2.4 Priority Queues

- API
- elementary implementations
- binary heaps
- heapsort
- event-driven simulation

- **API**
- elementary implementations
- binary heaps
- heapsort
- event-driven simulation

Priority queue

Collections. Insert and delete items. Which item to delete?

Stack. Remove the item most recently added.

Queue. Remove the item least recently added.

Randomized queue. Remove a random item.

Priority queue. Remove the largest (or smallest) item.

operation	argument	return value
insert	P	
insert	Q	
insert	E	
remove max		Q
insert	X	
insert	A	
insert	M	
remove max		X
insert	P	
insert	L	
insert	E	
remove max		P

Priority queue API

Requirement. Generic items are `Comparable`.

Key must be Comparable
(bounded type parameter)

`public class MaxPQ<Key extends Comparable<Key>>`

	`MaxPQ()`	*create an empty priority queue*
	`MaxPQ(Key[] a)`	*create a priority queue with given keys*
`void`	`insert(Key v)`	*insert a key into the priority queue*
`Key`	`delMax()`	*return and remove the largest key*
`boolean`	`isEmpty()`	*is the priority queue empty?*
`Key`	`max()`	*return the largest key*
`int`	`size()`	*number of entries in the priority queue*

Priority queue applications

- Event-driven simulation. [customers in a line, colliding particles]
- Numerical computation. [reducing roundoff error]
- Data compression. [Huffman codes]
- Graph searching. [Dijkstra's algorithm, Prim's algorithm]
- Computational number theory. [sum of powers]
- Artificial intelligence. [A* search]
- Statistics. [maintain largest M values in a sequence]
- Operating systems. [load balancing, interrupt handling]
- Discrete optimization. [bin packing, scheduling]
- Spam filtering. [Bayesian spam filter]

Generalizes: stack, queue, randomized queue.

Priority queue client example

Challenge. Find the largest M items in a stream of N items (N huge, M large).

- Fraud detection: isolate $$ transactions.
- File maintenance: find biggest files or directories.

Constraint. Not enough memory to store N items.

```
% more tinyBatch.txt
Turing        6/17/1990   644.08
vonNeumann    3/26/2002  4121.85
Dijkstra      8/22/2007  2678.40
vonNeumann    1/11/1999  4409.74
Dijkstra     11/18/1995   837.42
Hoare         5/10/1993  3229.27
vonNeumann    2/12/1994  4732.35
Hoare         8/18/1992  4381.21
Turing        1/11/2002    66.10
Thompson      2/27/2000  4747.08
Turing        2/11/1991  2156.86
Hoare         8/12/2003  1025.70
vonNeumann   10/13/1993  2520.97
Dijkstra      9/10/2000   708.95
Turing       10/12/1993  3532.36
Hoare         2/10/2005  4050.20
```

```
% java TopM 5 < tinyBatch.txt
Thompson     2/27/2000   4747.08
vonNeumann   2/12/1994   4732.35
vonNeumann   1/11/1999   4409.74
Hoare        8/18/1992   4381.21
vonNeumann   3/26/2002   4121.85
```

sort key

Priority queue client example

Challenge. Find the largest M items in a stream of N items (N huge, M large).

use a min-oriented pq

```
MinPQ<Transaction> pq = new MinPQ<Transaction>();

while (StdIn.hasNextLine())
{
   String line = StdIn.readLine();
   Transaction item = new Transaction(line);
   pq.insert(item);
   if (pq.size() > M)
      pq.delMin();
}
```

pq contains largest M items

Transaction data type is Comparable (ordered by $$)

order of growth of finding the largest M in a stream of N items

implementation	time	space
sort	N log N	N
elementary PQ	M N	M
binary heap	N log M	M
best in theory	N	M

- API
- **elementary implementations**
- binary heaps
- heapsort
- event-driven simulation

Priority queue: unordered and ordered array implementation

operation	*argument*	*return value*	*size*	*contents (unordered)*							*contents (ordered)*						
insert	P		1	P							P						
insert	Q		2	P	Q						P	Q					
insert	E		3	P	Q	E					E	P	Q				
remove max		Q	2	P	E						E	P					
insert	X		3	P	E	X					E	P	X				
insert	A		4	P	E	X	A				A	E	P	X			
insert	M		5	P	E	X	A	M			A	E	M	P	X		
remove max		X	4	P	E	M	A				A	E	M	P			
insert	P		5	P	E	M	A	P			A	E	M	P	P		
insert	L		6	P	E	M	A	P	L		A	E	L	M	P	P	
insert	E		7	P	E	M	A	P	L	E	A	E	E	L	M	P	P
remove max		P	6	E	M	A	P	L	E		A	E	E	L	M	P	

A sequence of operations on a priority queue

Priority queue: unordered array implementation

```
public class UnorderedMaxPQ<Key extends Comparable<Key>>
{
   private Key[] pq;    // pq[i] = ith element on pq
   private int N;       // number of elements on pq

   public UnorderedMaxPQ(int capacity)
   {  pq = (Key[]) new Comparable[capacity];  }

   public boolean isEmpty()
   {  return N == 0; }

   public void insert(Key x)
   {  pq[N++] = x;  }

   public Key delMax()
   {
      int max = 0;
      for (int i = 1; i < N; i++)
         if (less(max, i)) max = i;
      exch(max, N-1);
      return pq[--N];
   }
}
```

no generic array creation

`less()` and `exch()` similar to sorting methods

null out entry to prevent loitering

Priority queue elementary implementations

Challenge. Implement all operations efficiently.

order-of-growth of running time for priority queue with N items

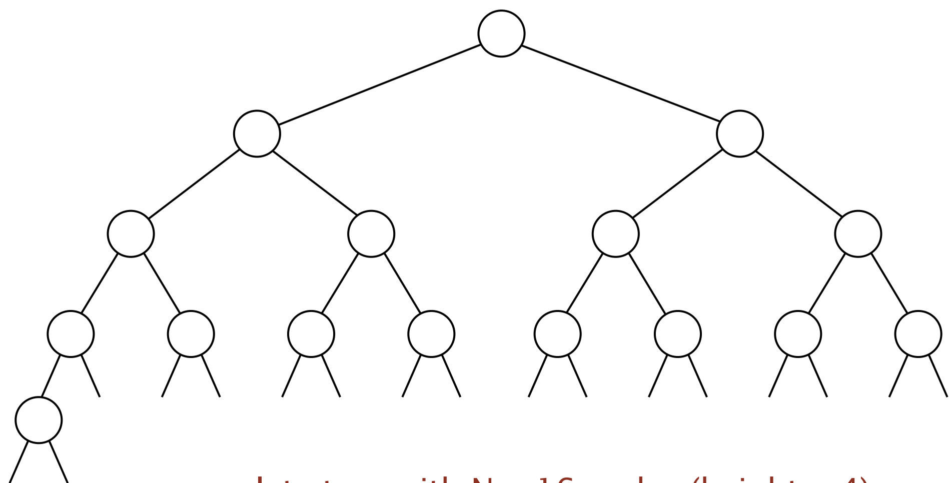
implementation	insert	del max	max
unordered array	1	N	N
ordered array	N	1	1
goal	log N	log N	log N

- API
- elementary implementations
- **binary heaps**
- heapsort
- event-driven simulation

Binary tree

Binary tree. Empty or node with links to left and right binary trees.

Complete tree. Perfectly balanced, except for bottom level.



complete tree with N = 16 nodes (height = 4)

Property. Height of complete tree with N nodes is $\lfloor \lg N \rfloor$.

Pf. Height only increases when N is a power of 2.

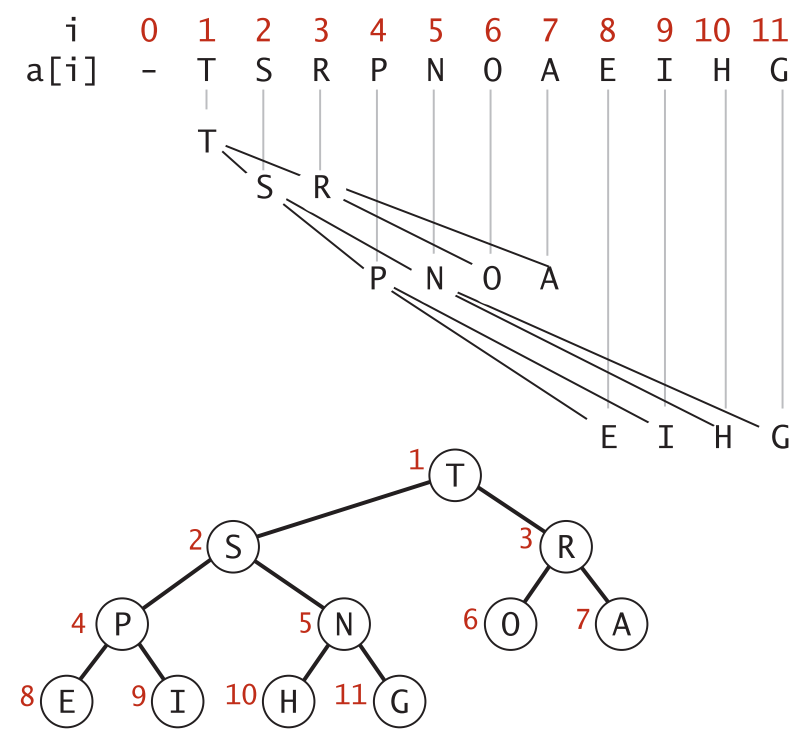
A complete binary tree in nature

Binary heap representations

Binary heap. Array representation of a heap-ordered complete binary tree.

Heap-ordered binary tree.
- Keys in nodes.
- Parent's key no smaller than children's keys.

Array representation.
- Take nodes in level order.
- No explicit links needed!



Heap representations

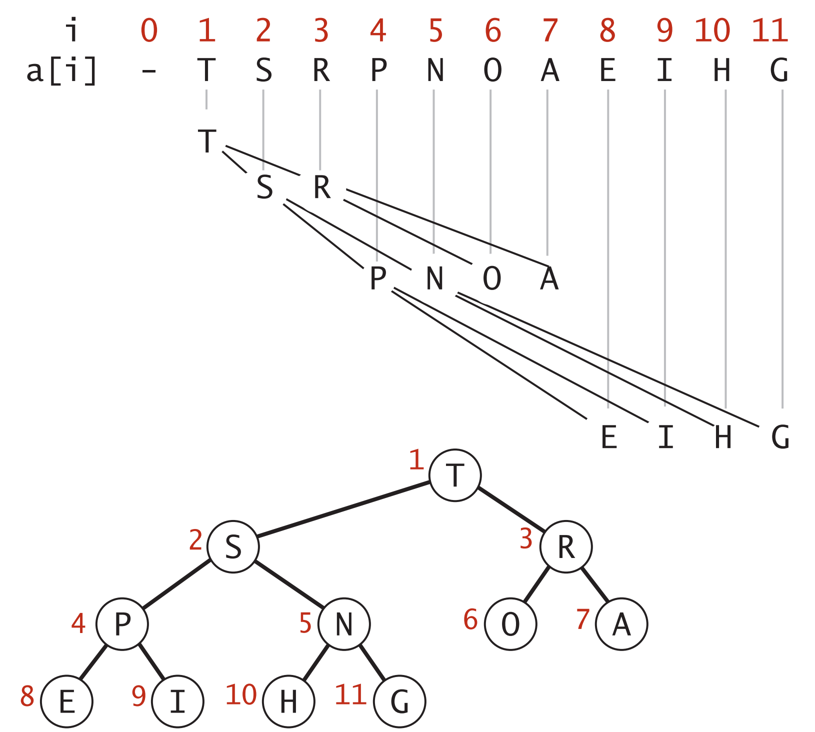
Binary heap properties

Proposition. Largest key is `a[1]`, which is root of binary tree.

indices start at 1

Proposition. Can use array indices to move through tree.

- Parent of node at `k` is at `k/2`.
- Children of node at `k` are at `2k` and `2k+1`.



Heap representations

Promotion in a heap

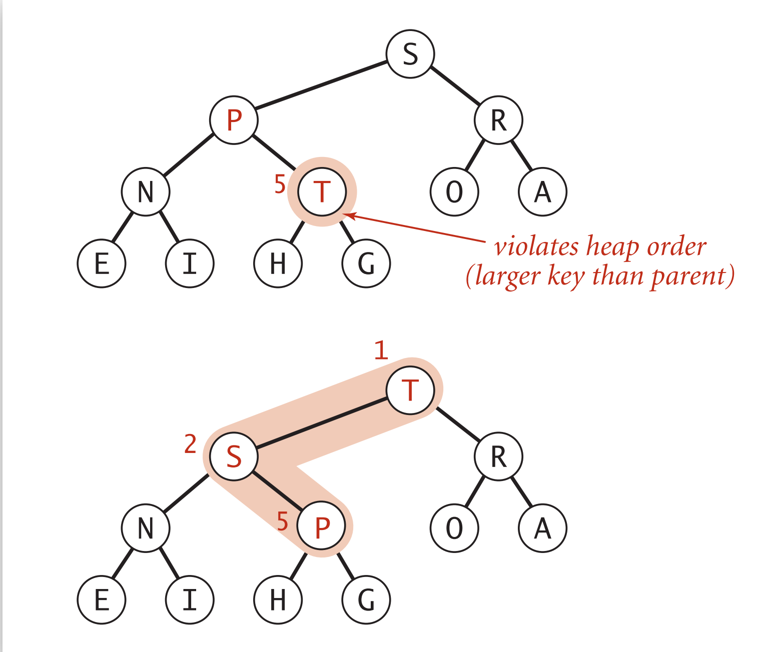
Scenario. Child's key becomes larger key than its parent's key.

To eliminate the violation:

- Exchange key in child with key in parent.
- Repeat until heap order restored.

```
private void swim(int k)
{
   while (k > 1 && less(k/2, k))
   {
      exch(k, k/2);
      k = k/2;
   }
}
```

parent of node at k is at k/2



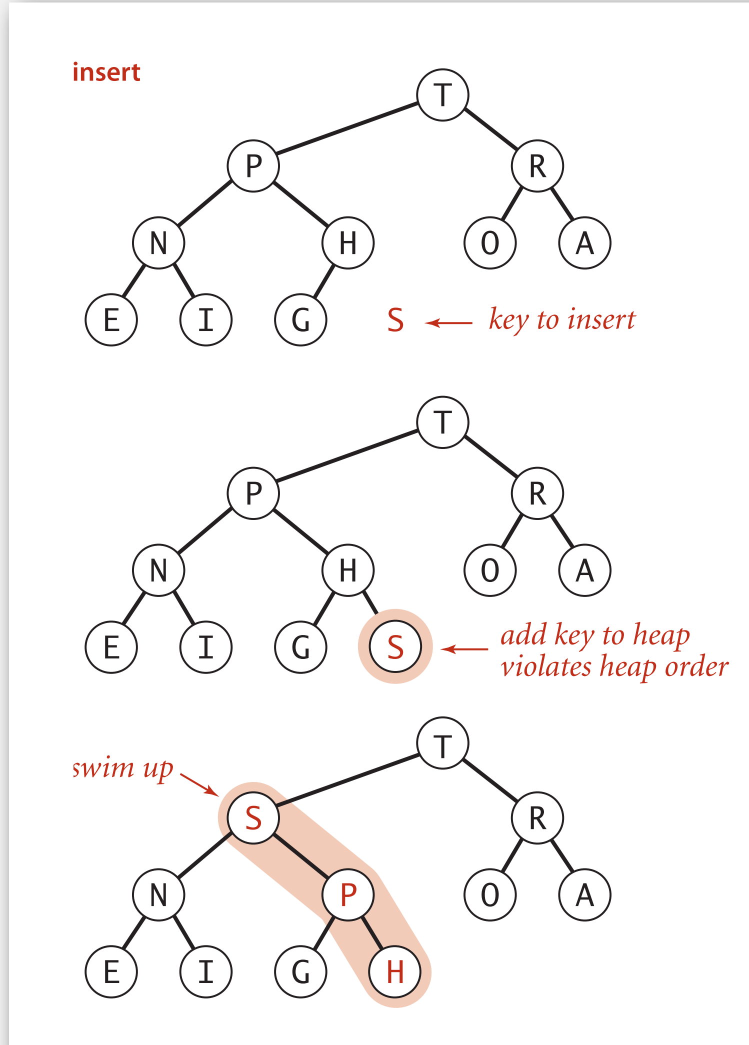
Peter principle. Node promoted to level of incompetence.

Insertion in a heap

Insert. Add node at end, then swim it up.

Cost. At most $1 + \lg N$ compares.

```
public void insert(Key x)
{
   pq[++N] = x;
   swim(N);
}
```



Demotion in a heap

Scenario. Parent's key becomes smaller than one (or both) of its children's keys.

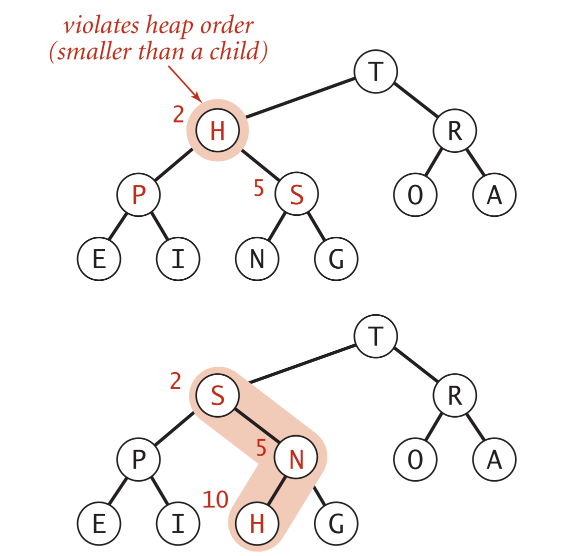
To eliminate the violation:

why not smaller child?

- Exchange key in parent with key in larger child.
- Repeat until heap order restored.

```
private void sink(int k)
{
   while (2*k <= N)
   {
      int j = 2*k;
      if (j < N && less(j, j+1)) j++;
      if (!less(k, j)) break;
      exch(k, j);
      k = j;
   }
}
```

children of node at k are 2k and 2k+1



Top-down reheapify (sink)

Power struggle. Better subordinate promoted.

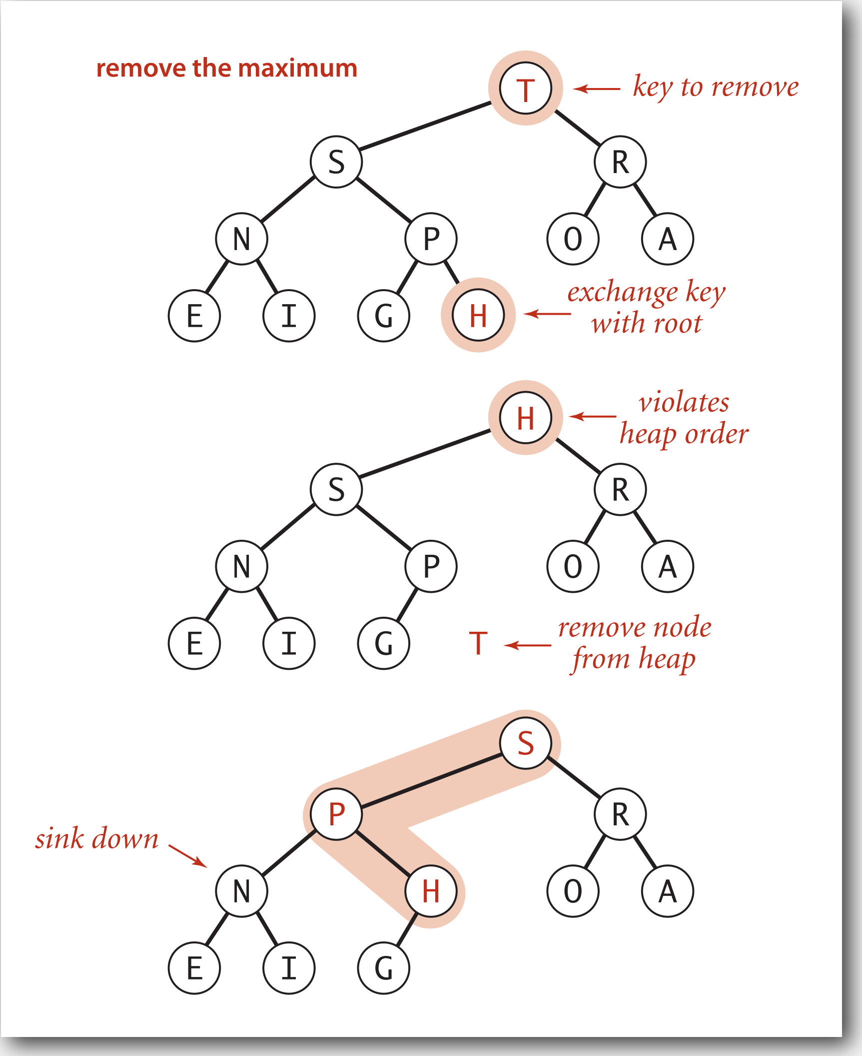
Delete the maximum in a heap

Delete max. Exchange root with node at end, then sink it down.

Cost. At most $2 \lg N$ compares.

```
public Key delMax()
{
   Key max = pq[1];
   exch(1, N--);
   sink(1);
   pq[N+1] = null;
   return max;
}
```

prevent loitering



Binary heap demo

Binary heap: Java implementation

```
public class MaxPQ<Key extends Comparable<Key>>
{
   private Key[] pq;
   private int N;

   public MaxPQ(int capacity)
   {  pq = (Key[]) new Comparable[capacity+1];  }

   public boolean isEmpty()
   {   return N == 0;   }
   public void insert(Key key)
   {   /* see previous code */   }
   public Key delMax()
   {   /* see previous code */   }

   private void swim(int k)
   {   /* see previous code */   }
   private void sink(int k)
   {   /* see previous code */   }

   private boolean less(int i, int j)
   {   return pq[i].compareTo(pq[j] < 0;   }
   private void exch(int i, int j)
   {   Key t = pq[i]; pq[i] = pq[j]; pq[j] = t;   }
}
```

PQ ops

heap helper functions

array helper functions

Priority queues implementation cost summary

order-of-growth of running time for priority queue with N items

implementation	insert	del max	max
unordered array	1	N	N
ordered array	N	1	1
binary heap	log N	log N	1
d-ary heap	$\log_d N$	$d \log_d N$	1
Fibonacci	1	log N †	1
impossible	1	1	1

← why impossible?

† amortized

Binary heap considerations

Immutability of keys.

- Assumption: client does not change keys while they're on the PQ.
- Best practice: use immutable keys.

Underflow and overflow.

- Underflow: throw exception if deleting from empty PQ.
- Overflow: add no-arg constructor and use resizing array.

leads to log N amortized time per op

Minimum-oriented priority queue.

- Replace `less()` with `greater()`.
- Implement `greater()`.

Other operations.

- Remove an arbitrary item.
- Change the priority of an item.

can implement with `sink()` and `swim()` [stay tuned]

Immutability: implementing in Java

Data type. Set of values and operations on those values.

Immutable data type. Can't change the data type value once created.

```
public final class Vector {
   private final int N;
   private final double[] data;

   public Vector(double[] data) {
      this.N = data.length;
      this.data = new double[N];
      for (int i = 0; i < N; i++)
         this.data[i] = data[i];
   }

   …
}
```

can't override instance methods

all instance variables private and final

defensive copy of mutable instance variables

instance methods don't change instance variables

Immutable. **String**, **Integer**, **Double**, **Color**, **Vector**, **Transaction**, **Point2D**.

Mutable. **StringBuilder**, **Stack**, **Counter**, Java array.

Immutability: properties

Data type. Set of values and operations on those values.

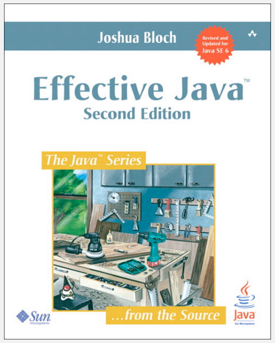
Immutable data type. Can't change the data type value once created.

Advantages.

- Simplifies debugging.
- Safer in presence of hostile code.
- Simplifies concurrent programming.
- Safe to use as key in priority queue or symbol table.

Disadvantage. Must create new object for each data type value.

> *" Classes should be immutable unless there's a very good reason to make them mutable.… If a class cannot be made immutable, you should still limit its mutability as much as possible. "*
>
> *— Joshua Bloch (Java architect)*



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Heapsort

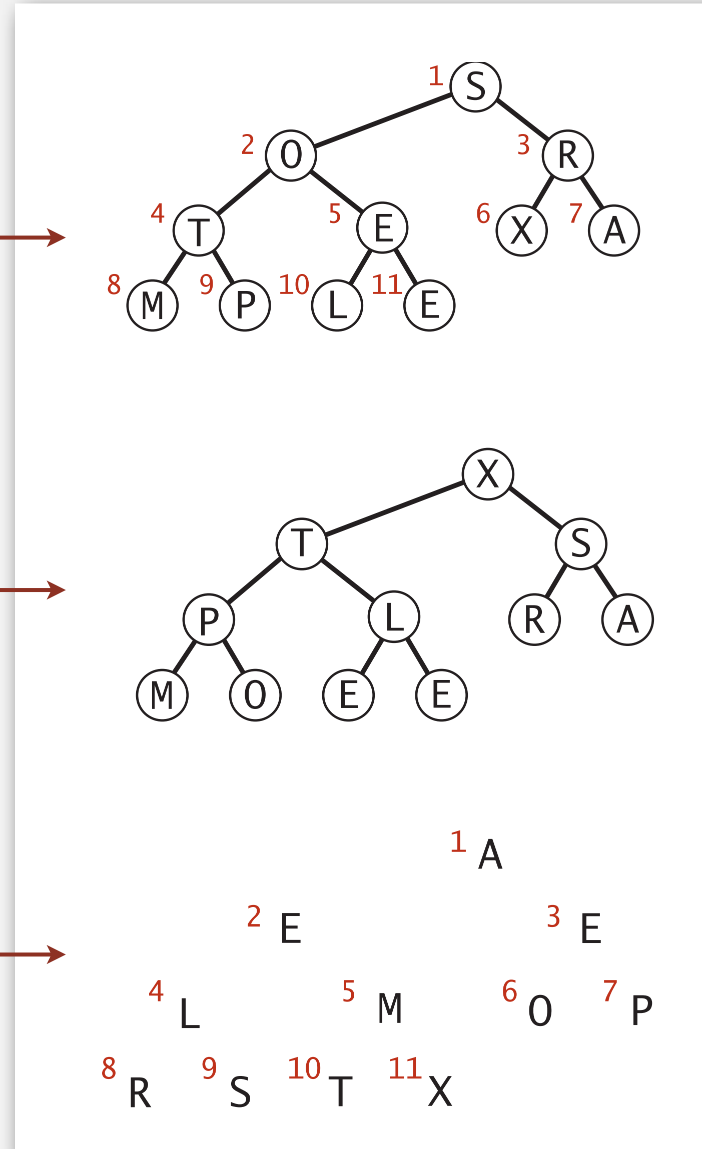
Basic plan for in-place sort.

- Create max-heap with all N keys.
- Repeatedly remove the maximum key.

start with array of keys
in arbitrary order

build a max-heap
(in place)

sorted result
(in place)

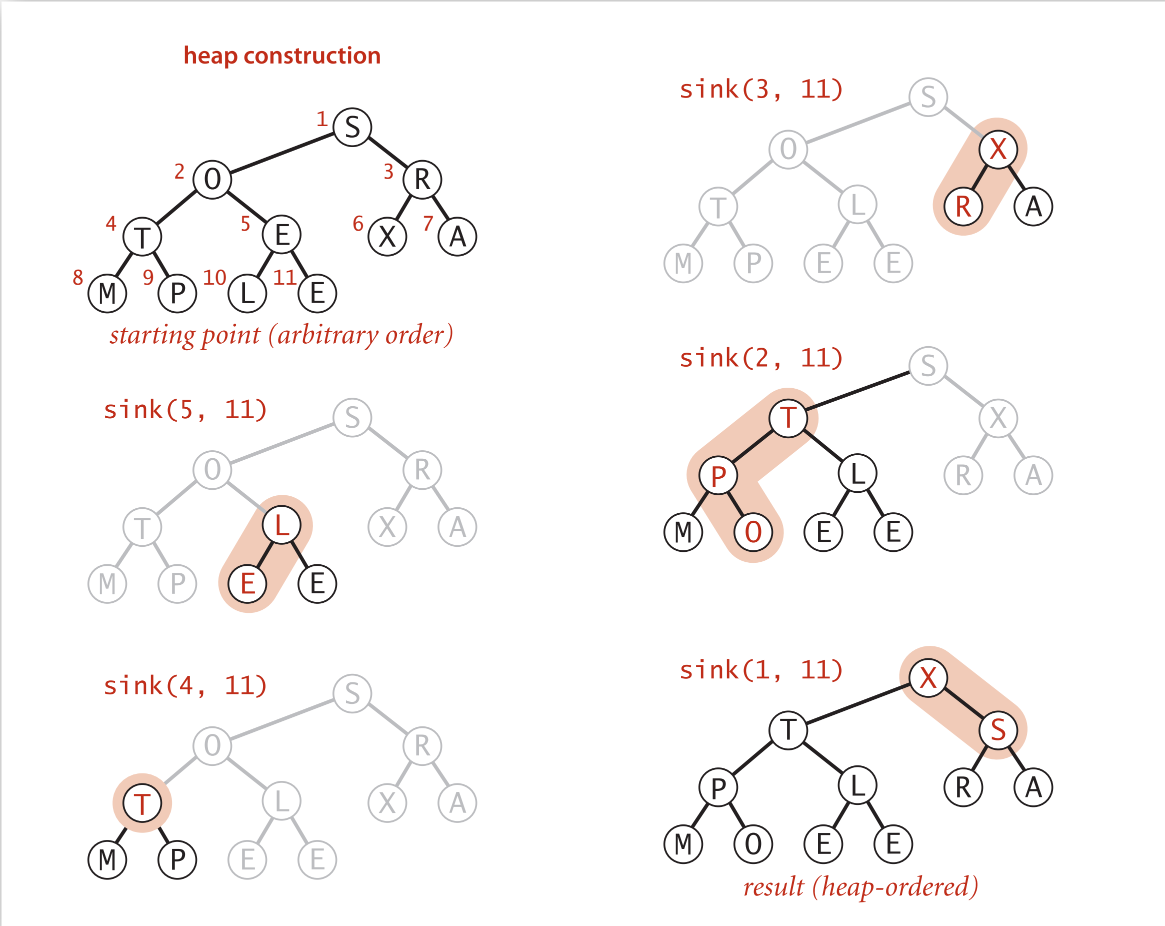


Heapsort demo

Heapsort: heap construction

First pass. Build heap using bottom-up method.

```
for (int k = N/2; k >= 1; k--)
   sink(a, k, N);
```

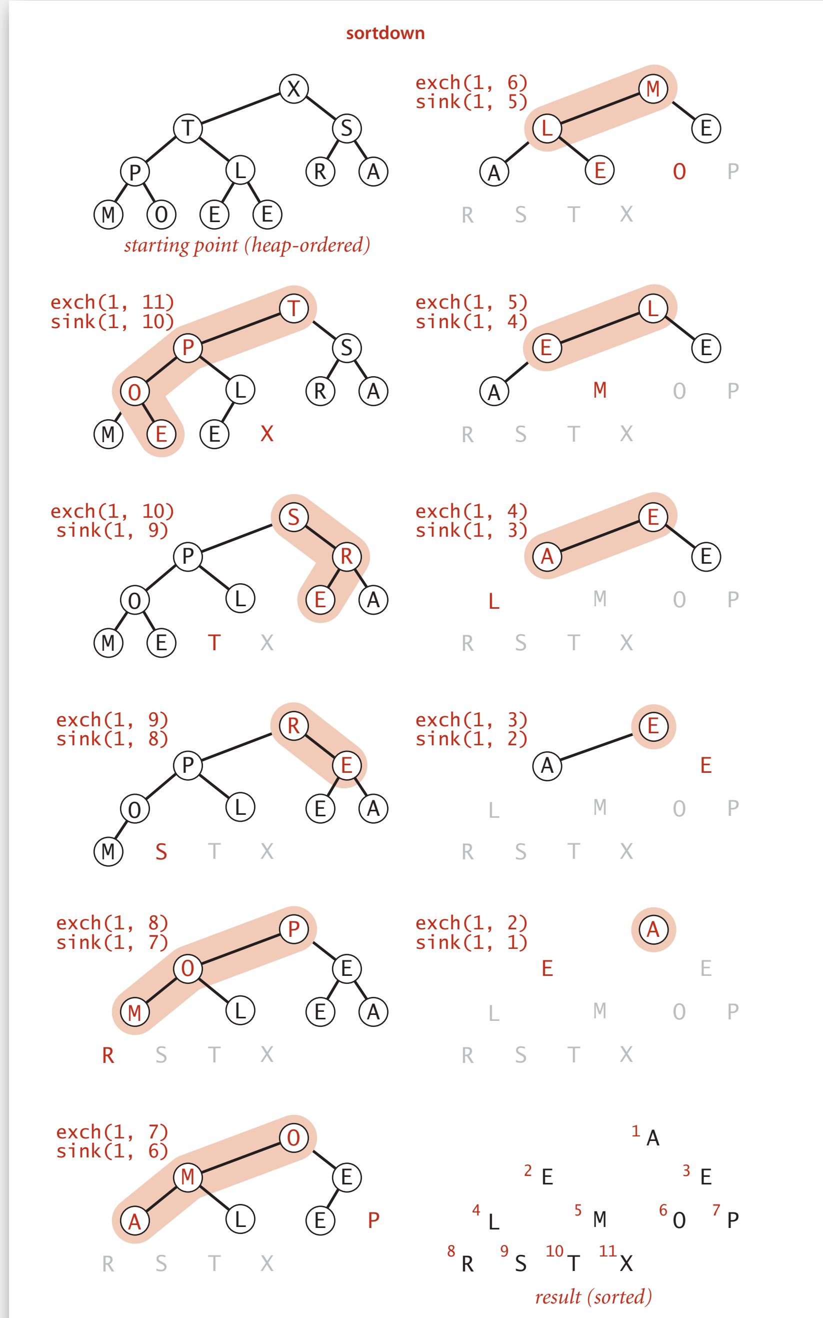


Heapsort: sortdown

Second pass.

- Remove the maximum, one at a time.
- Leave in array, instead of nulling out.

```
while (N > 1)
{
   exch(a, 1, N--);
   sink(a, 1, N);
}
```



Heapsort: Java implementation

```
public class Heap
{
   public static void sort(Comparable[] pq)
   {
      int N = pq.length;
      for (int k = N/2; k >= 1; k--)
         sink(pq, k, N);
      while (N > 1)
      {
         exch(pq, 1, N);
         sink(pq, 1, --N);
      }
   }

   private static void sink(Comparable[] pq, int k, int N)
   {  /* as before */  }

   private static boolean less(Comparable[] pq, int i, int j)
   {  /* as before */  }

   private static void exch(Comparable[] pq, int i, int j)
   {  /* as before */  }

}
```

but convert from
1-based indexing to
0-base indexing

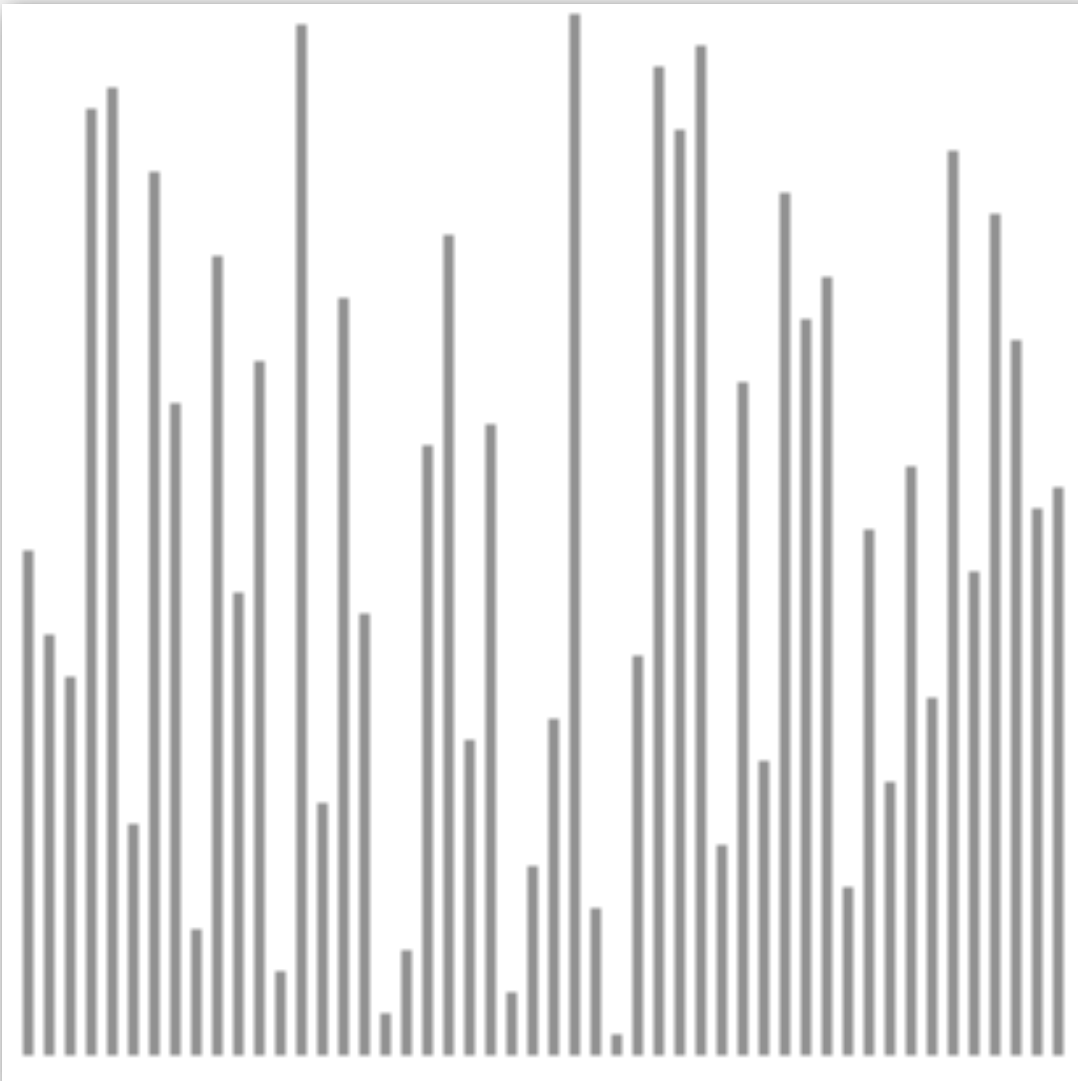
Heapsort: trace

N	k	a[i] 0	1	2	3	4	5	6	7	8	9	10	11
initial values			S	O	R	T	E	X	A	M	P	L	E
11	5		S	O	R	T	L	X	A	M	P	E	E
11	4		S	O	R	T	L	X	A	M	P	E	E
11	3		S	O	X	T	L	R	A	M	P	E	E
11	2		S	T	X	P	L	R	A	M	O	E	E
11	1		X	T	S	P	L	R	A	M	O	E	E
heap-ordered			X	T	S	P	L	R	A	M	O	E	E
10	1		T	P	S	O	L	R	A	M	E	E	X
9	1		S	P	R	O	L	E	A	M	E	T	X
8	1		R	P	E	O	L	E	A	M	S	T	X
7	1		P	O	E	M	L	E	A	R	S	T	X
6	1		O	M	E	A	L	E	P	R	S	T	X
5	1		M	L	E	A	E	O	P	R	S	T	X
4	1		L	E	E	A	M	O	P	R	S	T	X
3	1		E	A	E	L	M	O	P	R	S	T	X
2	1		E	A	E	L	M	O	P	R	S	T	X
1	1		A	E	E	L	M	O	P	R	S	T	X
sorted result			A	E	E	L	M	O	P	R	S	T	X

Heapsort trace (array contents just after each sink)

Heapsort animation

50 random items



http://www.sorting-algorithms.com/heap-sort

algorithm position

in order

not in order

Heapsort: mathematical analysis

Proposition. Heap construction uses fewer than $2N$ compares and exchanges.

Proposition. Heapsort uses at most $2N \lg N$ compares and exchanges.

Significance. In-place sorting algorithm with $N \log N$ worst-case.

- Mergesort: no, linear extra space. ← in-place merge possible, not practical
- Quicksort: no, quadratic time in worst case. ← N log N worst-case quicksort possible, not practical
- Heapsort: yes!

Bottom line. Heapsort is optimal for both time and space, but:

- Inner loop longer than quicksort's.
- Makes poor use of cache memory.
- Not stable.

Sorting algorithms: summary

	inplace?	stable?	worst	average	best	remarks
selection	x		$N^2 / 2$	$N^2 / 2$	$N^2 / 2$	N exchanges
insertion	x	x	$N^2 / 2$	$N^2 / 4$	N	use for small N or partially ordered
shell	x		?	?	N	tight code, subquadratic
quick	x		$N^2 / 2$	$2 N \ln N$	$N \lg N$	N log N probabilistic guarantee fastest in practice
3-way quick	x		$N^2 / 2$	$2 N \ln N$	N	improves quicksort in presence of duplicate keys
merge		x	$N \lg N$	$N \lg N$	$N \lg N$	N log N guarantee, stable
heap	x		$2 N \lg N$	$2 N \lg N$	$N \lg N$	N log N guarantee, in-place
???	x	x	$N \lg N$	$N \lg N$	$N \lg N$	holy sorting grail

- API
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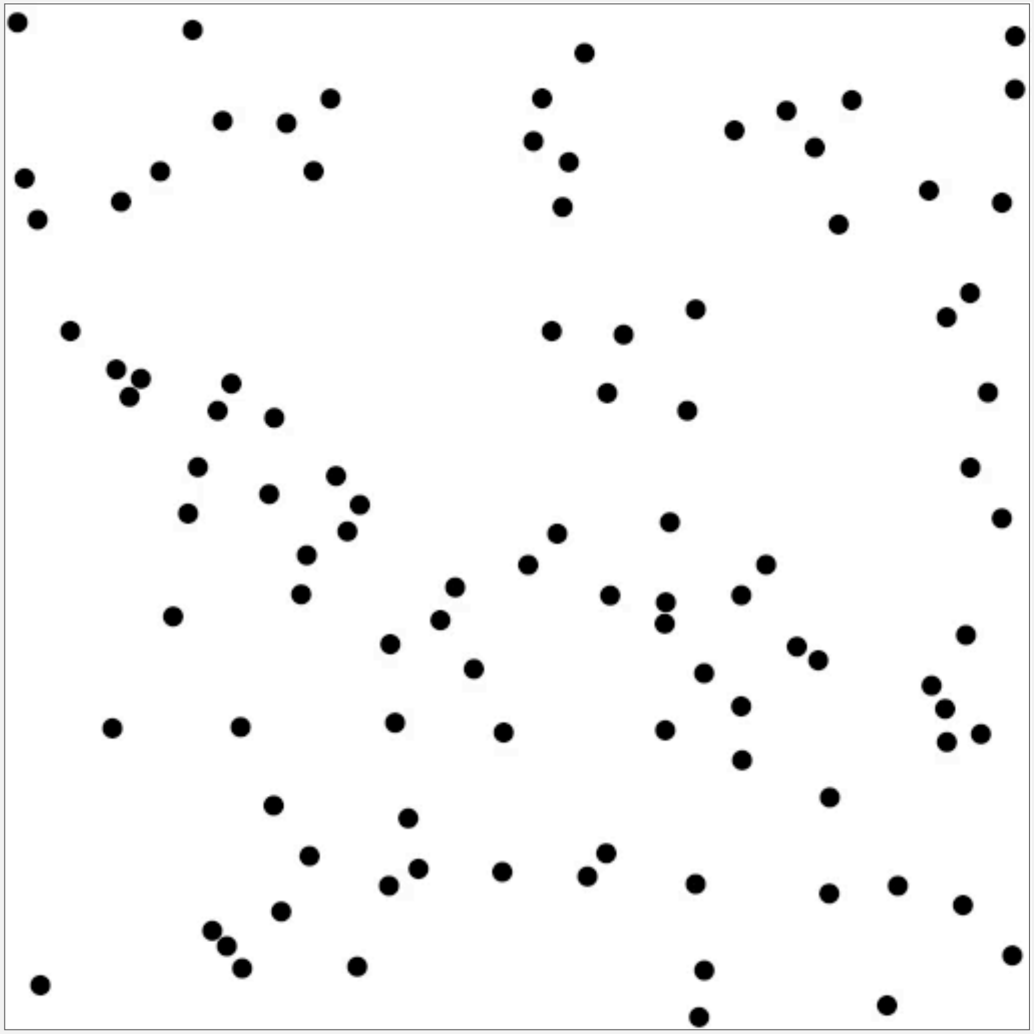
- **event-driven simulation**

Molecular dynamics simulation of hard discs

Goal. Simulate the motion of N moving particles that behave according to the laws of elastic collision.



Molecular dynamics simulation of hard discs

Goal. Simulate the motion of N moving particles that behave according to the laws of elastic collision.

Hard disc model.

- Moving particles interact via elastic collisions with each other and walls.
- Each particle is a disc with known position, velocity, mass, and radius.
- No other forces.

temperature, pressure, diffusion constant

motion of individual atoms and molecules

Significance. Relates macroscopic observables to microscopic dynamics.

- Maxwell-Boltzmann: distribution of speeds as a function of temperature.
- Einstein: explain Brownian motion of pollen grains.

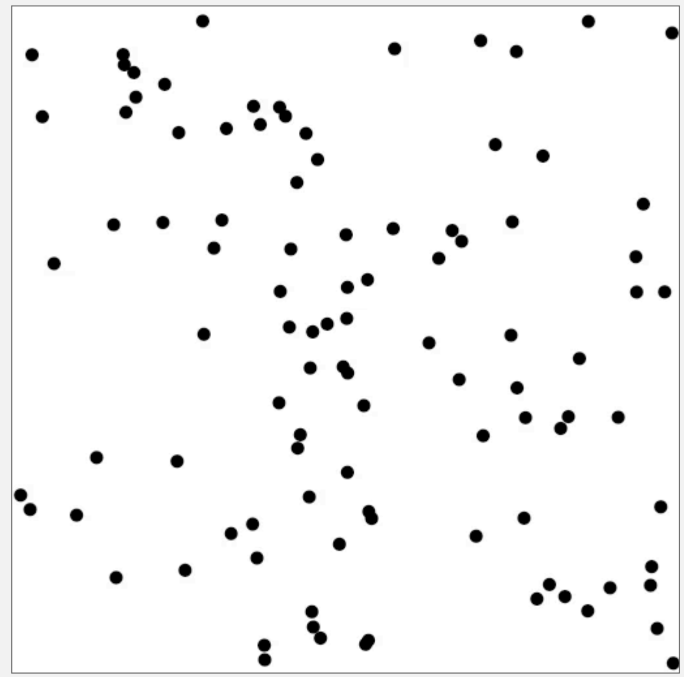
Warmup: bouncing balls

Time-driven simulation. N bouncing balls in the unit square.

```
public class BouncingBalls
{
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      Ball balls[] = new Ball[N];
      for (int i = 0; i < N; i++)
         balls[i] = new Ball();
      while(true)
      {
         StdDraw.clear();
         for (int i = 0; i < N; i++)
         {
            balls[i].move(0.5);
            balls[i].draw();
         }
         StdDraw.show(50);
      }
   }
}
```

main simulation loop

```
% java BouncingBalls 100
```



Warmup: bouncing balls

```
public class Ball
{
    private double rx, ry;         // position
    private double vx, vy;         // velocity
    private final double radius;   // radius
    public Ball()
    {  /* initialize position and velocity */  }

    public void move(double dt)
    {
        if ((rx + vx*dt < radius) || (rx + vx*dt > 1.0 - radius)) { vx = -vx; }
        if ((ry + vy*dt < radius) || (ry + vy*dt > 1.0 - radius)) { vy = -vy; }
        rx = rx + vx*dt;
        ry = ry + vy*dt;
    }
    public void draw()
    {  StdDraw.filledCircle(rx, ry, radius);  }
}
```

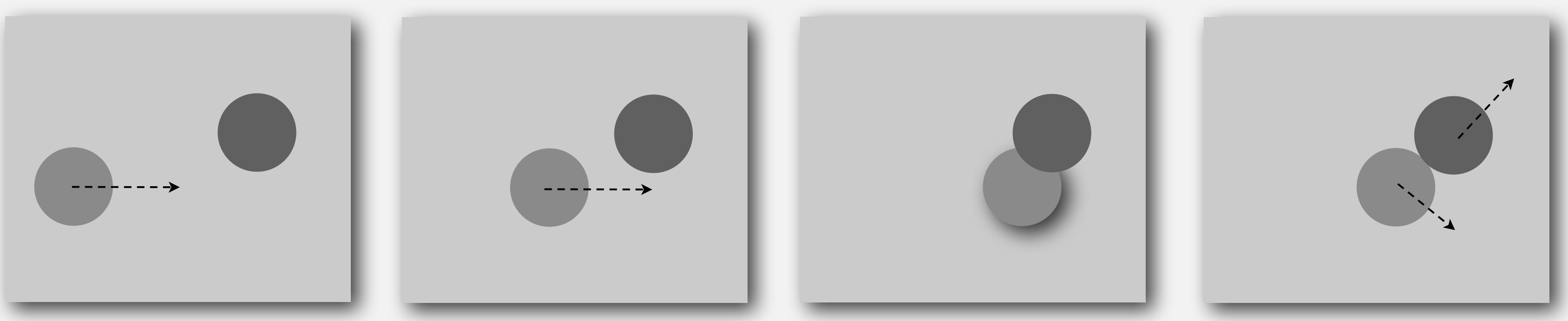
check for collision with walls

Missing. Check for balls colliding with each other.

- Physics problems: when? what effect?
- CS problems: which object does the check? too many checks?

Time-driven simulation

- Discretize time in quanta of size dt.
- Update the position of each particle after every dt units of time, and check for overlaps.
- If overlap, roll back the clock to the time of the collision, update the velocities of the colliding particles, and continue the simulation.



t

t + dt

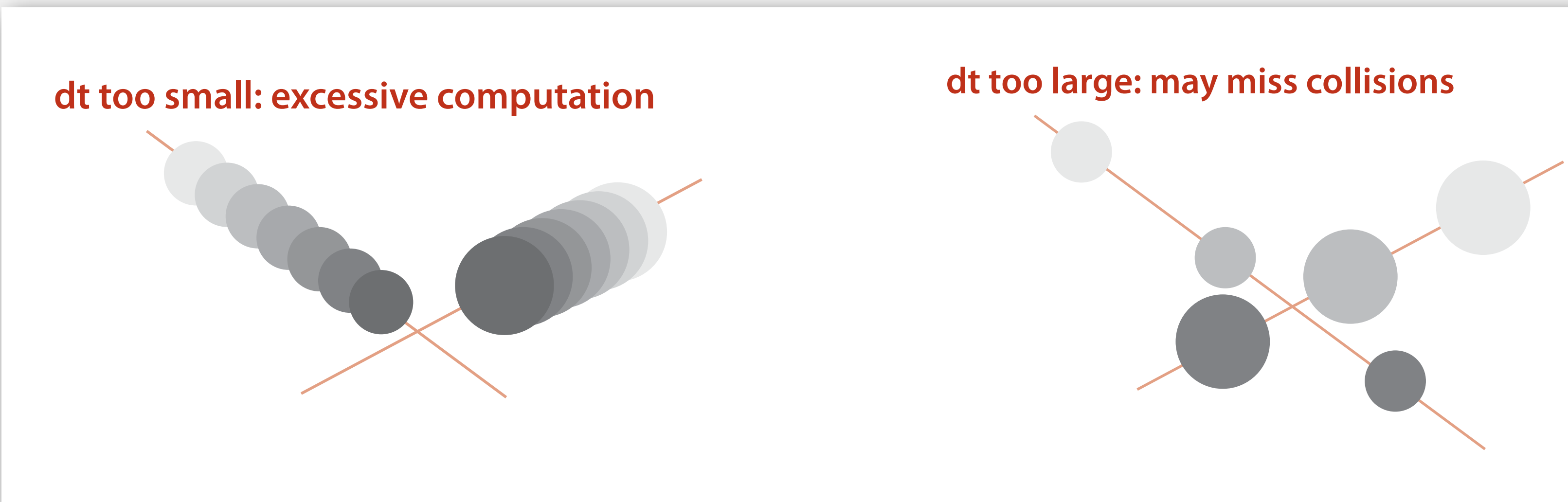
t + 2 dt
(collision detected)

t + Δt
(roll back clock)

Time-driven simulation

Main drawbacks.

- $\sim N^2 / 2$ overlap checks per time quantum.
- Simulation is too slow if dt is very small.
- May miss collisions if dt is too large.
  (if colliding particles fail to overlap when we are looking)



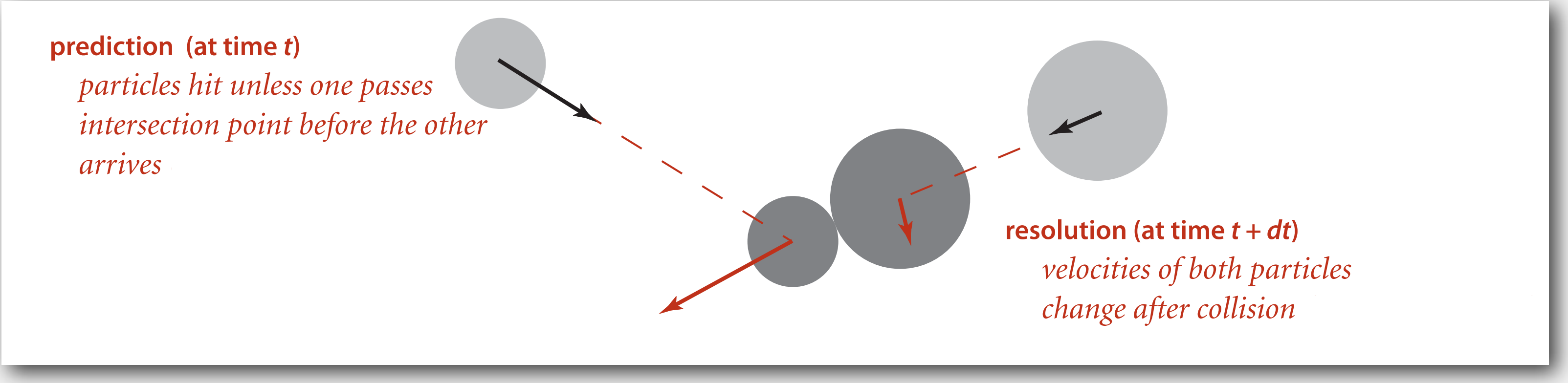
Event-driven simulation

Change state only when something happens.

- Between collisions, particles move in straight-line trajectories.
- Focus only on times when collisions occur.
- Maintain PQ of collision events, prioritized by time.
- Remove the min = get next collision.

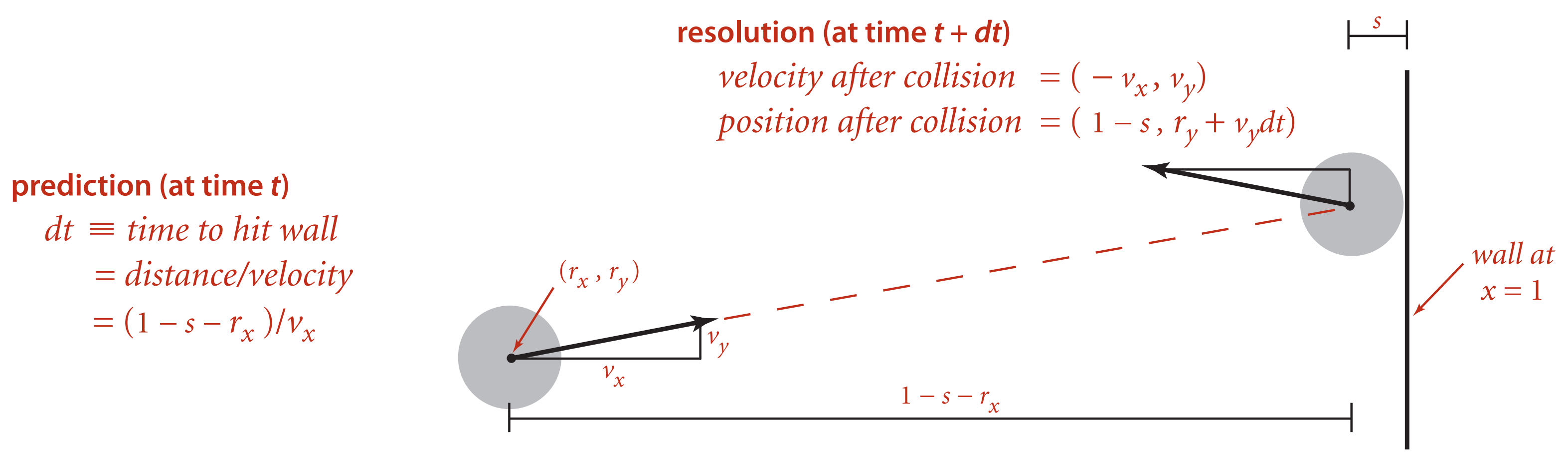
Collision prediction. Given position, velocity, and radius of a particle, when will it collide next with a wall or another particle?

Collision resolution. If collision occurs, update colliding particle(s) according to laws of elastic collisions.



Collision prediction and resolution.

- Particle of radius s at position (rx, ry).
- Particle moving in unit box with velocity (vx, vy).
- Will it collide with a vertical wall? If so, when?

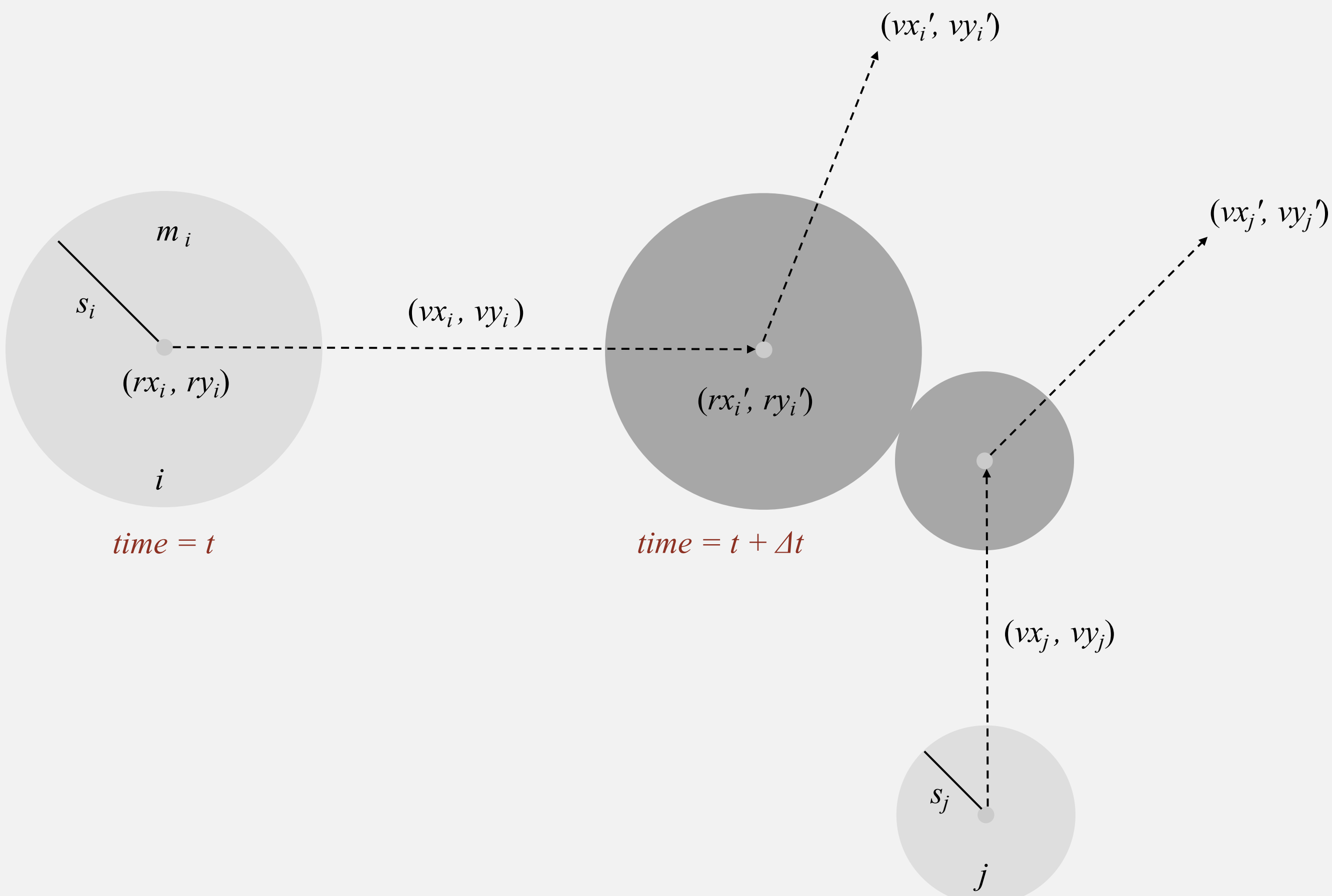


Predicting and resolving a particle-wall collision

Particle-particle collision prediction

Collision prediction.

- Particle i: radius s_i, position (rx_i, ry_i), velocity (vx_i, vy_i).
- Particle j: radius s_j, position (rx_j, ry_j), velocity (vx_j, vy_j).
- Will particles i and j collide? If so, when?



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$$\Delta t = \begin{cases} \infty & \text{if } \Delta v \cdot \Delta r \geq 0 \\ \infty & \text{if } d < 0 \\ -\dfrac{\Delta v \cdot \Delta r + \sqrt{d}}{\Delta v \cdot \Delta v} & \text{otherwise} \end{cases}$$

$$d = (\Delta v \cdot \Delta r)^2 - (\Delta v \cdot \Delta v)\,(\Delta r \cdot \Delta r - \sigma^2) \qquad \sigma = \sigma_i + \sigma_j$$

$$\Delta v = (\Delta vx,\ \Delta vy) = (vx_i - vx_j,\ vy_i - vy_j)$$
$$\Delta r = (\Delta rx,\ \Delta ry) = (rx_i - rx_j,\ ry_i - ry_j)$$

$$\Delta v \cdot \Delta v = (\Delta vx)^2 + (\Delta vy)^2$$
$$\Delta r \cdot \Delta r = (\Delta rx)^2 + (\Delta ry)^2$$
$$\Delta v \cdot \Delta r = (\Delta vx)(\Delta rx) + (\Delta vy)(\Delta ry)$$

Important note: This is high-school physics, so we won't be testing you on it!

Particle-particle collision resolution

Collision resolution. When two particles collide, how does velocity change?

$$
\begin{aligned}
vx_i' &= vx_i + Jx / m_i \\
vy_i' &= vy_i + Jy / m_i \\
vx_j' &= vx_j - Jx / m_j \\
vy_j' &= vy_j - Jy / m_j
\end{aligned}
$$

Newton's second law
(momentum form)

$$Jx = \frac{J\,\Delta rx}{\sigma}, \quad Jy = \frac{J\,\Delta ry}{\sigma}, \quad J = \frac{2\,m_i\,m_j\,(\Delta v \cdot \Delta r)}{\sigma(m_i + m_j)}$$

impulse due to normal force
(conservation of energy, conservation of momentum)

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Particle data type skeleton

```
public class Particle
{
    private double rx, ry;         // position
    private double vx, vy;         // velocity
    private final double radius;   // radius
    private final double mass;     // mass
    private int count;             // number of collisions

    public Particle(...) { }

    public void move(double dt) { }
    public void draw()          { }

    public double timeToHit(Particle that)   { }
    public double timeToHitVerticalWall()    { }
    public double timeToHitHorizontalWall()  { }

    public void bounceOff(Particle that)     { }
    public void bounceOffVerticalWall()      { }
    public void bounceOffHorizontalWall()    { }

}
```

predict collision with particle or wall

resolve collision with particle or wall

Particle-particle collision and resolution implementation

```
public double timeToHit(Particle that)
{
   if (this == that) return INFINITY;
   double dx  = that.rx - this.rx, dy  = that.ry - this.ry;
   double dvx = that.vx - this.vx; dvy = that.vy - this.vy;
   double dvdr = dx*dvx + dy*dvy;
   if( dvdr > 0) return INFINITY;
   double dvdv = dvx*dvx + dvy*dvy;
   double drdr = dx*dx + dy*dy;
   double sigma = this.radius + that.radius;
   double d = (dvdr*dvdr) - dvdv * (drdr - sigma*sigma);
   if (d < 0) return INFINITY;
   return -(dvdr + Math.sqrt(d)) / dvdv;
}
```

no collision

```
public void bounceOff(Particle that)
{
   double dx  = that.rx - this.rx, dy  = that.ry - this.ry;
   double dvx = that.vx - this.vx, dvy = that.vy - this.vy;
   double dvdr = dx*dvx + dy*dvy;
   double dist = this.radius + that.radius;
   double J = 2 * this.mass * that.mass * dvdr / ((this.mass + that.mass) * dist);
   double Jx = J * dx / dist;
   double Jy = J * dy / dist;
   this.vx += Jx / this.mass;
   this.vy += Jy / this.mass;
   that.vx -= Jx / that.mass;
   that.vy -= Jy / that.mass;
   this.count++;
   that.count++;
}
```

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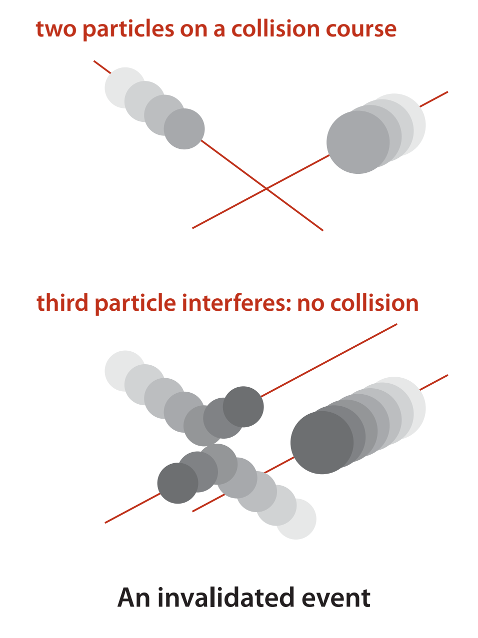
Collision system: event-driven simulation main loop



An invalidated event

Initialization.

- Fill PQ with all potential particle-wall collisions.
- Fill PQ with all potential particle-particle collisions.

"potential" since collision may not happen if some other collision intervenes

Main loop.

- Delete the impending event from PQ (min priority = t).
- If the event has been invalidated, ignore it.
- Advance all particles to time t, on a straight-line trajectory.
- Update the velocities of the colliding particle(s).
- Predict future particle-wall and particle-particle collisions involving the colliding particle(s) and insert events onto PQ.

Event data type

Conventions.

- Neither particle `null` $\Rightarrow$ particle-particle collision.
- One particle `null` $\Rightarrow$ particle-wall collision.
- Both particles `null` $\Rightarrow$ redraw event.

```
private class Event implements Comparable<Event>
{
    private double time;         // time of event
    private Particle a, b;       // particles involved in event
    private int countA, countB;  // collision counts for a and b

    public Event(double t, Particle a, Particle b) { }

    public int compareTo(Event that)
    {   return this.time - that.time;   }

    public boolean isValid()
    {   }
}
```

create event

ordered by time

invalid if intervening collision

Collision system implementation: skeleton

```
public class CollisionSystem
{
    private MinPQ<Event> pq;          // the priority queue
    private double t  = 0.0;          // simulation clock time
    private Particle[] particles;     // the array of particles

    public CollisionSystem(Particle[] particles) { }

    private void predict(Particle a)
    {
       if (a == null) return;
       for (int i = 0; i < N; i++)
       {
          double dt = a.timeToHit(particles[i]);
          pq.insert(new Event(t + dt, a, particles[i]));
       }
       pq.insert(new Event(t + a.timeToHitVerticalWall()  , a, null));
       pq.insert(new Event(t + a.timeToHitHorizontalWall(), null, a));
 }

    private void redraw()  { }

    public void simulate() {  /* see next slide */  }
}
```

add to PQ all particle-wall and particle-particle collisions involving this particle

Collision system implementation: main event-driven simulation loop

```
public void simulate()
{
   pq = new MinPQ<Event>();
   for(int i = 0; i < N; i++) predict(particles[i]);
   pq.insert(new Event(0, null, null));

   while(!pq.isEmpty())
   {
      Event event = pq.delMin();
      if(!event.isValid()) continue;
      Particle a = event.a;
      Particle b = event.b;

      for(int i = 0; i < N; i++)
         particles[i].move(event.time - t);
      t = event.time;

      if      (a != null && b != null) a.bounceOff(b);
      else if (a != null && b == null) a.bounceOffVerticalWall()
      else if (a == null && b != null) b.bounceOffHorizontalWall();
      else if (a == null && b == null) redraw();

      predict(a);
      predict(b);
   }
}
```

initialize PQ with collision events and redraw event

get next event

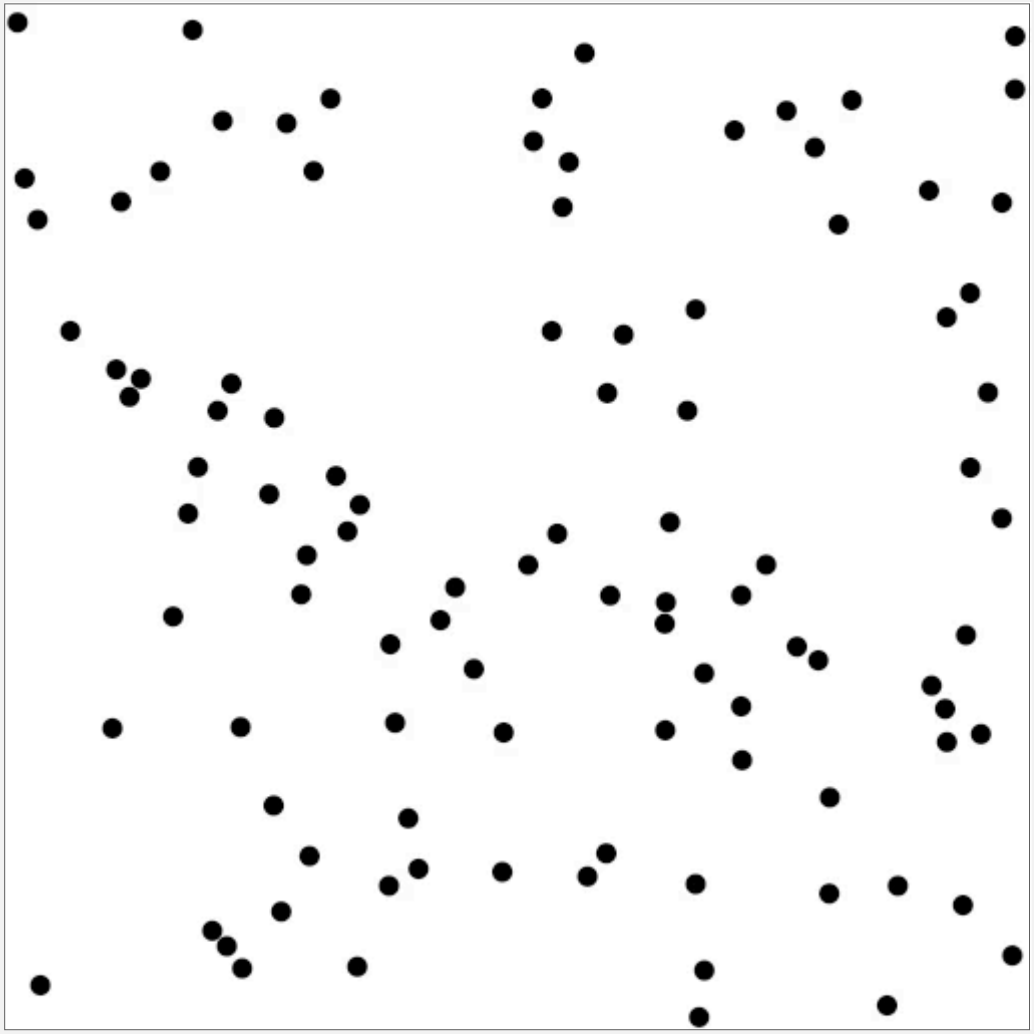
update positions and time

process event

predict new events based on changes

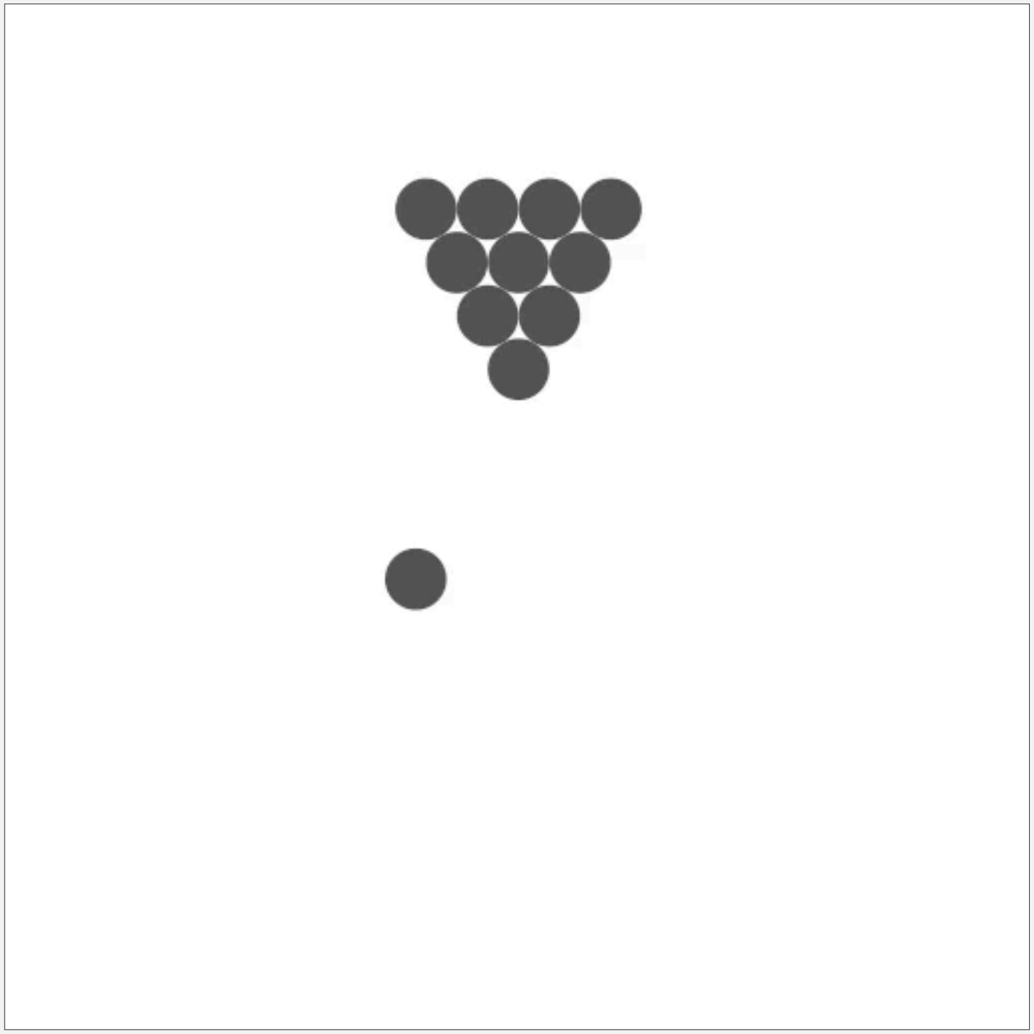
Particle collision simulation example 1

```
% java CollisionSystem 100
```



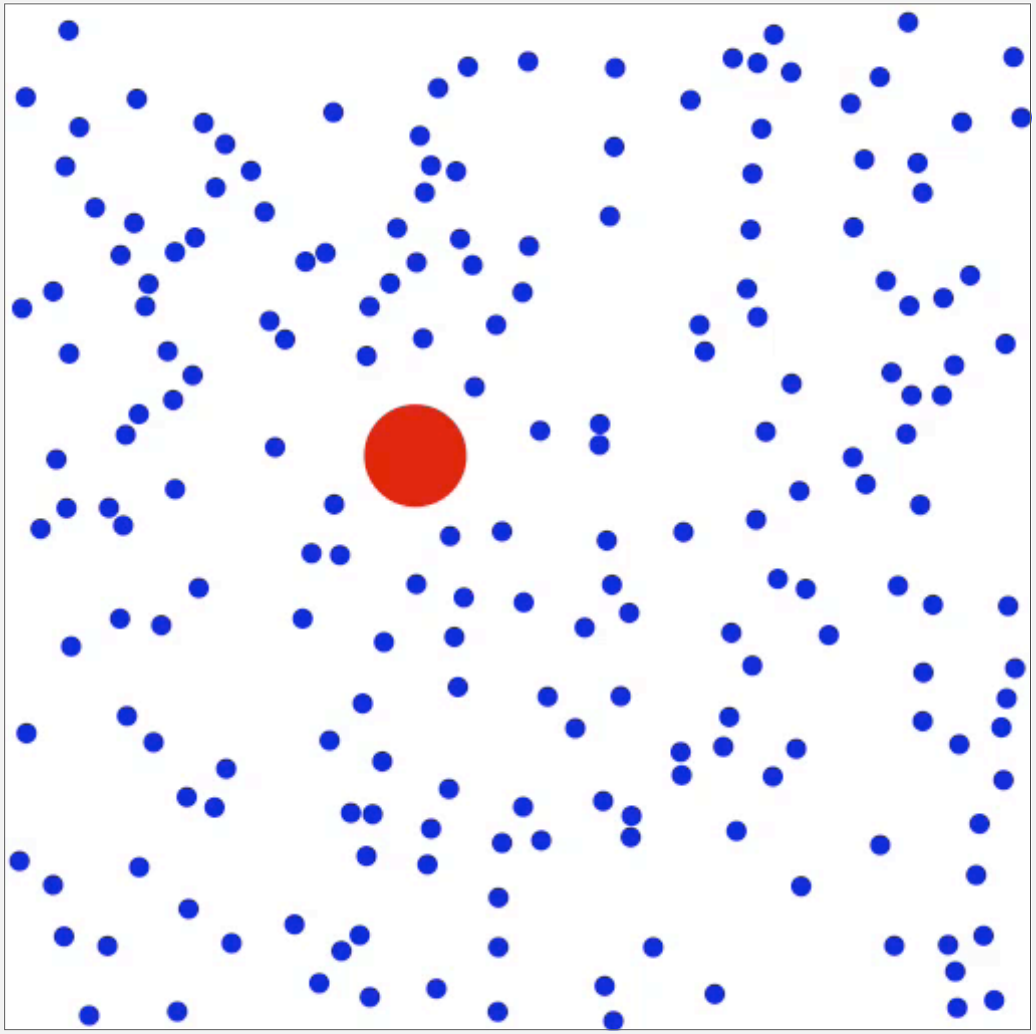
Particle collision simulation example 2

`% java CollisionSystem < billiards.txt`



Particle collision simulation example 3

```
% java CollisionSystem < brownian.txt
```



Particle collision simulation example 4

```
% java CollisionSystem < diffusion.txt
```