What computers just *cannot* do. (Part II)

COS 116: 3/1/2011

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Administrivia

- Midterm - in-class 3/10
  - Review session Wed during lab slot.
  - Past midterm available on website for self-study.

Recap from last time

- Turing-Post computational model:
  - Greatly simplified model
  - Infinite tape, each cell contains 0/1
  - Program = finite sequence of instructions (only 6 types!)
  - Unlike pseudocode, no conditionals or loops, only “GOTO”
  - code(*P*) = binary representation of program *P*

What does this program do?

1. GO RIGHT
2. GO TO STEP 1 IF 1 SCANNED
3. GO TO STEP 1 IF 0 SCANNED
4. STOP

Example: doubling program

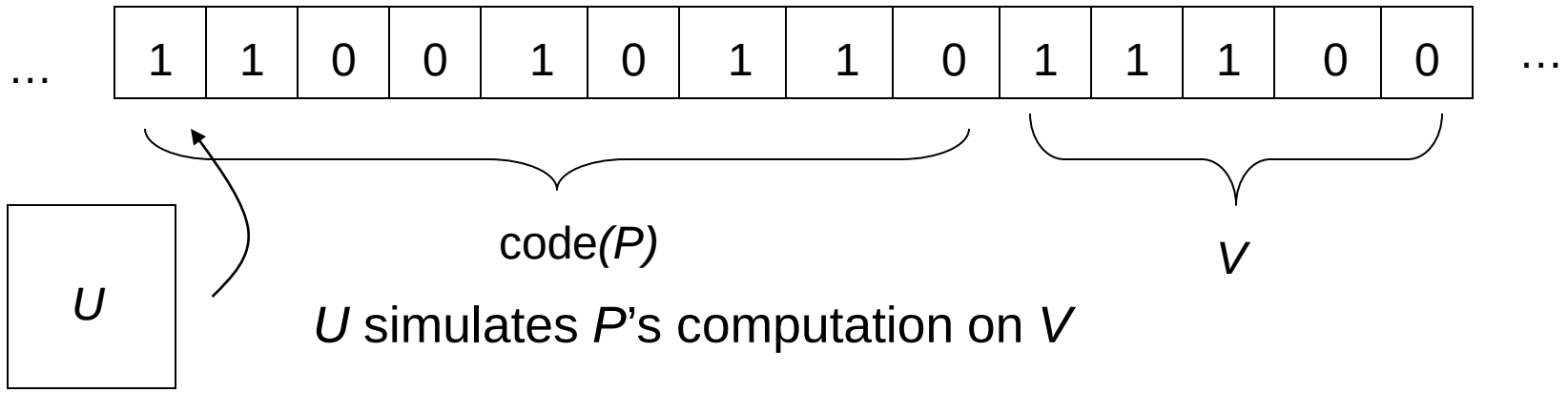
1. PRINT 0
2. GO LEFT
3. GO TO STEP 2 IF 1 SCANNED
4. PRINT 1
5. GO RIGHT
6. GO TO STEP 5 IF 1 SCANNED
7. PRINT 1
8. GO RIGHT
9. GO TO STEP 1 IF 1 SCANNED
10. STOP

Program said to halt on this input data if STOP is executed in a finite number of steps

Some facts

- Fact 1: Every pseudocode program can be written as a T-P program, and vice versa

- Fact 2: There is a universal T-P program

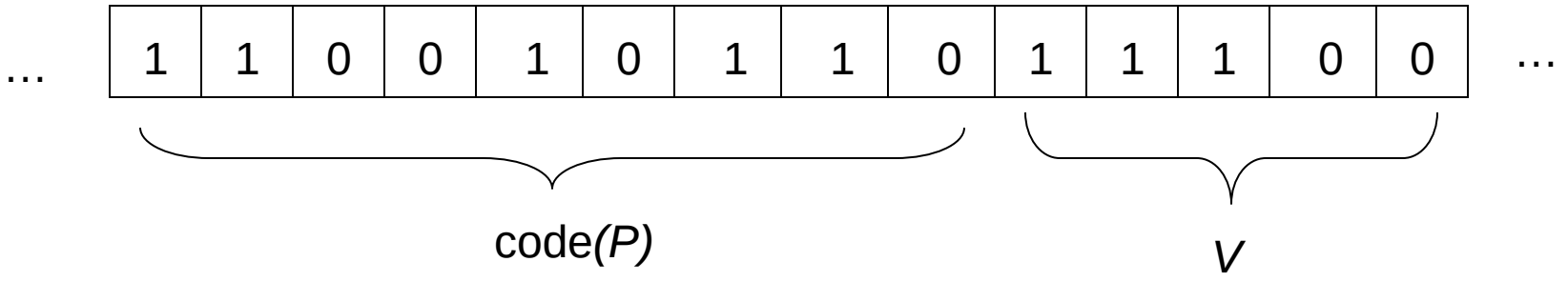


Discussion Time

How would you write a universal program for T-P programs?

What are some examples of universal programs in real life?

Halting Problem



- Decide whether *P* halts on *V* or not
- Cannot be solved! Turing proved that *no Turing-Post program can solve Halting Problem for all inputs (code(P), V).*

Makes precise something quite intuitive: “Impossible to demonstrate a negative”

Suppose program P halts on input V. How can we detect this in finite time?

“Just simulate.”

Intuitive difficulty: If P does not actually halt, no obvious way to detect this after just a finite amount of time.

Turing’s proof makes this intuition concrete.

Ingredients of Turing's proof…..

Ingredient 1: “Proof by contradiction”

Fundamental assumption:
A mathematical statement is either true or false

“When something’s not right, it’s wrong.”

Bob Dylan

Aside: Epimenides Paradox

- *Κρῆτες ἀεί ψεύσται*
- “Cretans, always liars!”
- But Epimenides was a Cretan!’

  (can be resolved…)

- More troubling: “This sentence is false.”

Ingredient 2:

Suppose you have programs A and B.

What is the program whose net effect is "Run A first and then B?"

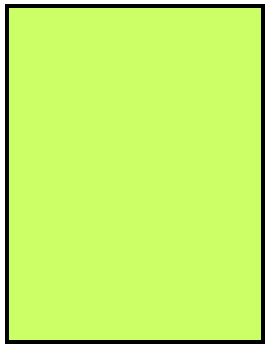
Suppose you are given some T-P program P

How would you turn P into a T-P program that does NOT halt on all inputs that P halts on?

Finally, the proof…

Suppose program H solves Halting Problem on ALL inputs of the form code(P), V.

H



Consider program D

1. On input V, check if it is code of a T-P program.
2. If no, HALT immediately.
3. If yes, use doubling program to create the bit string V, V
4. Run H on bit string V,V.
5. If H says “Doesn’t Halt”, HALT immediately.
6. If H says “Halts”, go into infinite loop

If H halts on every input, so does D

Gotcha! Does D halt on the input code(D)?

Lessons to take away

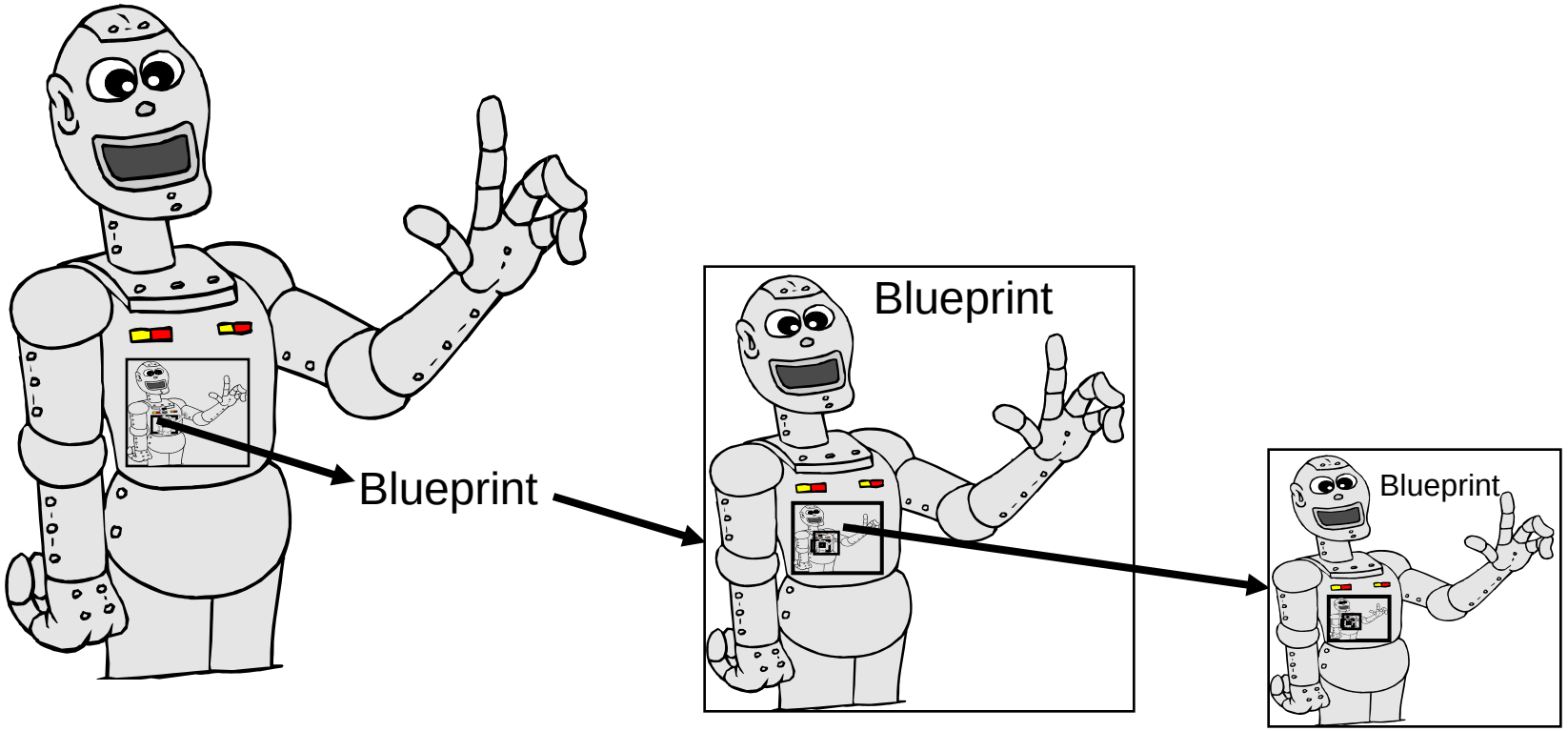
- Computation is a very simple process; 6 simple types of operations! (Later: can arise in unexpected places)
- Universal Program
- No real boundary between hardware, software, and data. (basis of much of modern computer technology, eg JAVA)
- No program that decides whether or not mathematical statements are theorems.
- Many tasks are uncomputable; e.g. “If we start Game of life in this configuration, will cell (100, 100) ever have a critter?”

Age-old mystery: Self-reproduction.

How does the seed encode the whole?

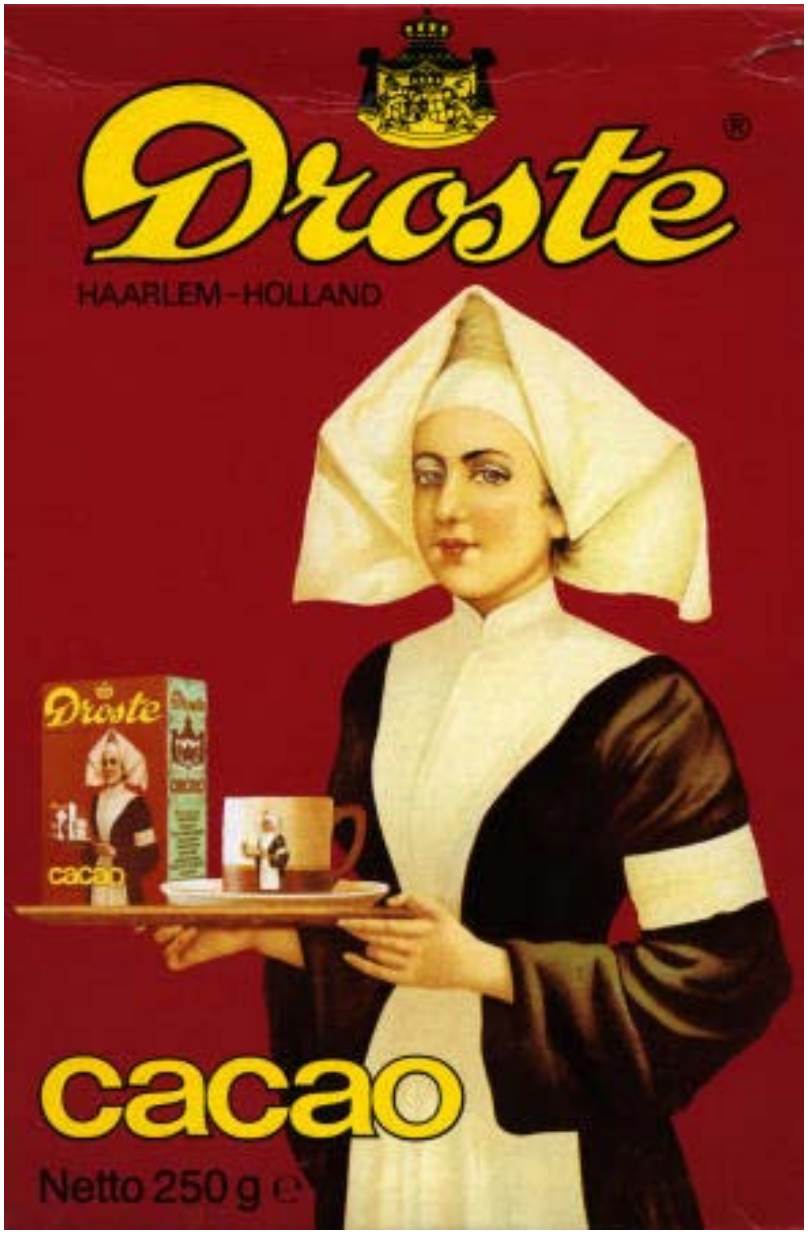
Self-Reproduction

Fallacious argument for impossibility:



M.C. Escher

Print Gallery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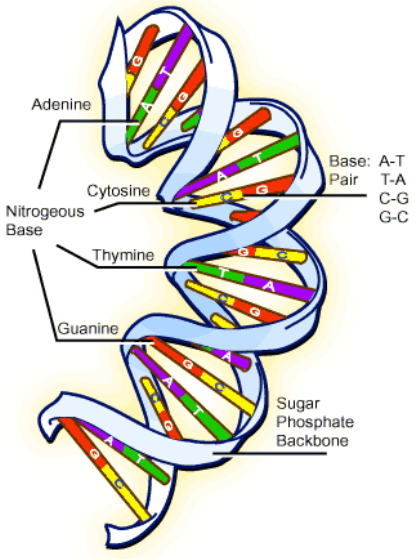
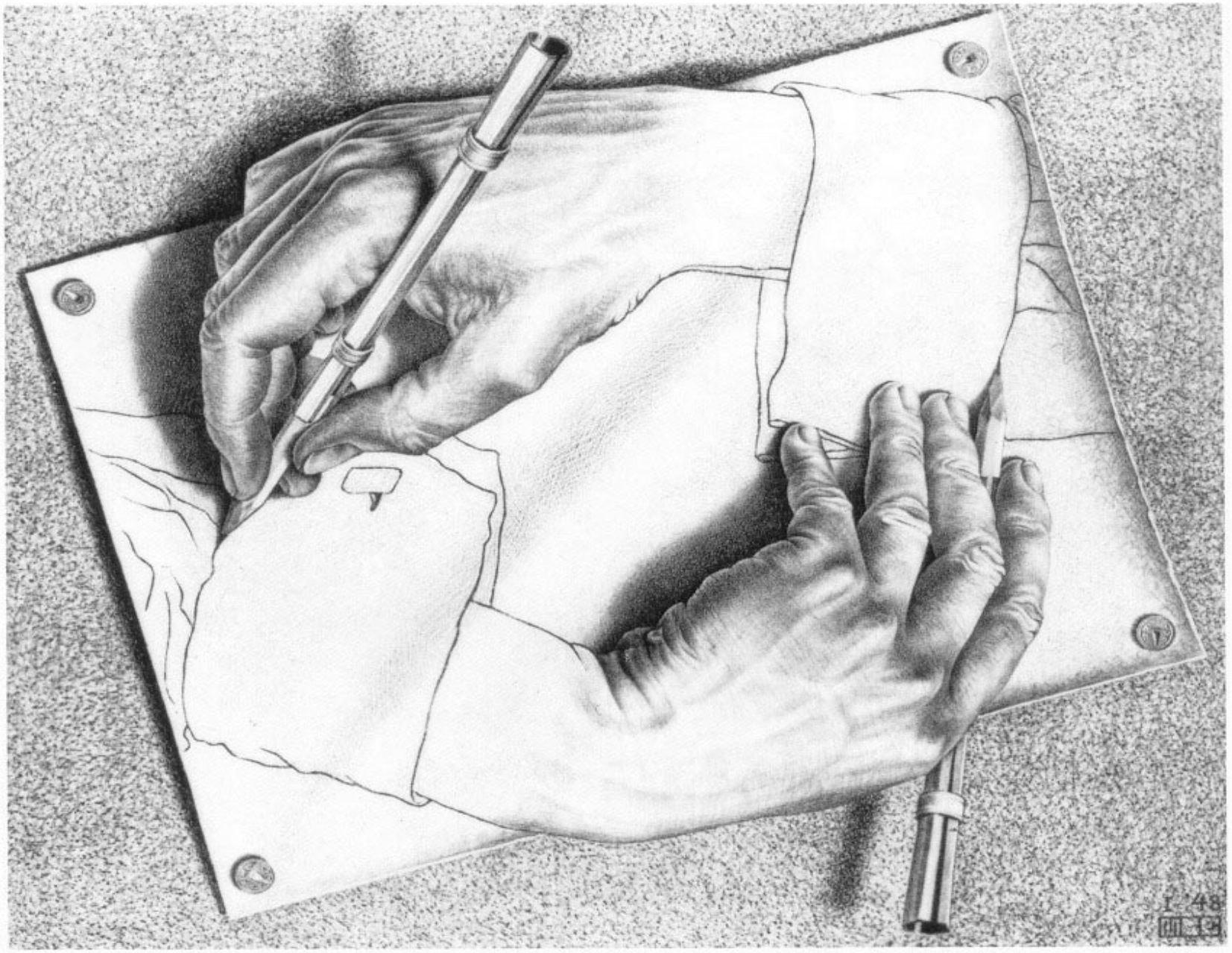
Discussion Time

What is “reproduction” at the molecular level?



Fallacy Resolved: “Blueprint” can involve some computation; need not be an exact copy!

Print this sentence twice, the second time in quotes. “Print this sentence twice, the second time in quotes.”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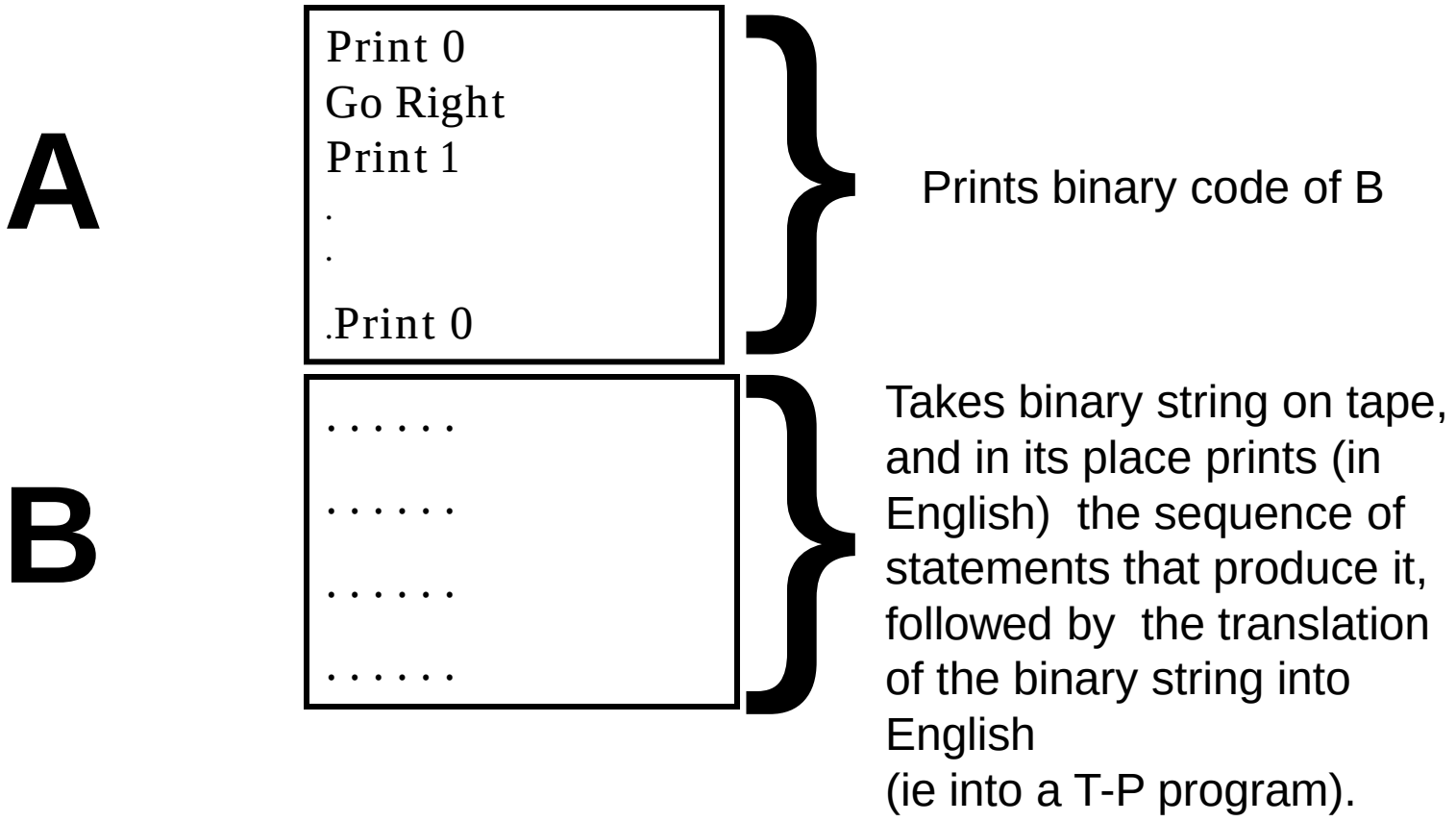
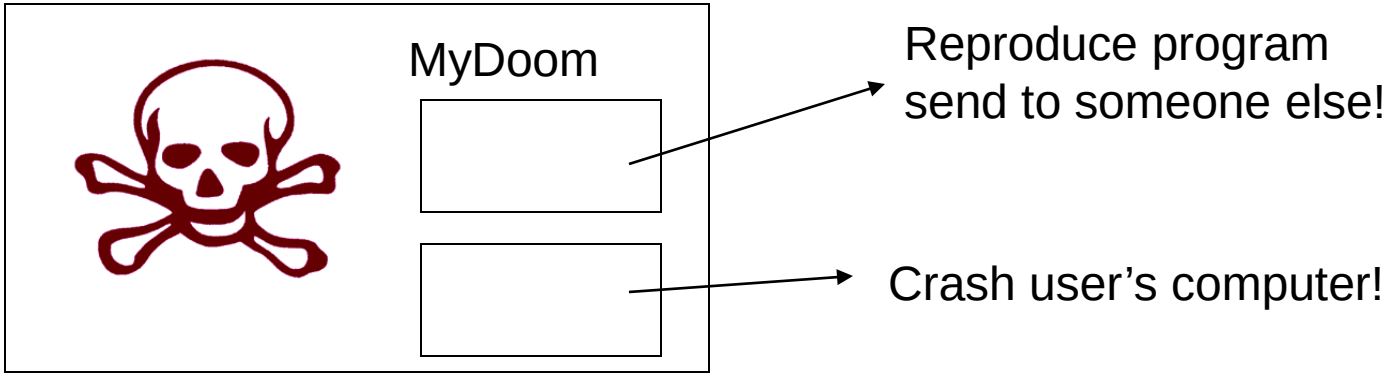
High-level description of program that self-reproduces



Self-reproducing programs



- Fact: for every program *P*, there exists a program *P'* that has the exact same functionality except at the end it also prints code(*P'*) on the tape