Social Networks and Ranking

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Social Networks

- Represent relationship between entities
  - paper cites paper
  - html page links to html page
  - A supervises B

  directed graph

  - A and B are friends
  - papers share an author
  - A and B are co-workers

  undirected graph

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Hypertext

- document or part of document links to other parts or other documents
  - construct documents of interrelated pieces
  - relate documents to each other
- pre-dates Web
- Web "killer app."

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How use links to improve information search?

- use structure to compute score
- include more objects to score
- can deal with objects of mixed types
  - images, PDF, …

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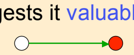
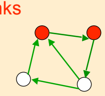
Scoring using structure

- Ideas
  1. link to object suggests it valuable object



  2. distance between objects in graph represents degree of relatedness

reachable by all in 2 links



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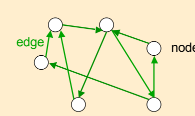
Pursuing linking and value

- Intuition: when Web page points to another Web page, it confers status/authority/popularity to that page
- Find a measure that captures intuition



- Not just web linking
  - Citations in books, articles
  - Doctors referring to other doctors

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Indegree

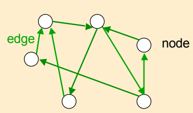
- Indegree = number of links into a node
- Most obvious idea:
  higher indegree => better node
- Doesn't work well
- Need some feedback in system
- Leads us to Page and Brin's PageRank

PageRank

- Algorithm that gave Google the leap in quality
- Used link structure between pages in fundamental way to score pages
  – link structure centerpiece of scoring
- published
  Page, Larry and Sergey Brin, R. Motwani, T. Winograd, *The PageRank Citation Ranking: Bringing Order to the Web*, Stanford Digital Library Technologies Project TR, Jan. 1998.

PageRank framework

- Given a directed graph with n nodes
- Assign each node a score that represents its importance in structure
  – Call score PageRank: pr(node)



Conferring importance

Core ideas:

- ➢ A node should confer some of its importance to the nodes to which it points
  – If a node is important, the nodes it links to should be important
- ➢ A node should not transfer more importance than it has

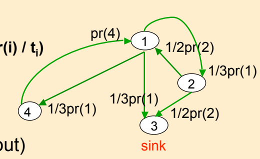
Attempt 1

Refer to nodes by numbers 1, … , n (arbitrary numbering)
Let t_i denote the number of edges out of *node i* (outdegree)
Node i transfers $1/t_i$ of its importance on each edge out of it

Define

$$\mathbf{pr_{new}(k)} = \Sigma_{\text{i with edge from i to k}} \mathbf{(pr(i) / t_i)}$$

Iterate until converges



Problems

- Sinks (nodes with no edges out)
- Cyclic behavior

Attempt 2

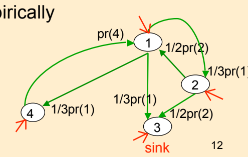
Random walk model

- Attempt 1 gives movement from node to linked neighbor with probability 1/outdegree
- Add random jump to any node

$$\mathbf{pr_{new}(k)} = \alpha/n + (1-\alpha)\Sigma_{\text{i with edge from i to k}} \mathbf{(pr(i) / t_i)}$$

– α parameter chosen empirically

- Break cycles
- Escape from sinks



Normalized?

- Would like $\Sigma_{1 \le k \le n}$ **(pr (k)) = 1**
- Consider $\Sigma_{1 \le k \le n}$ **(pr_{new}(k))**

$= \Sigma_{1 \le k \le n}(\ \alpha/n + (1-\alpha)\Sigma_{\text{i with edge from i to k}}(pr(i) / t_i)\ )$

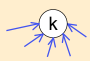
$= \Sigma_{1 \le k \le n}(\ \alpha/n) + \Sigma_{1 \le k \le n}((1-\alpha)\Sigma_{\text{i with edge from i to k}}(pr(i) / t_i))$ *

$= \quad \alpha \quad + (1-\alpha)\Sigma_{1 \le k \le n}\ \Sigma_{\text{i with edge from i to k}}(pr(i) / t_i)$

$= \quad \alpha \quad + (1-\alpha)\Sigma_{1 \le i \le n}\Sigma_{k \text{ with edge from i to k}}(pr(i) / t_i)$ *

$= \quad \alpha \quad + (1-\alpha)\Sigma_{\text{i with edge from i}}\ pr(i))$

*inner sum Σ_i over incoming edges for one k



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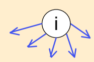

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Problem for desired normalization

- Have

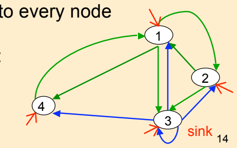
  $\Sigma_{1 \le k \le n}$ **(pr_{new}(k)) = α + (1-α)$\Sigma_{\text{i with edge from i}}$ pr(i))**

- Missing **pr(i)** for nodes with no edges from them
  - sinks!
- Solution: add n edges out of every sink
  - Edge to every node including self
  - Gives 1/n contribution to every node

Gives desired normalization:

If $\Sigma_{1 \le k \le n}$ ($pr_{initial}$(k)) = 1

then $\Sigma_{1 \le k \le n}$ (pr(k)) = 1



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Matrix formulation

- Let E be the n by n adjacency matrix

  E(i,k) = 1 if there is an edge from node i to node k

  = 0 otherwise

- Define new matrix L:

  For each row i of E ($1 \le i \le n$)

  If row i contains $t_i > 0$ ones, $L(i,k)=(1/ t_i)\, E(i,k)$, $1 \le k \le n$

  If row i contains 0 ones, $L(i,k) = 1/n$, $1 \le k \le n$

- Vector ***pr*** of PageRank values defined by

  $\boldsymbol{pr} = (\alpha/n, \alpha/n, \ldots \alpha/n)^T + (1-\alpha)\, L^T\, \boldsymbol{pr}$

- has a solution representing the steady-state values pr(k)

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Calculation

- Choose α
  - No single best value
  - Page and Brin originally used α=.15
- Simple iterative calculation
  - Initialize $pr_{initial}(k) = 1/n$ for each node k

    so $\Sigma_{1 \le k \le n}$ ($pr_{initial}$(k)) = 1
  - $pr_{new}(k) = \alpha/n + (1-\alpha)\Sigma_{1 \le i \le n} L(i,k) pr(i)$
- Converges
  - Has necessary mathematical properties
  - In practice, choose convergence criterion
    - Stops iteration

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PageRank Observations

- Can be calculated for *any* directed graph
- Google calculates on entire Web graph
  - query independent scoring
- Huge calculation for Web graph
  - precomputed
  - 1998 Google published:
    - 52 iterations for 322 million links
    - 45 iterations for 161 million links
- PageRank must be combined with query-based scoring for final ranking
  - Many variations
  - What Google exactly does secret
  - Can make some guesses by results

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HITS

Hyperlink **I**nduced **T**opic **S**earch

- Second well-known algorithm
- By Jon Kleinberg while at IBM Almaden Research Center
- Same general goal as PageRank
- Distinguishes **2 kinds of nodes**
  - **Hubs:** resource pages
    - **Point to many authorities**
  - **Authorities:** good information pages
    - **Point to many hubs**

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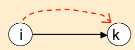
Mutual reinforcement

- Authority weight node j: a(j)
  - Vector of weights ***a***
- Hub weight node j: h(j)
  - Vector of weights ***h***
- Update:

$a_{new}(k) = \Sigma_{\text{i with edge from i to k}} (h(i))$



$h_{new}(k) = \Sigma_{\text{j with edge from k to j}} (a(j))$

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Matrix formulation

Steady state:

$\boldsymbol{a} = E^T\boldsymbol{h}$ $\qquad$ $\boldsymbol{a} = E^TE\boldsymbol{a}$

$\boldsymbol{h} = E\boldsymbol{a}$ $\qquad$ $\boldsymbol{h} = EE^T\boldsymbol{h}$

Interpretation:
- E^TE(i,j): number nodes point to both node i and node j
  - "Co-citation"
- EE^T(i,j): number nodes pointed to by both node i and node j
  - "Bibliographic coupling"

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Iterative Calculation

$\boldsymbol{a} = \boldsymbol{h} = (1, \ldots, 1)^T$

While (not converged) {

$\boldsymbol{a}_{new} = E^t\boldsymbol{h}$

$\boldsymbol{h}_{new} = E\boldsymbol{a}$

$\boldsymbol{a} = \boldsymbol{a}_{new} / ||\boldsymbol{a}_{new}||$ normalize to unit vector

$\boldsymbol{h} = \boldsymbol{h}_{new} / ||\boldsymbol{h}_{new}||$ normalize to unit vector

}

Provable convergence by linear algebra

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Use of HITS

- Actual use of HITS by IBM people was **after** find Web pages satisfying query:
  1. Retrieve documents satisfy query and rank by term-based techniques
  2. Keep top *c* documents: root set of nodes
     - *c* a chosen constant - tunable
  3. Make base set:
     1. Root set
     2. *Plus* nodes pointed to by nodes of root set
     3. *Plus* nodes pointing to nodes of root set

     using links to expand matches!
  4. Make base graph: base set plus edges from Web graph between these nodes
  5. Apply HITS to base graph

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Results using HITS

- Documents ranked by authority score a(doc) and hub score h(doc)
  - Authority score primary score for search results
- Heuristics:
  - delete all links between pages in same domain
  - Keep only pre-determined number of pages linking into root set (~200)
- Findings (original paper)
  - Number iterations in original tests ~50
  - most authoritative pages **do not** contain initial query terms
    - Compare LSI "concepts"

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Observations

- HITS can be applied to any directed graph
- Base graph **much smaller** than Web graph
- Kleinberg identified bad phenomena
  - Topic diffusion: generalizes topic when expand root graph to base graph
    - Want *compilers* - generalized to *programming*

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PageRank and HITS

- designed independently around 1997
- indicates time was ripe for this kind of analysis
- lots of embellishments by others

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