

Proposition. *PRIME* linear-time reduces to *COMPOSITE*.

```
public static boolean isPrime(BigInteger x)
{
    if (isComposite(x)) return false;
    else                 return true;
}
```

147573952589676412931

prime

147573952589676412927

composite

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Primality testing

PRIME. Given an integer x (represented in binary), is x prime?

COMPOSITE. Given an integer x , does x have a nontrivial factor?

Proposition. *COMPOSITE* linear-time reduces to *PRIME*.

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public static boolean isComposite(BigInteger x)
{
    if (isPrime(x)) return false;
    else           return true;
}
```

147573952589676412931

prime

147573952589676412927

composite

29

Caveat

PRIME. Given an integer x (represented in binary), is x prime?

COMPOSITE. Given an integer x , does x have a nontrivial factor?

Proposition. *PRIME* linear-time reduces to *COMPOSITE*.

Proposition. *COMPOSITE* linear-time reduces to *PRIME*.

Conclusion. *PRIME* and *COMPOSITE* have the same complexity.

best known deterministic algorithm
is about N^6 for N -bit integer

A possible real-world scenario.

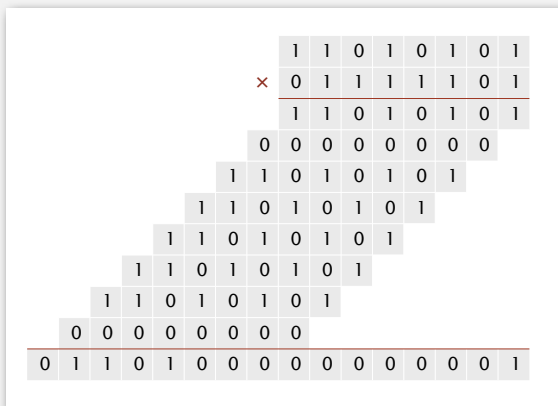
- System designer specs the APIs for project.
- Alice implements `isComposite()` using `isPrime()`.
- Bob implements `isPrime()` using `isComposite()`.
- Infinite reduction loop!
- Who's fault?

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Integer arithmetic reductions

Integer multiplication. Given two N -bit integers, compute their product.

Brute force. N^2 bit operations.



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Integer arithmetic reductions

Integer multiplication. Given two N -bit integers, compute their product.

Brute force. N^2 bit operations.

problem	arithmetic	order of growth
integer multiplication	$a \times b$	$M(N)$
integer division	$a / b, a \bmod b$	$M(N)$
integer square	a^2	$M(N)$
integer square root	$\lfloor \sqrt{a} \rfloor$	$M(N)$

integer arithmetic problems with the same complexity as integer multiplication

Q. Is brute-force algorithm optimal?

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Complexity of integer multiplication history

year	algorithm	order of growth
1962	Karatsuba-Ofman	$N^{1.585}$
1963	Toom-3, Toom-4	$N^{1.465}$, $N^{1.404}$
1966	Toom-Cook	$N^{1+\epsilon}$
1971	Schönhage-Strassen	$N \log N \log \log N$
2007	Fürer	$N \log N 2^{\log^* N}$
?	?	N

number of bit operations to multiply two N -bit integers

used in Maple, Mathematica, gcc, cryptography, ...

Remark. GNU Multiple Precision Library uses one of five different algorithm depending on size of operands.

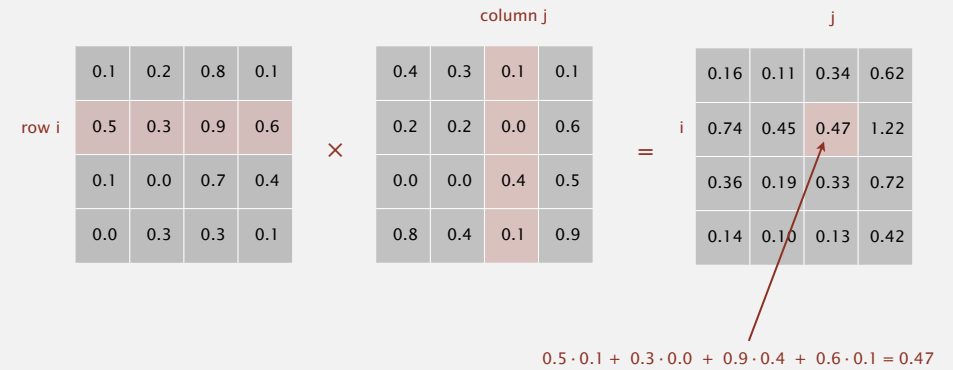


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Linear algebra reductions

Matrix multiplication. Given two N -by- N matrices, compute their product.

Brute force. N^3 flops.



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Linear algebra reductions

Matrix multiplication. Given two N -by- N matrices, compute their product.

Brute force. N^3 flops.

problem	linear algebra	order of growth
matrix multiplication	$A \times B$	$MM(N)$
matrix inversion	A^{-1}	$MM(N)$
determinant	$ A $	$MM(N)$
system of linear equations	$Ax = b$	$MM(N)$
LU decomposition	$A = LU$	$MM(N)$
least squares	$\min \ Ax - b\ _2$	$MM(N)$

numerical linear algebra problems with the same complexity as matrix multiplication

Q. Is brute-force algorithm optimal?

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Complexity of matrix multiplication history

year	algorithm	order of growth
1969	Strassen	$N^{2.808}$
1978	Pan	$N^{2.796}$
1979	Bini	$N^{2.780}$
1981	Schönhage	$N^{2.522}$
1982	Romani	$N^{2.517}$
1982	Coppersmith-Winograd	$N^{2.496}$
1986	Strassen	$N^{2.479}$
1989	Coppersmith-Winograd	$N^{2.376}$
2010	Strother	$N^{2.3737}$
2011	Williams	$N^{2.3727}$
?	?	$N^{2+\epsilon}$

number of floating-point operations to multiply two N -by- N matrices

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- ▶ designing algorithms
- ▶ establishing lower bounds
- ▶ classifying problems
- ▶ intractability

Bird's-eye view

Def. A problem is **intractable** if it can't be solved in polynomial time.

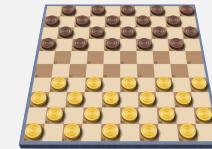
Desiderata. Prove that a problem is intractable.

Two problems that provably require exponential time.

- Given a constant-size program, does it halt in at most K steps?
- Given N -by- N checkers board position, can the first player force a win?

input size = $c + \lg K$

using forced capture rule



Frustrating news. Very few successes.

3-satisfiability

Literal. A boolean variable or its negation.

$$x_i \text{ OR } \neg x_i$$

Clause. An *or* of 3 distinct literals.

$$C_1 = (\neg x_1 \vee x_2 \vee x_3)$$

Conjunctive normal form. An *and* of clauses.

$$\Phi = (C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5)$$

3-SAT. Given a CNF formula Φ consisting of k clauses over n literals, does it have a satisfying truth assignment?

$$\Phi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_2 \vee x_3 \vee x_4) \\ \wedge (\neg T \vee T \vee F) \wedge (T \vee \neg T \vee F) \wedge (\neg T \vee \neg T \vee \neg F) \wedge (\neg T \vee \neg T \vee T) \wedge (\neg T \vee F \vee T)$$

yes instance $\begin{matrix} x_1 & x_2 & x_3 & x_4 \\ T & T & F & T \end{matrix}$

Applications. Circuit design, program correctness, ...

3-satisfiability is conjectured to be intractable

Q. How to solve an instance of 3-SAT with n variables?

A. Exhaustive search: try all 2^n truth assignments.

Q. Can we do anything substantially more clever?



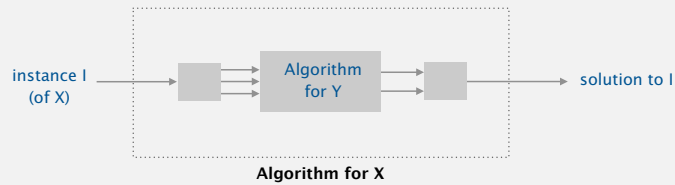
Conjecture ($P \neq NP$). 3-SAT is intractable (no poly-time algorithm).

consensus opinion

Polynomial-time reductions

Problem X **poly-time (Cook) reduces** to problem Y if X can be solved with:

- Polynomial number of standard computational steps.
- Polynomial number of calls to Y .



Establish intractability. If 3-SAT poly-time reduces to Y , then Y is intractable. (assuming 3-SAT is intractable)

Mentality.

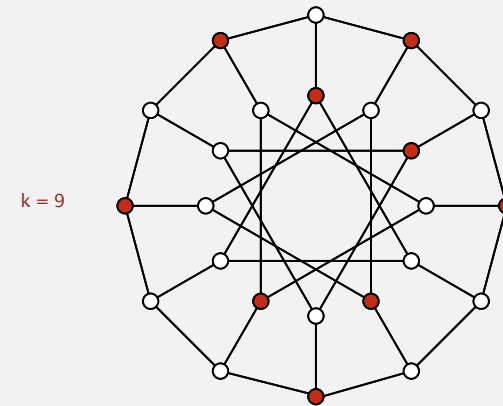
- If I could solve Y in poly-time, then I could also solve 3-SAT in poly-time.
- 3-SAT is believed to be intractable.
- Therefore, so is Y .

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Independent set

An **independent set** is a set of vertices, no two of which are adjacent.

IND-SET. Given a graph G and an integer k , find an independent set of size k .



Applications. Scheduling, computer vision, clustering, ...

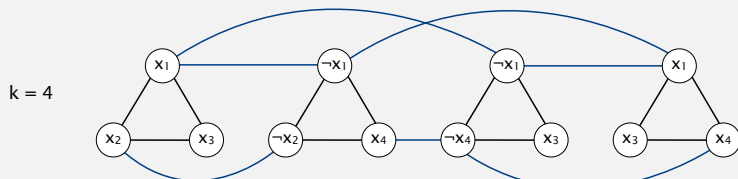
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3-satisfiability reduces to independent set

Proposition. 3-SAT poly-time reduces to $IND\text{-SET}$.

Pf. Given an instance Φ of 3-SAT , create an instance G of $IND\text{-SET}$:

- For each clause in Φ , create 3 vertices in a triangle.
- Add an edge between each literal and its negation.



$$\Phi = (x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee x_3 \vee \neg x_4) \wedge (x_1 \vee x_3 \vee x_4)$$

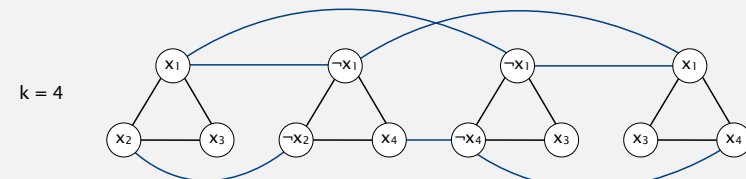
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3-satisfiability reduces to independent set

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- G has independent set of size $k \Rightarrow \Phi$ satisfiable.

↑
set literals corresponding to k vertices in independent set to true
(set remaining literals in any consistent manner)

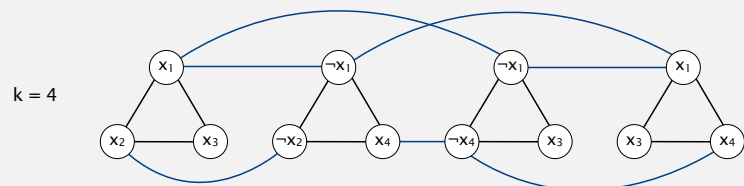
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3-satisfiability reduces to independent set

Proposition. 3-SAT poly-time reduces to IND-SET.

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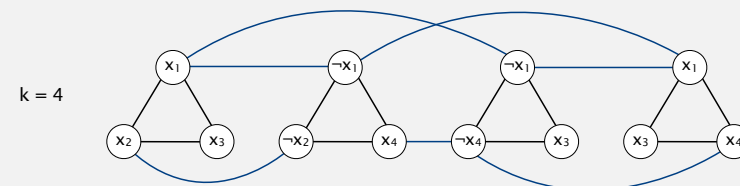
- G has independent set of size $k \Rightarrow \Phi$ satisfiable.
- Φ satisfiable $\Rightarrow G$ has independent set of size k .

↑
for each of k clauses, include in independent set one vertex corresponding to a true literal

3-satisfiability reduces to independent set

Proposition. 3-SAT poly-time reduces to IND-SET. ← lower-bound mentality: if I could solve IND-SET efficiently, I could solve 3-SAT efficiently

Implication. Assuming 3-SAT is intractable, so is IND-SET.



$$\Phi = (x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee x_3 \vee \neg x_4) \wedge (x_1 \vee x_3 \vee x_4)$$

Integer linear programming

ILP. Given a system of linear inequalities, find an **integral** solution.

$$\begin{aligned} 3x_1 + 5x_2 + 2x_3 + x_4 + 4x_5 &\geq 10 \\ 5x_1 + 2x_2 + 4x_4 + 1x_5 &\leq 7 \\ x_1 + x_3 + 2x_4 &\leq 2 \\ 3x_1 + 4x_3 + 7x_4 &\leq 7 \\ x_1 + x_4 &\leq 1 \\ x_1 + x_3 + x_5 &\leq 1 \\ \text{all } x_i &= \{0, 1\} \end{aligned}$$

linear inequalities

integer variables

yes instance: $x_1 \ x_2 \ x_3 \ x_4 \ x_5$
0 1 0 1 1

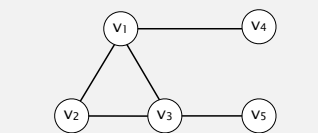
Context. Cornerstone problem in operations research.

Remark. Finding a real-valued solution is tractable (linear programming).

Independent set reduces to integer linear programming

Proposition. IND-SET poly-time reduces to ILP.

Pf. Given an instance $\{G, k\}$ of IND-SET, create an instance of ILP as follows:



is there an independent set of size 3?

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 3 && \leftarrow \text{number of vertices selected} \\ x_1 + x_2 &\leq 1 \\ x_2 + x_3 &\leq 1 \\ x_1 + x_3 &\leq 1 \\ x_1 + x_4 &\leq 1 \\ x_3 + x_5 &\leq 1 \\ \text{all } x_i &= \{0, 1\} && \leftarrow \text{binary variables} \end{aligned}$$

at most one vertex selected from each edge

is there a feasible solution?

Intuition. $x_i = 1$ if and only if vertex v_i is in independent set.

3-satisfiability reduces to integer linear programming

Proposition. *3-SAT* poly-time reduces to *IND-SET*.

Proposition. *IND-SET* poly-time reduces to *ILP*.

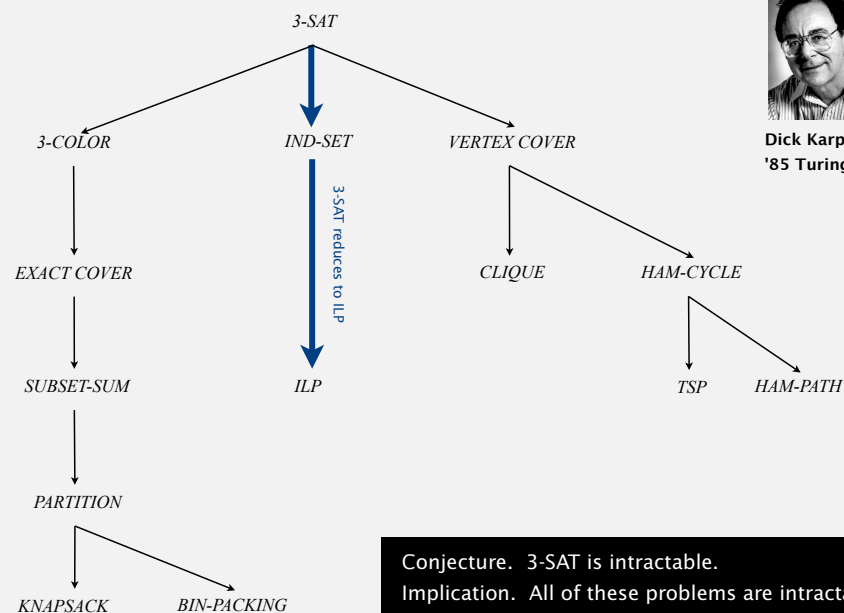
Transitivity. If *X* poly-time reduces to *Y* and *Y* poly-time reduces to *Z*, then *X* poly-time reduces to *Z*.

Implication. Assuming *3-SAT* is intractable, so is *ILP*.

lower-bound mentality:
if I could solve ILP efficiently,
I could solve *IND-SET* efficiently;
if I could solve *IND-SET* efficiently,
I could solve *3-SAT* efficiently

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More poly-time reductions from 3-satisfiability



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Implications of poly-time reductions from 3-satisfiability

Establishing intractability through poly-time reduction is an important tool in guiding algorithm design efforts.

Q. How to convince yourself that a new problem is (probably) intractable?

A1. [hard way] Long futile search for an efficient algorithm (as for *3-SAT*).

A2. [easy way] Reduction from *3-SAT*.

Caveat. Intricate reductions are common.

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Search problems

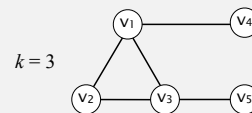
Search problem. Problem where you can check a solution in poly-time.

Ex 1. *3-SAT*.

$$\Phi = (x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee x_3 \vee \neg x_4) \wedge (x_1 \vee x_3 \vee x_4)$$

$x_1 = \text{true}, x_2 = \text{true}, x_3 = \text{true}, x_4 = \text{true}$

Ex 2. *IND-SET*.



$\{v_2, v_4, v_5\}$

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P vs. NP

P. Set of search problems solvable in poly-time.

Importance. What scientists and engineers can compute feasibly.

NP. Set of search problems.

Importance. What scientists and engineers aspire to compute feasibly.

Fundamental question.



Consensus opinion. No.

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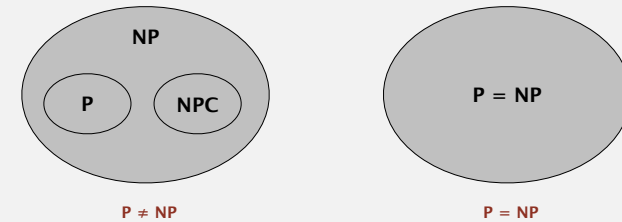
Cook's theorem

An **NP** problem is **NP-complete** if all problems in **NP** poly-time to reduce to it.

Cook's theorem. 3-SAT is NP-complete.

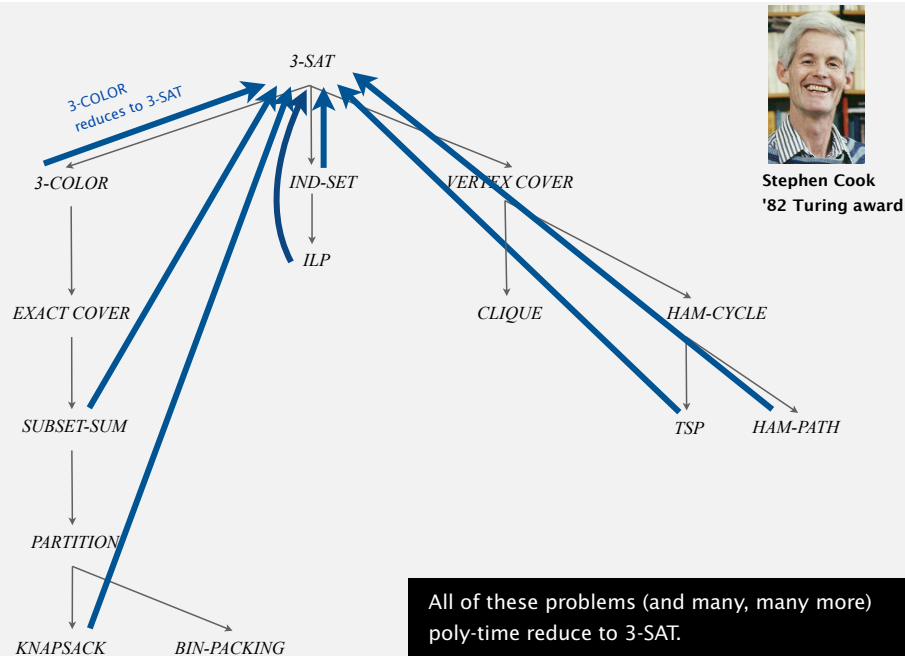
Corollary. 3-SAT is tractable if and only if $P = NP$.

Two worlds.



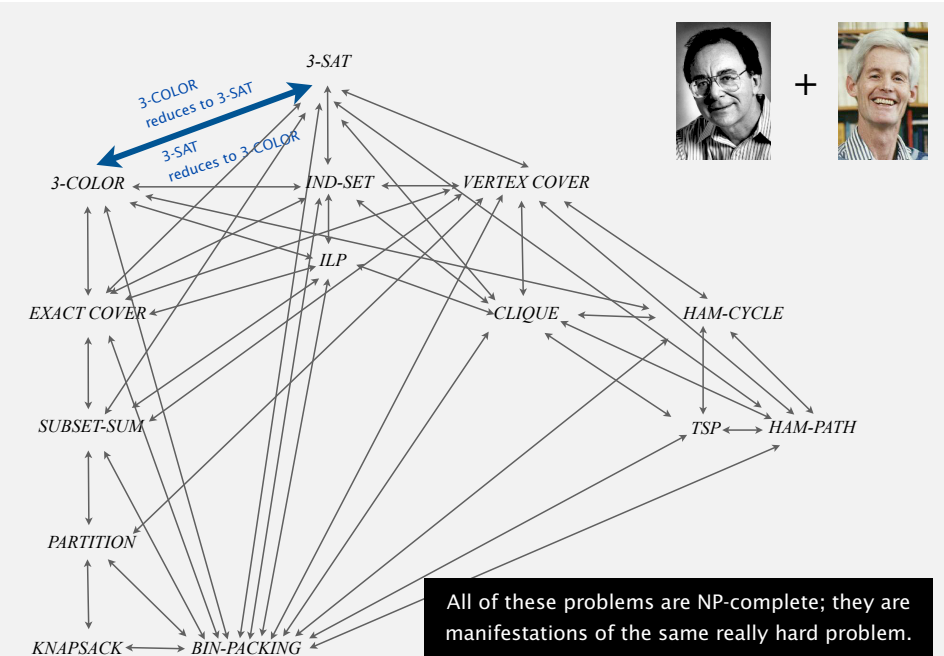
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Implications of Cook's theorem



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Implications of Karp + Cook



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Desiderata. Classify **problems** according to computational requirements.

complexity	order of growth	examples
linear	N	min, max, median, Burrows-Wheeler transform, ...
linearithmic	$N \log N$	sorting, convex hull, closest pair, farthest pair, ...
quadratic	N^2	???
\vdots	\vdots	\vdots
exponential	c^N	???

Frustrating news. Huge number of problems have defied classification.

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Complexity zoo

Complexity class. Set of problems that share some computational property.



http://qwiki.stanford.edu/index.php/Complexity_Zoo

Bad news. Lots of complexity classes.

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Desiderata. Classify **problems** according to computational requirements.

complexity	order of growth	examples
linear	N	min, max, median, Burrows-Wheeler transform, ...
linearithmic	$N \log N$	sorting, convex hull, closest pair, farthest pair, ...
$M(N)$	$?$	integer multiplication, division, square root, ...
3-SUM complete	probably N^2	3-SUM, 3-COLLINEAR, 3-CONCURRENT, ...
$MM(N)$	$?$	matrix multiplication, $Ax = b$, least square, determinant, ...
\vdots	\vdots	\vdots
NP-complete	probably not N^b	3-SAT, IND-SET, ILP, ...

Good news. Can put many problems into equivalence classes.

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Summary

Reductions are important in theory to:

- Establish tractability.
- Establish intractability.
- Classify problems according to their computational requirements.

Reductions are important in practice to:

- Design algorithms.
- Design reusable software modules.
 - stacks, queues, priority queues, symbol tables, sets, graphs
 - sorting, regular expressions, Delaunay triangulation
 - MST, shortest path, maxflow, linear programming
- Determine difficulty of your problem and choose the right tool.
 - use exact algorithm for tractable problems
 - use heuristics for intractable problems

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