Creating new worlds inside the computer

COS 116: 2/10/2011

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Pseudocode

- Simple instructions: involve +, -, ×, ÷
- Compound instructions
  - Conditionals
  - Loops

- No need to sweat over exact wording during exams (unless it changes meaning!)

Algorithm defn; revisited

"Pseudocode for turning a set of inputs into outputs in a finite amount of time"

Questions to think about:

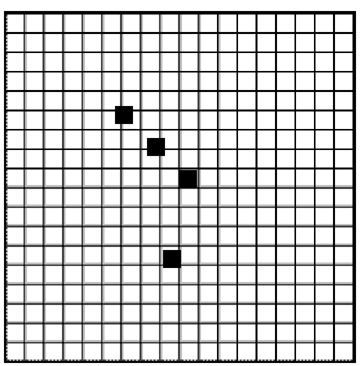
- What class of computational tasks can be solved by algorithms?
- How dependent is this class on the exact definition of pseudocode?

Today’s topic: Creating new worlds inside the computer.

“simulation”

Conway's Game of life

- Rules: At each step, in each cell
  - Survival: Critter survives if it has 2 or 3 neighbors.
  - Death: Critter dies if it has 1 or fewer neighbors, or more than 3.
  - Birth: New critter is born if cell is currently empty and 3 neighboring cells have critters.



Discussion Time

How would you write pseudocode that simulates Game of Life?

Should use: n x *n* array *A (for desired n)*

A[i, j] = 1 means critter lives in square, 0 means empty square

Pseudocode for each step

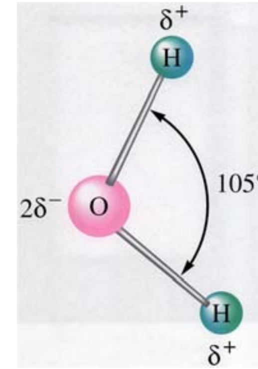
```
Do for i = 1 to n
{
    Do for  j = 1 to n
    {
        neighbors ← A[i – 1, j - 1] + A[i – 1, j] + A[i – 1,j + 1] +
                    A[i, j – 1] + A[i, j + 1] + A[i + 1, j – 1] +
                    A[i + 1, j] + A[i + 1, j + 1]
        if ( neighbors = 2 OR neighbors = 3 ) then
                { B[i, j] ← 1 }
        else if ( neighbors = 1 …)
                …etc. //see handout; Example 3//
    }
}
Do for i = 1 to n
{
    Do for  j = 1 to n
        { A[i,j] ←B[i,j] }
}
```

Moral of the Game of Life?

- Simple local behavior can lead to complex global behavior

(cf. Brian Hayes article on blackboard)

Physics of snow crystals



Water vapor → (cooling) →

Water → (further cooling) →

Snow crystals

- “Cooling” – reduce amount of molecular motion
- Crystal growth: capture of nearby floating molecules

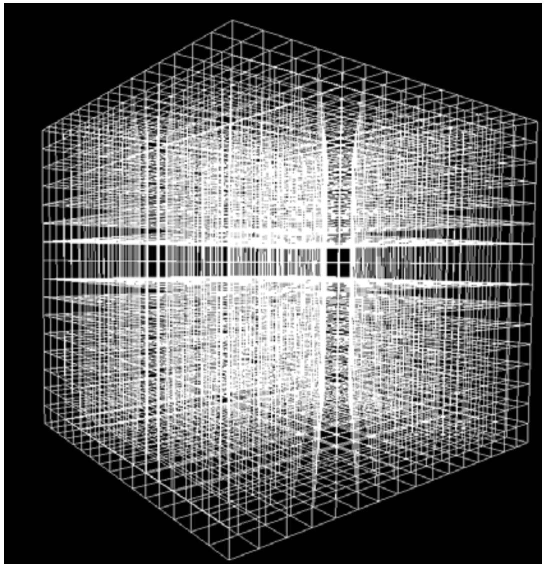
Next..

Discussion Time

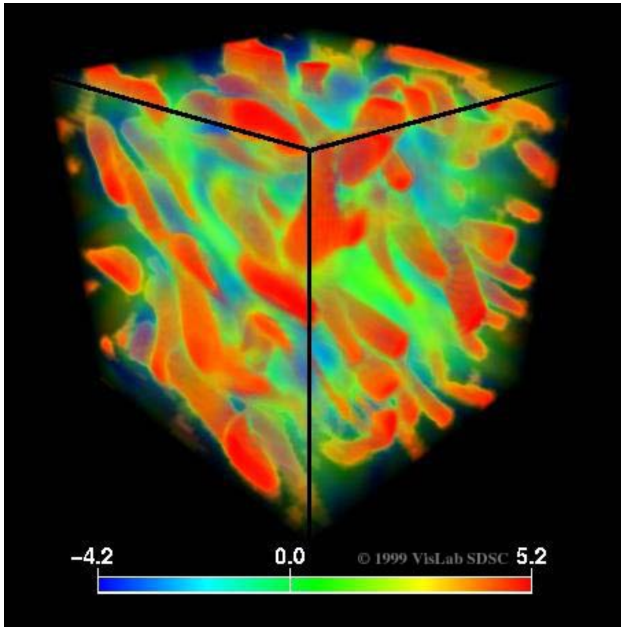
How does weather prediction happen?

Why can't we predict the weather a month from today?

Twister simulation



- Divide region into 3D grid
- Identify laws of physics for air



Navier Stokes equations:

How does a block of air move when certain pressure, temperature and velocity differentials exist on its boundary?

Simulator pseudocode

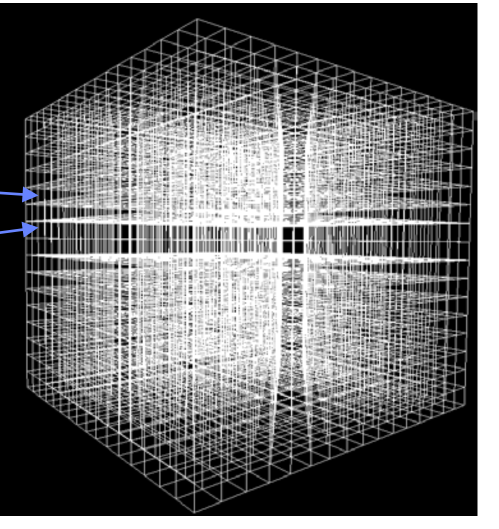
- Initialize Grid using data from observations: surface and aircraft measurements, radar (NEXRAD) readings, etc.

```
Do for i = 1 to n
{
    Do for j = 1 to n
    {
        Do for k = 1 to n
        { Update state of Grid[i, j, k] }
    }
}
```

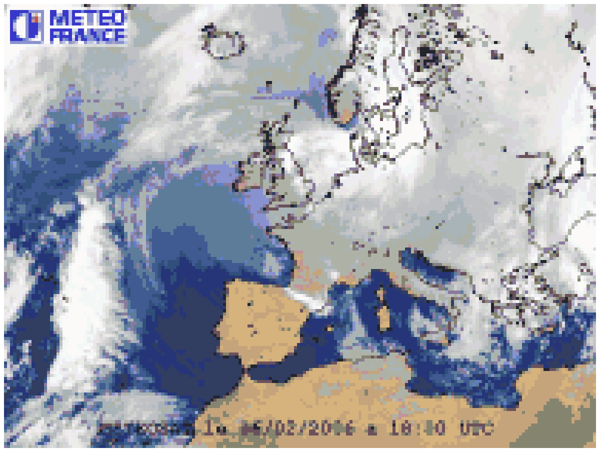
10ºC, 15 psi, 20% humidity

11ºC, 15 psi, 23% humidity

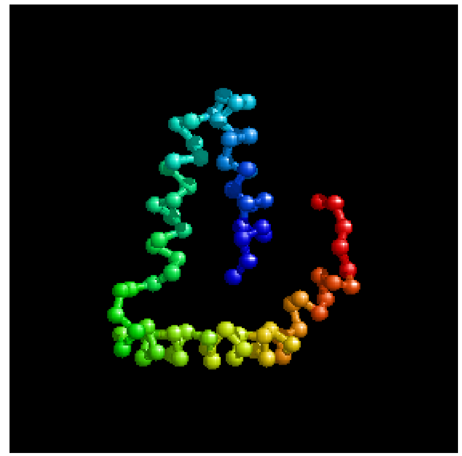
etc.



Other examples of simulation



Weather forecasting



Protein folding

[Turk 91] following:

[Turing 52] Turing, Alan, "The Chemical Basis of Morphogenesis," *Philosophical Transactions of the Royal Society B*, Vol. 237, pp. 37–72 (August 14, 1952).

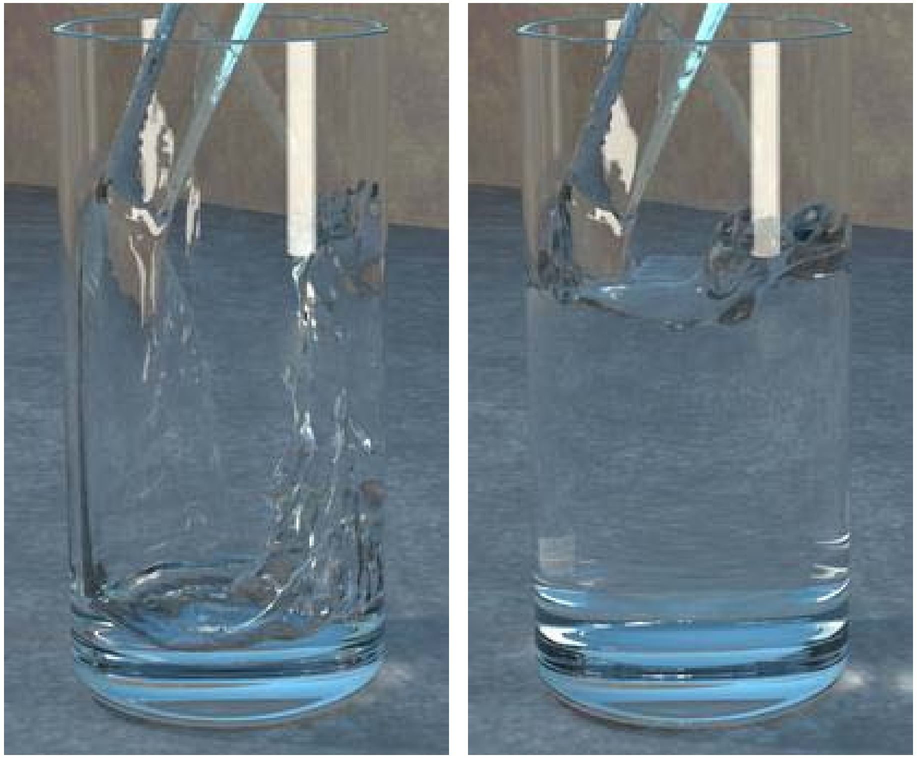
How patterns arise in plants and animals

Animation

Display

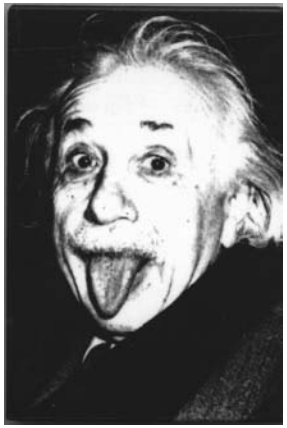
Q: How to display result of simulation?

A: Computer graphics (later in course)



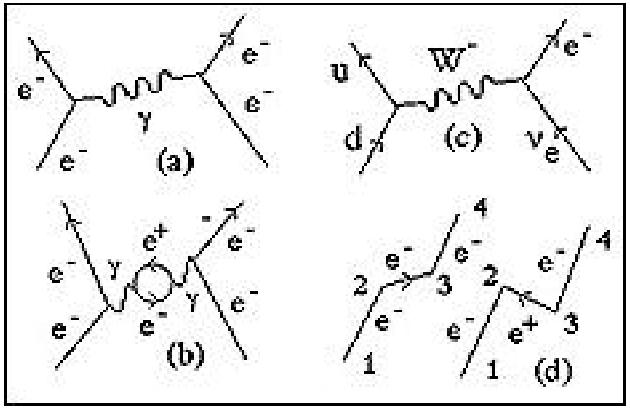
[Enright and Fedkiw 02]

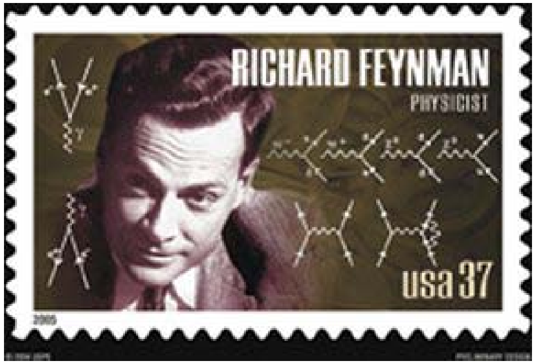
Bigger questions

Alan Turing   Albert Einstein

- Can computer simulation be replaced by a "theory of weather"? A "theory of tornadoes"?
- Is there a "theory" that answers this type of problem:
  - Given: A starting configuration in the game of life
  - Output: "Yes" if the cell at position (100, 100) is ever occupied, "No" otherwise

Actually, reverse trend: “theory of matter” (particle physics) is becoming computational.





1670   F = ma

Today

Hayes (reading this week): The universe as a “cellular automaton”

Peeking ahead:

A computer can simulate another computer (e.g., a Classic Mac simulator on a PC). Will explore the implications of this in a future lecture.

Game of life is actually a “computer.”

What does this pseudocode do?
Write on a piece of paper and hand it in.

- *n* items, stored in array *A*
- Variables are *i*, *S.*

```
S← 0
Do for i = 1 to [n/2]
   {
         S ← S + A[2*i];
   }
```

Stable Matching Problem

Problem:

Given *N* men & *N* women, find “suitable” matching

- Everyone lists their preferences from best to worst.

Men’s Preference List

Man	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erika	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

↑ best ↑ worst

Stable Matching Problem

<u>Problem</u>:

Given *N* men & *N* women, find "suitable" matching

- Everyone lists their preferences from best to worst.

Women's Preference List

Woman	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

↑ best ↑ worst

Stable matching: definition

- There is no pair such that they prefer each other more than their current partners.

A man & woman are currently unstable if they prefer each other more than their current partner

Men's Preference List

Man	1st	2nd	3rd
Xavier	A	B	C
Yancey	B	A	C
Zeus	A	B	C

Women's Preference List

Woman	1st	2nd	3rd
Amy	Y	X	Z
Bertha	X	Y	Z
Clare	X	Y	Z

- Lavender assignment is a possible matching. Are there any unstable pairs?

  Yes. Bertha and Xavier form an unstable pair. They would prefer each other to current partners.

Example

Men's Preference List

Man	1^{st}	2^{nd}	3^{rd}
Xavier	A	B	C
Yancey	B	A	C
Zeus	A	B	C

Women's Preference List

Woman	1^{st}	2^{nd}	3^{rd}
Amy	Y	X	Z
Bertha	X	Y	Z
Clare	X	Y	Z

- Green assignment is a stable matching.

Example

Men's Preference List

Man	1st	2nd	3rd
Xavier	A	B	C
Yancey	B	A	C
Zeus	A	B	C

Women's Preference List

Woman	1st	2nd	3rd
Amy	Y	X	Z
Bertha	X	Y	Z
Clare	X	Y	Z

- Gray assignment is also a stable matching.

Propose-And-Reject Algorithm

- Guarantees a stable matching.

Gale-Shapley Algorithm (men propose)

```
Initialize each person to be free.
while (some man m is free and hasn't proposed to every woman)
{
    w = first woman on m's list to whom he has not yet proposed
    if (w is free)
        assign m and w to be engaged
    else if (w prefers m to her fiancé f)
        assign m and w to be engaged, and f to be free
    else
        w rejects m

}
```

Extensions

- Unacceptable partners
  - Every woman is not willing to marry every man, and vice versa.
  - Some participants declare others as “unacceptable.”
- Sets of unequal size
  - Unequal numbers of men and women, e.g. 100 men & 90 women
- Limited Polygamy
  - e.g., Bill wants to be matched with 3 women.

Matching Residents to Hospitals

- Hospitals ~ Men (limited polygamy allowed).
- Residents ~ Women (more than hospitals)
- Started just after WWII (before computer usage).
- Ides of March, 13,000+ residents are matched.
- Rural hospital dilemma.
  - Certain hospitals (mainly in rural areas) were unpopular and declared unacceptable by many residents.
  - How to find stable matching that benefits rural hospitals?

Assignment for Valentine's day

(write on piece of paper and bring to class on Tues; participation grade)

- Try Gale-Shapley algorithm for previously-shown Amy-Erica / Victor-Zeuss preference lists, but vary the order of choosing man *m*. Does this affect the outcome?
- Try the version where women propose. Does this affect the outcome?
- Bonus question: Try to justify this statement *The Gale-Shapley algorithm finishes at some point, and when it finishes, there are no unstable pairs.*

Other homework

Readings for this week (on blackboard):
(i) Brian Hayes article; first 5 pages
(ii) Brooks 99-126.

HW 1: Will be on website and has to be submitted in
Class next Thurs