# 3.2 Creating Data Types

# Data Types

Data type. Set of values and operations on those values.

Basic types.

| Data Type | Set of Values | Some Operations |
|---|---|---|
| `boolean` | `true, false` | not, and, or, xor |
| `int` | $-2^{31}$ to $2^{31} - 1$ | add, subtract, multiply |
| `String` | sequence of Unicode characters | concatenate, compare |

Last time. Write programs that use data types.

Today. Write programs to create our own data types.

# Defining Data Types in Java

To define a data type, define:

- Set of values.
- Operations defined on them.

Java class. Allows us to define data types by specifying:

- Instance variables. (set of values)
- Methods. (operations defined on them)
- Constructors. (create and initialize new objects)

# Point Charge Data Type

**Goal.** Create a data type to manipulate point charges.

**Set of values.** Three real numbers. [position and electrical charge]

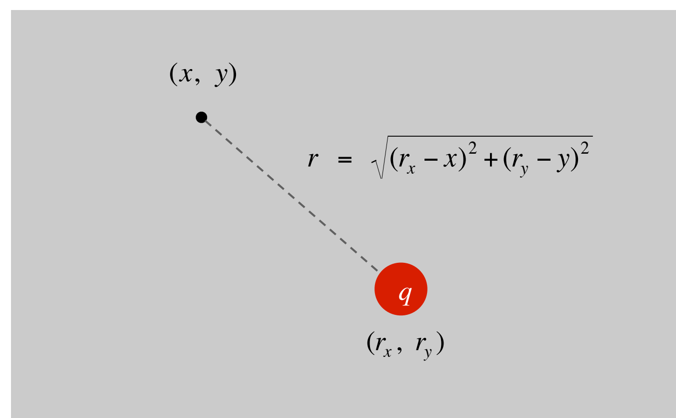
**Operations.**

- Create a new point charge at $(r_x, r_y)$ with electric charge $q$.
- Determine electric potential $V$ at $(x, y)$ due to point charge.
- Convert to `String`.

$$V = k \frac{q}{r}$$

$r$ = distance between $(x, y)$ and $(r_x, r_y)$

$k$ = electrostatic constant = $8.99 \times 10^{9}\ \mathrm{N \cdot m^2 / C^2}$



# Point Charge Data Type

Goal. Create a data type to manipulate point charges.

Set of values. Three real numbers. [position and electrical charge]

API:

| public class Charge | | |
|---|---|---|
| | Charge(double x0, double y0, double q0) | |
| double | potentialAt(double x, double y) | *electric potential at (x, y) due to charge* |
| String | toString() | *string representation* |

# Charge Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args)
{
   double x = Double.parseDouble(args[0]);
   double y = Double.parseDouble(args[1]);
   Charge c1 = new Charge(.51, .63, 21.3);
   Charge c2 = new Charge(.13, .94, 81.9);
   double v1 = c1.potentialAt(x, y);
   double v2 = c2.potentialAt(x, y);
   StdOut.println(c1);
   StdOut.println(c2);
   StdOut.println(v1 + v2);
}
```

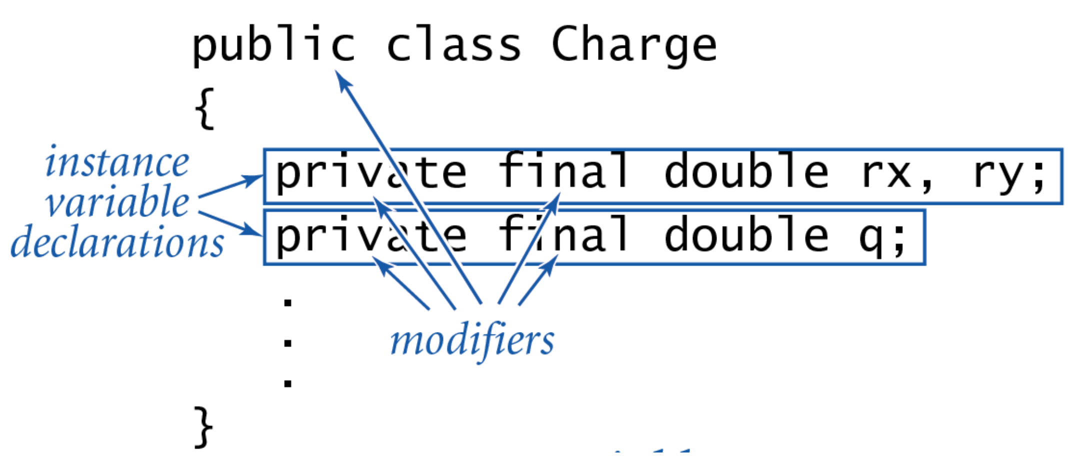
automagically invokes the `toString()` method

```
% java Charge .50 .50
21.3 at (0.51, 0.63)
81.9 at (0.13, 0.94)
2.74936907085912e12
```

# Anatomy of Instance Variables

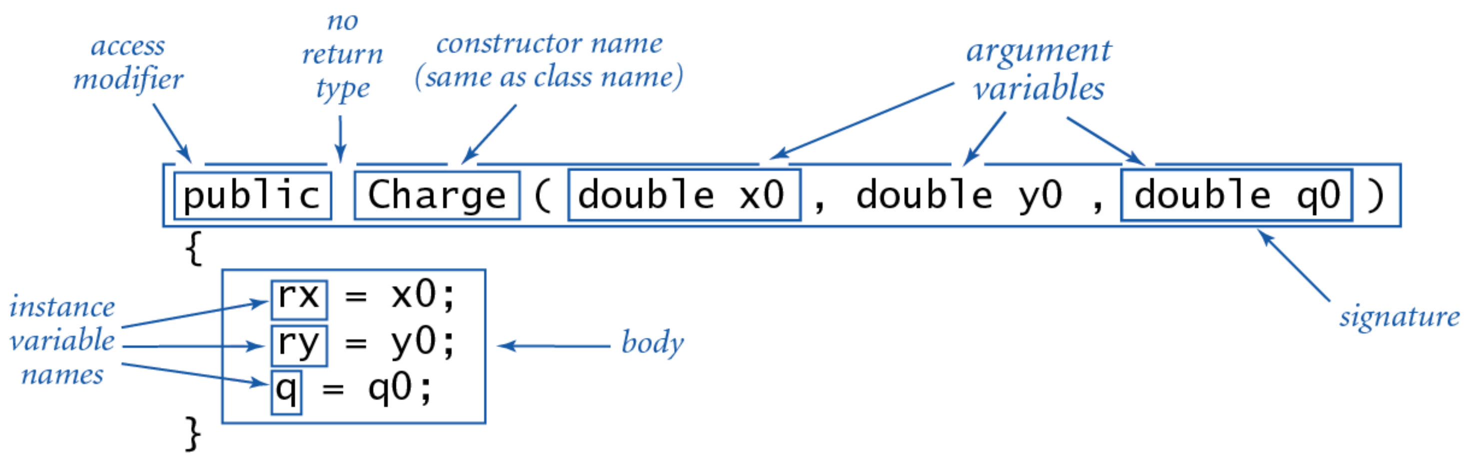
Instance variables. Specifies the set of values.

- Declare outside any method.
- Always use access modifier **`private`**. ← makes data type abstract
- Use modifier **`final`** with instance variables that never change. ← makes objects immutable (stay tuned)



# Anatomy of a Constructor

**Constructor.** Specifies what happens when you create a new object.



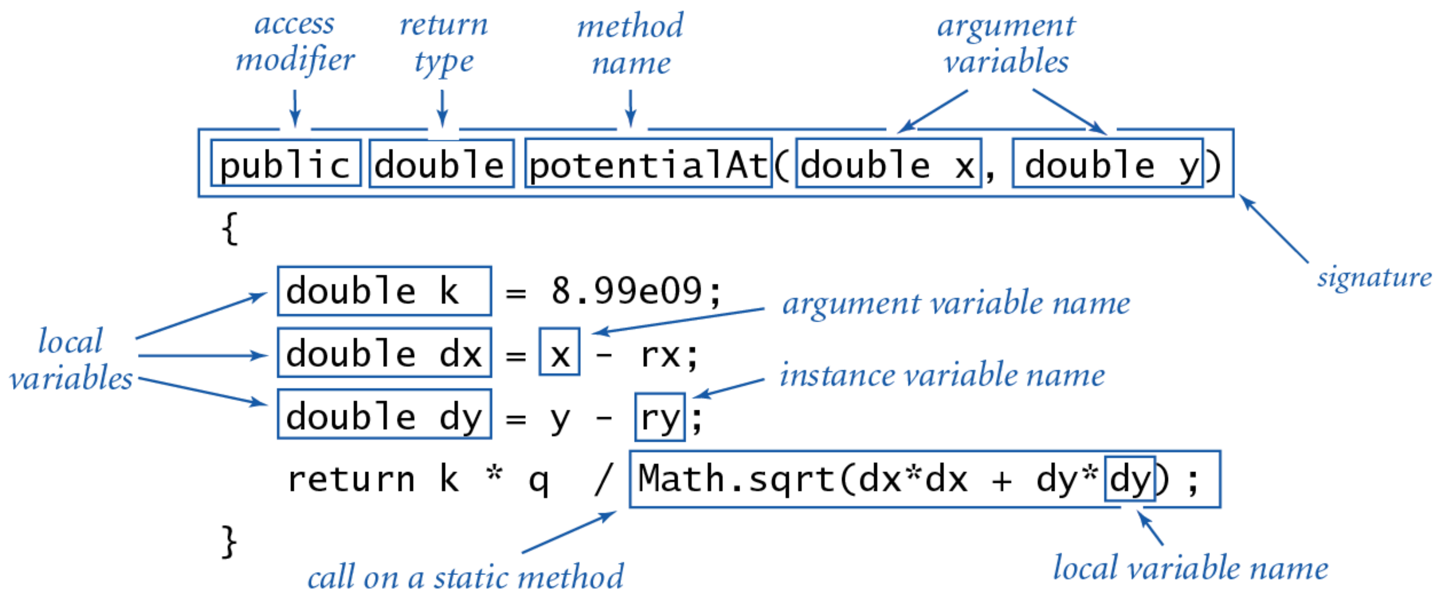

**Invoking a constructor.** Use `new` operator to create a new object.

```
Charge c1 = new Charge(.51, .63, 21.3);
Charge c2 = new Charge(.13, .94, 81.9);
```

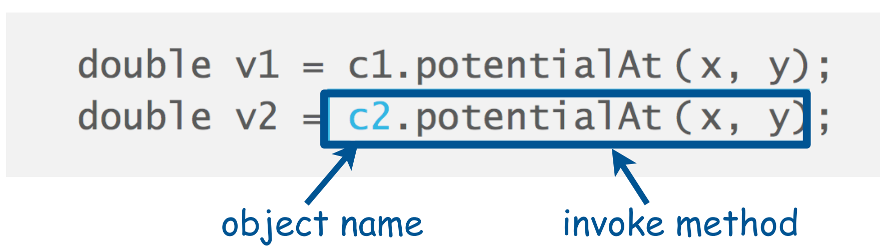
invoke constructor

# Anatomy of a Data Type Method

**Method.** Define operations on instance variables.




**Invoking a method.** Use dot operator to invoke a method in client code.



# Anatomy of a Class

```
public class Charge
{
   private final double rx, ry;
   private final double q;

   public Charge(double x0, double y0, double q0)
   {  rx = x0; ry = y0; q = q0;  }

   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }

   public String toString()
   {  return q +" at " + "("+ rx + ", " + ry +")";  }

   public static void main(String[] args)
   {
      double x = Double.parseDouble(args[0]);
      double y = Double.parseDouble(args[1]);
      Charge c1 = new Charge(.51, .63, 21.3);
      Charge c2 = new Charge(.13, .94, 81.9);
      double v1 = c1.potentialAt(x, y);
      double v2 = c2.potentialAt(x, y);
      StdOut.printf("%.1e\n", (v1 + v2));
   }
}
```

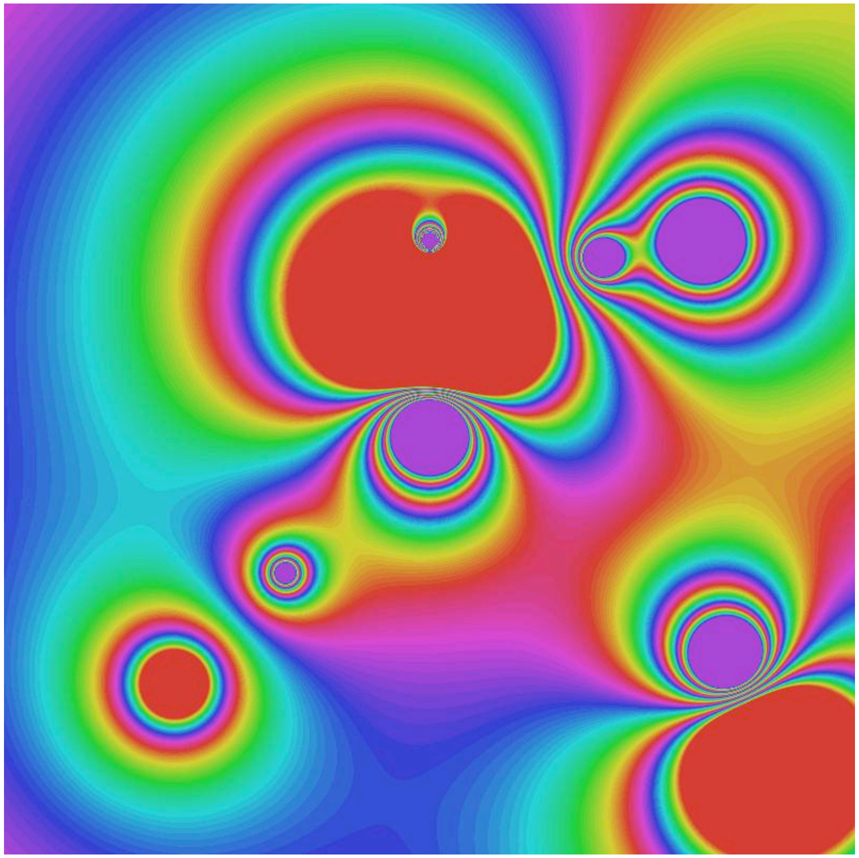
class name
instance variables
constructor
instance methods
instance variable names
test client
create and initialize object
invoke constructor
object name
invoke method

# Charge Client Example: Potential Visualization

Potential visualization. Read in N point charges from a file; compute total potential at each point in unit square.

```
% more charges.txt
9
.51 .63 -100
.50 .50   40
.50 .72   10
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50
```

```
% java Potential < charges.txt
```



# Potential Visualization

Arrays of objects. Allocate memory for the array; then allocate memory for each individual object.

```
// Read in the data.
int N = StdIn.readInt();
Charge[] a = new Charge[N];
for (int i = 0; i < N; i++)
{
   double x0 = StdIn.readDouble();
   double y0 = StdIn.readDouble();
   double q0 = StdIn.readDouble();
   a[i] = new Charge(x0, y0, q0);
}
```

# Potential Visualization

```
// Plot the data.
int SIZE = 512;
Picture pic = new Picture(SIZE, SIZE);
for (int col = 0; col < SIZE; col++)
   for (int row = 0; row < SIZE; row++)
   {
      double V = 0.0;
      for (int i = 0; i < N; i++)
      {
         double x = 1.0 * col / SIZE;
         double y = 1.0 * row / SIZE;
         V += a[i].potentialAt(x, y);
      }
      Color color = getColor(V);  // Arbitrary double-Color map.
      pic.set(col, SIZE-1-row, color);
   }
pic.show();
```

$$V = \sum_i \left( k\, q_i \,/\, r_i \right)$$

(0, 0) is upper left

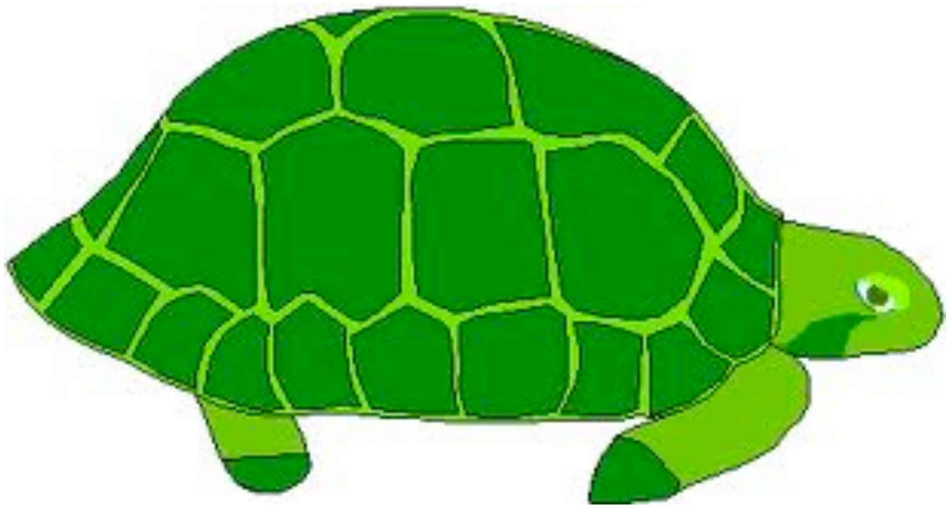
# Data Type Challenge

[easy if you read Exercise 3.2.5]

Fix the serious bug in the following code.

```
public class Charge
{
   private double rx, ry;
   private double q;
   public Charge (double x0, double y0, double q0)
   {
      double rx = x0;
      double ry = y0;
      double q = q0;
   }
}
```

# Turtle Graphics



# Turtle Graphics

Goal. Create a data type to manipulate a turtle moving in the plane.
Set of values. Location and orientation of turtle.

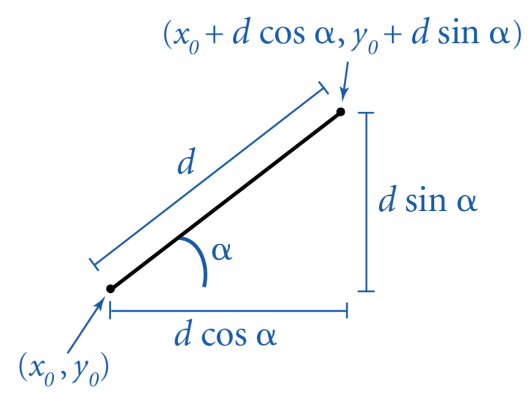
API. `public class Turtle`

| | | |
|---|---|---|
| | `Turtle(double x0, double y0, double a0)` | *create a new turtle at* $(x_0, y_0)$ *facing* $a_0$ *degrees counterclockwise from the x-axis* |
| `void` | `turnLeft(double delta)` | *rotate* `delta` *degrees counterclockwise* |
| `void` | `goForward(double step)` | *move distance* `step`*, drawing a line* |

```
// Draw a square.
Turtle turtle = new Turtle(0.0, 0.0, 0.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
```

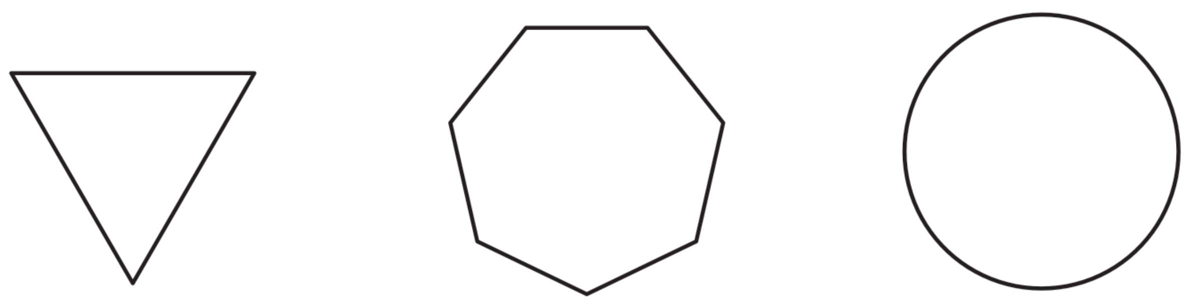
# Turtle Graphics Implementation

```
public class Turtle
{
   private double x, y;   // turtle is at (x, y)
   private double angle;  // facing this direction

   public Turtle(double x0, double y0, double a0)
   {
      x = x0;
      y = y0;
      angle = a0;
   }

   public void turnLeft(double delta)
   {
      angle += delta;
   }

   public void goForward(double d)
   {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }
}
```



# Turtle client example: N-gon

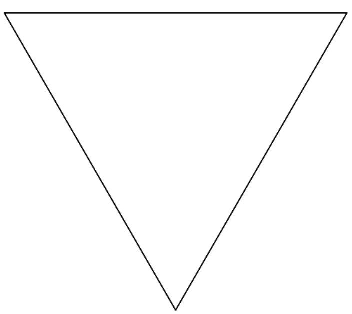
```
public class Ngon
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < N; i++)
      {
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```



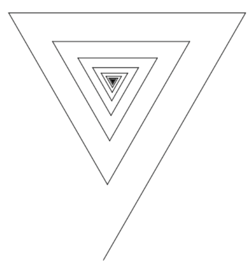
3 7 1440

# Turtle client example: Spira Mirabilis

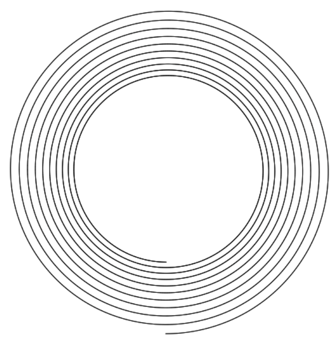
```
public class Spiral
{
   public static void main(String[] args)
   {
      int N          = Integer.parseInt(args[0]);
      double decay = Double.parseDouble(args[1]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < 10 * N; i++)
      {
         step /= decay;
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```



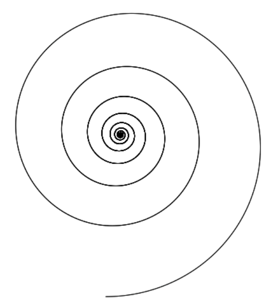
3 1.0



3 1.2

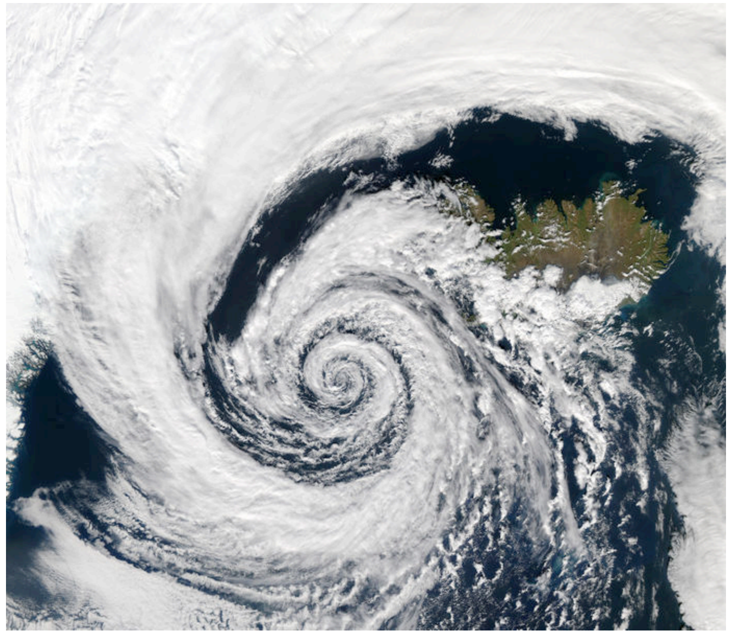


1440 1.00004

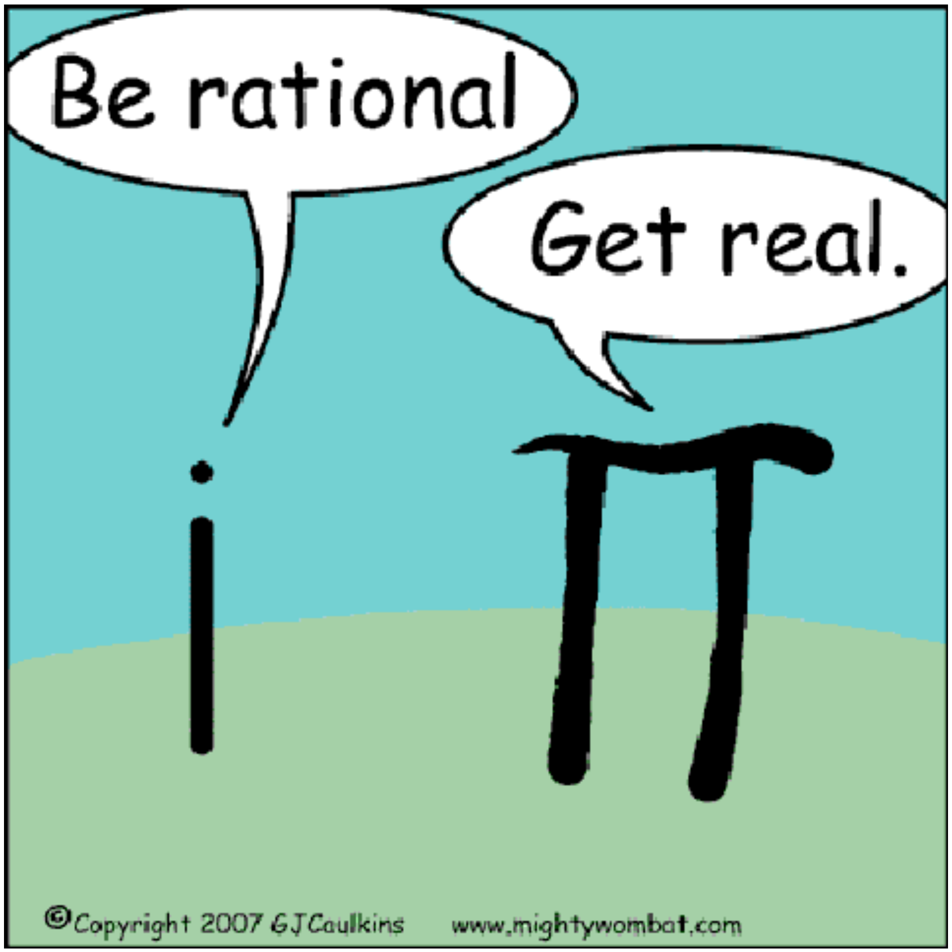


1440 1.0004

# Spira Mirabilis in Nature



# Complex Numbers



# Complex Number Data Type

Goal. Create a data type to manipulate complex numbers.

Set of values. Two real numbers: real and imaginary parts.

API.

| public class Complex | | |
|---|---|---|
| | `Complex(double real, double imag)` | |
| `Complex` | `plus(Complex b)` | *sum of this number and* `b` |
| `Complex` | `times(Complex b)` | *product of this number and* `b` |
| `double` | `abs()` | *magnitude* |
| `String` | `toString()` | *string representation* |

$a = 3 + 4i, \;\; b = -2 + 3i$

$a + b = 1 + 7i$

$a \times b = -18 + i$

$| a | = 5$

# Applications of Complex Numbers

Relevance. A quintessential mathematical abstraction.

Applications.

- Fractals.
- Impedance in RLC circuits.
- Signal processing and Fourier analysis.
- Control theory and Laplace transforms.
- Quantum mechanics and Hilbert spaces.
- …

# Complex Number Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args)
{
   Complex a = new Complex( 3.0, 4.0);
   Complex b = new Complex(-2.0, 3.0);
   Complex c = a.times(b);
   StdOut.println("a = " + a);
   StdOut.println("b = " + b);
   StdOut.println("c = " + c);
}
```

result of `c.toString()`

```
% java TestClient
a = 3.0 + 4.0i
b = -2.0 + 3.0i
c = -18.0 + 1.0i
```

Remark. Can't write `a = b*c` since no operator overloading in Java.

# Complex Number Data Type: Implementation

```java
public class Complex
{

   private final double re;
   private final double im;

   public Complex(double real, double imag)
   {
      re = real;
      im = imag;
   }

   public String toString()
   {  return re + " + " + im + "i"; }

   public double abs()
   {  return Math.sqrt(re*re + im*im); }

   public Complex plus(Complex b)
   {
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);
   }

   public Complex times(Complex b)
   {
      double real = re * b.re – im * b.im;
      double imag = re * b.im + im * b.re;
      return new Complex(real, imag);
   }

}
```

instance variables

constructor

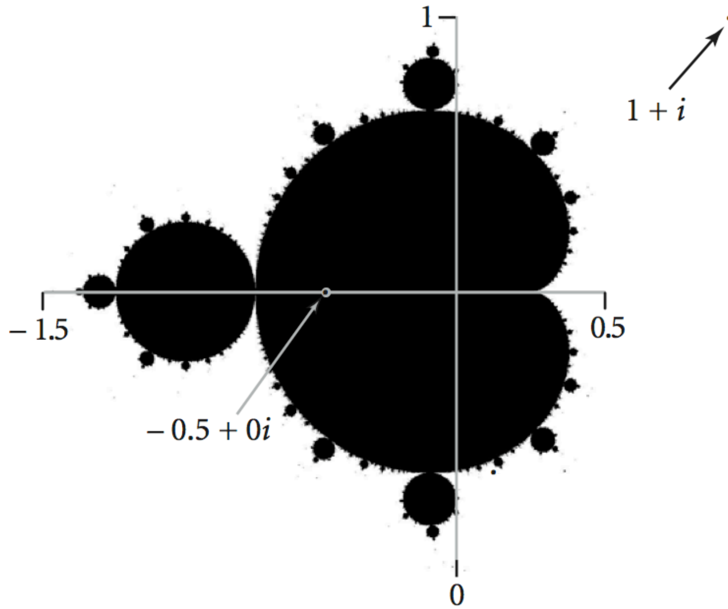
methods

refers to b's instance variables

# Mandelbrot Set

Mandelbrot set. A particular set of complex numbers.

Plot. Plot $(x, y)$ black if $z = x + y\,i$ is in the set, and white otherwise.

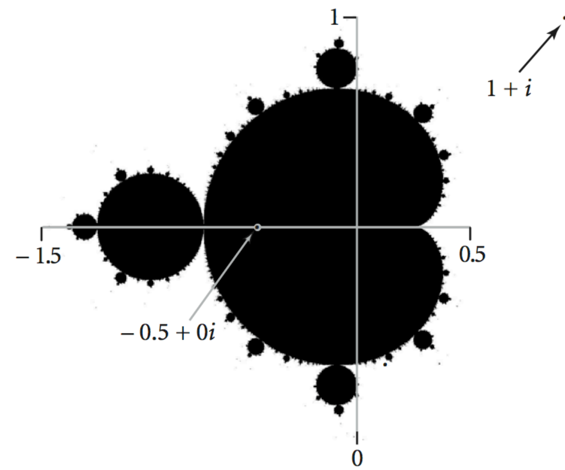



- No simple formula describes which complex numbers are in set.
- Instead, describe using an algorithm.

# Mandelbrot Set

**Mandelbrot set.** Is complex number $z_0$ in set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ diverges to infinity, then $z_0$ not in set; otherwise $z_0$ is in set.



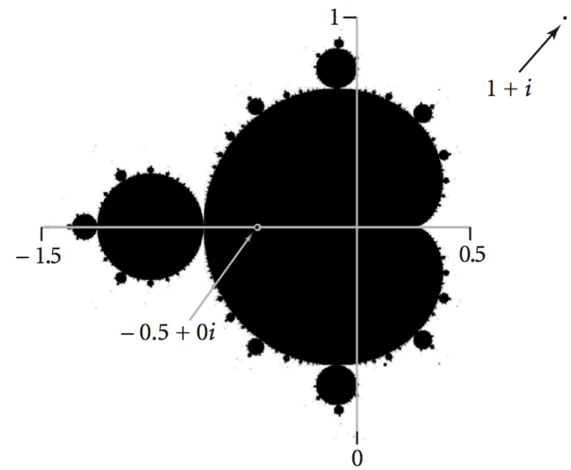

| t | $z_t$ |
|---|---|
| 0 | -1/2 + 0i |
| 1 | -1/4 + 0i |
| 2 | -7/16 + 0i |
| 3 | -79/256 + 0i |
| 4 | -26527/65536 + 0i |
| 5 | -1443801919/4294967296 + 0i |

z = -1/2 is in Mandelbrot set

| t | $z_t$ |
|---|---|
| 0 | 1 + i |
| 1 | 1 + 3i |
| 2 | -7 + 7i |
| 3 | 1 – 97i |
| 4 | -9407 – 193i |
| 5 | 88454401 + 3631103i |

z = 1 + i not in Mandelbrot set

# Plotting the Mandelbrot Set

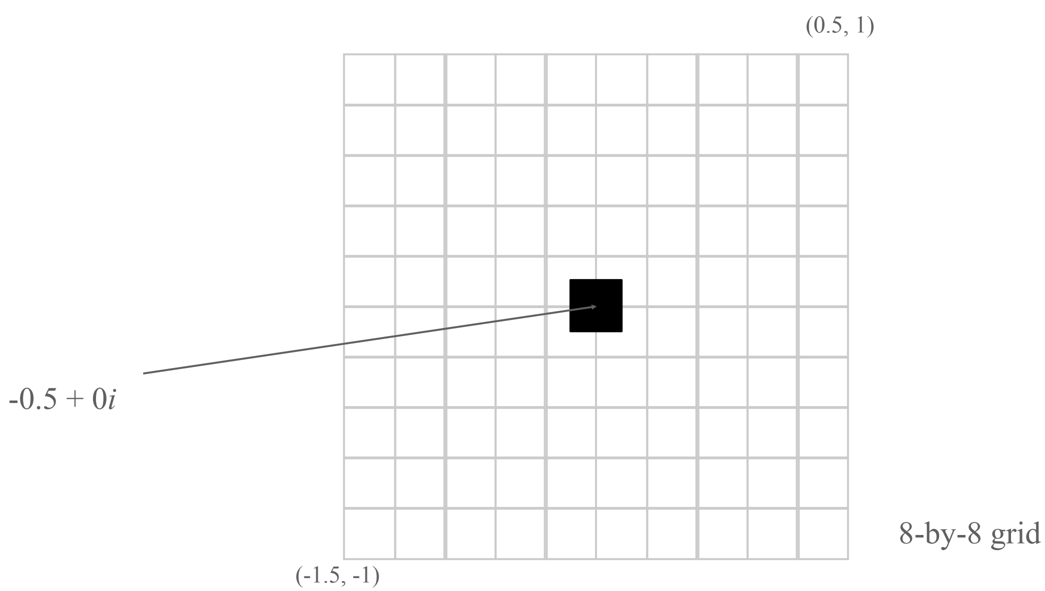



**Practical issues.**

- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.

**Approximate solution.**

- Sample from an $N$-by-$N$ grid of points in the plane.
- Fact: if $|z_t| > 2$ for any $t$, then $z$ not in Mandelbrot set.
- Pseudo-fact: if $|z_{255}| \leq 2$ then $z$ "likely" in Mandelbrot set.



# Complex Number Data Type: Another Client

Mandelbrot function with complex numbers.

- Is $z$ in the Mandelbrot set?
- Returns white (definitely no) or black (probably yes).

```
public static Color mand(Complex z0)
{
   Complex z = z0;
   for (int t = 0; t < 255; t++)
   {
      if (z.abs() > 2.0) return Color.WHITE;
      z = z.times(z);
      z = z.plus(z0);
   }
   return Color.BLACK;
}
```

$z = z^2 + z_0$

More dramatic picture: replace `Color.WHITE` with grayscale or color.

`new Color(255-t, 255-t, 255-t)`

# Complex Number Data Type: Another Client

Plot the Mandelbrot set in gray scale.

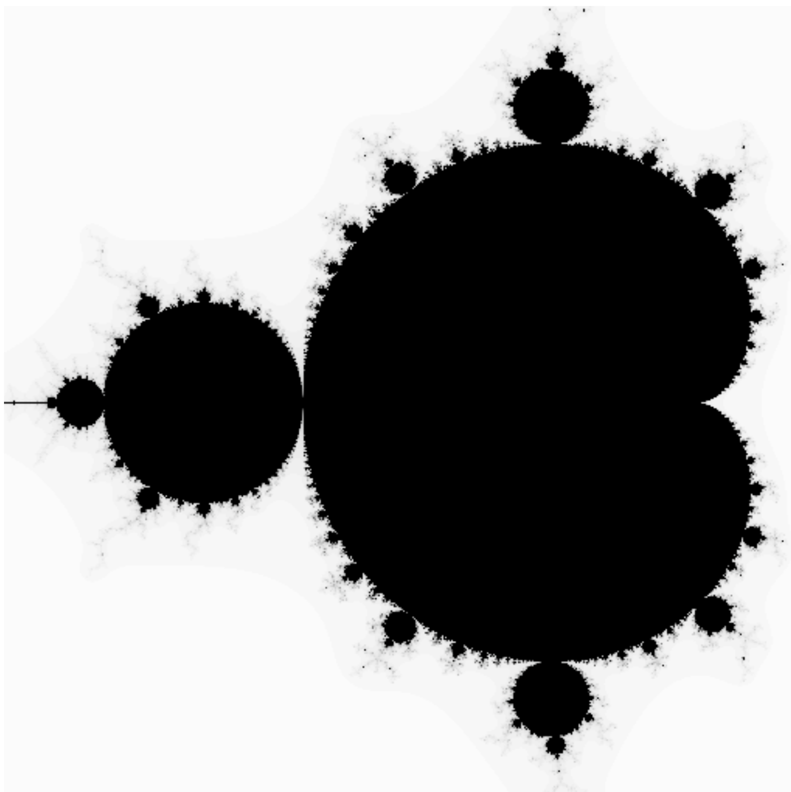
```
public static void main(String[] args)
{
   double xc   = Double.parseDouble(args[0]);
   double yc   = Double.parseDouble(args[1]);
   double size = Double.parseDouble(args[2]);
   int N = 512;
   Picture pic = new Picture(N, N);

   for (int x = 0; x < N; x++)
      for (int y = 0; y < N; y++)
      {
         double x0 = xc - size/2 + size*x/N;
         double y0 = yc - size/2 + size*y/N;
         Complex z0 = new Complex(x0, y0);
         Color color = mand(z0);
         pic.set(x, N-1-y, color);
      }
   pic.show();
}
```

scale to screen coordinates

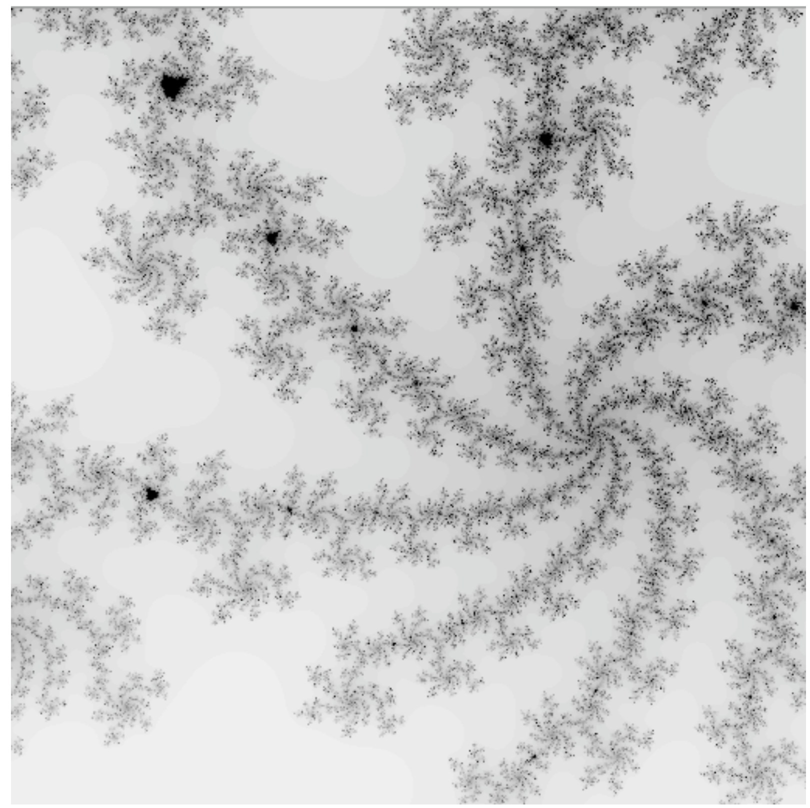
(0, 0) is upper left

# Mandelbrot Set

`% java Mandelbrot -.5 0 2`

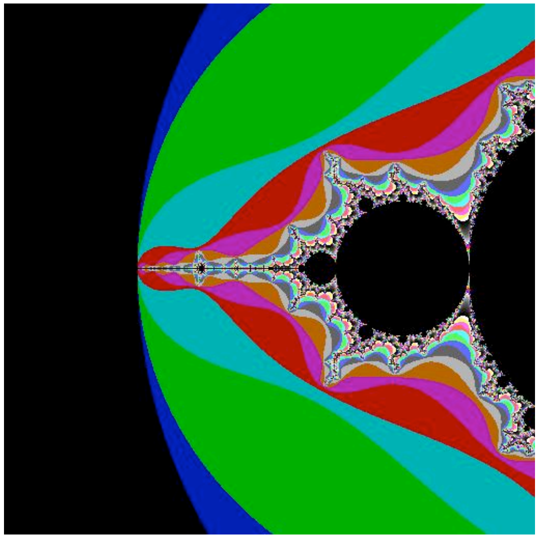


`% java Mandelbrot .1045 -.637 .01`

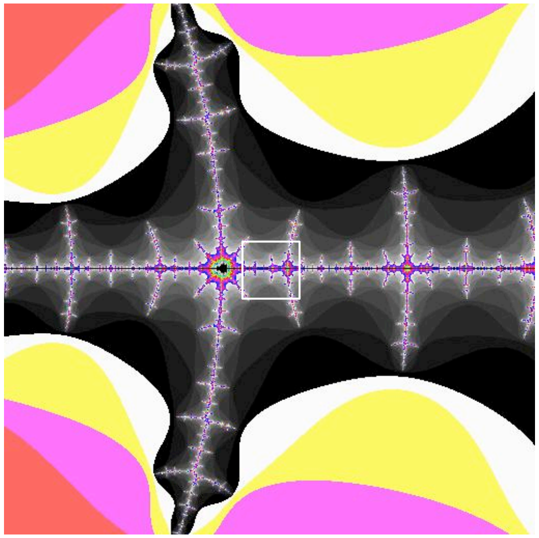


# Mandelbrot Set

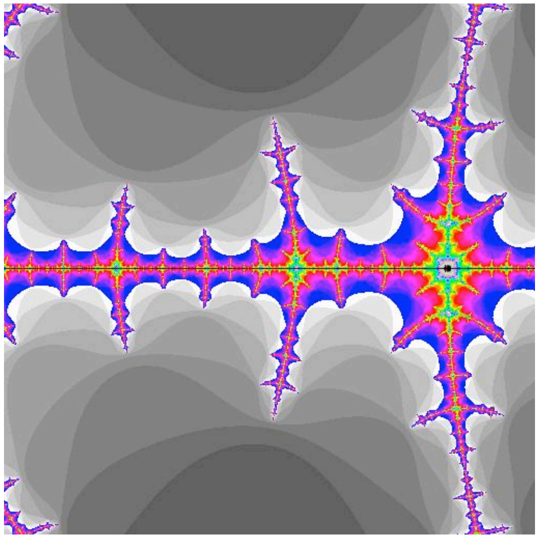
```
% java ColorMandelbrot -1.5 0 2 < mandel.txt
```



**-1.5 0 .02**



**-1.5 0 .002**



# Applications of Data Types

Data type. Set of values and collection of operations on those values.

Simulating the physical world.
- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Ex: charged particle, molecule, COS 126 student, ….

Extending the Java language.
- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Ex: complex, vector, polynomial, matrix, ....