From Greek philosophers to circuits: An introduction to boolean logic.

COS 116, Spring 2011

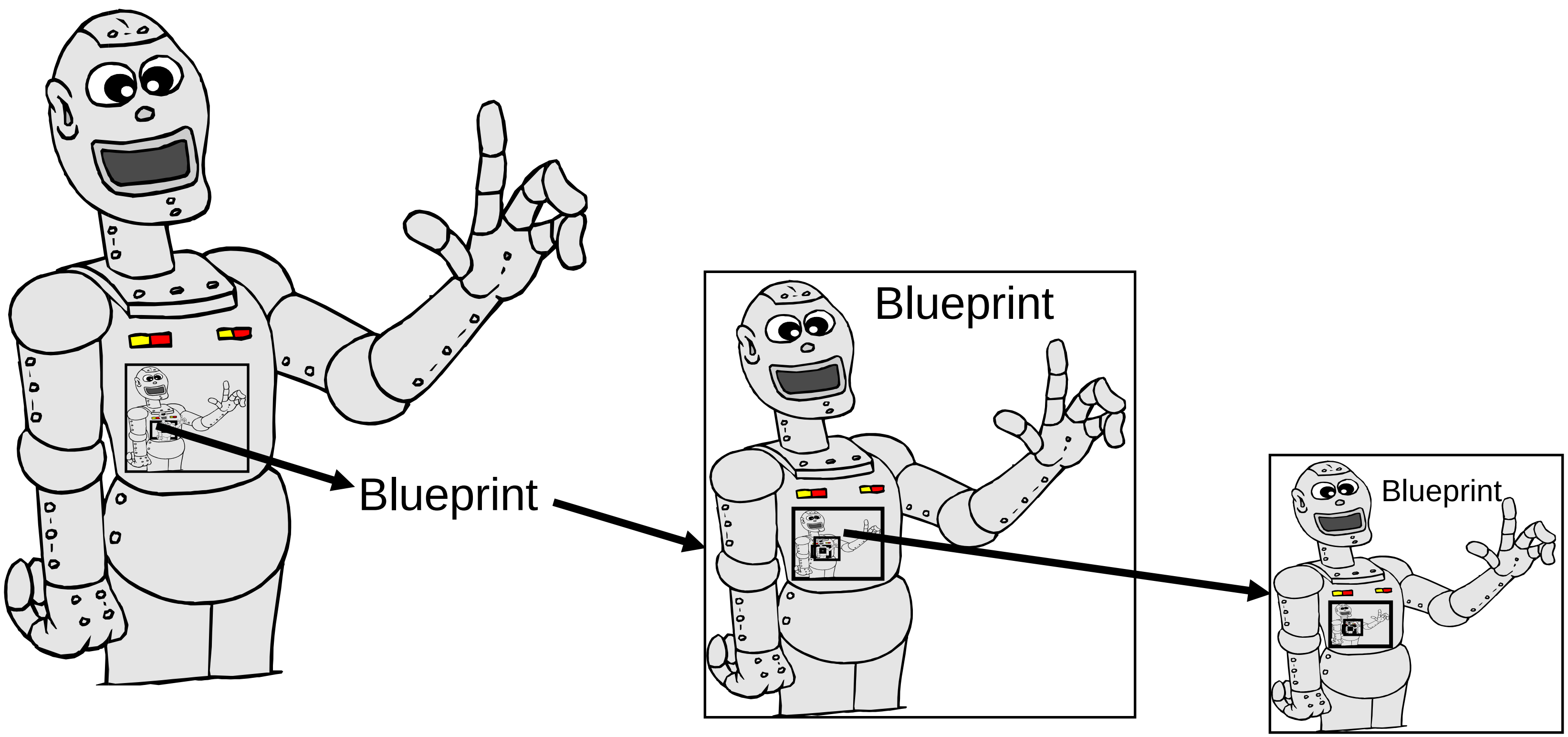
Sanjeev Arora

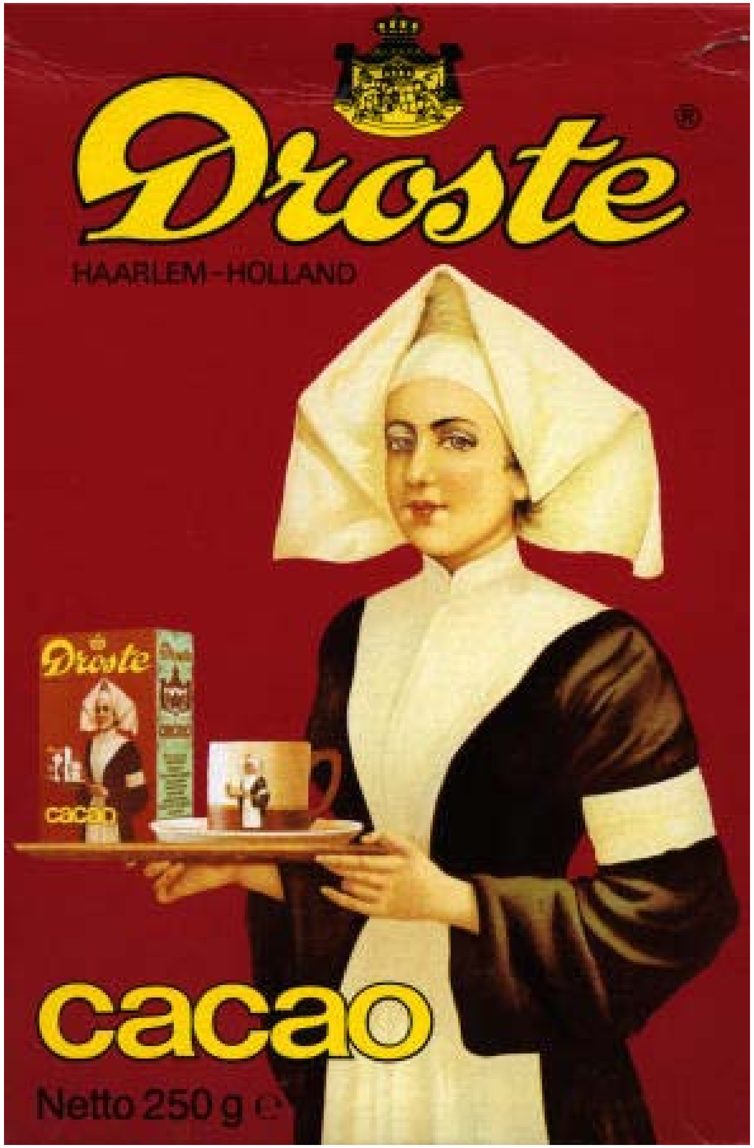
Midterm

- One week from today – in class Mar 10
- Covers
  - lectures, labs, homework, readings to date
  - You can use the pseudocode handout during exam; no other material.

Recap: Self-Reproduction

Fallacious argument for impossibility:





"Droste Effect"

Fallacy Resolved: “Blueprint” can involve *computation*; need not be an exact copy!

Print the following sentence twice, the second time in quotes. “Print the following sentence twice, the second time in quotes.”

High-level view of self-reproducing program

A

Print 0
Go Right
Print 1
.
.
.Print 0

Prints binary code of B

B

. . . . . .

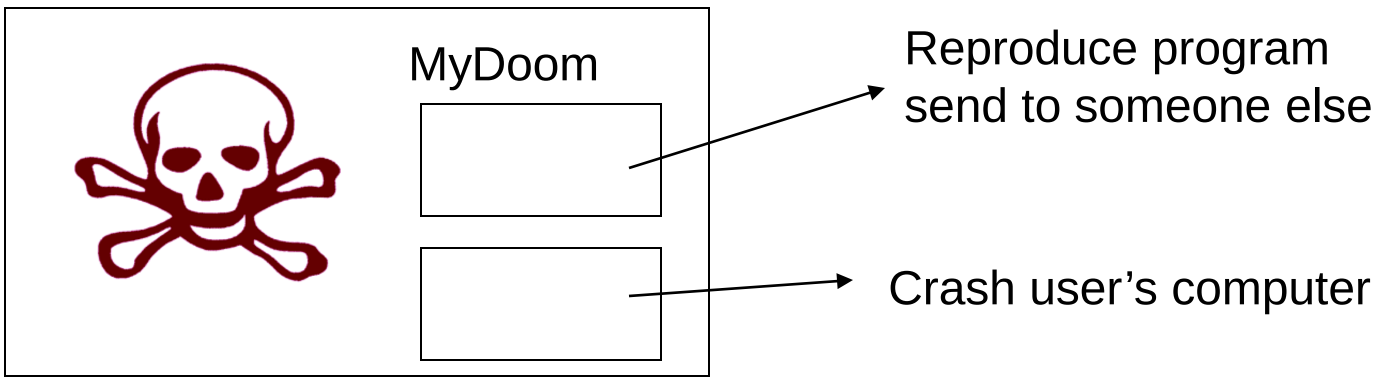
. . . . . .

. . . . . .

. . . . . .

Takes binary string on tape, and in its place prints (in English) the sequence of statements that produce it, followed by the translation of the binary string into English.

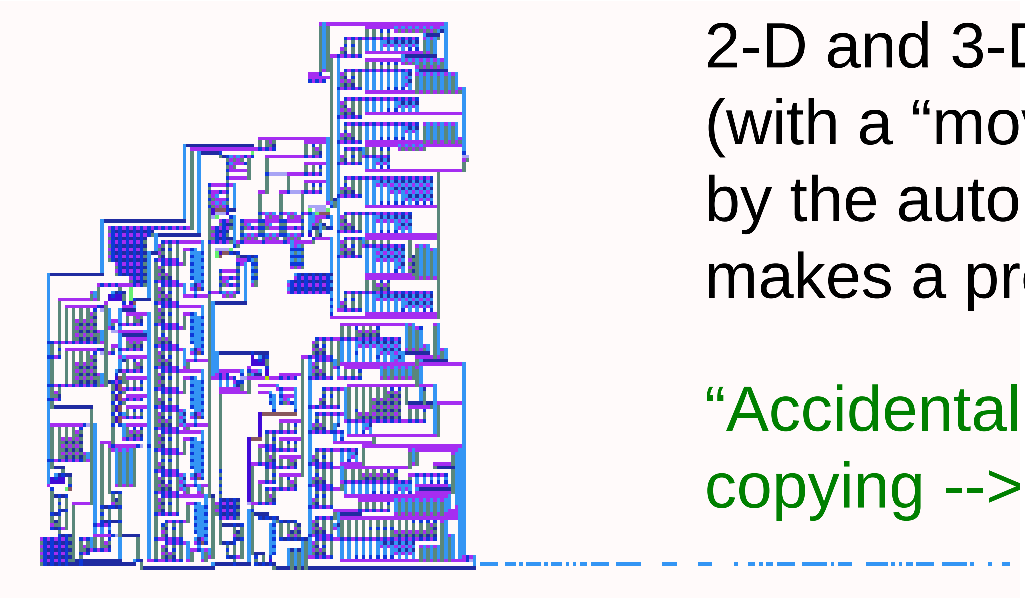
Self-reproducing programs



- Fact: for every program *P*, there exists a program *P'* that has the exact same functionality except at the end it also prints code(*P'*) on the tape

Self-reproducing machines

[John von Neumann, 1940s]



2-D and 3-D cellular automata (with a “moving arm” controlled by the automaton itself) that makes a precise copy of itself.

“Accidental changes” during copying --> mutations, evolution

This and related ideas of Pauli motivated discovery of the molecular basis of life on earth (DNA, RNA etc.)

Moving on to part 2…
Upcoming lectures: Computational Hardware

- Boolean logic and Boolean circuits
- Sequential circuits (circuits with memory)
- Clocked circuits and Finite State Machines
- CPUs
- Operating System
- Networks, Internet

Discussion Time

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up late (he hates to bike in the rain). But if there's an exam that day he'll bike if he overslept, even in the rain.

It is raining today, Ben overslept, and there's an exam. Will Ben bike today?

"Logical reasoning", "*Propositional logic*."

Propositional Logic: History

- Aristotle – Law of excluded middle,
  Law of contradiction.
- Stoic Philosophers (3rd century BC) –
  Basic inference rules (*modus ponens* etc.)
- Some work by medieval philosophers
- De Morgan and Boole (19th century):
  Symbolic logic – “automated”, “mechanical”
- C. Shannon (1930s) –
  Proposal to use digital hardware

Example

Ed goes to the party if Dan does not and Stella does.

Choose "Boolean variables" for 3 events:

E: Ed goes to party
D: Dan goes to party
S: Stella goes to party

Each is either TRUE or FALSE

E = S **AND** (**NOT** D)

Alternately: E = S **AND** $\overline{\text{D}}$

Logical "OR"

Ed goes to the party if Dan goes **or** Stella goes

E = D **OR** S

E is TRUE if one or both of D and S are TRUE

Note:

Different from everyday meaning of OR!

Example: You can eat an orange or an apple

Boolean expressions

Composed of **boolean variables**, **AND**, **OR**, **NOT**

Examples:

D **AND** (P **OR** (**NOT** Q))

C **OR** D **OR** E

Truth table

Lists the truth value of the Boolean expression for all combinations of values for the variables.

Boolean Expression $E = S \text{ AND } \overline{D}$

Truth table

0 = FALSE

1 = TRUE

For all possible values of D, S, write corresponding value of E

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0

Let’s work an example…

Boolean Expression $E = D \text{ OR } \overline{S}$

What are x and y ?!?

D	S	E
0	0	1
0	1	x
1	0	y
1	1	1

Possible answers:
x=0, y=0
x=0, y=1
x=1, y=0
X=1, y=1

Ben Revisited

Ben only rides to class if he overslept.
But even then if it is raining he'll walk and show up late (he hates to bike in the rain).
But if there's an exam that day he'll bike if he overslept, even in the rain.

B: Ben Bikes
R: It is raining
E: There is an exam today
O: Ben overslept

Break up in groups of three and come up with (a) Truth table for B in terms of values of R, E, O (b) Boolean expression for B in terms of R, E and O.

Fail-safe method to work out the expression: the truth table

Expression for O

= **OR** of all input combinations that make O TRUE

O	R	E	B
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Boolean "algebra"

A **AND** B written as A • B

$0 \bullet 0 = 0$

$0 \bullet 1 = 0$

$1 \bullet 1 = 1$

A **OR** B written as A + B

$0 + 0 = 0$

$1 + 0 = 1$

$1 + 1 = 1$

Funny arithmetic

Will provide readings on this…

Truth table → Boolean expression

Use **OR** of all input combinations that lead to TRUE

$$B = O \cdot \overline{R} \cdot \overline{E} + O \cdot \overline{R} \cdot E + O \cdot R \cdot E$$

O	R	E	B
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Note:
AND, OR, and NOT gates suffice to implement every Boolean function (basis of the implementation of universal computer in silicon chips)!

Sizes of representations

- For k variables:

k	10	20	30
2^k	1024	1048576	1073741824

$$2^k \left\{ \begin{array}{|c|c|c|c|} \hline A & B & \ldots & X \\ \hline 0 & 0 & \ldots & 0 \\ \hline 0 & 0 & \ldots & 0 \\ \hline 0 & 1 & \ldots & 0 \\ \hline 0 & 1 & \ldots & 1 \\ \hline \ldots & \ldots & \ldots & \ldots \\ \hline \ldots & \ldots & \ldots & \ldots \\ \hline 1 & 1 & \ldots & 1 \\ \hline \end{array} \right.$$

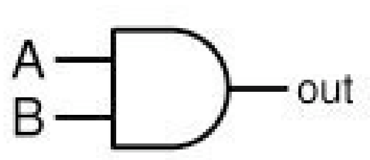
$$\underbrace{\hspace{12em}}_{k + 1}$$

For an arbitrary function,
expect roughly half of X's to be 1
(for 30 inputs roughly 1/2 billion!)

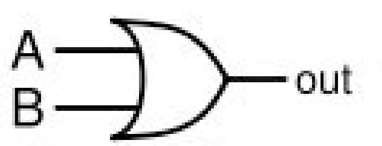
Tools for reducing size:
(a) circuit optimization (b) modular design

Boolean circuit

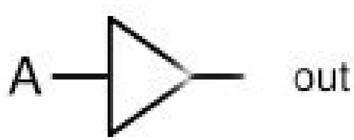
Pictorial representation of Boolean expression using special symbols for AND, OR and NOT



A AND B



A OR B

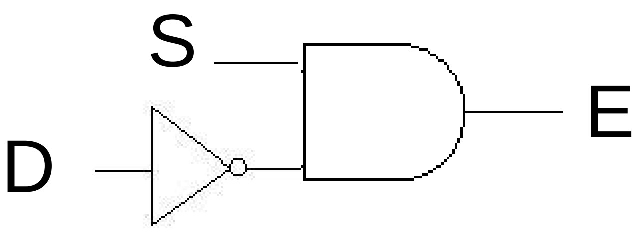


$\overline{A}$

Three Equivalent Representations

Boolean Expression $E = S \text{ AND } \overline{D}$

Boolean Circuit



Truth table:
Value of E for every possible D, S.
TRUE=1; FALSE= 0.

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0

Next time: Boolean circuits, the basic components of the digital world

Midterm will have a question on boolean logic.

Ed goes to the party if

Dan doesn't AND Stella doesn't

$E = \overline{D}$ **AND** $\overline{S}$

Is this equivalent to:

Ed goes to the party if

NOT (Dan goes OR Stella goes)

….?

(De Morgan's Laws)

Readings for Tues on website

- “Theory of everything.”
- Boole’s reformulation of Clarke’s “proof” of existence of God
- To hand in next time (participation grade): What you understood from boole’s proof and your reaction to it. (1 para)